Spin splitting and spin current in strained bulk semiconductors

B. Andrei Bernevig and Shou-Cheng Zhang
Department of Physics, Stanford University, Stanford, California 94305, USA
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We present an analysis for two recent experiments in bulk strained semiconductors and show that a new, previously overlooked, strain spin-orbit coupling term may play a fundamental role. We propose simple experiments that could clarify the origin of strain-induced spin-orbit coupling terms in inversion asymmetric semiconductors. We predict that a uniform magnetization parallel to the electric field will be induced for specific directions of the applied electric field. We also propose special geometries to detect spin currents in strained semiconductors.

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Spin manipulation in semiconductors has seen remarkable theoretical and experimental interest in recent years with the advent of spin electronics and with the realization that strong spin-orbit coupling in certain materials can influence the transport of carriers in so-called spintronics devices.[1] In particular, the issue of creating spin polarization of carriers in nonmagnetic semiconductors with spin-orbit coupling using only electric fields has caused a flurry of theoretical and experimental activity.[2–14]

Two kinds of theories of spin-polarization under the action of an electric field have been put forward. The first kind, dating back since the mid 1980's,[7] predicts the existence of a spatially homogeneous net spin polarization perpendicular to the applied electric current in two dimensional samples with spin-orbit interaction. This effect is dissipative and has been recently observed experimentally.[15] There also exist two very recent[2,3] theories predicting nondissipative, intrinsic spin currents with the spin polarization and flow direction perpendicular to each other and to the electric field. This effect does not create a bulk magnetization but, if observed, can be used for spin injection, and its validity is being experimentally tested at the present time. One of the theories[3] predicts a spin current polarized out of plane and flowing perpendicular to the in-plane electric field applied on a two-dimensional semiconductor sample exhibiting Rashba spin-orbit coupling. As long as the Rashba spin splitting is large enough (larger than the disorder energy), the spin conductivity is "universal" $(e/8\pi\hbar)$ in the sense that it does not depend on the value of the coupling. The other effect[2] appears in the valence band of the bulk samples and is proportional to the spin-orbit splitting of the valence bands (to the difference between the Fermi momenta of the heavy and light-hole bands).

In the first part of this paper we analyze the theory behind two recent experiments in bulk strained semiconductors[16,17] where an electric-field-induced uniform homogeneous spin polarization upon an applied electric field is observed. We make the case that the observed spin-splitting (whose origin is puzzling) and spin polarization is due to a previously overlooked strain-spin-splitting term, and propose easy experimental checks of our theory.

In the second part of this letter we predict the appearance of an intrinsic spin polarized spin current in *n*-doped bulk (and two-dimensional) strained semiconductors (GaAs, GaSb, InSb, InGaAs, AlGaAS, etc.) under the influence of an electric field. The spin conductance is "universal," in the sense that it does not depend on the value of strain (for large enough strain), but it is proportional to the average Fermi momentum of the conduction band. The effect is due to the spin-orbit splitting of the conduction band under strain and is hence absent in strain-free semiconductors. We propose an experimental technique using the already existing setup in Refs. 16 and 17 to measure the spin current and to differentiate between the intrinsic spin current and the uniform magnetization effects.

In Ref. 16 nine samples of *n*-doped $(n=3\times10^{16}\ \mathrm{cm}^{-3})\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As}(x=5\%-7\%)$ of thicknesses between 200 and 1500 nm, grown in the [001] direction on undoped GaAs substrate, are used to probe the electron spin dynamics through time and spatially resolved Faraday rotation (FR). The length and width of the samples are roughly 300 μm $\times$ 80 μm. The lattice mismatch provides for diagonal strain in the x,y,z=[100], [010], [001] directions of 0.04%–0.46% (Ref. 18) (contrary to claims in Ref. 31, the lattice constants in x and y directions are also strained, this being a generic feature of [001] growth). Moreover, anisotropic shear strain develops in all directions (xy,xz,yz =[110], [101], [011]) due to different direction-dependent strain relaxation rates at the growth temperature of around 500 °C. For this and more on strained sample growth, the reader should consult Ref. 19. This guarantees that all the components of the strain tensor ϵ_{ij}, $i,j=x,y,z$ are nonzero and of the same order of magnitude. The magnitude of the strain components is given in Table I.

