Half-skyrmion picture of a single-hole-doped CuO_2 plane

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Based on the Zhang-Rice singlet picture, it is argued that the half-skyrmion is created by the doped hole in the single-hole-doped high-T_c cup rates with Néel ordering. The spin configuration around the Zhang-Rice singlet, which has the form of superposition of the two different d-orbital hole spin states, is studied within the nonlinear σ model and the CP^1 model. The spin configurations associated with each hole spin state are obtained, and we find that the superposition of these spin configurations turns out to be the half-skyrmion that is characterized by a half of the topological charge. The excitation spectrum of the half-skyrmion is obtained by making use of Lorentz invariance of the effective theory and is qualitatively in good agreement with angle-resolved photoemission spectroscopy on the parent compounds. Estimated values of the parameters contained in the excitation spectrum are in good agreement with experimentally obtained values. The half-skyrmion theory suggests a picture for the difference between the hole-doped compounds and the electron-doped compounds.

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I. INTRODUCTION

In the phase diagram of high-temperature superconductors, there are a variety of phases: the Néel ordering phase, the spin-glass-like phase, the d-wave superconducting phase, and the pseudogap phase.[1] This rich phase diagram is controlled by the hole doping concentration x and temperature. If we focus on the ground state properties, then the fundametal parameter is x. In order to understand the physics of high-temperature superconductivity, it is necessary to figure out how to describe the doped holes. The goal of this approach is, of course, to find a unfied description of the doped holes over the whole range of x.

In this paper, we consider the simplest case: We focus on the single-hole-doped system. The motivation is the following. Although d-wave superconductivity occurs in a moderately doping region, and there is the intriguing pseudogap phase, it is quite hard to find a reliable description of the doped holes because of the strong correlation effects between the holes and background spin fluctuations. In contrast, the physics of the undoped compound is well established compared with other phases. The system is described by the $S=1/2$ antiferromagnetic Heisenberg model on the square lattice and the ground state is the Néel ordered state. Considering one hole doping upon this well-established phase would be the first step in understanding the effects of doped holes in the high-temperature superconductors.

Experimentally it has been known that doped holes occupy oxygen p orbitals in the CuO_2 plane. Between an oxygen p-orbital hole and the nearest neighbor copper d-orbital holes, there are strong correlations of forming a singlet pair. The resulting singlet state is called a Zhang-Rice singlet.[2] Based on this picture, the t-J model was proposed.

In terms of the t-J model, the single-hole problem has been discussed extensively.[3] However, the focus is mainly on the frustration effect induced by hopping of the doped hole. Not so much attention has been paid on the spin configuration around the Zhang-Rice singlet state. In this paper, we study the spin configuration created around the Zhang-Rice singlet within effective field theory approaches to the Heisenberg antiferromagnet.

As an effective theory for the $S=1/2$ Heisenberg antiferromagnet on the square lattice, the nonlinear σ model (NLσM) has been studied extensively:[4]

$$S=\frac{\rho_s}{2}\int_0^\beta d\tau\int d^2\mathbf{r}\left[(\boldsymbol{\nabla}\mathbf{n})^2+\frac{1}{c_{\rm sw}^2}\left(\frac{\partial\mathbf{n}}{\partial\tau}\right)^2\right], \tag{1}$$

where ρ_s is the spin stiffness and $c_{\rm sw}$ is the antiferromagnetic spin wave velocity. (Hereafter we take the unit of $\hbar=1$.) Here τ is the imaginary time and $\beta=(k_BT)^{-1}$, with T being temperature. The unit vector $\mathbf{n}$ represents the staggered moment. This model is derived from the Heisenberg antiferromagnet by applying Haldane's mapping.[5] The theoretical formula for the antiferromagnetic correlation length $\xi_{\rm AF}$ based on the renormalization group analysis of this model is in quite good agreement with experimentally obtained $\xi_{\rm AF}$.

