Transport in a spin-incoherent Luttinger liquid

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We theoretically investigate transport in a spin-incoherent one-dimensional electron system, which may be realized in quantum wires at low-electron density and finite temperature. Both the pure and disordered cases are considered, both in finite wires and in the thermodynamic limit. The effect of Fermi-liquid leads attached to the finite-length system is also addressed. In the infinite system, we find a phase diagram identical to that obtained for a spinless Luttinger liquid, provided we make the identification $g=2g_c$, where g is the interaction parameter in a spinless Luttinger liquid and g_c is the interaction parameter of the charge sector of a Luttinger-liquid theory for electrons with spin. For a finite-length wire attached to Fermi-liquid leads, the transport depends on the details of the disorder in the wire. A simple picture for the crossover from the spin-incoherent regime to the spin-coherent regime as the temperature is varied is also discussed, as well as some physical implications.

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I. INTRODUCTION

Low-dimensional electron systems have attracted much attention in recent years because they provide an opportunity to realize exceptionally rich physics not readily found in higher dimensions. One such example is the so-called Luttinger-liquid (LL) state of an electron gas.[1,2] The LL state is characterized by gapless excitations and, in the case of an electron gas, by the spin-charge separation realized by the separate collective spin and charge excitations, each with its distinct propagation velocity. The tunneling density of states also exhibits a characteristic power-law suppression at low energies as a result of an "orthogonality catastrophe" at zero energy from the rearranging of the wave functions of the electrons to accommodate the new (tunneling) particle. Experimental evidence for the LL state in one-dimensional electron systems is by now irrefutable with measurements showing both the characteristic power-law suppression of the tunneling density of states and measurements of the spectral function providing direct evidence of spin-charge separation, including quantitative measures of the respective collective mode velocities.[3–6]

Real samples measured in experiments are not perfectly clean, and it is important to understand the effects of impurities on the electronic properties, such as transport, in one-dimensional (1D) interacting electron systems. Work by Kane and Fisher,[7,8] and by Furusaki and Nagaosa[9,10] established the central results for transport in a single and double impurity system in a LL, both at weak and strong impurity strengths. Since then, beautiful numerical studies have confirmed these results in detail[11] and extensions to include finite magnetic fields have been made.[12] For a single impurity in a spinless LL, the main result is that for repulsive electron interactions the impurity "cuts" the LL into two semi-infinite sections, while for attractive electron interactions, the impurity is irrelevant in the renormalization group (RG) sense and scales to zero at low energies. When spin is introduced, the transport can be more complicated, with the spin and charge sectors possibly behaving differently. For example, one possibility is that the spin could pass easily through the impurity while the charge would be reflected. The double impurity system exhibits even richer behavior, including zero-width resonances at zero temperature. At finite temperature, the resonance line shapes exhibit a characteristic non-Lorentzian shape, in contrast to the case for noninteracting electrons. When Fermi-liquid leads are attached to the end of a finite-length 1D LL wire, a new length scale is introduced beyond which the leads play an important role in the physics.[13–20] For example, the dc conductance of a clean system is completely determined by the leads and, therefore, has a value of $2e^2/h$ independent of the strength of the electron interactions in the wire.

LL theory is based on a picture of interacting electrons in which the interaction strength is not too great. In this regime of not too strong electron interactions, the characteristic exchange energy of two electrons—defined by $\hbar$ times the inverse of the time required for two electrons initially in position eigenstates separated by the average interelectron distance to undergo a spin flip—is typically of the same order as the Fermi energy. However, at low densities, the potential energy grows relative to the kinetic energy and eventually dominates it for sufficiently low densities when $na_B \ll 1$, with n being the average density of the electrons and the Bohr radius $a_B=\epsilon\hbar^2/me^2$, with ϵ the dielectric constant, m the mass of the electron, and e its charge. At these low densities, there is a separation of energy scales between the magnetic exchange energy,[21,22] $J\sim E_F e^{(-C/\sqrt{na_B})}$, and the Fermi energy, $E_F=(\pi\hbar n)^2/8m$. Here C is a positive constant of order unity. When the interactions between electrons are very strong, they must tunnel through one another to exchange, leading to an exponentially small J and a situation where $J \ll E_F$. For $na_B \ll 1$ it is possible to reach a regime where the temperature T is much larger than the magnetic exchange energy, but still much less than the Fermi energy: $J \ll T \ll E_F$. We refer to this energy scale hierarchy as the spin-

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incoherent or magnetically incoherent regime.

In this paper, we revisit the transport problem in an interacting 1D electron gas, with an eye toward understanding the behavior in the magnetically incoherent regime. Recently the magnetically incoherent regime has been investigated for a clean, infinite system by studying the one particle Green's function[23–25] and the momentum-distribution function,[26] and for finite systems by studying the influence of incoherent magnetic degrees of freedom on the momentum-resolved tunneling[27] and on the conductance of a clean quantum-point contact.[28,29] Our main result is that the transport in an infinite magnetically incoherent electron gas is very much like that of a spinless LL, except that all the quantum-phase transitions of Kane and Fisher,[7,8] and Furusaki and Nagaosa[9,10] (understood to be in the limit $J\rightarrow 0$, then $T\rightarrow 0$) are obtained by replacing g by $2g_c$, where g is the interaction parameter of the spinless LL and g_c is the interaction parameter of the charge sector of a LL theory for electrons with spin. This result can be understood in the following physical terms: The condition $T\gg J$ means that the spin degrees of freedom become nondynamical in that, within the "thermal coherence time" $t_{th}\sim\hbar/k_BT$, the spin quantum numbers of individual electrons remain unchanged, since a spin-flip transition requires a time $t_J\sim\hbar/J\gg t_{th}$ to occur. Moreover, because the energy splittings between distinct spin states are negligible compared to k_BT, all spin states are excited with equal probability. Hence, dynamically, the electron gas behaves in a "spinless" fashion, since the spin degrees of freedom are static and random, and do not couple to the electron coordinates. More specifically, the charge degrees of freedom behave as a LL (because $T\ll E_F$, only low-energy charge excitations are important) only with effective interaction parameter $2g_c$. This result can be established at the level of the Hamiltonian so that the correspondence $g=2g_c$ in the spin-incoherent regime is actually quite general and applies to any particle-conserving operator. (The single-particle Green's function *does not* involve particle-conserving operators and, therefore, has a qualitatively different form from a spinless[23–25] LL.)

For a finite-length wire in the magnetically incoherent regime, the transport is more subtle. Matveev has argued[28,29] that for a clean wire, the conductance drops to $\frac{1}{2}$ of its zero-temperature value giving e^2/h rather than $2e^2/h$. When disorder is present in the spin-incoherent region of the wire, more careful considerations are needed. We distinguish between two cases: (i) weak and (ii) strong backscattering and discuss features of each.