Pump-probe FR beams measure the total magnetization of the optically injected electron spins in the growth direction z when the samples are placed in an electric field on the [110] and [1$\bar{1}$0] directions, respectively. The dynamics of the spin packet is mainly described by a precession around a *total* magnetic field $\vec{B}_{\mathrm{tot}}=\vec{B}_{\mathrm{int}}+\vec{B}_{\mathrm{ext}}$ where $\vec{B}_{\mathrm{ext}}$ is an externally applied magnetic field whereas $\vec{B}_{\mathrm{int}}$ is the momentum-dependent internal magnetic field caused by the spin-orbit coupling. The precession around a $\vec{B}_{\mathrm{int}}$ is the main feature of most of the spintronics devices, starting with the Das-Datta spin-field transistor.[22] Under an applied electric field, the average particle momentum acquires a nonzero value, parallel

TABLE I. Strain components (in %) in Ref. 16 converted to cartesian coordinates [C and D are compositionally graded films (Ref. 16)]. The conversion equations, obtained after a simple coordinate transformation, are: $\epsilon_{zz}=\epsilon_{[001]}, \epsilon_{xx}=\epsilon_{yy}=\frac{1}{2}(\epsilon_{[110]}+\epsilon_{[1\bar{1}0]}), \epsilon_{xy}=\frac{1}{2}(\epsilon_{[110]}-\epsilon_{[1\bar{1}0]})$. The measured spin splitting slope values $\beta_{\mathrm{SIA}}^{\mathrm{exp}}$ and $\beta_{\mathrm{BIA}}^{\mathrm{exp}}$ as well as the theoretical value $\beta_{\mathrm{SIA}}^{\mathrm{th}}$ obtained using the Hamiltonians H_2 for the SIA - type splitting are given in neV nS $\mu\mathrm{m}^{-1}$, the units of Ref. 16. The value of the deformation potential D in the term H_3, determined as explained in Eq. (4) is given for each sample. $\beta_{\mathrm{BIA}}^{\mathrm{exp}}/(\epsilon_{zz}-\epsilon_{xx})$ is measured in 10^2 neV nS $\mu\mathrm{m}^{-1}$, while D is measured in 10^4 m/s.

Sample	ϵ_{zz}	$\epsilon_{xx}=\epsilon_{yy}$	ϵ_{xy}	$\beta_{\mathrm{SIA}}^{\mathrm{exp}}$	$\beta_{\mathrm{SIA}}^{\mathrm{th}}$	$\beta_{\mathrm{BIA}}^{\mathrm{exp}}$	$\frac{\beta_{\mathrm{BIA}}^{\mathrm{exp}}}{\epsilon_{zz}-\epsilon_{xx}}$	$D/\hbar$
A	0.46	−0.16	0.2	−24	604	75	121	1.59
B	0.14	−0.2	0.08	−26	241	13	38	0.5
E	0.13	−0.42	0.18	69	543	43	78	1.03
F	0.07	−0.32	0.12	54	362	31	79	1.04
G	0.04	−0.32	−0.04	44	−121	31	86	1.13
H	0.13	−0.42	0.26	−2	785	24	43	0.56
I	0.04	−0.16	0.2	65	604	23	115	1.51

to the electric field. The internal magnetic field is caused by the spin-orbit coupling: The electric field acts on the particle momentum which in turn couples to the spin. The signal at the probe beam can be fitted to $\cos(g\mu_B|\vec{B}_{\mathrm{int}}+\vec{B}_{\mathrm{ext}}|\Delta t/\hbar)$ where μ_B is the Bohr magneton, g is the electron g-factor while Δt is the temporal delay between the pump and probe pulses. This fit gives the direction and value of $\vec{B}_{\mathrm{int}}$ which turns out to be perpendicular to the applied electric field E and the $\hat{z}$ axis (for E in-plane); the value of $\vec{B}_{\mathrm{int}}$ is used to determine the spin splitting $\Delta_0=g\mu_B B_{\mathrm{int}}$ and a phenomenological relation $\Delta_0=\beta v_d$ is observed where v_d is the spin-drift velocity and β is a constant of proportionality that is the focus of the experiment.[16] Experiments find that v_d is linearly proportional to the electric field E.