Another well-known approach is the Schwinger boson mean field theory (SBMFT).[6] In this theory, the spin $S=1/2$ is represented by

$$\mathbf{S}_j=\frac{1}{2}(z_{j\uparrow}^\dagger \quad z_{j\downarrow}^\dagger)\boldsymbol{\sigma}\begin{pmatrix} z_{j\uparrow}\\ z_{j\downarrow}\end{pmatrix}, \tag{2}$$

with the constraint $\Sigma_\sigma z_{j\sigma}^\dagger z_{j\sigma}=1$. The components of $\boldsymbol{\sigma}$ are Pauli matrices. The antiferromagnetic correlations are described by a mean field $A_{ij}=\langle z_{i\uparrow}z_{j\downarrow}-z_{i\downarrow}z_{j\uparrow}\rangle$. This mean field describes also the pairing correlations of the Schwinger bosons.[7–9] The quasiparticle excitation spectrum of the Schwinger bosons is given by

$$\omega_k=\sqrt{\lambda^2-4J^2A^2\gamma_k^2}, \tag{3}$$

where $\gamma_k=(\sin k_x\pm\sin k_y)/2$, where the plus sign is for $k_xk_y>0$ and the minus sign is for $k_xk_y<0$. The parameter λ is originally introduced as a Lagrange multiplier to impose the constraint. In the mean field approximation, the Lagrange multiplier is taken to be uniform. For the ground state, λ

$=2JA$. Bose-Einstein condensation of the Schwinger bosons at $\mathbf{k}=(\pm\pi/2,\pm\pi/2)$ leads to Néel ordering.[10]

In order to consider fluctuations about Bose-Einstein condensate of Schwinger bosons, it is convenient to use a gauge field description. Introducing a variable $x_0=c_{\rm sw}\tau$, the action reads as $S=1/2g\int d^3x(\partial_\mu \mathbf{n})^2$, with $g=c_{\rm sw}/\rho_s$. In terms of complex field $\bar{\zeta}_\sigma$ and ζ_σ with $\Sigma_\sigma\bar{\zeta}_\sigma\zeta_\sigma=1$, the vector $\mathbf{n}$ is represented by $\mathbf{n}=\bar{\zeta}\boldsymbol{\sigma}\zeta$. Substituting this into the action, we obtain

$$S=\frac{2}{g}\int d^3x[(\partial_\mu\bar{\zeta})(\partial_\mu\zeta)+(\bar{\zeta}\,\partial_\mu\zeta)^2]. \tag{4}$$

Performing a Stratonovich-Hubbard transformation at the interaction term, we obtain

$$S=\frac{2}{g}\int d^3x\sum_\sigma|(\partial_\mu-i\alpha_\mu)\zeta_\sigma(x)|^2. \tag{5}$$

Note that (4) is invariant under a local U(1) gauge transformation, $\zeta\to\zeta e^{i\theta(x)}$ and $\bar{\zeta}\to\bar{\zeta}e^{-i\theta(x)}$. The field α_μ is a gauge field associated with this gauge invariance. As explicitly shown in Ref. 7, the CP^1 model is also derived from SBMFT. The relation between the fields ζ and $\bar{\zeta}$ and the Schwinger boson fields (2) is given by $\zeta_{j\sigma}=z_{j\sigma}$ for one sublattice and $\zeta_{j\sigma}=\bar{z}_{j\sigma}$ for the other sublattice.

In addition to these bosonic models, there is a fermionic theory. For $S=1/2$ spins, one can represent them by fermions,

$$\mathbf{S}_j=\frac{1}{2}(f_{j\uparrow}^\dagger\quad f_{j\downarrow}^\dagger)\boldsymbol{\sigma}\begin{pmatrix}f_{j\uparrow}\\ f_{j\downarrow}\end{pmatrix}, \tag{6}$$

with the constraint $\Sigma_\sigma f_{j\sigma}^\dagger f_{j\sigma}=1$. Based on a mean field theory, the π-flux phase was proposed by Affleck and Marston.[11] The excitation spectrum of the quasiparticles in the π-flux phase is given by

$$\epsilon_k=\pm v\sqrt{\cos^2 k_x+\cos^2 k_y}, \tag{7}$$

where v is a mean field parameter of the theory. Fluctuations about this mean field state can be taken into account through a gauge field. For the two-dimensional $S=1/2$ quantum Heisenberg antiferromagnet, the condition for the dynamical mass generation is satisfied,[12,13] and so the spectrum is modified as

$$\epsilon_k=\pm\sqrt{v^2(\cos^2 k_x+\cos^2 k_y)+m^2}. \tag{8}$$

The mass m is associated with the ordered staggered moment.[12]

