Pseudogap, superconducting energy scale, and Fermi arcs of underdoped cuprate superconductors

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Through the measurements of magnetic field dependence of specific heat in La_{2-x}Sr_xCuO₄ in zero temperature limit, we determined the nodal slope v_{Δ} of the quasiparticle gap. It is found that v_{Δ} has a very similar doping dependence of the pseudogap temperature T^* or value Δ_p . Meanwhile the virtual maximum gap at $(\pi, 0)$ derived from v_{Δ} is found to follow the simple relation $\Delta_q = 0.46k_BT^*$ upon changing the doping concentration. This strongly suggests a close relationship between the pseudogap and superconductivity. It is further found that the superconducting transition temperature is determined by both the residual density of states of the pseudogap phase and the nodal gap slope in the zero temperature limit, namely, $T_c \approx \beta v_{\Delta} \gamma_n(0)$, where $\gamma_n(0)$ is the extracted zero temperature value of the normal state specific heat coefficient which is proportional to the size of the residual Fermi arc k_{arc} . This shows that the superconductivity may be formed by forming a new gap on the Fermi arcs near nodes below T_c . These observations mimic the key predictions of the SU(2) slave boson theory based on the general resonating-valence-bond picture.

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I. INTRODUCTION

Since the discovery of the cuprate superconductors, 19 years have elapsed without a consensus about its mechanism. Many exotic features beyond the Bardeen-Cooper-Schrieffer theory have been observed. One of them is the observation of a pseudogap in the electron spectral function near the antinodal points $(\pi, 0)$ and $(0, \pi)$ at a temperature $T^* \gg T_c$.¹ In a conventional BCS superconductor, this gapping process occurs simultaneously with the superconductivity at T_{c} . It has been heavily debated about the relationship between the pseudogap and the superconductivity in cuprates. One scenario assumes that the pseudogap Δ_p marks only a competing or coexisting order with the superconductivity and it has nothing to do with the pairing origin. However another picture, namely the Anderson's resonating-valencebond (RVB) (Ref. 2) model (and its offspring)³ predict that the spin-singlet pairing in the RVB state (which causes the formation of the pseudogap) may lend its pairing strength to the mobile electrons and cause them to naturally pair and then to condense at T_c . According to this picture there should be a close relationship between the pseudogap and the superconductivity.

In order to check whether this basic idea is correct, we need to collect the information for the pseudogap and the superconducting energy scale, especially their doping dependence. The pseudogap values Δ_p (or its corresponding temperature $k_B T^* \sim \Delta_p$) and its doping dependence have been measured through experiments.¹ To determine the superconducting energy scale, we note that the normal state Fermi surface is formed by four small arcs near the nodal points.⁴ As temperature is lowered below T_c , a new gap opens on these arcs. To illustrate this point more clearly, in Fig. 1 we present a schematic plot for different gaps or energy scales. The dotted line represents the gap structure of the normal

state, assuming the presence of Fermi arcs near the nodal points. The region of zero gap corresponds to the Fermi arc. The dashed line and the solid line represent two possible gap structures for superconducting states at T=0. The solid line is the standard *d*-wave gap with maximum gap value Δ_p at $(\pi,0)$ and $(0,\pi)$. From this picture, we see that the nodal gap slope, which is defined as $v_{\Delta} = [d\Delta_s/d\theta]_{node}/\hbar k_F$, can be used to determine the superconducting energy scale. The relationship between the nodal gap slope v_{Δ} and the maximum pseudogap Δ_p remains a mystery. In particular, the two quantities may be independent of each other if the superconductivity is not induced by the formation of the pseudogap. Therefore, to measure the nodal gap slope near the nodal