As a first step, let us theoretically address the question of origin of β. By group theory, inversion symmetry breaking bulk strained semiconductors exhibit three main types of spin splitting:[23]

$$H=\frac{\hbar^2}{2m}k^2+H_1+H_2+H_3,$$

$$H_1=\lambda[\sigma_x k_x(k_y^2-k_z^2)+\sigma_y k_y(k_z^2-k_x^2)+\sigma_z k_z(k_x^2-k_y^2)]$$

$$H_2=\frac{1}{2}C_3[\sigma_x(\epsilon_{xy}k_y-\epsilon_{xz}k_z)+\sigma_y(\epsilon_{yz}k_z-\epsilon_{yx}k_x)+\sigma_z(\epsilon_{zx}k_x-\epsilon_{zy}k_y)]$$

$$H_3=D[\sigma_x k_x(\epsilon_{zz}-\epsilon_{yy})+\sigma_y k_y(\epsilon_{xx}-\epsilon_{zz})+\sigma_z k_z(\epsilon_{yy}-\epsilon_{xx})] \quad (1)$$

$m=0.0665\ m_0$ is the effective electron mass in the conduction band,[24] $\lambda, C_3, D>0$ are material constants, $\sigma_{x,y,z}$ are the 3 spin-Pauli matrices,and ϵ_{ij}, $i,j=x,y,z$ are the components of the symmetric strain tensor.

All three Hamiltonians can be written as the coupling of a fictitious k-dependent internal magnetic field $\vec{B}_{\mathrm{int}}$ to the electron spin, $\vec{B}_{\mathrm{int}}(k)\vec{\sigma}=B_x(k)\sigma_x+B_y(k)\sigma_y+B_z(k)\sigma_z$ (an overall factor of $g\mu_B$ has been absorbed into the definition of B to simplify notation). The directions of $\vec{B}_{\mathrm{int}}$ as dependent on the directions of $\vec{k}$ are shown in Fig. 1. The SIA-type term gives a $\vec{B}_{\mathrm{int}}$ that keeps its relative orientation with respect to $\vec{E}$ as $\vec{k}(\|\vec{E})$ is rotated between the [110] and $[1\bar{1}0]$ directions, while both BIA-type $\vec{B}_{\mathrm{int}}$ coming from H_1 and H_3 change their sign between [110] and $[1\bar{1}0]$. The difference between H_1 and H_3 is that the latter has a finite $\vec{B}_{\mathrm{int}}$ when $\vec{E}\|x,y$ whereas the former has zero B_{int} for the same directions.

In Ref. 16 the values of the splitting coefficient β are measured on the [110] and $[1\bar{1}0]$ directions and because of the sign-changing properties of $\vec{B}_{\mathrm{int}}$ mentioned above, the

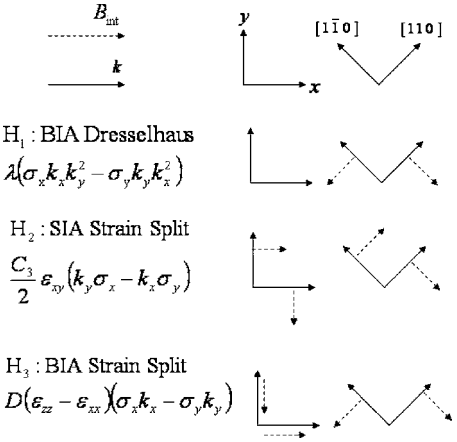

FIG. 1. Direction of the internal magnetic field $\vec{B}_{\mathrm{int}}$ for the three spin splitting Hamiltonians H_1, H_2, H_3, $(\lambda, C_3, D>0)$ considering the electric field E (and hence the average momentum) to be in plane. It was also assumed that $\epsilon_{xx}=\epsilon_{yy}$, as appropriate in the experiment (Ref. 16). The experimental SIA data cannot be explained by the term H_2, but nevertheless, the SIA-type B_{int} seen in the experiment has the same direction as the one plotted here. When $E\|x$ or $E\|y$, the BIA term H_3 induces a $B_{\mathrm{int}}\|E$ whereas the BIA term H_1 does not induce spin splitting for these directions. This constitutes a simple check of the experiment

BIA and the SIA contributions to β can be obtained as follows: $\beta_{\mathrm{BIA}}=(\beta_{[110]}-\beta_{[1\bar{1}0]})/2$, $\beta_{\mathrm{SIA}}=(\beta_{[110]}+\beta_{[1\bar{1}0]})/2$. Surprisingly, the spin splitting is more of a BIA-type rather than an SIA-type, contradicting the conventional knowledge that an SIA-type term described by H_2 is responsible for spin splitting in strained semiconductors.[23,25–29]