For the description of the doped hole, we shall assume that the hole forms a Zhang-Rice singlet with a copper hole that was proposed by Zhang and Rice[2] from the analysis of the d-p model,

$$H=\sum_{j\sigma}\epsilon_d d_{j\sigma}^\dagger d_{j\sigma}+\sum_{\ell\sigma}\epsilon_p p_{\ell\sigma}^\dagger p_{\ell\sigma}+U\sum_j d_{j\uparrow}^\dagger d_{j\uparrow}d_{j\downarrow}^\dagger d_{j\downarrow}$$
$$-\sum_{j,\ell,\sigma}\epsilon_{j\ell}t_{pd}d_{j\sigma}^\dagger p_{\ell\sigma}+\text{H.c.}, \tag{9}$$

where $\epsilon_{j,j-\hat{x}/2}=\epsilon_{j,j-\hat{y}/2}=-\epsilon_{j,j+\hat{x}/2}=-\epsilon_{j,j+\hat{y}/2}=1$. Here $d_{j\sigma}^\dagger$ and $p_{\ell\sigma}^\dagger$ are the hole creation operator at the copper d-orbital state and the oxygen p-orbital state, respectively. The vacuum is defined as Cu($3d$) (Ref. 10) and O($2p$).[6] Applying a canonical transformation and omitting unimportant terms, we obtain

$$H_K=\sum_j 2\left(\frac{t_{pd}^2}{U-\Delta}+\frac{t_{pd}^2}{\Delta}\right)(d_j^\dagger\boldsymbol{\sigma}d_j)(P_j^\dagger\boldsymbol{\sigma}P_j), \tag{10}$$

where $P_{j\sigma}=\Sigma_{\{\ell\}\in j}\epsilon_{j\ell}p_{\ell\sigma}/2$. Constructing the Wannier wave functions,[2] the Zhang-Rice singlet state is created by the following operator:

$$\phi_{\mathrm{ZR}j}^\dagger=\frac{1}{\sqrt{2}}(d_{j\uparrow}^\dagger\phi_{j\downarrow}^\dagger-d_{j\downarrow}^\dagger\phi_{j\uparrow}^\dagger), \tag{11}$$

where

$$\phi_{j\sigma}=\sum_{j'}\left\{\frac{1}{N}\sum_{\mathbf{k}}\frac{\exp[i\mathbf{k}\cdot(\mathbf{R}_j-\mathbf{R}_{j'})]}{\sqrt{1-\frac{1}{2}(\cos k_x+\cos k_y)}}\right\}P_{j'\sigma}. \tag{12}$$

In this paper, we study the spin configuration around a Zhang-Rice singlet. We shall show that a spin texture that is called a half-skyrmion characterized by half of a topological charge is created around the Zhang-Rice singlet. The dispersion of the half-skyrmion is given by the same form as that of the quasiparticle in the π-flux phase. The rest of this paper is organized as follows: In Sec. II, we consider a static Zhang-Rice singlet. We argue that the spin configuration around such a static Zhang-Rice singlet is a half-skyrmion. In Sec. III, we construct the moving half-skyrmion state by making use of Lorentz invariance of the NLσM. The lattice action is derived in Sec. IV. We shall compare the dispersion of the half-skyrmion with the result of angle-resolved photoemission spectroscopy (ARPES) on the parent compounds. The effective theory of the half-skyrmion is obtained by applying duality mapping in Sec. V. Section VI is devoted to a discussion. Finally, we summarize the results in Sec. VII.

II. HALF-SKYRMION SOLUTION FOR STATIC ZHANG-RICE SINGLET

In this section, we consider the spin configuration induced around a static Zhang-Rice singlet. The spin configuration for a moving Zhang-Rice singlet shall be discussed in Sec. III based on the results of this section.

The creation operator for a Zhang-Rice singlet residing at the site j defined by Eq. (11) suggests that the spin configuration around a static Zhang-Rice singlet is given by superposition of the two spin configurations: One is the spin configuration created by a spin-up state at the d-orbital state and the other is the spin configuration created by a spin-down state at the d-orbital state. We study these states separately; then we consider the superposition of these states. We assume that the spin state of $\phi_{j\sigma}$ does not play an important role. We shall discuss its possible effects in Sec. VI.

