Superfluid phase of ^{3}He-B near the boundary

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Having analyzed old and some of the most recent transverse sound experiments in superfluid ^{3}He-B we are now able to solve the old problem of superfluid quantum liquids in confined geometry—that is, the boundary state of ^{3}He-B. We pay attention to the difference between the transverse sound experiments data and those of longitudinal sound experiments. We consider several possible explanations of the above experimental data: the existence of a new superfluid phase in the vicinity of the boundary, excitation of different branches of the squashing mode by longitudinal and transverse sound, and deformation of B phase near the boundary. We have come to the conclusion that the last possibility seems the most likely and that the boundary state of ^{3}He-B is indeed a deformed B phase, as suggested by Brusov and Popov 20 years ago for the case of the presence of external perturbations such as magnetic and electric fields. Our result implies that the influence of the wall or, generally speaking, of confined geometry does not lead to the existence of a new phase near the boundary, as has been suggested up to now, but like other external perturbations (e.g., magnetic and electric fields) the wall deforms the order parameter of the B phase and this deformation leads to some important consequences. In particular, the frequencies of collective modes in the vicinity of the boundary change by up to 20%. One other evidence in support of our conclusion with regard to the realization of the deformed B-phase in ^{3}He-B phase near the boundary is the peak in transverse ultrasound impedance observed at a temperature of 0.94 T_c and interpreted by the authors as a manifestation of the transition from a supercooled A-phase to B-phase, which is really the transition between A-phase and deformed B-phase.

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I. INTRODUCTION

The problem of the boundary state of a superfluid quantum liquid with complex order parameter such as superfluid ^{3}He is quite interesting and has a long history.[1–3] One of the problems here is a particular superfluid phase realizing in the vicinity of the boundary.

Ultrasound experiments using longitudinal sound have played an important role in the investigation of superfluid ^{3}He. In these experiments, all collective modes in both A and B phases of ^{3}He (18 modes in each phase) were observed.[4,5] Longitudinal sound experiments also helped identify superfluid phases in bulk ^{3}He. While these experiments provide information for the whole volume of liquid, transverse sound experiments (because of the strong damping of transverse sound) serve as a reliable probe of the state in the vicinity of the boundary. Thus, the problem of the boundary state could be solved by transverse sound experiments. In the past, several possibilities for the "boundary" state were considered: A phase (with the $\boldsymbol{l}$ vector perpendicular to the surface) and 2D-phase (hypothetical bulk phase existing in a magnetic field). It has been shown that the former phase is unlikely.

Here we consider several additional superfluid phases as well as one alternative picture that exploits an idea of Brusov and Popov[6] concerning deformation of a gap in the B phase. Our analysis of the existing experimental data leads us to conclude that the deformed B phase is realized near the boundary.

II. TRANSVERSE SOUND EXPERIMENTS

There are two types of transverse sound experiments: the propagation ones, where the attenuation and frequency shift are measured, are one of them. Because of the high attenuation of the transverse sound, propagation experiments require a rather short sound path length. An alternative to propagation experiments is an acoustic impedance technique, where a complex acoustic impedance of the liquid is measured. The former type has been used in Japanese experiments,[7] while a later method in Northwestern experiments.[8,9]

Let us start from the earlier experiments, where some qualitative data have been obtained. Over ten years ago Kalbfeld *et al.*[8,9] developed transverse acoustic impedance technique and used it to study the superfluid ^{3}He-B. During that time, interest in transverse sound in superfluid ^{3}He followed theoretical predictions by Moorse and Sauls[10] that the collective modes with $J=2^-$ (squashing modes) can provide an additional mechanism of support of a transverse response. Kalbfeld *et al.* used a fixed sound frequency and swiped temperature. They were able to simultaneously perform measurements of the longitudinal and transverse responses that allowed them to compare data from these two measurements. It turned out that while there was a correlation between the longitudinal and transverse responses they were not identical: there were peaks in the imaginary transverse acoustic impedance at temperatures above the squashing (sq) mode as well as below. At temperatures above the *sq*-mode these features may be interpreted as due to the standing-wave pattern.[8] Based on this assumption, the authors[8] were able to obtain the change in the phase velocity associated with these oscillations. The nature of the contribution at temperatures below the *sq*-mode still remains unclear. In this paper we will concentrate on our interpretation of the last features, taking into account that they have been observed in recent transverse sound experiments as well.[7] These recent Japanese experiments are essential since they provide some numerical data that allow us to distinguish between different possibilities. The key results of these experiments are summarized below.