FIG. 1. Schematic plot for the pseudogap energy, superconducting energy scale, and nodal gap slope. The solid line represents a standard *d*-wave gap $\Delta = \Delta_p \cos 2\theta$ with maximum gap value Δ_p near $(\pi, 0)$ and θ the angle starting from k_x . The dotted line shows the pseudogap near nodes if the superconductivity would be suppressed completely (based on the Fermi arc picture). The dashed line shows a possible quasiparticle gap near nodes. The nodal gap slope is defined as $v_{\Delta} = [d\Delta_s/d\theta]_{node}/\hbar k_F$. The nodal gap slope v_{Δ} and the maximum gap Δ_p near $(\pi, 0)$ may not be related if the superconductivity (which controls the gap structure near nodes) has nothing to do with the pseudogap.

point in the zero temperature limit becomes highly desired. When combined with the known results on the pseudogap Δ_p , this will allow us to detect the relation between the pseudogap and the superconductivity.

Some previous results using, for example, angle-resolved photo-emission (ARPES) (Ref. 5) or superfluid density seem to be inconclusive due to either energy resolution (ARPES above 10 meV) or unexpected difficulty in analyzing the data (e.g., a so-called Fermi liquid correction factor α_{FL} is inevitably involved in analyzing the low temperature data of superfluid density). In this paper, we report the evidence of a proportionality between the nodal gap slope v_{Δ} and the pseudogap temperature T^* . Remarkably a simple relation, namely $\Delta_q = 0.46T^*$, between the virtual maximum quasiparticle gap (Δ_q) derived from v_{Δ} and the pseudogap temperature T^* is found. We also find that T_c is determined by both the nodal gap slope v_{Δ} and the size of the Fermi arcs (k_{arc}) in the underdoped normal state. Both observations are anticipated by the SU(2) slave boson theory⁶ based on the general RVB picture.

II. EXPERIMENT

We determine the properties of the nodal quasiparticles by measuring the low temperature electronic specific heat. $La_{2-r}Sr_rCuO_4$ single crystals measured in this work were prepared by the traveling solvent floating-zone technique. Samples with seven different doping concentrations as follows: p=0.063 ($T_c=9$ K, nominal x=0.063, post-annealed in Ar gas at 800 °C for 48 h), 0.069 ($T_c = 12$ K, as-grown sample with x=0.063), 0.075 ($T_c=15.6$ K, nominal x=0.07and post-annealed in O₂ gas at 750 °C for 12 h), 0.09 (T_c =24.4 K, as grown, x=0.09), 0.11 (T_c =29.3 K, as grown, x=0.11), 0.15 (T_c =36.1 K, nominal x=0.15), and 0.22 (T_c =27.4 K, nominal x=0.22) have been investigated. The quality of our samples has been characterized by x-ray diffraction, and R(T) data showing a narrow transition $\Delta T_c \leq 2$ K. The samples have also been checked by ac and dc magnetization showing also quite narrow transitions. The full squares in Fig. 6 represent the transition temperatures of our samples. The heat capacity presented here was measured with the relaxation method based on an Oxford cryogenic system Maglab-EXA-12. In all measurements the magnetic field was applied parallel to the c axis. As also observed by other groups for La-214 systems, the anomalous upturn of C/T due to the Schottky anomaly of free spins is very weak. This avoids the complexity in the data analysis. Details about the sample characterization, the specific heat measurement, the residual linear term, and extensive analysis are reported in a recent paper.⁷

It has been widely perceived that the pairing symmetry in the hole doped cuprate superconductors is of *d* wave symmetry with line nodes in the gap function. In the mixed state, due to the presence of vortices, Volovik⁸ pointed out that supercurrents around a vortex core lead a Doppler shift to the quasiparticle excitation spectrum. This will dominate the low energy excitation and the specific heat (per mol) behaves as^{8,9} $C_{vol} = A \sqrt{H}$ with $A \propto 1/v_{\Delta}$. This square-root relation has been verified by many measurements which were taken as