In order to study the spin configuration, we use the NLσM. Note that we cannot apply a linear response theory, such as a spin wave theory, to obtain the spin configuration. For the d-orbital spin state at the site j that is the same as that of the Néel ordered state of the parent compound before introducing the hole, the surrounding spins are not so much affected by the fixed spin state at the site j. However, for the d-orbital spin state that is in the opposite direction to that of the Néel ordered state of the parent compound before introducing the hole, the surrounding spins can change their directions, which are beyond the range of the description of the linear response theory.

We assume that before introducing the doped hole, the spin state at the site j is the up-spin state and the staggered magnetization is in the z direction. Under this assumption, the spin-up state does not change the directions of the neighborhood spins. If we include quantum fluctuation effects, then the staggered moments of neighboring spins would be enhanced because we fix the state at the site j to spin up and quantum fluctuations is quenched at this site. In other words, the state is in the subspace of eigenstates of the system with a spin-up state at the site j d orbital. However, for simplicity we do not consider quantum fluctuation effects here, and we restrict ourselves to a classical spin configuration. Quantum fluctuation effects shall be taken into account through the renormalization of the parameters. Thus, the spin configuration associated with the spin-up state is $\mathbf{n}(\mathbf{R}_\ell)=+\hat{e}_z$ for any site ℓ.[14]

The spin configuration for the spin-down state at the site j d orbital is nontrivial. We would like to obtain the spin configuration satisfying the following boundary conditions:

$$\mathbf{n}(\mathbf{r}) \to +\hat{e}_z \quad (r \to \infty), \tag{13}$$

and

$$\mathbf{n}(\mathbf{R}_j) = -\hat{e}_z. \tag{14}$$

Since we consider the static spin configuration, we are interested in the spin configuration that minimizes the energy,

$$E = \frac{\rho_s}{2} \int d^2\mathbf{r} (\nabla \mathbf{n})^2. \tag{15}$$

The analysis given below follows a general argument for skyrmion excitations in the NLσM.[15] We include the constraint $|\mathbf{n}|^2=1$ through a Lagrange multiplier:

$$E = \frac{\rho_s}{2} \int d^2\mathbf{r}[(\nabla \mathbf{n})^2 + \lambda(|\mathbf{n}|^2 - 1)]. \tag{16}$$

Taking variation with respect to $\mathbf{n}$, we obtain

$$\nabla^2 \mathbf{n} - \lambda \mathbf{n} = 0. \tag{17}$$

The Lagrange multiplier λ is eliminated by using this equation and $\mathbf{n}^2=1$:

$$\nabla^2 \mathbf{n} - (\mathbf{n} \cdot \nabla^2 \mathbf{n})\mathbf{n} = 0. \tag{18}$$

By using the identity[16]

$$\int d^2\mathbf{r}^2[\partial_\mu \mathbf{n} \pm \epsilon_{\mu\nu}(\mathbf{n} \times \partial_\mu \mathbf{n})]^2 \geqslant 0, \tag{19}$$

with $\epsilon_{xx}=\epsilon_{yy}=1$ and $\epsilon_{xy}=-\epsilon_{yx}=1$, we find

$$E \geqslant 4\pi\rho_s|Q|, \tag{20}$$

where

$$Q = \frac{1}{4\pi} \int d^2\mathbf{r}\, \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n}), \tag{21}$$

is called the topological charge. The equality in Eq. (20) is satisfied if and only if

$$\partial_\mu \mathbf{n} \pm \epsilon_{\mu\nu}(\mathbf{n} \times \partial_\mu \mathbf{n}) = 0. \tag{22}$$

The solution of Eq. (18) is divided into sectors with different Q values. Since the energy in each sector has the lower bound determined by Eq. (20), it is enough to solve Eq. (22) for our purpose. To solve this equation, it is convenient to use a variable $w=(n_x+in_y)/(1-n_z)$. In terms of w, Eq. (22) is

$$\partial_x w = -i\, \partial_y w, \quad \partial_x w = i\, \partial_y w. \tag{23}$$

Introducing $z=x+iy$ and $\bar{z}=x-iy$, we find

$$\partial_z w = 0, \quad \partial_{\bar{z}} w = 0. \tag{24}$$

Therefore, the solution satisfies the Cauchy-Rieman eqation. The analytic function of z or $\bar{z}$ is the solution of Eq. (22). In terms of w and $\bar{w}$, the vector $\mathbf{n}$ is described by

$$\mathbf{n} = \left(\frac{w+\bar{w}}{|w|^2+1}, -i\frac{w-\bar{w}}{|w|^2+1}, \frac{|w|^2-1}{|w|^2+1} \right). \tag{25}$$