Theoretically, the Dresselhaus term H_1 is a bulk-inversion asymmetry term that appears even in the absence of strain. As observed in the experiment, the fictitious internal magnetic field $\vec{B}_{\mathrm{int}}$ is perpendicular to the momentum $\vec{k}$: $\vec{B}_{\mathrm{int}}(k)\vec{k}=0$, where $B_x=\lambda k_x(k_y^2-k_z^2)$, B_y and B_z being obtained by cubic permutation. For GaAs, the constant $\lambda=22$ eV Å^3. However, we believe this term is not responsible for the spin splitting observed in the experiment.[16] The observed splitting is linear in momentum k, inconsistent with the $H_1\sim k^3$. Experiments performed on InSb,[29] another material with inversion asymmetry, support this conclusion and point strongly to the fact that strained InSb is described by H_2. In Ref. 29 stress of up to 4 kbar is applied mechanically on a $1\times1\times10$ mm^3 sample and Shubnikov-de-Haas oscilations are used to probe the band structure. Without applied strain, the conduction band exhibits a spin-splitting that is small and cubic in k, described by H_1. In Ref. 29 the application of diagonal strain does not induce any observable spin splitting whereas the application of shear strain induces a splitting linear in k, described by H_2. Relatively large stress-induced splitting of the Fermi surfaces occurs in the lower concentration ($n=1.4\times10^{15}-2.0\times10^{17}$ cm^{-3}) samples.[29] The energy splitting dispersion switches from k^3 in the unstrained case to k when strain (stress) above 1 kbar is applied, in accordance to H_2 becoming dominant over H_1. From a theoretical estimate, at $n=3\times10^{16}$ cm^{-3}, H_1 should be of the same size as H_3 and roughly one order of magnitude lower than H_2. The spin splitting at the Fermi wavevector $k_F=0.96\times10^8$ m^{-1} due to H_1 is less than 10^{-5} eV. By contrast the spin splitting due to H_2 is $C_3\epsilon_{xy}k_F=5.06\times10^{-2}\epsilon_{xy}$ eV $=5\times10^{-5}-15\times10^{-5}$ eV for $\epsilon_{xy}=0.1\%-0.3\%$ as in Ref. 16 (see Table I for conversion of strain components from Ref. 16 to the orthogonal system ϵ_{xy}). An experimental value of $C_3/\hbar=8\times10^5$ m/s for GaAs was used.[30] Contrary to previous remarks,[31] there is hence no theoretical or experimental *a priori* reason to disregard the strain-dependent spin splitting terms in favor of the Dresselhaus k^3 term for the doping values in the experiment.[16]

There are a number of experimental reasons in Ref. 16 hinting the marginal significance of the k^3 term. In Ref. 16 strain plays a critical role in generating the spin-orbit coupling $\vec{B}_{\mathrm{int}}$. Samples prepared from the same wafer but unstrained show a reduction by an order of magnitude of B_{int} along both the [110] and the $[1\bar{1}0]$ directions. If H_1 were responsible for spin splitting, its value would remain unchanged upon varying strain. Strain could only enter the system through the variation of the effective electron mass in the x,y,z directions, as Ref. 31 points out. However, these variations with strain are of a maximum 2% to 3%[24,31] thereby not accounting for the order of magnitude variation of the spin-splitting between the strained and the unstrained cases observed in Ref. 16.

The term H_2 is a structural inversion asymmetry (SIA)-type term that has its origin in the acoustic phonon interaction of the valence band with the conduction band.[23] In the framework of the Kane's 8×8 matrix (2×2 for the conduction and split-off band and 4×4 for the valence band) the conduction band couples to the valence band. In systems with inversion symmetry where the selection rules for **L** are satisfied, it is impossible to couple spin-0 ($|s\rangle$) with spin-1 ($|z\rangle$) through a spin-2 term (ϵ_{ij}) and hence $\langle s|\epsilon_{xy}|z\rangle=0$. However, when inversion symmetry is broken, the forementioned term need not be zero as the **L** selection rule need not apply. Upon straining, the matrix elements between the conduction and valence band have the form $\langle s|\epsilon_{xy}|z\rangle$ (plus cyclic permutations) where $|s\rangle$ is the s-orbital and $|z\rangle$ is one of the p orbitals. Through perturbation theory, one can compute the effect of this valence-conduction band interaction when projected to the conduction band and obtain the conduction band effective Hamiltonian H_2.[23,26–28] Taking into account that the electric field is in-plane ($\langle k_z\rangle=0$) and that $\epsilon_{xy}\neq0$ (see Table I), in H_2 the components of the internal magnetic field (which due to the rescaling by $g\mu_B$ has units of energy) are: $B_x=\frac{1}{2}C_3\epsilon_{xy}k_y$, $B_y=-\frac{1}{2}C_3\epsilon_{yx}k_x=-\frac{1}{2}C_3\epsilon_{xy}k_x$. Switching coordinates to the [110] and $[1\bar{1}0]$ directions, $B_{[110]}=\frac{1}{2}C_3\epsilon_{xy}k_{[1\bar{1}0]}$, $B_{[1\bar{1}0]}=-\frac{1}{2}C_3\epsilon_{xy}k_{[110]}$ (see Fig. 1. Since H_2 is an SIA term, the spin splitting β will be of SIA type $\beta_{\mathrm{SIA}}^{\mathrm{th}}$ (th stands for the theoretical estimate). Since $\langle k\rangle=(1/\hbar)mv_d$ where v_d is the drift velocity of the spin packed due to the electric field, we get a simple formula for the