This paper is organized in the following way. In Sec. II, we discuss important physical models, parameters, and limits for quantum wires with low-electron density. In particular, we show the physics of the incoherent regime is independent of the *range* of electron interactions; the interactions need only be sufficiently strong to achieve $J\ll E_F$. In Sec. III, we establish the equivalence, summarized by $g=2g_c$, between a spinless LL and a spin-incoherent LL for particle-conserving charge properties. In Sec. IV, we discuss what the effects of Fermi-liquids leads are on the transport through a finite-length quantum wire, then, in Sec. V, we discuss some details of the crossover from the magnetically incoherent LL to the familiar spin-coherent LL regime. Finally, in Sec. VI, we present our main conclusions.

II. QUANTUM WIRES AT LOW-ELECTRON DENSITY

It is useful here, in the beginning, to outline the physical situations where we expect the spin-incoherent regime to be realized. The physics we discuss in this paper is expected to be present provided the interactions between electrons is sufficiently strong that there is a separation of magnetic and nonmagnetic energy scales in the problem: $J\ll E_F$. (As we show in Appendix A, this regime can also be understood in terms of a separation of scale in the spin and charge velocities.) It is possible to reach this regime with either short-range or long-range interactions, so it is not necessary to have a Wigner solidlike picture in mind. (Strictly speaking, quantum fluctuations destroy the long-range charge order unless the interactions are of a longer range than a Coulomb.[30]) However, the Wigner solid picture does often provide a convenient physical picture for a one-dimensional electron gas, and we will use it to discuss the low-density limit. In fact, as we discuss below, the effective Hamiltonian in the Wigner solid limit turns out to be quite universal, in the sense that its form is independent of the range of the electron interactions, even down to zero-range interactions.

A. Charge and spin Hamiltonians

With a Wigner solid picture in mind, we can understand the physical state of the electron gas in classical terms as being dominated by the Coulomb repulsion between electrons, which forces the electrons to occupy discrete, evenly spaced positions. A finite but small kinetic energy of the electrons implies small displacements from their equilibrium positions, and the lowest-energy displacements are long wavelength sound modes or "phonons." These displacements can be described within elasticity theory in terms of the displacement $u(x)$ from equilibrium of the solid at point x, and the momentum density $p(x)$.[29] Adding these two contributions to the energy gives the total energy of the elastic medium

$$H_{\text{elastic}} = \int dx\left[\frac{p^2}{2mn} + \frac{1}{2}mns^2(\partial_x u)^2\right], \tag{1}$$

where $s=\sqrt{(n/m)(\partial^2E/\partial n^2)}$ is the sound velocity of the phonons.[31] Here E is the energy of the resting medium per unit length.

In order to obtain a quantum theory, the Hamiltonian (1) can be quantized by imposing the commutation relations $[u(x),p(x')]=i\hbar\delta(x-x')$. Then new fields can be identified as

$$u(x) = \frac{\sqrt{2}}{n\pi}\theta_c(x), \quad p(x) = \frac{n\hbar}{\sqrt{2}}\partial_x\phi_c(x), \tag{2}$$

which satisfy the commutation relations $[\theta_c(x),\partial_{x'}\phi_c(x')]=i\pi\hbar\delta(x-x')$. In terms of these new fields, the Hamiltonian (1) becomes

$$H_c = \hbar v_c\int\frac{dx}{2\pi}\left[\frac{1}{g_c}[\partial_x\theta_c(x)]^2 + g_c[\partial_x\phi_c(x)]^2\right], \tag{3}$$

where

$$v_c = s, \quad g_c = \frac{v_F}{s}, \tag{4}$$

with $v_F = \hbar \pi n / 2m$. The attentive reader will immediately notice that Eq. (3) is just the charge sector Hamiltonian that is familiar from LL theory. Since it is well known that Eq. (3) can be derived for weakly interacting electrons by linearizing the kinetic energy about the Fermi points, the discussion above shows that the Hamiltonian (3) is actually valid for arbitrary strength interactions. In the rest of this paper, we will assume that low-energy charge states are adequately described by Eq. (3).

So far, we have neglected the spin of the electrons. Returning to the Wigner solid picture again, we see that the spin degrees of freedom will act like a Heisenberg spin chain with lattice spacing equal to that of the electron spacing. Virtual hopping of electrons from one site to another (occupied site) requires the electrons to have an opposite spin, resulting in an effective magnetic exchange for the spin chain that is antiferromagnetic. Therefore, the Hamiltonian of the spin sector behaves as

$$H_s = \sum_l J \vec{S}_l \cdot \vec{S}_{l+1}, \tag{5}$$

where $\vec{S}_l$ is the spin of the lth electron and $J > 0$ is the nearest-neighbor exchange energy. The spin chain (5) can be bosonized[32,33] and the low-energy spin excitations can be computed within a LL theory for the spin sector. However, since here we are concerned with the high-energy situation $T \gg J$ (from the point of view of the spin degrees of freedom), we do not pursue that direction.

B. Long-range versus short-range interactions for $T \gg J$

All quantum wires are "quasi-1D" since there is usually some finite width to the wire w. This provides a short-range cutoff for the electron interactions at $x \sim w$, so that for $x \lesssim w, V(x) \sim 1/w$. On the other hand, quantum wires are often gated so that electron separations that are large compared to the distance to the metallic gate d (which is always present in experiments), electrons induce image charges in the gate to produce a dipolar electron-electron interaction for $x \gtrsim d, V(x) \sim d^2/|x|^3$. A potential consistent with this form is[34]

$$V(x) = \frac{e^2}{\epsilon}\left(\frac{1}{\sqrt{x^2 + w^2}} - \frac{1}{\sqrt{x^2 + w^2 + (2d)^2}}\right), \tag{6}$$

where ϵ is the dielectric constant. Since we are interested in the low-density limit where $n^{-1} \gg w$, we can set $w = 0$ to obtain an approximate form of $V(x)$.

In the low-density limit, we can then argue along the lines of Ref. 29. For $n \ll a_B/d^2$, Eq. (6) shows that the interaction between two particles at a typical distance of n^{-1} is small compared to their kinetic energy $\sim E_F$. Here $a_B = \epsilon \hbar^2/me^2$. On the other hand, when the distance between electrons is sufficiently short, $|x| \lesssim n^{-1}(nd^2/a_B)^{1/3} \ll n^{-1}$, the potential energy dominates the kinetic energy, $V(x) \gtrsim E_F$. As a result, at low densities the potential (6) can be modeled by the short-range potential

$$V^{\text{eff}}(x) = \mathcal{V}\delta(x), \tag{7}$$

where $\mathcal{V}$ is chosen to provide the same scattering phase shift as (6).