The boundary conditions of (13) and (14) are

$$|w| \to \infty \quad (r \to \infty), \tag{26}$$

and

$$w = 0 \quad (\mathbf{r} = \mathbf{R}_j), \tag{27}$$

respectively. The solution that satisfies these boundary condition is

$$w = \frac{z}{\lambda}, \tag{28}$$

or

$$w = \frac{\bar{z}}{\lambda}, \tag{29}$$

up to a phase factor that is associated with a global rotation of all spins in the plane. Here λ is a parameter that is associated with the size of the spin configuration and $\mathbf{R}_j$ is taken to be the origin to simplify the expressions. The vector $\mathbf{n}$ representation of Eqs. (28) and (29) is the following:

$$\mathbf{n} = \left(\frac{2\lambda x}{r^2+\lambda^2}, \frac{2\lambda y}{r^2+\lambda^2}, \frac{r^2-\lambda^2}{r^2+\lambda^2} \right), \tag{30}$$

and

$$\mathbf{n} = \left(\frac{2\lambda x}{r^2+\lambda^2}, -\frac{2\lambda y}{r^2+\lambda^2}, \frac{r^2-\lambda^2}{r^2+\lambda^2} \right). \tag{31}$$

The topological charge Q is given by

$$Q = \frac{1}{\pi}\int d^2\mathbf{r}^2 \frac{(\bar{\partial}\bar{w})(\partial w) - (\partial\bar{w})(\bar{\partial}w)}{(1+|w|^2)^2}. \tag{32}$$

For the solutions, Eqs. (28) and (29), the topological charges are $Q=1$ and $Q=-1$, respectively. These solutions are called skyrmion. The energy of these skyrmion solution is $E=4\pi\rho_s$, which is calculated from the following expression:

$$E = 4\rho_s \int d^2\mathbf{r} \frac{(\bar{\partial}\bar{w})(\partial w) + (\partial\bar{w})(\bar{\partial}w)}{(1+|w|^2)^2}. \tag{33}$$

Note that there is no solution in the $Q=0$ sector. In fact, solutions in this sector satisfy $\partial\omega=0$ and $\bar{\partial}\omega=0$, and we obtain ω=const. Obviously such a solution does not satisfy the boundary conditions.

Now we consider superposition of the uniform state and the skyrmion state. Unfortunately, the classical superposition of the two-spin configuration is not the solution of the field equation. However, these solutions suggest that the resultant spin configuration is chracterized by a topological charge Q with $0<|Q|<1$. The value of Q is determined by making use of the fact that the Néel ordering state is described by Bose-Einstein condensation of the Schwinger bosons.[10] In order to examine the value of Q, we use the following representation of Q by the CP^1 gauge field α_μ:

$$Q = \int \frac{d^2\mathbf{r}}{2\pi}(\partial_x \alpha_y - \partial_y \alpha_x). \tag{34}$$

From this expression, we see that the spin configuration with Q corresponds to the flux $2\pi Q$ in the condensate of the Schwinger bosons. Since the spin-1/2 bosons $\zeta_\sigma(x)$ are confined in the Néel state, all bosons are paired in the low-energy physics. Because pairs of the bosons carry the gauge charge two, the flux quantum is π, similar to the conventional BCS superconductors. Therefore, the flux value is not arbitrary and Q must be in the form of $Q=n\pi$, with n being an integer. Meanwhile, from the constraint $0<|Q|<1$, the flux associated with the spin configuration satisfies $0<2\pi|Q|<2\pi$. Thus, we conclude $2\pi|Q|=\pi$, or $|Q|=1/2$.[17] Since the topological charge is one-half, we call this spin configuration a half-skyrmion. The energy associated with the half-skyrmion spin texture is $4\pi\rho_s|Q|=2\pi\rho_s\equiv E_0$ because we can apply Eq. (22) outside the core region. Due to the limitation of the effective theories, we cannot determine the core energy. It would be determined from a calculation based on a microscopic model. The half-skyrmion solution is also discussed in the ferromagnetic Heisenberg model[18] in the context of quantum Hall systems. Since the value of $|Q|=1/2$ is obtained from the calculation of the topological charge using Eqs. (30) and (31) with excluding the core region $r<\lambda$, we may use Eqs. (30) and (31) for the expressions of the half-skyrmion and the anti-half-skyrmion spin textures, respectively. The half-skyrmion and anti-half-skyrmion spin textures are shown schematically in Figs. 1 and 2.