The authors[7] have studied shear horizontal surface acoustic waves. They obtained the transverse acoustic impedance measuring the velocity and damping of transverse sound. The experimental data were obtained for the two transverse sound frequencies 28 MHz and 47 MHz. While the temperature-dependent squashing-mode frequency is $\omega_{sq} = \sqrt{12/5}\Delta_0(T) \approx 1.55\Delta_0(T)$ [here $\Delta_0(T)$ is the gap in a single-particle spectrum, isotropic in the *B* phase; in the 2D phase and other phases, considered below, $\Delta_0(T)$ is the maximum gap in a single particle spectrum] at both frequencies, one has a peak at $\omega \approx 1.25\Delta_0(T)$ for the transverse sound frequency of 28 MHz and at $\omega \approx 1.35\Delta_0(T)$ for the transverse sound frequency of 47 MHz. We have considered several possibilities of interpreting the above experimental data: the existence of some new superfluid phases in the vicinity of the boundary, excitation of different branches of the squashing mode by longitudinal and transverse sounds, and deformation of the *B* phase due to the wall. We conclude that the last possibility seems the most likely. The authors[7] observed as well the peak in transverse ultrasound impedance at a temperature of 0.94 T_c. This value is lower than the well-known temperature $T_{\rm AB}=0.97\,T_c$ for the transition from A-phase to *B*-phase. The authors of Ref. 7 interpreted this fact as a manifestation of the transition from a supercooled *A*-phase to *B*-phase. However, it will become clear that this peak demonstrates the transition between *A*-phase and deformed *B*-phase with the observed difference in the transition temperature caused by the difference between the order parameter of the *B*-phase and that of the deformed B-phase.

III. POSSIBLE NEW PHASES NEAR THE BOUNDARY

For a theoretical description of the superfluid phases of ^{3}He one usually uses the order parameter value, nonzero below T_c and equal to zero above T_c. As the order parameter, we can use an anomalous Green function—wave function of Cooper pairs—$F_{\alpha\beta}(k)$, where α,β are the spin indexes and k is the momentum of the Cooper pairs. Because the wave function of the Cooper pairs is a symmetric spinor of rank 2, it can be decomposed by the basis of symmetric unitary second-order matrices $i(\sigma\sigma_y)_{\alpha\beta}$:

$$F_{\alpha\beta}(k) = id(k)(\sigma\sigma_y)_{\alpha\beta}, \tag{1}$$

where $\sigma=(\sigma_1,\sigma_2,\sigma_3)$ are Pauli matrices.

Here the vector $\boldsymbol{d}(\boldsymbol{k})$ depends on the direction in momentum space, $\mathbf{n}=\mathbf{k}/k_F$, only. Under the pairing with orbital moment $l=1$ this dependence is described by combinations of spherical harmonics with $l=1$, which can be considered as components of the unit vector n. Thus,

$$d_i = A_{ij} n_j. \tag{2}$$

Here j is a spin (isotope) index and i is a vector index. The complex matrix (3×3) A_{ij} is the order parameter in the superfluid phases of ^{3}He.

As possible "boundary" states we can consider the 2D phase and phases with order parameters proportional to

$$\begin{pmatrix}100\\0-10\\000\end{pmatrix}, \quad \begin{pmatrix}010\\100\\000\end{pmatrix}, \quad \begin{pmatrix}010\\-100\\000\end{pmatrix}, \quad \begin{pmatrix}110\\ii0\\000\end{pmatrix}. \tag{3}$$

2D phase

The two-dimensional (2D) phase with the order parameter

$$\begin{pmatrix}100\\010\\000\end{pmatrix}$$

exists in magnetic fields $H > H_C$.[11] In the absence of the magnetic fields in the vicinity of the wall the gradient terms in free energy can stabilize this phase; thus, it can be considered as a candidate for the "boundary" state.

Brusov and Popov have calculated the collective-mode spectrum in the 2D phase on the basis of what is known as the path integral technique.[11] In the presence of a magnetic field, the spectrum consists of 18 collective modes, among them six Goldstone modes, four clapping modes $E=(1.17-i0.13)\Delta_0$, two pair-breaking modes $E=(1.96-i0.31)\Delta_0$, and six modes depending on magnetic fields: two $E=2\mu H$, two $E^2=(1.96-i0.31)\Delta_0^2+4\mu^2H^2$, $E^2=(0.518)\Delta_0^2+4\mu^2H^2$, and $E^2=(0.495)\Delta_0^2+4\mu^2H^2$.