The model (7) is equivalent at low energies to the 1D Hubbard model. Starting from the Hubbard model, it can be shown[35] that in the low-density limit with $U/t \to \infty$, the spin sector takes the form (5) with the exchange energy given by[36]

$$J = \frac{4t^2}{U} n_e \left(1 - \frac{\sin 2\pi n_e}{2\pi n_e}\right), \tag{8}$$

where n_e is the average number of electrons per site. Recall that we originally motivated the spin Hamiltonian (5) within the Wigner solid picture, which relies on sufficiently long-range interactions. Here, we show that even short-range interactions lead to the spin Hamiltonian (5). We can thus view the Hamiltonian

$$H = H_c + H_s \tag{9}$$

as a general form, valid in the energy hierarchy $J \ll E_F$ for any temperature $T \ll E_F$, including both $J \ll T \ll E_F$ and $T \ll J \ll E_F$. However, the dependence of J on the density depends on the microscopic details[21,22,37] of the electron interactions with the form (8) for zero-range interactions and $J \sim E_F e^{(-C/\sqrt{na_B})}$ for Coulomb interactions.

In the remainder of this paper, we will study the implications of the Hamiltonian (9) in the limit $J \ll T \ll E_F$ on the electrical transport.

III. INFINITELY LONG WIRE

In this section, we show explicitly that in the limit of strong interactions and $J \ll T \ll E_F$ the electrons become effectively spinless (for quantities that do not directly probe spin) and are governed by a Hamiltonian of the form (3) with interaction parameter $g = 2g_c$. Here g is the interaction parameter of a spinless LL and g_c is the interaction parameter of the charge sector for an electron gas with spin and the same interaction strength as in the spinless LL. We discuss implications for the transport in such a magnetically incoherent LL when impurities are present.

A. The relation $g = 2g_c$ for $J \ll T \ll E_F$

We assume that $J \ll T \ll E_F$ and take the limit $J/T \to 0$. Since $T \gg J$, the spin degrees of freedom are nondynamical and the system behaves as if $J = 0$ identically [or equivalently $H_s = 0$ identically from (5)]. Thus, the only dynamics is in the charge sector of the theory. This implies that if we look at any quantity that does not depend explicitly on spin (conductance or compressibility, for example), the system behaves as if it were spinless.

To show this microscopically, we consider a particular basis of states for the Hilbert space of the system. We work in the canonical ensemble, i.e., with a fixed number of electrons. Of course, dynamics in the grand canonical ensemble can be obtained from this by summing over the sectors with each electron number, since the intrinsic physical Hamiltonian is anyway number conserving. For a fixed electron number, a convenient real-space basis set is given by states specifying the position x_n of each electron, and the spin projection on the $\hat{z}$ axis, m_n, in order, from left to right across the system

$$|x_1 \cdots x_N\rangle|m_1 \cdots m_N\rangle = c^{\dagger}_{m_1}(x_1) \cdots c^{\dagger}_{m_N}(x_N)|0\rangle, \qquad (10)$$

where $|0\rangle$ is the vacuum state (no particles).

The physics of the spin-incoherent regime is that, within the thermal coherence time, $t_{th} \sim \hbar/k_B T$, the probability of a transition between states with different values of $\{m_n\}$ is negligible. Hence, the physics is well approximated by neglecting off-diagonal matrix elements in these states. Moreover, in the same approximation, for spin-independent interactions, the matrix elements of H are independent of the $\{m_n\}$, i.e.,

$$\langle m_1' \cdots m_N'|\langle x_1' \cdots x_N'|H|x_1 \cdots x_N\rangle|m_1 \cdots m_N\rangle \approx \langle x_1' \cdots x_N'|H_{sl}|x_1 \cdots x_N\rangle \delta_{m_1',m_1} \cdots \delta_{m_N',m_N}, \qquad (11)$$

where H_{sl} is an effective spinless identical—and "hard core"—particle Hamiltonian that governs the (independent) dynamics within each spin sector. Note that it is manifestly identical, in first quantized form, to the original spinfull Hamiltonian, if the coordinates of all particles are treated equivalently (as some spinless particles).

It is important to recognize that this reduction to a spinless particle problem is extremely general. In particular, nowhere do need we to assume that the system is even spatially uniform, only that exchange processes are everywhere negligible, i.e., $J \ll T$ throughout the system; that there are no explicit spin-dependent interactions in the Hamiltonian; and that electrons are not added or removed from the system during the dynamics. The equivalence to a spinless problem continues to hold in the presence of arbitrary potentials, weak links, etc.

With this understanding, we now address the remaining question of exactly which spinless theory describes the charge dynamics in the bulk of the spin-incoherent wire. By the usual LL arguments (as given above for instance for the Wigner solid), the spinless particle system is described at low energies ($T, E \ll E_F, E_F r_s$) by the bosonized Hamiltonian

$$H_{\text{incoh}} = \hbar v \int \frac{dx}{2\pi}\left(\frac{1}{g}(\partial_x \theta)^2 + g(\partial_x \phi)^2\right), \qquad (12)$$

with characteristic "zero-sound" velocity v and interaction parameter g. Here we follow one standard convention, in which the normalization of the fields is fixed by the relation $\partial_x \theta(x) = \pi \rho(x)$ with $\rho(x)$ the fluctuation in electron density at position x, and the commutation relation $[\theta(x), \partial_{x'}\phi(x')] = i\pi\hbar\delta(x-x')$. In such a spinless gas, power-law charge density correlations occur at wave vectors which are multiples of $Q_{CDW} = 2\pi n$ (the reciprocal lattice vector of the incipient Wigner solid), and we define $Q_{CDW} = 2\tilde{k}_F$, which gives $\tilde{k}_F = \pi n$.

Upon crossing over from the spin-incoherent regime, $T \gg J$, to the ultimate low-energy limit, $T \ll J$, we expect the description of the system to change to the more "conventional" spinful LL theory. This theory exhibits, as is well known, spin-charge separation, so the charge dynamics can be discussed independently. Again, standard arguments give the bosonized effective charge Hamiltonian (including the effects of rather arbitrary interactions) exhibited in Eq. (3) with $v_c = v_F/g_c$, $v_F = \hbar k_F/m$, and $k_F = \pi n/2$. Normalization is fixed here by $\partial_x \theta_c(x) = \pi \rho_c(x)/\sqrt{2}$, and θ_c, ϕ_c are taken to obey the same commutation relations as θ, ϕ above. The value of the undetermined "LL parameter" g_c depends in detail upon the nature of interactions in the electron system, but takes the value $g_c = 1$ for independent electrons and generally decreases with the strength of repulsive interactions.