III. MOVING HALF-SKYRMION SOLUTION

In the last section, we have argued that the spin configuration around a static Zhang-Rice singlet is the half-skyrmion. In this section, we construct the moving half-skyrmion solution from the static half-skyrmion solution by making use of the Lorentz invariance of the NLσM and the CP^1 model.

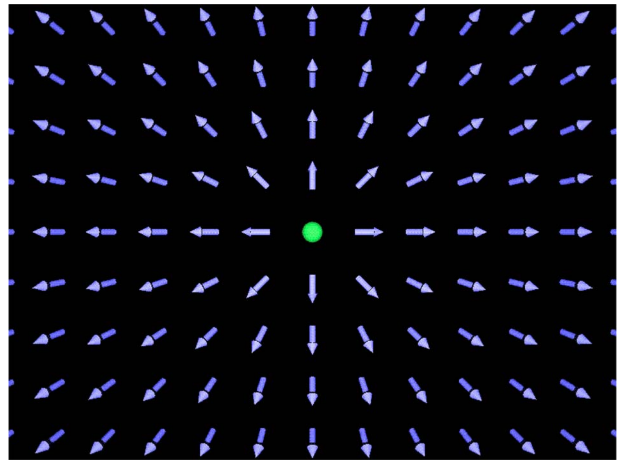

FIG. 1. (Color online) The half-skyrmion spin texture. Arrows indicate the staggered moment and the filled circle at the center represents the Zhang-Rice singlet formed site.

The action (1) is written in the Euclidean space time. The form in Minkowskii space time reads as

$$S = \frac{\rho_s}{2}\int dt \int d^2\mathbf{r}\left[\frac{1}{c_{\rm sw}^2}\left(\frac{\partial \mathbf{n}}{\partial t}\right)^2 - (\boldsymbol{\nabla}\mathbf{n})^2\right]. \tag{35}$$

Apparently the action is invariant under Lorentz transformations with $c_{\rm sw}$ being the speed of "light." Let us consider a following Lorentz transformation:

$$x' = \frac{x - vt}{\sqrt{1-(v/c_{\rm sw})^2}}, \tag{36}$$

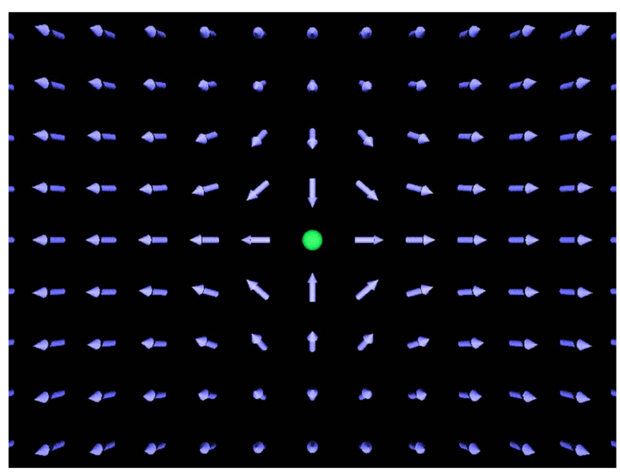

FIG. 2. (Color online) The anti-half-skyrmion spin texture. Arrows indicate the staggered moment and the filled circle at the center represents the Zhang-Rice singlet formed site.

$$t' = \frac{t - (v/c_{\rm sw}^2)t}{\sqrt{1-(v/c_{\rm sw})^2}}. \quad (37)$$

It is easy to check that the action is invariant under this Lorentz transformation.