FIG. 2. Field dependence of $\Delta \gamma = [C(H) - C(0)]/T$ normalized by the data at about 12 T in zero temperature limit. It is clear that Volovik's \sqrt{H} relation describes the data rather well for all samples. This indicates a robust *d*-wave superconductivity in all doping regimes.

evidence for *d* wave symmetry, for example, by specific heat,^{10–16} thermal conductivity,¹⁷ tunneling (to measure the Doppler shift of the Andreev bound states),¹⁸ etc. In this way one can determine the nodal gap slope (v_{Δ}) . Since the phonon part of the specific heat is independent on the magnetic field, this allows us to remove the phonon contribution by subtracting the C/T at a certain field as that at zero field, one has $\Delta \gamma = \Delta C/T = [C(H) - C(0)]/T = C_{vol}/T - \alpha T$ with α the coefficient for the quasiparticle excitations of a *d*-wave superconductor at zero field ($C_e = \alpha T^2$). In the zero temperature limit $\Delta \gamma = C_{vol}/T = A \sqrt{H}$ is anticipated.

III. RESULTS AND DISCUSSION

In order to get $\Delta \gamma$ in the zero temperature limit, we extrapolate the low temperature data of C/T vs T^2 (between 2 K and 4 K) to zero K. The data taken in this way and normalized at 12 T are presented in Fig. 2. It should be mentioned that the similar data have been published in our previous paper.⁷ Here for clarity we present the data again with more detailed analysis. It is clear that the Volovik's \sqrt{H} relation describes the data rather well for all doping concentrations. This is to our surprise since it has been questioned whether the Volovik relation is still obeyed in the underdoped regime¹⁴ especially when competing orders are expected to appear^{19–21} and impurity scattering is present. We attribute the success of using the Volovik relation here to three reasons.

(1) We use $\Delta \gamma = [C_{H\parallel c} - C_{H=0}]/T$ instead of using $\Delta \gamma = [C_{H\parallel c} - C_{H\perp c}]/T$. The latter may inevitably involve the unknown DOS contributions from other kinds of vortices (for example, Josephson vortices) when $H \perp C$.

(2) The contribution from a second competing order to $\Delta \gamma$ may be small compared to the Volovik's term in the zero temperature limit. This is reasonable when considering a contribution to the heat capacity by the competing order as $\sim T^{\nu}$ with $\nu > 1$. For example, the specific heat due to the spin correlation in the two-dimensional (2D) antiferromagnetic phase is $\sim T^2$. In zero temperature limit this term goes away.

(3) The DOS induced by the Doppler shift effect in our experiment is much stronger than that induced by the impu-



FIG. 3. (a) The typical original data of $\Delta \gamma$ vs. *T* for the underdoped sample p=0.069 at different magnetic fields. (b) The same set of data plotted as $\Delta \gamma / \sqrt{H}$ vs. *T*. One can clearly see that in zero temperature limit $\Delta \gamma / \sqrt{H}$ is a constant for all fields implying the validity of the Volovik's relation $\Delta \gamma = A \sqrt{H}$. From here one can also determine the value *A* which is about 0.28 mJ/mol K² T^{0.5} as marked by the thick bar.

rity scattering. We will further address this point in the forthcoming discussion. To have a self-consistent check of the \sqrt{H} relation found in the zero temperature limit, we plot the raw data of $\Delta \gamma / \sqrt{H}$ vs *T* at finite temperatures. A typical example for the very underdoped one (*p*=0.069) is shown in Figs. 3(a) and 3(b). One can see that in the low temperature region the data $\Delta \gamma / \sqrt{H}$ scale for all fields ranging from 1 T to 12 T, showing the nice consistency with the relationship $\Delta \gamma \propto \sqrt{H}$ for this sample in the zero temperature limit. From here one can also determine the prefactor *A* in $\Delta \gamma = A \sqrt{H}$ (here, for example, *A*=0.28 mJ/mol K² T^{0.5} for *p*=0.069) and then compare backwards the value determined from the data shown in the main panel leading to, of course, the same value. The same feature appears for all other doping concentrations. For clarity, they will not be shown here.