Comparing Eqs. (12) and (3), one sees a strong similarity. In fact, in the limit $J \ll E_F$, we expect that they can indeed be identified. This is because, with exchange energy so small compared to the characteristic energies of the individual electron (and hence charge) dynamics, we do not expect the presence of these weak additional exchange interactions to substantially modify the charge dynamics itself (except for the emergence of $2k_F$ correlations—see Sec. V—which is in fact a "weak" effect in this limit). Thus, up to corrections of $O(J^2/E_F^2)$, we expect that the same charge Hamiltonian should govern charge dynamics both for $T \ll J$ and $T \gg J$. Comparing the different normalization conventions [relating the fields to the physical charge density $\rho(x) = \rho_c(x)$], we see that we must equate

$$\theta = \sqrt{2}\theta_c, \qquad (13)$$

$$\phi = \phi_c/\sqrt{2}. \qquad (14)$$

Requiring identity of H_{sl} and H_{incoh}, we immediately find $v = v_c$ and $g = 2g_c$ as promised. We also obtain $\tilde{k}_F = 2k_F$, which implies the absence of $2k_F = \tilde{k}_F$ oscillating correlations in the charge density in the spin-incoherent regime. Their emergence at low temperature is discussed in Sec. V.

B. Transport through single and double impurities

Having established in Sec. III A that for properties that do not depend explicity on the spin, such as the conductance, a low-density electron gas in the regime $J \ll T \ll E_F$ behaves effectively as a spinless LL with $g = 2g_c$, we are now ready to address transport properties of an infinitely long wire. Fortunately, most of the work has been done for us already by Kane and Fisher[7,8] and by Furusaki and Nagaosa.[9,10] All that needs to be done is to substitute $g = 2g_c$ into their formulas for spinless electrons. For completeness, we summarize the important results here.

For a single impurity in a spinless LL, the result found earlier was that for repulsive (attractive) interactions, $g < 1$ $(g > 1)$ the impurity became relevant (irrelevant) in the renormalization group (RG) sense at the lowest energies. Thus, in the spin-incoherent regime the critical value of g_c is $\frac{1}{2}$. In particular, this implies that for $\frac{1}{2} < g_c < 1$ the transparency of a single barrier should *increase rather than decrease* under the RG flows for $J \ll T \ll E_F$. More generally, in the spin-incoherent regime, we expect that for weak tunneling (strong backscattering)

$$G(T) = \frac{dI}{dV} \propto T^{(1/g_c)-2}, \tag{15}$$

and for weak backscattering (strong tunneling)

$$G_0 - G(T) \propto T^{2(2g_c-1)}, \tag{16}$$

where $G_0 = 2g_c e^2/h$ is the bare conductance of the spin-incoherent ($J=0$) wire with no impurities.

For the two-impurity problem, the results can also be carried over directly. As in the case of the spinless LL with two impurities, the double impurity nature of the problem will only be relevant at energy scales below $\sim\hbar v_c/d$ where d is the separation between the two barriers, since at larger energies the two barriers will not add coherently. We remind the reader that the main results from the spinless LL case are that the double barrier exhibits behavior in striking contrast to the noninteracting electron liquid, which has temperature-independent resonances with a Lorentzian line shape at low T: For repulsive interactions the LL has non-Lorentzian resonances with a width that vanishes as $T\to 0$. These line shapes are determined by a universal scaling function[8]

$$G(T,\delta) = \tilde{G}(c\delta/T^{\lambda}), \tag{17}$$

where δ is a small parameter to tune away from the resonance, c is a dimensionful constant, and

$$\lambda = 1 - 2g_c. \tag{18}$$

Defining $X \equiv c\delta/T^{\lambda}$, we see that for $X\to 0$

$$\tilde{G}(X) = G_0[1 - X^2 + \mathcal{O}(X^4)], \tag{19}$$

while for $X\to\infty$,

$$\tilde{G}(X) \approx X^{-1/g_c}. \tag{20}$$

We note that at a low temperature, Eq. (20) applies, explicitly showing that the tails of the resonance are non-Lorentzian.

The most important results are summarized in the phase diagram shown in Fig. 1. As in the case of the spinless LL, we expect there to be a line of Kosterlitz-Thouless separatrices between the zero conductance and perfect conductance (on resonance) regions of the phase diagram.

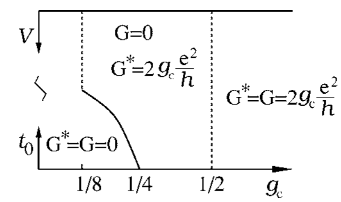

FIG. 1. Phase diagram (in the limit $J\to 0$, then $T\to 0$) for a spin-incoherent Luttinger liquid with a double barrier structure. Here V denotes the strength of the backscattering for weak impurities, and t_0 the strength of the tunneling for strong impurities. The conductance off resonance is denoted by G and the conductance on resonance is denoted by G^*. The dashed line at $g_c=\frac{1}{2}$ is the fixed line for transmission off resonance separating the regions where electrons will $\left(g_c>\frac{1}{2}\right)$ or will not $\left(g_c<\frac{1}{2}\right)$ propagate through the double barrier structure. The dashed line at $g_c=\frac{1}{8}$ separates the regions of zero $\left(g_c<\frac{1}{8}\right)$ and nonzero $\left(g_c>\frac{1}{8}\right)$ conductance on resonance at vanishing energies. The solid line between $g_c=\frac{1}{8}$ and $\frac{1}{4}$ is a line of Kosterlitz-Thouless separatrices. Compare with Fig. 3 of Ref. 8.

IV. FINITE-LENGTH WIRE CONNECTED TO FERMI-LIQUID LEADS

Transport through a finite-length segment of a spin-incoherent LL connected to Fermi-liquid (FL) leads is more subtle than the case discussed in Sec. III B in which the system is infinite in length. Here we have in mind a situation where the finite-length segment is adiabatically connected to the leads so that no backscattering occurs due to the slowly changing background potential itself. Matveev has recently argued[28,29] that for a clean wire adiabatically connected to FL leads the conductance is reduced by a factor of 2 when $J \ll T \ll E_F$ compared to $T \ll J, E_F$, resulting in a conductance of e^2/h rather than $2e^2/h$ for a single-mode wire. The physics of this result appears to be that when an electron with energy $\sim T$ from the lead enters the spin-incoherent LL portion the spin modes are reflected because there are no spin states of energy $\sim T \gg J$, while the charge modes have states up to energy $\sim E_F \gg T$ and, thus, are able to pass through.