This spectrum contains a clapping mode with frequency $E=(1.17-i0.13)\Delta_0$.

Since the *B* phase is mostly dominant in most of the samples and we have the 2D phase in the vicinity of the boundary, our signal of transverse (longitudinal) sound is a mediated signal from both regions. In bulk we have a squashing mode with frequency $\omega_{sq}=\sqrt{12/5}\Delta_0 \approx 1.55\Delta_0$ and a clapping mode of frequency $E=1.17\Delta_0$ in the vicinity of the boundary. It is easy to demonstrate that in case of sound propogation perpendicular to the surface detecting signal will have frequency depending on both squashing and clapping mode frequencies and the amplitudes of two these modes. If one has two signals of sound absorption $y_1=-a(x-x_0)^2+b$ and $y_2=-a'(x-x_0')^2+b'$ with peaks at frequencies of x_0 and x_0', then the resulting signal $y=-a(x-x_0)^2+b-a'(x-x_0')^2+b'$ will have a maximum at frequency $x=(ax_0+a'x_0')/(a+a'')$ depending on both frequencies and the amplitudes of both signals. For example, at $a=0$ (or $a \ll a'$) $x=x_0'$, at $a'=0$ (or $a' \ll a$) $x=x_0$ and at $a=2a'$ $x=(2x_0+x_0')/3$. If this picture is valid, there are two resonances in the sound absorption: one from the squashing mode in the bulk *B* phase and the other from the clapping mode in the boundary 2D phase. Due to the large uncertainty in the determination of the temperature of the peaks, these two peaks can be seen as one peak with features described above. For longitudinal sound experiments the major signal absorption takes place in bulk liquid (region of the "boundary" phase is much smaller than the bulk phase one): thus, the attenuation amplitude is peaked at the squashing mode frequency $\omega_{sq}=\sqrt{12/5}\Delta_0 \approx 1.55\Delta_0$. In the case of the transverse sound experiments, the "boundary" phase region becomes comparable to the bulk phase region; thus, one should expect that the attenuation amplitude peaked at a frequency somewhat between the squashing-mode frequency $\omega_{sq}=\sqrt{12/5}\Delta_0 \approx 1.55\Delta_0$ and the clapping-mode frequency $\omega_{cl}=1.17\Delta_0$. From this point of view, the observed peak frequencies $\omega \approx 1.25\Delta_0(T)$ and $\omega \approx 1.35\Delta_0(T)$ are understood. But let us now consider the dependence of the peak frequency on the sound frequency. The specific value of the peak frequency depends on the attenuation amplitudes of the sound into these two modes, which depend on the relative regions occupied by the bulk and the "boundary" phases. The result of the Japanese experiment[7] is that the peak frequency increases with sound frequency: this implies that part of the bulk liquid involved in the considered process increases with the frequency of the transverse sound. This fact contradicts the hydrodynamic attenuation mechanism of the transverse sound, which predicts increased attenuation with frequency as ω^2. Thus we should state that the suggested picture of having the B phase in bulk and the 2D phase near the boundary is inconsistent with the Japanese data[7] and such a possibility must be ruled out. (Note, here, that the geometry of the Japanese experiments, mentioned above (transverse sound propogates horizontally along the surface), can require more careful consideration.)

$$\textit{\textbf{Phases}} \begin{pmatrix} 100 \\ 0-10 \\ 000 \end{pmatrix}, \begin{pmatrix} 010 \\ 100 \\ 000 \end{pmatrix}, \textit{\textbf{and}} \begin{pmatrix} 010 \\ -100 \\ 000 \end{pmatrix}$$

Let us consider the above three phases. Their spectra are identical and the following set of equations (at zero momentum of excitations) was obtained:

$$\int_0^1 dx(1-x^2)\left(1+\frac{4\Delta^2}{\omega^2}\right)J = 0(2),$$

$$\int_0^1 dx(1-x^2)\left(1+\frac{6\Delta^2}{\omega^2}\right)J = 0(3),$$

$$\int_0^1 dx(1-x^2)\left(1+\frac{8\Delta^2}{\omega^2}\right)J = 0(1),$$

$$\int_0^1 dx(2-x^2)\left(1+\frac{4\Delta^2}{\omega^2}\right)(J-1) = 0(1),$$

$$\int_0^1 dx(2-x^2)\left(1+\frac{6\Delta^2}{\omega^2}\right)(J-1) = 0(2),$$

$$\int_0^1 dx(1-x^2)\left(1-\frac{2\Delta^2}{\omega^2}\right)J = 0(2),$$

$$\int_0^1 dx(1-x^2)J = 0(2),$$

$$\int_0^1 dx(1-x^2)\left(1-\frac{4\Delta^2}{\omega^2}\right)J = 0(2),$$

$$\int_0^1 dx x^2\left[\left(1-\frac{2\Delta^2}{\omega^2}\right)J - 1\right] = 0(2),$$