It is clear that the Volovik's \sqrt{H} relation describes the data rather well for all doping concentrations. This successful scaling of $\Delta \gamma$ vs \sqrt{H} makes it possible to derive the prefactor *A*, and one can further determine the nodal gap slope v_{Δ} . Figure 4(a) shows the doping dependence of the prefactor *A*. The error bar is obtained by fitting the extracted zero temperature data to $\Delta \gamma = A \sqrt{H}$. For a typical *d*-wave superconductor, by calculating the excitation spectrum near the nodes, it was shown that¹⁶

$$A = \alpha_p \frac{4k_B^2}{3\hbar l_c} \sqrt{\frac{\pi}{\Phi_0}} \frac{nV_{mol}}{v_\Delta} \tag{1}$$

here $l_c = 13.28$ Å is the c-axis lattice constant, $V_{mol} = 58$ cm³ (the volume per mol), α_p a dimensionless constant taking 0.5 (0.465) for a square (triangle) vortex lattice, n=2 (the number of Cu-O planes in one unit cell), and Φ_0 the flux quanta.



FIG. 4. (a) Doping dependence of the pre-factor A determined in present work (open circles). Here the point at p=0.19 was adopted from the work by Nohara *et al.* on a single crystal.¹⁴ (b) Doping dependence of the pseudogap temperature T^* (open symbols) summarized in Ref. 1 (see Fig. 26 there) and our data v_{Δ} (solid line). T^* -susceptibility refers to the pseudogap temperature determined from the maxima in the static susceptibility, and $T^* - \rho$ to the temperature at which there is a slope change in the DC resistivity (all in La_{2-x}Sr_xCuO₄). Above $T^* - \rho$ the resistivity has a linear temperature dependence. The full squares represent the calculated virtual maximum quasi-particle gap Δ_q derived from v_{Δ} without any adjusting parameters. Surprisingly both set of data are correlated through a simple relation ($\Delta_q \approx 0.46k_BT^*$) although they are determined in totally different experiments. This result implies a close relationship between the pseudogap Δ_p and the nodal gap slope v_{Δ} .

The v_{Δ} has then been calculated without any adjusting parameter (taking $\alpha_p = 0.465$) and shown in Fig. 4(b). It is remarkable that v_{Δ} has a very similar doping dependence as the pseudogap temperature T^* , indicating that $v_{\Delta} \propto T^* \propto \Delta_p$. If converting the data v_{Λ} into the virtual maximum quasiparticle gap (Δ_a) (Ref. 16) via $v_{\Delta} = 2\Delta_a/\hbar k_F$ [here $k_F \approx \pi/\sqrt{2}a$ is the Fermi vector of the nodal point with a=3.8 Å (the inplane lattice constant)], surprisingly the resultant Δ_a value [shown by the filled squares in Fig. 4(b)] is related to T^* in a simple way $(\Delta_a \sim 0.46 k_B T^*)$. It is important to emphasize that this result is obtained without any adjusting parameters. Counting the uncertainties in determining T^* and the value of α_p , this relation is remarkable since Δ_q and T^* are determined in totally different experiments. Because v_{Δ} (or Δ_a) reflect mainly the information near nodes which is predominantly contributed by the superconductivity, above discovery, i.e., $v_{\Delta} \propto T^* \propto \Delta_p$ (or $\Delta_q \sim 0.46 k_B T^*$) strongly suggests a close relationship between the superconductivity and the pseudogap. A similar conclusion was drawn in underdoped $YBa_2Cu_3O_{\nu}$ by analyzing the low temperature thermal conductivity.²² If the pseudogap is supposed to be caused by the formation of the RVB state,² our results here point to a fact that the RVB singlet pairing may be one of the unavoidable ingredients for superconductivity. It remains to be seen whether this conclusion holds also for the electron doped samples, since so far it is not clear yet whether the pseudogap exists in these N-type samples.