Here we discuss some additional considerations when there are impurities in the finite length spin-incoherent LL. We begin with the simplest case—a single impurity in the center of the wire. As in the infinite system, there are two limits to consider: (i) weak backscattering and (ii) weak tunneling. The weak tunneling limit is the most straightforward of the two. We also discuss the case of more than one impurity.

A. Weak tunneling through a finite wire

Let us first discuss the case of a very strong potential barrier in the center of the wire. In this limit, an infinite one-dimensional system is cut into two semi-infinite pieces and electrons tunnel between these two semi-infinite systems. The tunneling can be described by the Hamiltonian

$$H_{tun} = t_0 \sum_{\pm} [\Psi^{\dagger}_{1\pm}(0)\Psi_{2\pm}(0) + \text{H.c.}], \tag{21}$$

where t_0 is the tunneling amplitude for an electron to hop from side 1 to side 2. Using Eq. (21), the current I through the barrier can be computed via Fermi's Golden Rule as

$$I = \frac{et_0^2}{2\pi\hbar}\int d\omega[\varrho_1^{>}(\omega)\varrho_2^{<}(\omega - eV) - \varrho_1^{<}(\omega - eV)\varrho_2^{>}(\omega)], \tag{22}$$

where $\varrho^{>}(\varrho^{<})$ is the tunneling density of states for adding (removing) an electron *at the end of the wire*. The two are related by $\varrho^{<}(\omega) = \varrho^{>}(-\omega)$. The subscripts on the density of states refers to the two semi-infinite segments of the 1D system. Clearly then, computing the current depends on knowing the tunneling density of states at the end of the wires. The energy dependence will depend on the energy itself: For a wire of length L, for $\hbar\omega \gg \hbar v_c/L$ the Fermi-liquid leads will not be felt and we can use the tunneling density of states of an infinite spin-incoherent wire near a boundary, while for $\hbar\omega \ll \hbar v_c/L$ the energy dependence of the tunneling density of states will be dominated by the FL leads.

For $eV > \hbar v_c/L$, we can use the tunneling density of states in the spin-incoherent regime for a semi-infinite system computed in Ref. 27 from the Green's function, $\mathcal{G}_\sigma^>(0,\tau) = \langle \Psi_\sigma(0,\tau)\Psi_\sigma^\dagger(0,0)\rangle$,

$$\mathcal{G}_\sigma^>(0,\tau) \sim \langle e^{i[\phi_c(0,\tau)-\phi_c(0,0)]/\sqrt{2}}\rangle = e^{-\langle\tilde{\phi}_c^2\rangle/4} \sim \left(\frac{1}{\tau}\right)^{1/2g_c}, \tag{23}$$

where $\tilde{\phi}_c = \phi_c(0,\tau) - \phi_c(0,0)$. The correlator $\langle\tilde{\phi}_c^2\rangle$ was evaluated using the Hamiltonian (3) subject to the boundary condition $\partial_x\phi_c(x=0)=0$, i.e., that no current passes through the barrier. [Finite current comes from (21).] See Appendix B for details.

After Fourier transforming the Green's function (23) to the frequency domain, the frequency dependence of the tunneling density of states at the end of the wire is obtained

$$\varrho^>(\omega) \sim \mathrm{Re}[\mathcal{G}^>(0,\omega)] \sim \omega^{1/(2g_c)-1}. \tag{24}$$

Note that for $g_c > \frac{1}{2}$ the density of states diverges. This should be contrasted with the result obtained for the infinite system in which $\varrho^>(\omega) \sim \omega^{1/(4g_c)-1}$ and, therefore, diverges for $g_c > \frac{1}{4}$.[25] Substituting the result (24) into Eq. (22) gives the following result for the conductance of the wire (for $T \gtrsim V$, where V is the voltage):

$$G(T) = \frac{dI}{dV} \propto t_0^2 T^{(1/g_c)-2}, \quad T, eV \gtrsim \hbar v_c/L. \tag{25}$$

This reproduces the expected result of Eq. (15) obtained through the indentification $g=2g_c$.

Let us now suppose we go to sufficiently low temperatures and voltages that the "charge dephasing length" (this is not strictly proper terminology, but it will do) is longer than the spin-incoherent wire, i.e., $T, eV \ll \hbar v_c/L$, but still $J \ll T$. Then clearly, the charge excitations are modified on these energy scales by the absence of interactions in the leads. In an ordinary LL, where the spin and charge energy scales are comparable, the condition $T \ll \hbar v_c/L$ also implies $T \ll \hbar v_s/L$, so that in this regime all excitations become controlled by the leads. In that more familiar situation, the tunneling density of states crosses over to a constant value, as is appropriate for a Fermi liquid. This is not the case for the spin-incoherent wire. One may understand this by the fact that, in a time $\sim\hbar/T$, the spin disturbance created by the tunneling event has not propagated to the leads and remains within the spin-incoherent region.

A naïve approach to the behavior in this regime is simply to recalculate $\mathcal{G}_\sigma^>(0,\tau)$ in Eq. (23) by assuming a spatially dependent $g_c(x)$, with $g_c(x) \to 1$ for x outside the wire (in the leads). This gives a decay at long times, $\mathcal{G}_\sigma^>(0,\tau) \sim 1/\sqrt{\tau}$. Matching this to Eq. (25) for $T \sim \hbar v_c/L$, one finds

$$G = \frac{dI}{dV} \propto t_0^2 \left(\frac{\hbar v_c}{L}\right)^{1/g_c - 1} \frac{1}{T}, \quad T, eV \lesssim \hbar v_c/L, \tag{26}$$

valid so long as one is in the weak tunneling limit, $G \ll e^2/h$. Remarkably, the conductance does not become constant for such temperatures, but actually diverges more strongly with decreasing temperature than for an infinite spin-incoherent wire. This is because the spin excitation created by the tunneling event is still within the wire, so the enhancement of the density of states is still operative, while the competing suppression of the density of states due to "charging" is no longer in effect once the charge disturbance has exited the wire. A crossover to true Fermi-liquid behavior, therefore, is only expected once the spin disturbance has had time to reach the leads, requiring $T \ll J$.

Similar reasoning applies to the case when two impurities are present in the short wire, but it is more involved. We expect the same results as we found for the infinite spin-incoherent wire for $T, eV \gtrsim \hbar v_c/L$, that is the Kane-Fisher and Furusaki-Nagaosa results with $g=2g_c$. For low energies, $T, eV \lesssim \hbar v_c/L$, the leads again play an important role in the conductance, the precise nature of which remains to be determined.