$$\beta_{\mathrm{SIA}}^{\mathrm{th}}=\frac{C_3}{\hbar}\epsilon_{xy}m \tag{2}$$

By using the experimentally known value for $C_3/\hbar=8\times10^5$ m/2, the predicted values for $\beta_{\mathrm{SIA}}^{\mathrm{th}}$ are given in Table I. The theoretical values are larger than the observed ones by a factor of 3–30 and no matching trend between the data and the SIA term H_2 can be found. Moreover, as remarked in Ref. 16 no systematic correlation between the experimentally observed SIA contribution and the strain is observed. We hence come to the conclusion that the SIA spin splitting observed in[16] is not induced by the uniform shear strain (which would give the values $\beta_{\mathrm{SIA}}^{\mathrm{th}}$ and which have been confirmed in mechanical experiments) but borrows substantially from the dislocations and strain gradient inherent in growing such a thick sample through MBE techniques. This is not to say that the SIA term is negligible: As seen in Table I the SIA term is substantial and comparable in magnitude with the BIA term. However, the SIA term does not correlate with strain ad cannot be described by H_2.

The remaining spin splitting term is H_3. Although this term is allowed by group theory, it only shows up at higher order in perturbation theory than H_2 in the $k\cdot p$ method. We claim that in the experiment[16] this term is responsible for the spin-splitting observed, and determine the value of the constant D. We note, however, that this does not settle the theoretical puzzle of why the H_3 would be more significant than H_2 in this case, which might have to do with the conditions of the experiment such as low temperature, the appearance of dislocations and strain gradient. H_3 is a Hamiltonian of BIA-type and vanishes with vanishing strain, thereby satisfying

two of the experimental observations in Ref. 16. From Eq. (1), the internal magnetic field $\vec{B}_{\text{int}}$ reads: $B_x = Dk_x(\epsilon_{zz}-\epsilon_{yy})$, $B_y = Dk_y(\epsilon_{xx}-\epsilon_{xx})$, $B_z = Dk_z(\epsilon_{yy}-\epsilon_{xx})$, and since the in-plane electric field influences only the in-plane momentum, $\langle k_z\rangle = 0$ and only the two in-plane components of the internal magnetic field remain. In accordance with the experiment, we place $\epsilon_{xx}=\epsilon_{yy}$, and $\epsilon_{zz}-\epsilon_{xx}>0$ (Table I). We hence have $B_x = k_x D(\epsilon_{zz}-\epsilon_{xx})$, $B_y = k_y D(\epsilon_{xx}-\epsilon_{zz})$, $D(\epsilon_{zz}-\epsilon_{xx})>0$. Since $\vec{B}_{\text{int}}$ is not perpendicular to $\vec{k}\|\vec{E}$: $B_x k_x + B_y k_y = (k_x^2 - k_y^2)D(\epsilon_{zz}-\epsilon_{xx})$, one may think this term is incompatible with the observed $\vec{B}_{\text{int}} \perp \vec{E}\|\vec{k}$ in Ref. 16. This, however, would be hasty: The experiment is performed in only two directions, with $\vec{E}\|[110]$ and $\vec{E}\|[1\bar{1}0]$, for which $k_x = \pm k_y$. For these two directions only, the $\vec{B}_{\text{int}}$ in H_3 is perpendicular to the momentum and the electric field, hence satisfying a major constraint the experimental data poses on the theory. Since the value of the constant D is unknown from previous experimental studies (although it was suggested that they can be sometimes sizable[32]) there is no way of theoretically predicting the values of the spin-splitting from our model. However, we can check if the model is consistent with the experimental data and we can also obtain a value of the constant D which, being a material constant, should be similar on all the samples cited here. Since $\langle k\rangle = (m/\hbar)v_d$ where v_d is the spin drift velocity along the spin packet we find:

$$\beta^{\text{th}}_{[\text{BIA}]} = 2\frac{D}{\hbar}(\epsilon_{zz}-\epsilon_{xx})m \tag{3}$$