Now we apply the Lorentz transformation (36) and (37) to the static half-skyrmion solution (30),

$$\mathbf{n}' = \left(\frac{2\lambda x'}{x'^2 + y^2 + \lambda^2}, - \frac{2\lambda y}{x'^2 + y^2 + \lambda^2}, \frac{x'^2 + y^2 - \lambda^2}{x'^2 + y^2 + \lambda^2} \right). \quad (38)$$

The excitation spectrum of the half-skyrmion is obtained by calculating the energy-momentum tensors. We write Eq. (38) as $\mathbf{n}' = \mathbf{n}_s(x', y)$, where $x' = \gamma(x - vt)$ with $\gamma = 1/\sqrt{1-(v/c_{\rm sw})^2}$. The energy-momentum tensors are given by

$$T^{\mu}_{\nu} = \rho_s\, \partial^{\mu}\mathbf{n}\, \partial_{\nu}\mathbf{n} - \frac{\rho_s}{2} \partial^{\rho}\mathbf{n}\, \partial_{\rho}\mathbf{n}\, \delta^{\mu}_{\nu}, \quad (39)$$

where $A^{\mu}B_{\mu} = A_0B_0 - \mathbf{A}\cdot\mathbf{B}$. The energy is

$$E = \int d^2\mathbf{r}\, T^0_0 = \frac{\rho_s}{2} \int d^2\mathbf{r} \left(\frac{1}{c_{\rm sw}^2} (\partial_t \mathbf{n}')^2 + (\partial_x \mathbf{n}')^2 + (\partial_y \mathbf{n}')^2 \right)$$
$$= \frac{\rho_s}{2} \int d^2\mathbf{r} \left[\left(\frac{\gamma^2 v^2}{c_{\rm sw}^2} + \gamma^2 \right) (\partial_{x'} \mathbf{n}_s)^2 + (\partial_y \mathbf{n}_s)^2 \right]$$
$$= \gamma\rho_s \int dx'\, dy (\partial_{x'}\mathbf{n}_s)^2 = \gamma E_0, \quad (40)$$

where we have used that $\partial_t \mathbf{n}' = -\gamma v\, \partial_{x'}\mathbf{n}_s$, $\partial_x \mathbf{n}' = \gamma\, \partial_{x'}\mathbf{n}_s$, $dx = \gamma\, dx'$, and $(\partial_{x'}\mathbf{n}_s)^2 = (\partial_y \mathbf{n}_s)^2$. The x component of the momentum is calculated as follows:

$$P_x = \frac{1}{c_{\rm sw}} \int d^2\mathbf{r}\, T^0_1 = \frac{\rho_s}{c_{\rm sw}^2} \int dx\, dy (\partial_t \mathbf{n}_s) \cdot (\partial_x \mathbf{n}_s) = -\beta\gamma E_0/c_{\rm sw}, \quad (41)$$

$$P_y = \frac{1}{c_{\rm sw}} \int d^2\mathbf{r}\, T^0_2 = \frac{\rho_s}{c_{\rm sw}^2} \int dx\, dy (\partial_t \mathbf{n}_s) \cdot (\partial_y \mathbf{n}_s) = 0. \quad (42)$$

From Eqs. (40)–(42), we find

$$E^2 = c_{\rm sw}^2 P_x^2 + E_0^2. \quad (43)$$

By considering general Lorentz transformations, we find that the following relation holds:

$$E^2 = c_{\rm sw}^2 (P_x^2 + P_y^2) + E_0^2. \quad (44)$$

Therefore, the excitation spectrum of the half-skyrmion is given by

$$E_k = \pm \sqrt{c_{\rm sw}^2 k^2 + E_0^2}. \quad (45)$$

Having obtained the relativistic dispersion (45), we consider the action of the half-skyrmion. Since the half-skyrmion is a topological spin texture, one might expect that Berry phases affect the statistice of the half-skyrmion, as in the fractional quantum Hall systems. In the fractional quantum Hall systems, the effective theories are characterized by topological field theory. The Berry phase effect determines the statistics of quasiparticles. In constrast, the system of the single-hole-doped antiferromagnet is not characterized by topological field theory. Indeed, a gauge field that describes the Berry phase effects is massive due to Bose-Einstein condensation. Therefore, the leading term of the gauge field is the mass term. In such a situation, we do not expect that Berry phases play an important role in determining the statistics of quasiparticles. In the absence of the Berry phase effects, the statistice of the half-skyrmion is fermion simply because the doped hole that obeys the fermionic statistics sits at the core.