B. Weak backscattering in a finite wire

For $J \ll T \ll E_F$ and a weak impurity, we are asking about corrections to the simpler problem, attacked by Matveev, for the clean wire. It is clear that, even in the clean case, the spin-incoherent region constitutes a strong deviation from the usual regime of ideal conduction quantization, $G_{\mathrm{ideal}} = 2e^2/h$. Matveev has given arguments that suggest the conductance is reduced to

$$G(T) = \frac{e^2}{h} + \frac{e^2}{h}F(J/T), \tag{27}$$

where $F(x) \to 0$ for $x \to 0$ and $F(x) \to 1$ for $x \to \infty$, so that $G(T) \approx e^2/h$ for $J \ll T$ deep in the spin-incoherent limit.

Strictly speaking, the impurity corrections in this limit should be calculated using perturbation theory starting from known correlation functions of the clean "Matveev problem." Unfortunately, it is not clear from the analysis of Refs. 28 and 29 how to carry out such a perturbative calculation. Lacking this, we do not discuss this problem in detail. A general remark is that, in this limit, the corrections to the conductance due to the impurity are small and, hence, require precision to observe. Once the corrections are no longer small (which will occur at a low enough temperature for $g_c < \frac{1}{2}$, provided the thermal length does not first exceed the wire length), the perturbative approach has broken down.

Can we guess the nature of the first perturbative correction? Let us assume that the (charge) thermal length is less than the length of the (spin-incoherent) wire, $\hbar v_c/T < L$, for simplicity. A naïve extension of the arguments of Refs. 28 and 29 then gives a suggestion. The arguments, therein, proceed by determining the power radiated to infinity by the spin-charge separated modes in a bosonized formulation of the leads. This dissipated power, calculated for an imposed (charge) current I, is proportional, by definition, to I^2R, where R is the physical resistance. In this formulation, it appears that the charge and spin resistances are added together, $R=R_c+R_s$, since the charge and spin sectors provide separate channels for energy to radiate, and we are interested in the total rate of radiation. Naïvely, a weak impurity in the spin-incoherent region, which has the form of a potential, couples only to the charge density. Therefore, we may naïvely estimate its effect by calculating the leading order increase in R_c by the impurity. Thus, one expects a (small) additive contribution to the resistance, whose scaling form (e.g., power-law dependence on temperature, etc.) is that of a spinless LL with spinless Luttinger parameter $g=2g_c$, i.e, the right-hand side of Eq. (16).

V. CROSSOVER TO SPIN-COHERENT REGIME

In order to better understand the physics (including and beyond electrical transport) to be expected in the spin-incoherent LL regime, it is worthwhile to elaborate on some features of the crossover between $J \ll T \ll E_F$ and $T \ll J \ll E_F$. A discussion of the approach to the spin-incoherent LL from a finite-temperature LL is discussed in Appendix A. A discussion of the changes to the spectral function (obtained by Fourier transforming the Green's function, for example) are given in Refs. 25 and 27. The main result is that the propagating spin mode of the LL theory is lost and the charge mode excitation is broadened in momentum space to an amount of the order of the Fermi wave vector. For short-range interactions, there is also a shift in the "edge" of the momentum distribution from $k_F=\pi n/2$ to $\tilde{k}_F=2k_F=\pi n$ as T goes from below to above J.[26] Moreover, there is also a crossover from $2k_F$ to $4k_F$ oscillations in the density-density correlation function.[38] This can be seen by introducing magnetoelastic coupling in the spin Hamiltonian (5) by making the magnetic exchange J depend on the displacement from equilibrium of neighboring electrons (assuming the Wigner solid picture we discussed earlier), $u_{l+1}-u_l$, as

$$J_l = J_0 + J_1(u_{l+1} - u_l) + \mathcal{O}[(u_{l+1} - u_l)^2]. \qquad (28)$$

To the second order in J_1 one finds

$$\langle \rho(x)\rho(x')\rangle^{(2)} \propto (J_1)^2 \sum_{l,l'} \int_0^{\beta} d\tau_1 \int_0^{\tau_1} d\tau_2 \langle \vec{S}_{l+1}\cdot\vec{S}_l(\tau_1)\vec{S}_{l'+1}\cdot\vec{S}_{l'}(\tau_2)\rangle^{(0)}\langle \rho(x)\rho(x')\rangle^{(0)}, \qquad (29)$$

where $\langle \rho(x)\rho(x')\rangle^{(0)}$ is the density-density correlation function for $J=J_0$ identically and $\langle \vec{S}_{l+1}\cdot\vec{S}_l(\tau_1)\vec{S}_{l'+1}\cdot\vec{S}_{l'}(\tau_2)\rangle^{(0)}$ is the dimer correlation function for $J=J_0$ identically and $\rho(x)=\sum_l \delta(x-al-u_l)$. In the limit of strong interactions ($J\ll E_F$) considered here, $\langle \rho(x)\rho(x')\rangle^{(0)}$ contains only $4k_F$ oscillations. However, at the second order the dimer correlation function enters and for $T\ll J$ a small lattice distortion can lower the energy by allowing singlet pairs to form on adjacent pairs of sites. This produces a $2k_F$ oscillation in the dimer correlation function which enters the density-density correlation function at the second order and, thus, produces $2k_F$ oscillations in that quantity as well. When $T\gg J$ the dimer correlations are lost and only the $4k_F$ oscillations will remain in the density-density correlation function.

The loss of $2k_F$ oscillations in the density-density correlation function as T rises above J will have implications for electrical transport and drag between parallel quantum wires. In particular, if there is some potential $V(x)$ acting on the electrons in the wire (either static due to impurities or dynamic due to electrons in another nearby wire in the drag geometry) with significant $2k_F$ Fourier components coupling to the electron density, these components of the density oscillations will be lost when $T\gg J$. This could lead to a sharp temperature dependence of the electrical transport.

VI. CONCLUSIONS

The main conclusion of this paper is that the physics of particle-conserving quantities that do not explicity depend on spin are described by a spinless Hamiltonian with $g=2g_c$, in the regime $J\ll T\ll E_F$ where g is the interaction parameter of the spinless LL and g_c is the interaction parameter of the charge sector in the usual LL theory of electrons with spin that describes when $T\ll J$. Physically this follows from the condition $J\ll T$, which renders the spins effectively nondynamical. The condition $T\ll E_F$ allows the charge sector to be described by an effective low-energy LL theory.

As an application, we discuss single and double impurity problems of an infinitely long 1D electron gas and a finite-length system coupled to Fermi-liquid leads. In the infinite case, all of the phase diagrams for a spinless LL can be directly used to obtain the behavior of the spin-incoherent 1D system by using the identification $g=2g_c$. For a finite-length spin-incoherent LL of length L, and for energies larger than $\hbar v_c/L$ the transport behaves much like the infinite case. However, for lower energies the effects of the FL leads dominate the transport.