We can determine the value of D from the experimental data for β and strain ϵ:

$$\frac{D}{\hbar} = \frac{1}{2m}\frac{\beta^{\text{exp}}_{[\text{BIA}]}}{\epsilon_{zz}-\epsilon_{xx}} \tag{4}$$

As a consistency check, since D is a material constant, $\beta^{\text{exp}}_{[\text{BIA}]}/(\epsilon_{zz}-\epsilon_{xx})$ should be quasi-constant between the samples quoted in the experiment. In 5 out of the 7 samples studied in Ref. 16, the values of $\beta^{\text{exp}}_{[\text{BIA}]}/(\epsilon_{zz}-\epsilon_{xx})$ are close together to within 30%, lumped in two groups (samples A, I are very close to each other, and within 30% of the value for E, F, G which are again very close between themselves). The samples E, F, G were grown in the same day. The deviant samples B, H were also grown in the same day, and hence the variation of the constant coefficient D within a sample set that was grown on the same day is less than 15%.[21] Different growth conditions are most likely responsible for the (still small) variations between samples grown in different days. The consistency check is further proof that H_3 is the term responsible to the spin-splitting in Ref. 16. The values obtained for D are given in Table I.

We showed that H_3 is a BIA-type Hamiltonian vanishing with vanishing strain, with an internal magnetic field that is perpendicular to the applied electric field for the two experimental directions $[110]$ and $[1\bar{1}0]$ and which is consistent with the reported data for the spin splitting. On the other hand, H_1 and H_2, the previously known spin splitting terms, fail to reproduce the data on more than several counts. It is easy to experimentally prove, using the setup in Ref. 16, that H_3 is responsible for the spin splitting is easy: One would measure the internal magnetic field due to BIA on the x or y direction. In this case, an H_3 term would give an internal magnetic field parallel to $\vec{E}$ (of course, there will also be an internal $\vec{B}$ from an SIA term that is still perpendicular to $\vec{E}$, but a component of $\vec{B}_{\text{int}}$ parrallel to the electric field should be easily detectable from the fact that the FR probe signal should be shifted when $\vec{B}_{\text{ext}}\|\vec{E}$ in the same way as in Fig. 2(a) of Ref. 16.

In another beautiful experiment, Kato *et al.* measure through Farraday Rotation (FR) a nonzero uniform magnetization ρ_{el} induced by driving an electric current (electric field) through the sample E of their previous experiment.[16] It has been long predicted[7,8] that semiconductors with spin-orbit coupling will exhibit a uniform magnetization when placed in an electric field generating a charge current. This can be trivially understood by a simple argument: Writing the spin-orbit Hamiltonian as a $\vec{k}$ dependent magnetic field Zeeman coupled to spin, $\vec{B}_{\text{int}}(k)\vec{\sigma}(k)$, the application of an electric field $\vec{E}$ will make the average value of the momentum be nonzero $\langle\vec{k}\rangle = (e/m)\vec{E}\tau$ where τ is the momentum relaxation time. This creates a nonzero average $\langle\vec{B}_{\text{int}}\rangle = \vec{B}(\langle\vec{k}\rangle)$ which orients the spins along its direction through the Zeeman-type coupling of the spin-orbit term.