$$\int_0^1 dx x^2(J-1) = 0(1). \tag{4}$$

Here

$$J = \frac{1}{\sqrt{1+4\Delta^2/\omega^2}} \ln \frac{1-\sqrt{1+4\Delta^2/\omega^2}}{1+\sqrt{1+4\Delta^2/\omega^2}} = 0$$

$x=\cos\vartheta$, $\omega=\omega/\Delta_0$ is dimensionless collective mode frequency and the index in the brackets shows the degeneracy of the branch.

Having solved these equations numerically, we get the following high-frequency modes:

$$E = \Delta_0(T)(1.83 - i0.06), \quad E = \Delta_0(T)(1.58 - i0.04),$$

$$E = \Delta_0(T)(1.33 - i0.10),$$

$$E = \Delta_0(T)(1.33 - i0.08), \quad E = \Delta_0(T)(1.28 - i0.04),$$

$$E = \Delta_0(T)(1.09 - i0.22),$$

$$E = \Delta_0(T)(0.71 - i0.05), \quad E = \Delta_0(T)(0.33 - i0.34),$$

$$E = \Delta_0(T)(0.23 - i0.71). \tag{5}$$

The last two modes have imaginary parts of the same order as the real ones. This implies that these modes are damped very strongly and cannot be considered as resonances. At the same time, there are modes in the spectrum with the frequencies close to the observed $\omega \approx 1.25\Delta_0(T)$ and $\omega \approx 1.35\Delta_0(T)$. These are well-defined modes (the imaginary part in the frequency is substantially smaller than the real part)

$$E = \Delta_0(T)(1.33 - i0.10), \quad E = \Delta_0(T)(1.33 - i0.08),$$

$$E = \Delta_0(T)(1.28 - i0.04). \tag{6}$$

But there is no reason why at one sound frequency one branch should be excited while at another frequency another branch would also be excited. Thus we are obliged to rule out these phases as possible candidates for the "boundary" state.

$$\textit{\textbf{Phase}} \begin{pmatrix} 110 \\ ii0 \\ 000 \end{pmatrix}$$

For the spectrum of collective modes in the above phase the following set of equations (at zero momentum of excitations) is true:

$$\int_0^1 dx x^2 \left(1 + \frac{2\Delta^2}{\omega^2}\right)(J-1) = 0(6),$$

$$\int_0^1 dx (1-x^2) \left(1 + \frac{2\Delta^2}{\omega^2}\right) J = 0(4),$$

$$\int_0^1 dx (1-x^2) \left(1 + \frac{\Delta^2}{\omega^2}\right) J = 0(4),$$

$$\int_0^1 dx (1-x^2) \left(1 + \frac{3\Delta^2}{\omega^2}\right) J = 0(4). \qquad (7)$$

From these equations the following high-frequency modes were obtained:

$$E = \Delta_0(T)(1.55 - i0.32), \quad E = \Delta_0(T)(1.2 - i0.06),$$

$$E = \Delta_0(T)(0.62 - i0.05),$$

$$E = \Delta_0(T)(0.4 - i0.55), \quad E = \Delta_0(T)(0.3 - i1.0). \qquad (8)$$

The last two modes have imaginary parts of the same order as the real ones. They are damped very strongly and therefore cannot be considered as resonances.

Among the collective modes in this state there is only one mode $E=\Delta_0(T)(1.2-i0.06)$, with frequency close to that of one of the observed peaks, $\omega \approx 1.25\Delta_0(T)$. Thus we cannot explain the existence of the peak at $\omega \approx 1.35\Delta_0(T)$. Therefore, similar to the previous case, we may rule out this phase as a candidate for the "boundary" state.