We have discussed how the condition $J\ll E_F$ implies that the electron interactions must be very strong and that this separation of magnetic and nonmagnetic energy scales does not depend on the range of the interactions. We have also discussed how a 1D Wigner solid picture of electrons at low density provides a clear physical picture of how the physics we discuss in this paper may arise, and we have shown how the physical results of interest obtained within the Wigner solid picture are actually quite general since they can also be shown to hold for models with very strong, but short-range interactions, such as the Hubbard model.

We have also discussed general features of the spin-incoherent regime and which properties are expected to change as a function of temperature when $J\ll T$ or when $J \gg T$. For example, the $2k_F$ oscillations of the density-density correlation function are lost at $T\gg J$ and this may affect the coupling of the density to a background potential and, hence, the transport or any other quantity that depends on density variations in the electron gas.

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APPENDIX A: FINITE-TEMPERATURE LUTTINGER-LIQUID THEORY FOR $v_s \ll v_c$

The approach to the spin-incoherent regime $J \ll T \ll E_F$ from the LL state can be understood in the limit of vanishing spin velocity[39] relative to charge velocity $v_s/v_c \to 0$ at finite temperature.

We assume that the spin and charge Hamiltonians are given by the LL forms

$$H_c = \hbar v_c \int \frac{dx}{2\pi}\left[\frac{1}{g_c}[\partial_x\theta_c(x)]^2 + g_c[\partial_x\phi_c(x)]^2\right], \quad \text{(A1)}$$

$$H_s = \hbar v_s \int \frac{dx}{2\pi}\left[\frac{1}{g_s}[\partial_x\theta_s(x)]^2 + g_s[\partial_x\phi_s(x)]^2\right], \quad \text{(A2)}$$

where the charge (c) and spin (s) fields are

$$\theta_c = \frac{1}{\sqrt{2}}(\theta_\uparrow + \theta_\downarrow), \quad \theta_s = \frac{1}{\sqrt{2}}(\theta_\uparrow - \theta_\downarrow), \quad \text{(A3)}$$

$$\phi_c = \frac{1}{\sqrt{2}}(\phi_\uparrow + \phi_\downarrow), \quad \phi_s = \frac{1}{\sqrt{2}}(\phi_\uparrow - \phi_\downarrow), \quad \text{(A4)}$$

so that $[\partial_x\theta_\alpha(x), \phi_\beta(x')] = -i\pi\delta(x-x')\,\delta_{\alpha,\beta}$, where $\alpha,\beta = c$ or s.

Let us consider the one particle Green's function in imaginary time

$$\mathcal{G}_\sigma(x,\tau) = \langle \Psi_\sigma(x,\tau)\Psi_\sigma^\dagger(0,0)\rangle, \quad \text{(A5)}$$

where the average is taken at finite temperature. Neglecting the rapidly oscillating pieces coming from $R \to L$ and $L \to R$ scattering, we have

$$\mathcal{G}_\sigma(x,\tau) = \langle \Psi_\sigma^R(x,\tau)\Psi_\sigma^{R\dagger}(0,0)\rangle + \langle \Psi_\sigma^L(x,\tau)\Psi_\sigma^{L\dagger}(0,0)\rangle. \quad \text{(A6)}$$

Substituting

$$\Psi_\pm^R(x) = \frac{1}{\sqrt{2\pi a}} e^{i(k_F x + \theta_c(x)/\sqrt{2})} e^{i\phi_c(x)/\sqrt{2}} e^{\pm i(\theta_s(x)/\sqrt{2})} e^{\pm i\phi_s(x)/\sqrt{2}}, \quad \text{(A7)}$$

$$\Psi_\pm^L(x) = \frac{1}{\sqrt{2\pi a}} e^{-i(k_F x + \theta_c(x)/\sqrt{2})} e^{i\phi_c(x)/\sqrt{2}} e^{\mp i(\theta_s(x)/\sqrt{2})} e^{\pm i\phi_s(x)/\sqrt{2}}, \quad \text{(A8)}$$

where + (−) refers to spin ↑ (↓), using the imaginary time path integral representation, and the Gaussian action that follows from (A1) and (A2), we find

$$\begin{aligned}\langle \Psi_\pm^R(x,\tau)\Psi_\pm^{R\dagger}(0,0)\rangle &\sim e^{ik_F x} e^{-(1/4)\langle(\tilde{\theta}_c(x,\tau)^2\rangle} e^{-(1/4)\langle(\tilde{\phi}_c(x,\tau)^2\rangle} e^{-(1/2)\langle\tilde{\theta}_c(x,\tau)\tilde{\phi}_c(x,\tau)\rangle} \\ &\times e^{-(1/4)\langle(\tilde{\theta}_s(x,\tau)^2\rangle} e^{-(1/4)\langle(\tilde{\phi}_s(x,\tau)^2\rangle} e^{\mp(1/2)\langle\tilde{\theta}_s(x,\tau)\tilde{\phi}_s(x,\tau)\rangle},\end{aligned} \quad \text{(A9)}$$

where the $\tilde{\theta}_c(x,\tau) = \theta_c(x,\tau) - \theta_c(0)$, etc. The corresponding formula for $\langle \Psi_\pm^L(x,\tau)\Psi_\pm^{L\dagger}(0,0)\rangle$ has $k_F \to -k_F$ and $\langle\tilde{\theta}(x,\tau)\tilde{\phi}(x,\tau)\rangle \to -\langle\tilde{\theta}(x,\tau)\tilde{\phi}(x,\tau)\rangle$ for both the spin and charge sectors. All of the correlators in (A9) can be evaluated by doing Gaussian integrals. They are

$$\begin{aligned}\langle\tilde{\theta}_c(x)^2\rangle &= 2\pi\hbar g_c v_c \int \frac{dk}{2\pi} T\sum_{\omega_n} \frac{1 - e^{i(kx-\omega_n\tau)}}{\omega_n^2 + v_c^2k^2} \\ &= \frac{g_c}{2}\ln\left[\frac{\cosh(2\pi Tx/v_c) - \cos(2\pi T\tau)}{(2\pi T/v_c\Lambda)^2}\right],\end{aligned} \quad \text{(A10)}$$

where Λ is a large momentum cutoff. The other correlators are computed likewise

$$\langle\tilde{\phi}_c(x)^2\rangle = \frac{1}{2g_c}\ln\left[\frac{\cosh(2\pi Tx/v_c) - \cos(2\pi T\tau)}{(2\pi T/v_c\Lambda)^2}\right], \quad \text{(A11)}$$

and

$$\langle\tilde{\theta}_c(x)\tilde{\phi}_c(0)\rangle = \frac{1}{2}\ln\left[\frac{\tanh(\pi Tx/v_c) + i\tan(\pi T\tau)}{\tanh(\pi Tx/v_c) - i\tan(\pi T\tau)}\right], \quad \text{(A12)}$$

with the corresponding formulas for the spin sector obtained by replacing $v_c \to v_s$ and $g_c \to g_s$.