We now try to numerically estimate the value of the uniform magnetization ρ_{el} using the BIA-type H_3. From Ref. 17, the BIA contribution to the uniform magnetization can be obtained as $\rho^{\text{BIA}}_{\text{el}} = \frac{1}{2}(\rho^{\vec{E}\|[1\bar{1}0]}_{\text{el}} - \rho^{\vec{E}\|[110]}_{\text{el}})$ and is around 3×10^{18} m^{-3} for $E=10^4$ V/m. We will now try to estimate this from first principles using H_3 as the main BIA term and using the value of D for sample E deduced in Table I. A simple linear response calculation of the magnetization σ_i to the electric current J_j (due to the applied electric field E_j) gives:

$$\rho^i_{\text{el}} = \frac{2\pi e\tau}{\hbar}Q_{ij}E_j,$$

$$Q_{ij} = \langle T\sigma_i J_j\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{n_{E_-}-n_{E_+}}{B^2}\left(B_i\frac{\partial B}{\partial k_j} - B\frac{\partial B_i}{\partial k_j}\right), \tag{5}$$

where $i, j = x, y, z$ and $B_i(k)$, are the components of the internal magnetic field for H_3, $B = \sqrt{\Sigma_{i=x,y,z}B_iB_i}$, $n_{E_\pm}$ is the Fermi function of the spin-split energies $E_\pm = (\hbar^2/2m)k^2 \pm B$ for H_3. For the Hamiltonian H_3, considering $\epsilon_{xx}=\epsilon_{yy}$ we obtain for:

$$\rho^{\text{BIA}}_{\text{el}} = \frac{e\tau m k_F D(\epsilon_{zz}-\epsilon_{yy})}{\pi\hbar^3}E. \tag{6}$$

As previously pointed out the magnetization is parallel to the electric field for $\vec{E}\|\hat{x}$ or $\vec{E}\|\hat{y}$. This provides an important and easy check of the above assumption that the observed strain spin splitting comes from H_3. The only two directions where

$\vec{\rho}_{\rm el}$ is perpendicular to the electric field are [110] and [1$\bar{1}$0], the directions on which the experiment is performed. Considering a sample of mobility $\mu=0.6\ {\rm m}^2/{\rm V\ s}$ (Ref. 18) we obtain an estimate for $\rho_{\rm el}=3.45\times10^{18}\ {\rm m}^{-3}$ for a field $E=10^4\ {\rm V/m}$, compared to an experimental value of $3\times10^{18}\ {\rm m}^{-3}$ for the same value of the electric field. The theoretical value obtained is within the experiment's error margins.

Finally, using the current setup in Refs. 16 and 17 we propose an experiment to test the prediction of dissipationless spin current. For spin-1/2 two-dimensional systems, the initial prediction[3] is subject to some sort of controversy,[12,33,34] as the introduction of impurities apparently makes the spin current vanish. We here adopt the alternative view and propose a clear-cut experiment which can see the spin accumulation due to the spin current. Similar to the two-dimensional (2D) case, in the present case, the application of an electric field E_j to a semiconductor with spin orbit coupling will create a spin current S_l^i flowing perpendicular to the electric field and polarized perpendicular to both the field and the direction of flow. Using linear response, the expression for the spin conductance is[35]

$$J_i^l=\sigma_{ij}^l E_j,$$

$$\sigma_{ij}^l=\frac{\hbar^2}{2m}\int\frac{d^3k}{(2\pi)^3}\frac{n_{E_-}-n_{E_+}}{B^3}k_i\epsilon_{lnm}B_n\frac{\partial B_m}{\partial k_j},\tag{7}$$

where $i,j,l,m,n=x,y,z$, ϵ_{lnm} is the totally antisymmetric tensor in three dimensions and $B_i(k)$ are the components of the internal magnetic field. For H_3 and for $\epsilon_{xx}=\epsilon_{yy}$ the only nonzero components of the spin conductance are

$$\sigma_{21}^3=-\sigma_{12}^3=\frac{e}{\hbar}\frac{1}{D(\epsilon_{zz}-\epsilon_{yy})}\frac{\hbar^2}{m}\int\frac{d^3k}{(2\pi)^3}\frac{n_{E_-}-n_{E_+}}{(k_x^2+k_y^2)^{3/2}}k_y^2$$

$$=\frac{e}{\hbar}\frac{1}{D(\epsilon_{zz}-\epsilon_{yy})}\frac{\hbar^2}{2m}\frac{1}{(2\pi)^3}\int_0^{2\pi}d\phi\int_0^{\pi}d\theta\sin^2\phi(k_-^2-k_+^2),\tag{8}$$

where ϕ,θ are the polar angles of $\vec{k}$ and where k_-,k_+ are the Fermi momenta of the two bands. When both bands are occupied (positive Fermi energy), we find $k_--k_+=(2m/\hbar^2)(\lambda(k)/k),\lambda(k)=D(\epsilon_{zz}-\epsilon_{xx})k\sin\theta$. Usually the spin splitting is much smaller than the Fermi energy, and we can define an average Fermi momentum $k_F=\frac{1}{2}(k_-+k_+)\approx(3\pi^2n)^{1/3}$, n being the dopant density. With this, we find that the spin-conductivity will be independent of the value of the strain