In above discussion, we see the consistency between our low temperature specific heat data and the Volovik's square root relation $\Delta \gamma = A \sqrt{H}$. This seems surprising since the temperature range we considered here is about several kelvin. At such an energy scale, the impurity scattering will strongly alter the DOS in the low energy region by generating some new quasiparticles. However, by applying a magnetic field, the Doppler shift of the quasiparticle excitation spectrum will contribute a new part to DOS. This energy shift is actually not small compared to the temperature. We can give a simple estimation on the energy shift ΔE . It is known that $\Delta E = \alpha_{FL} \sqrt{3}/2\pi (v_F \hbar/l_B)$, here l_B is the magnetic length which is defined as $l_B = (c\hbar/eB)^{1/2}$, and $0 < \alpha_{FL} < 1$ is a Fermi liquid correction term. Taking $v_F = 2.73 \times 10^7$ cm/s,²³ we have $\Delta E = 3.67 \alpha_{FL} \sqrt{B/1T}$ meV. For example, taking the maximum field (12 T) in our experiment, we get ΔE =12.2 α_{FL} meV which is actually a relatively big energy scale compared to the temperature T since $\alpha_{FL} \sim 1$. This may explain why the Volovik's simple square-root relation $\Delta \gamma$ $=A\sqrt{H}$ can be easily observed in our single crystals with an inevitable certain amount of impurities.

In the following we will investigate what determines T_c . Keeping the doping dependence of v_{Δ} in mind, it is easy to understand that $v_{\Delta}\hbar k_F$ should not be a good estimate of the superconducting energy scale for the underdoped samples since T_c and v_{Δ} have opposite doping dependences. The basic reason is that the normal-state Fermi surfaces are small arcs of length k_{arc} near the nodal points. The superconducting transition occurs by forming extra gaps on the Fermi arcs. So the effective superconducting energy scale should be estimated as $E_s \sim \frac{1}{2} v_{\Delta} \hbar k_{arc}$. From the normal state electronic specific heat $C_{ele} = \gamma_n T$, we have $\gamma_n = 4nk_B^2 k_{arc} V_{mol}/\hbar v_F l_c$. Assuming $E_s \sim k_B T_c$ we find

$$T_c = \alpha_s \frac{\hbar^2 v_F l_c \gamma_n v_\Delta}{8nk_B^3 V_{mol}} = \beta \gamma_n v_\Delta, \qquad (2)$$

where α_s is a dimensionless constant in the order of unity; v_F is the nodal Fermi velocity normal the Fermi surface. The value of $\gamma_n(0)$ can be estimated from specific heat,^{24,25} or indirectly by ARPES (Ref. 26) or NMR.²⁷ Here we take the values for $\gamma_n(0)$ summarized by Matsuzaki *et al.*²⁴ and fit it (in units of mJ/mol K²) with a formula $\gamma_n = \zeta (p - p_c)^{\eta}$ yielding $\zeta = 182.6$, $p_c = 0.03$, $\eta = 1.54$. In Fig. 5 we present the doping dependence of the zero temperature specific heat coefficient $\gamma_n(0)$ and k_{arc} . One can see that k_{arc} becomes smaller than $2\pi/a$ in an underdoped region showing the selfconsistency of the picture of Fermi arcs. In Fig. 6 we present the doping dependence of the truly measured T_c (filled squares) and the calculated value (open squares) by Eq. (2) with $\beta = 0.7445 \text{ K}^3 \text{ mol s/Jm}$. In underdoped region, the truly measured and calculated T_c values coincide rather well implying the validity of Eq. (2). In the overdoped region, γ_n will gradually become doping independent, therefore one expects $T_c \propto v_{\Delta}$. So the energy scale of the superconductivity is not given by $v_{\Delta}\hbar k_F \sim \Delta_p$, but by $\frac{1}{2}v_{\Delta}\hbar k_{arc}$ or more precisely by Eq. (2) in the underdoped region.