As the density of the electron gas is lowered, the ratio $v_s/v_c \to 0$ with decreasing density for $n^{-1} = a \gg a_B$. Consider the equal time Green's function where $\tau = 0$. According to Eqs. (A10)–(A12),

$$\begin{aligned}\mathcal{G}_\sigma(x,0) &\sim (e^{ik_F x} + e^{-ik_F x}) e^{-(1/4)\langle(\tilde{\theta}_c(x)^2\rangle} e^{-(1/4)\langle(\tilde{\phi}_c(x)^2\rangle} \\ &\times e^{-(1/4)\langle(\tilde{\theta}_s(x)^2\rangle} e^{-(1/4)\langle(\tilde{\phi}_s(x)^2\rangle} \\ &= (e^{ik_F x} + e^{-ik_F x})\left(\frac{(2\pi T/v_c\Lambda)^2}{\cosh(2\pi Tx/v_c) - 1}\right)^{[(g_c+g_c^{-1})/8]} \\ &\times \left(\frac{(2\pi T/v_s\Lambda)^2}{\cosh(2\pi Tx/v_s) - 1}\right)^{[(g_s+g_s^{-1})/8]}.\end{aligned} \quad \text{(A13)}$$

For very small temperatures we recover the LL result,

$$\mathcal{G}_\sigma(x,0) \sim (e^{ik_F x} + e^{-ik_F x})\left(\frac{(1/\Lambda)^2}{x^2}\right)^{[(g_c+g_c^{-1})/8]} \times \left(\frac{(1/\Lambda)^2}{x^2}\right)^{[(g_s+g_s^{-1})/8]}. \quad \text{(A14)}$$

For finite temperatures, the Green's function is cut off at large x when $2\pi Tx/v \gg 1$ where v is either the charge or spin velocity. Since for low-electron density, we have $v_s \ll v_c$, it is possible to have $2\pi Tx/v_s \gg 1$, but $2\pi Tx/v_c \ll 1$. In this parameter range, the Green's function behaves as

$$\mathcal{G}_\sigma(x,0) \sim \left(\frac{(1/\Lambda)^2}{x^2}\right)^{[(g_c+g_c^{-1})/8]}\left(\frac{2\pi T}{v_s\Lambda}\right)^{[(g_s+g_s^{-1})/4]} e^{-|x|/\xi_s}, \tag{A15}$$

where the spin-coherence length is given by $\xi_s=4v_s/[\pi(g_s+g_s^{-1})T]$. Luttinger-liquid theory is valid only on length scales that are long compared to the lattice spacing, which in the present case is the mean electron separation. Thus, we expect that the minimum coherence length occurs when $\xi_s^* \approx a$. This implies a $T^* \sim v_s/a \sim J$ where J is the characteristic interparticle exchange energy. LL theory, therefore, only applies down to temperatures in the range $J \sim T \ll E_F$. When $J \ll T$ and $T \ll E_F$, we expect qualitatively new physics. It is precisely this regime that we focus on in this paper.

APPENDIX B: EVALUATION OF CORRELATORS APPEARING IN BOSONIZED FORMULAS FOR INFINITE AND SEMI-INFINITE SYSTEMS

For an infinite system, the θ_c and ϕ_c correlators are readily evaluated using the Hamiltonian (A1) and then going to a path integral representation which results in Gaussian integrals. The finite-temperature correlators have already been computed in Eqs. (A10)–(A12), from which the zero-temperature results (valid for $T \ll E_F$) are readily extracted

$$\langle \tilde{\theta}_c(x)^2 \rangle = \frac{g_c}{2}\ln[(x^2+(v_c\tau)^2)/a^2], \tag{B1}$$

$$\langle \tilde{\phi}_c(x)^2 \rangle = \frac{1}{2g_c}\ln[(x^2+(v_c\tau)^2)/a^2], \tag{B2}$$

$$\langle \tilde{\theta}_c(x)\tilde{\phi}_c(0) \rangle = \frac{1}{2}\ln\left[\frac{x+iv_c\tau}{x-iv_c\tau}\right]. \tag{B3}$$

An important correlator that appears in the problem of strong barriers is $\langle \tilde{\phi}_c(x)^2 \rangle$ at the end of a semifinite wire. This correlator can be readily evaluated using the following expansion of the fields in a Fourier series:

$$\theta_c(x,\tau) = \sum_{m=1}^{\infty} i\sqrt{\frac{g_c}{m}}\sin\left(\frac{m\pi x}{L}\right)(b_m e^{-\omega_m\tau} - b_m^\dagger e^{\omega_m\tau}) + \theta^{(0)}(x), \tag{B4}$$

$$\phi_c(x,\tau) = \sum_{m=1}^{\infty} \sqrt{\frac{1}{g_c m}}\cos\left(\frac{m\pi x}{L}\right)(b_m e^{-\omega_m\tau} + b_m^\dagger e^{\omega_m\tau}) + \Phi_c, \tag{B5}$$

where the zero-mode term $\theta_c^{(0)}(x) \equiv (x/L)(\pi/\sqrt{2})N$, the b_m satisfy $[b_m, b_{m'}^\dagger]=\delta_{mm'}$ and the operators N and Φ_c satisfy $[N,\Phi_c]=1$. We have assumed that the electrons are confined to a wire of length L. The fields must satisfy the boundary conditions $\partial_x\phi_c(x=0,L)=0$ and $\theta_c(L)-\theta_c(0)=\pi N/\sqrt{2}$ where N is the total number of electrons.

Using Eq. (B5) it is readily found that at the boundary

$$\langle \tilde{\phi}_c(x=0)^2 \rangle^{\text{boundary}} = \frac{2}{g_c}\ln[v_c\tau/a]. \tag{B6}$$

By comparing the boundary value of the correlator (B6) with the expression for the infinite system for $x=0$ (B2), we see that the effect of the boundary is to "double" the exponent of $e^{-\langle \tilde{\phi}_c(x=0)^2 \rangle}$. This exponent "doubling" can be understood quite simply by going to a chiral fermion basis valid on the whole real axis, rather than the basis we have used here so far, which is only valid for the positive half line. The resulting chiral action contains an overall factor of $\frac{1}{2}$ the difference from the nonchiral case and this factor translates into the factor of 2 that "doubles" the exponent of the infinite case.

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