IV. DIFFERENT BRANCHES OF THE SQUASHING MODE

Kalbfield *et al.*[8] suggested one other possible reason for the difference in the longitudinal and transverse sound experiments data. They proposed that, in accordance with Ref. 10, the longitudinal and transverse sounds can be coupled to different branches of the squashing mode: that said, in the absence of a magnetic field the transverse sound couples to $J_Z=\pm 1$ modes while the longitudinal sound couples to the $J_Z=0$ mode. While the frequencies of all branches of the squashing mode are the same (at zero momentum of collective mode) and are equal to $\omega_{sq}=\sqrt{12/5}\Delta_0 \approx 1.55\Delta_0$, dispersion corrections for branches with different projections of the total moment of Cooper pairs J appear to be different. These dispersion corrections have been calculated by a number of authors, with complete calculations for all 18 collective modes in the B phase made by Brusov and Popov,[11] who obtained the following results for considering branches of the squashing mode:

$$\omega_{sq}^2 = 12/5\Delta_0^2(T) + 0.418 c_F^2 k^2 (2, \pm 1, i) \text{ and } \omega_{sq}^2 = 12/5\Delta_0^2(T) + 0.502 c_F^2 k^2 (2,0,i).$$

Here the indexes in brackets are (J, J_Z, i). From these results it follows that the longitudinal resonance should take place at higher frequencies than the transverse one and this explains the attenuation peak at temperatures below the squashing mode in the experiments on the transverse sound attenuation.

However, if one compares the real splitting of the squashing-mode branches via dispersion corrections (via nonzero momentum of the Cooper pairs) and difference between the squashing-mode peak (from the longitudinal experiments) and the transverse sound data, it appears that the former is much smaller than the latter. To estimate the dispersion-induced splitting of the squashing-mode branches we can refer to a similar splitting of the real squashing mode observed by Shivaram *et al.*[12] This splitting is of the order $0.01T_C$. While the splitting of the squashing mode is a little bit larger it should be of the same order of magnitude. On the other side, the difference between the squashing-mode peak (from the longitudinal experiments) and the transverse sound data is $0.2\Delta_0(T)-0.3\Delta_0(T)$ depending on the transverse sound frequency. Taking into account that T_C is of the same order of the magnitude as $\Delta_0(0)$, we see that the splitting of the squashing mode is of the order $0.01\Delta_0(T)$, which is too small compared to the difference between the longitudinal and the transverse sound data, which is of the order of $0.2\Delta_0(T)-0.3\Delta_0(T)$. Thus we come to a conclusion that the coupling of the longitudinal and transverse sounds to different branches of squashing mode cannot explain the observed features.

V. DEFORMED B PHASE

The presence of a boundary leads to deformation of the order parameter. One component of vector $\boldsymbol{d}$, perpendicular to the boundary, becomes zero at the boundary in the case of a mirror, as well as diffusive reflection of atoms. In other words, Cooper pairs tend to move in the plane parallel to the boundary. We will consider x-y plane as a boundary.

While the A phase in the slab geometry should have the same order parameter as in the bulk case (only $\boldsymbol{l}$ should be parallel to z), in the case of the B-phase order parameter of the bulk B phase cannot satisfy the boundary condition if it remains nondeformed. This deformation is coupled to the appearance of an additional gradient energy and decreasing the condensation energy.

Twenty years ago Brusov and Popov[11] investigated the influence of gap distortion, caused by a dipole interaction or by different types of external perturbations such as magnetic or electric fields, on the order parameter collective modes in the B phase. They demonstrated that the consequences of such an influence were significant: it changed the frequencies of all collective modes and, most importantly, split the pair breaking, the squashing and the real squashing modes at a zero momentum $\boldsymbol{q}$ of the collective excitations. At a nonzero $\boldsymbol{q}$ they predicted a branch crossing of these modes with different J_Z.

Let us summarize their results and see what we have for the squashing mode.

In the presence of external perturbations such as the magnetic or electric fields or the boundary, the order parameter takes the following form:

$$A_{ij} = \lfloor \Lambda^{1/2} R(\hat{n}, \theta) \rfloor_{ij} e^{i\varphi}; \tag{9}$$

here, R is matrix of rotation around **n**-axis on angle θ and Λ is diagonal matrix with elements

$$\lambda_1, \lambda_1, \lambda_2 \quad \text{and} \quad \lambda_1 = \Delta_1^2 = \Delta^2 + \Omega^2, \quad \lambda_2 = \Delta_2^2 = \Delta^2 + \alpha\Omega^2. \tag{10}$$

We can rewrite the order parameter as

$$A_{ij} \approx \begin{pmatrix} \Delta_1 \cos\theta & -\Delta_1 \sin\theta & 0 \\ \Delta_1 \sin\theta & \Delta_1 \cos\theta & 0 \\ 0 & 0 & \Delta_2 \end{pmatrix}. \tag{11}$$

Here $\theta = \arccos(-1/4)$. In the case of a dipole interaction and the electric fields, $\alpha = -2$; in the case of the magnetic fields, $\alpha = -4$. In the case of confined geometry, the boundary will suppress the gap in the single-particle spectrum in a perpendicular direction; thus, we assume α to be negative in the vicinity of the boundary. Thus, the gap Δ_1 along the boundary will be larger than the gap Δ in the bulk liquid, while the gap Δ_2 perpendicular boundary will be smaller than Δ.