To have a framework about the experimental results, in the following, we will review one particular explanation



FIG. 5. (a) The doping dependence of $\gamma_n(0)$ derived from specific heat (open squares²⁴) based on entropy conservation, ARPES (open circles²⁶), and Knight shift (up-triangles²⁷) in La_{2-x}Sr_xCuO₄. The solid line is a fit to the data with $\gamma_n = \zeta(p-p_c)^{\eta}$ yielding $\zeta = 182.6$, $p_c = 0.03$, $\eta = 1.54$. (b) A comparison between k_{arc} calculated from $\gamma_n(0)$ and $2\pi/a$. One can see that the k_{arc} becomes smaller than $2\pi/a$ in the underdoped region showing the relevance of the Fermi arcs in the pseudogap phase.

based on the slave-boson approach although there are many theories about the ground state of the pseudogap.²⁸ Within the SU(2) slave-boson theory,⁶ the pseudogap metallic state is viewed as a doped algebraic spin liquid (ASL).²⁹ A doped ASL is described by spinons (neutral spin- $\frac{1}{2}$ Dirac fermions) and holons (spinless charge-*e* boson) coupled with a U(1) gauge field. Due to the attraction between the spinons and the holons caused by the U(1) gauge field, a spinon and a holon recombine into an electron at low energies.^{6,29} Due to the spin-charge recombination, the pseudogap metallic state is described by electronlike quasiparticles at low energy. Since the binding between the spinon and the holon is weak, the large pseudogap near the antinodal points (π ,0) and (0, π) is not affected. So the Fermi surface of the recombined



FIG. 6. Doping dependence of the truly measured superconducting transition temperature T_c (full squares) and that calculated by $T_c = \beta v_{\Delta} \gamma_n(0)$ (open squares) with $\beta = 0.7445$ K³ mol s/Jm. The solid line represents the empirical relation $T_c/T_c^{max} = 1-82.6(p - 0.16)^2$ with $T_c^{max} = 38$ K.

electrons cannot form a large closed loop. A simple theoretical calculation⁶ suggests that the Fermi surface of the recombined electrons forms four small arcs near the nodal points $(\pm \pi/2, \pm \pi/2)$. Thus the SU(2) slave-boson theory⁶ contains two key features: the pseudogap due to spin singlet pairing and the Fermi arcs due to the spin-charge recombination.²⁹ And the superconductivity arises from the coherent motion of the quasiparticles on the arcs, thus one expects that T_c is proportional to the gap on the Fermi arc: $k_B T_c \approx \frac{1}{2} v_\Delta \hbar k_{arc}$, instead of the pseudogap Δ_p near the antinodal points. Meanwhile, since the spin pairing is responsible for both the pseudogap Δ_p near the antinodal points and the nodal gap slope v_Δ , it is reasonable to see the proportionality between v_Δ and $T^*(\propto \Delta_p)$ or $\Delta_q \approx 0.46 k_B T^*$. This is exactly what we found in the experiment.

IV. CONCLUDING REMARKS

In summary, the Volovik's relation of the *d*-wave pairing symmetry has been well demonstrated by low temperature specific heat in wide doping regime in $La_{2-x}Sr_xCuO_4$. Based on this analysis the nodal gap slope v_{Δ} is derived and is found to follow the same doping dependence of the pseudogap Δ_p . This strongly indicates the close relationship between the pseudogap and the superconductivity. Meanwhile it is found that the superconducting transition temperature T_c is determined by $v_{\Delta}\gamma_n(0)$ instead of v_{Δ} . This discovery may suggest the importance of Fermi arcs near the nodal region and the superconductivity is induced by the formation of a new gap on these arcs. Both observations are consistent with the SU(2) slave-boson theory based on the general RVB picture.

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