$$\sigma_{21}^3=-\sigma_{12}^3=\frac{e}{\hbar}\frac{k_f}{4\pi^2}\tag{9}$$

The result for the spin conductance is intermediate between the 2D spin-1/2 spin current and the 3D spin-3/2 spin current. Similar to Ref. 2 but unlike Ref. 3 the spin conductance depends on the Fermi momentum, a characteristic of the 3D. Unlike Ref. 2, but similar to Ref. 3, the spin conductance does not depend on the strength of spin-orbit coupling. Even though the spin conductance does not depend on the value of strain, it is essential that spin-orbit splitting (due to strain in this case) be present. Upon the application of an electric field on the x axis, a spin current will flow on the y axis spin polarized in the z direction. For $n=10^{16}\ {\rm cm}^{-3}$, and a field $E=10^4\ {\rm V/m}$ we estimate a spin current $j_{\rm spin}=(e/\hbar)(k_f/4\pi^2)E=2\times10^{21}(\mu_B/{\rm cm}^2\ {\rm s})$ where μ_B is a Bohr magneton. Since spin conductivity varies as $n^{1/3}$ and charge conductivity varies as n, for low values of n the spin conductance will overtake the charge conductance and the spin current will be larger than the charge current caused by the electric field. The density at which this happens is $n^{2/3}<[2e/\hbar\mu][(6\pi^2)^{1/3}/8\pi^2]$, where μ is the mobility in the sample, or for a sample of mobility $\mu=0.6\ {\rm m}^2/{\rm V\ s}$.

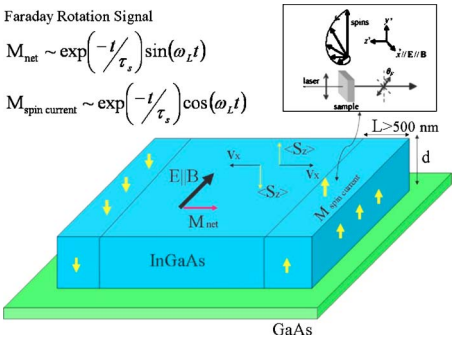

FIG. 2. (Color online) An electric field will cause both an observed net uniform bulk magnetization and a proposed spin current. The spin current will accumulate at the edges over a spin diffusion length of more than 500 nm thus making its detection practical with a beam slightly more focused than in Ref. 17. The uniform magnetization and the spin current spin accumulation are $\pi/2$ out of phase.

The flow will result in accumulation on the opposite zx faces of the crystal (see Fig. 2). For the present experiment, we estimate this spin accumulation of the order of $J_{\rm spin}\tau_S=10^{13}\mu_B/{\rm cm}^2$. Due to the extremely spin life time of above 1 ns, the distance from the edge of the sample, the spin diffusion length is very large, of the order $L=500\ {\rm nm}-1\ \mu{\rm m}$. The FR beam used in Ref. 17 has a resolution of 4.7 μm/9.7 μ_m on the x and y axis, respectively, but focusing the beam within 1 μm is possible.[20,36] Then, if the spin current prediction is right, applying the FR beam on the edge of the sample should give a clear signal (larger than the uniform magnetization in the bulk). Since the uniform magnetization and the spin accumulation due to spin hall current are perpendicular to each other, in time-resolved FR experiments, the spin current spin accumulation and the uniform magnetization are out of phase by $\pi/2$ (see Fig. 2)

In conclusion, we have analyzed two very recent experiments[16,17] and proved that conventional spin splitting terms and strain spin splitting terms do not explain the data. We have introduced a previously largely unknown term and made the case as to why it explains the observed features in Refs. 16 and 17. We have proposed further simple experiments to verify our assertions. If true, our proposal gives rise to the clear possibility of obtaining a uniform magnetization parallel to the applied electric field, as opposed to the one perpendicular to it that has been observed so far. Along with

predicting a 3D spin current, we have also proposed a way to test the spin currents in spin-1/2 systems.

Note added. After the submission of a web preprint version of the paper but prior to its publication in physical form, the spin Hall effect has been experimentally discovered[37] in the samples whose spin splitting is characterized in this paper. Further research is needed to clarify whether its origin is intrinsic or extrinsic.

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