Results obtained by Brusov and Popov[11] for the squashing mode are as below:

$$E^2 = (12/5)\Delta_0^2(T) + (6\alpha + 11)\Omega^2/10 \quad \text{for } (2, \pm 1, i) \text{ branches},$$

$$E^2 = (12/5)\Delta_0^2(T) + (2\alpha + 3)\Omega^2/2 \quad \text{for } (2, 0, i) \text{ branch},$$

$$E^2 = (12/5)\Delta_0^2(T) + 3(\alpha + 3)\Omega^2/5 \quad \text{for } (2, \pm 2, i) \text{ branches}. \tag{12}$$

The results for the $(2,0,i)$ branch are insufficient because in the longitudinal sound experiments a signal comes from the whole volume of liquid while the influence of the region near the boundary is small or even negligible in this case. Thus in this case we have "bulk" squashing mode frequency $\omega_{sq} = \sqrt{12/5}\Delta_0 \approx 1.55\Delta_0$.

Essential for us is a result for the $(2, \pm 1, i)$ branches, which are excited in the transverse sound experiments. This result allows us to explain the observed features of the transverse sound experiments: resonant absorption of the transverse sound below the squashing-mode frequency, dependence of the absorption frequency on the transverse sound one. Thus, from results for the $(2, \pm 1, i)$ branches it follows that $(6\alpha + 11)\Omega^2/10 < 0$, thus $\alpha < -11/6 \approx -2$. We can estimate the value of the extra term in the squashing-mode spectrum appearing via the boundary influence (through the gap distortion). From the experiments[7] it follows that $|(6\alpha + 11)\Omega^2/8\sqrt{15}\Delta|$ should be of the order of $0.3\Delta(T)$ at a sound frequency of 28 MHz and of the order of $0.2\Delta(T)$ at a sound frequency of 47 MHz.

VI. CONCLUSION

We have analyzed both old and recent transverse sound experiments in superfluid ^{3}He-B, where some peaks in the transverse sound absorption have been observed. These peaks are in disagreement with the squashing-mode frequency, obtained from longitudinal sound experiments. We considered several possible explanations of the above experimental data: the existence of a new superfluid phase in the vicinity of the boundary, excitation of different branches of the squashing mode by the longitudinal and transverse sounds, and deformation of the B phase by the wall. Our analysis of existing experimental data leads us to a conclusion that the deformed B phase is realized near the boundary. This important conclusion solves the considered issue. Our conclusion implies that the influence of the wall or, speaking in general, of any confined geometry does not lead to the existence of a new phase near the boundary, as had been assumed for many years up until now, but instead, similarly to other external perturbations (such as the magnetic and electric fields), the wall deforms the order parameter of the B phase and this deformation leads to some important consequences. In particular, frequencies of the collective modes in the vicinity of the boundary change by up to 20%. Their branches could cross at a nonzero momentum of the collective excitations. Knowledge of the collective-mode spectrum together with experimental data on the transverse sound allows us to estimate parameters of the gap deformation, caused by the boundary. One other evidence in support of our conclusion with regard to the realization of the deformed B-phase in ^{3}He-B phase near the boundary follows from the fact that, in Ref. 7, the peak in transverse ultrasound impedance was observed at a temperature of 0.94 T_c. This value is lower than the well-known temperature $T_{AB} = 0.97\ T_c$ for the transition from A-phase to B-phase. The authors of Ref. 7 interpreted this fact as a manifestation of the transition from a supercooled A-phase to B-phase. However, it becomes clear now that this peak demonstrates the transition between A-phase and deformed B-phase with the observed difference in the transition temperature caused by the difference between the order parameter of the B-phase and that of the deformed B-phase.

Further transverse sound experiments at different sound frequencies would be highly desirable in order to get the full picture. The already carried out experiments proved important in terms of further investigation of the superfluid ^{3}He in confined geometry.

The discussed problem is important and is in very close connection with the study of superfluid ^{3}He in aerogel.[13]

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