The statistics of the Zhang-Rice singlet is infered from its field operator. Since the d-orbital hole states constitute localized spin-1/2 moments and one can choose either a fermionic description or a bosonic description for the spins, the statistics of the Zhang-Rice singlet is either fermion or boson. However, now we are interested in the Néel ordering phase. Bosonic descriptions, such as Schwinger boson mean field theory or NLσM, are suitable for the description of the Néel ordered state. In this case, the statistics of the Zhang-Rice singlet is fermion.

The fermion field obeying the relativistic excitation spectrum (45) is described by a Dirac fermion action. The action of the half-skymion may be written as

$$\mathcal{L} = \sum_{s=\pm} \bar{\psi}_s (\gamma_\mu \partial_\mu + mc_{\rm sw}^2) \psi_s, \quad (46)$$

with $\bar{\psi}_s = \psi_s^\dagger \gamma_0$ and $mc_{\rm sw}^2 = E_0$. The index s is for the sign of the topological charge. That is, $s=+$ is for the half-skyrmion and $s=-$ is for the anti-half-skyrmion. The Dirac fermion ψ has four components: There are positive and negative energy states. These states obey the dispersion (45) with the origin either at $\mathbf{k}_1 = (\pi/2, \pi/2)$ or $\mathbf{k}_2 = (-\pi/2, \pi/2)$. This is suggested from the fact that the Schwinger bosons are gapless at four zone centers $(\pm\pi/2, \pm\pi/2)$. Because $\mathbf{Q} = (\pi, \pi)$ connects two points in the diagonal directions, there are two independent points $\mathbf{k}_1$ and $\mathbf{k}_2$. The γ matrices are 4×4 matrices and satisfy $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$.

IV. HALF-SKYRMION ON THE LATTICE

The expression on the lattice is derived by discretizing the continuum Lagrangian, Eq. (46). After Fourier transformations, we obtain

$$\mathcal{L} = \sum_{a=1,2} \sum_{\sigma} \bar{\psi}_{a\sigma}(k) \times \begin{pmatrix} mc_{\rm sw}^2 + \partial_\tau & \sin k_x + i \sin k_y \\ -\sin k_x + i \sin k_y & mc_{\rm sw}^2 - \partial_\tau \end{pmatrix} \psi_{a\sigma}(k). \quad (47)$$

By shifting the origin in the momentum space to either $\mathbf{k}_1$ or $\mathbf{k}_2$, we obtain

$$\mathcal{L} = \sum_{\sigma} \left[\bar{\psi}_{1\sigma}(k) \begin{pmatrix} mc_{\rm sw}^2 + \partial_\tau & \cos k_x + i \cos k_y \\ -\cos k_x + i \cos k_y & mc_{\rm sw}^2 - \partial_\tau \end{pmatrix} \psi_{1\sigma}(k) + \bar{\psi}_{2\sigma}(k) \begin{pmatrix} mc_{\rm sw}^2 + \partial_\tau & -\cos k_x + i \cos k_y \\ \cos k_x + i \cos k_y & mc_{\rm sw}^2 - \partial_\tau \end{pmatrix} \psi_{2\sigma}(k) \right]. \quad (48)$$

Diagonalization of the matrix leads to the following expression:

$$\epsilon_k^{\pm} = \pm \sqrt{c_{\rm sw}^2(\cos^2 k_x + \cos^2 k_y) + (mc_{\rm sw}^2)^2}. \quad (49)$$

Note that there is no fermion-doubling problem, which occurs when one formulates a Dirac fermion on a lattice.[19]

The parameters $c_{\rm sw}$ and $mc_{\rm sw}^2 = 2\pi\rho_s$ are determined from the values for the Heisenberg antiferromagnet. We use $Z_c = 1.17$ and $Z_\rho = 0.72$, which are estimated from quantum Monte Carlo simulations[20] and a series expansion technique.[21] Substituting these values into Eq. (49), we find that the bandwidth is ~ 1.5J and $m/J \sim 1.13$. Meanwhile, an experimentally estimated bandwidth by Wells *et al.* is $\sim 2.2J$.[22,23] This discrepancy would be associated with the deviation of the real system from the NLσM. For the mass value, Ronning *et al.* evaluated it by using the form $\epsilon_k^- - mc_{\rm sw}^2$. The result is $m/J \sim 1.3$.[24] Some of this discrepancy might be associated with the mass renormalization due to antiferromagnetic spin fluctuations. This shall be discussed in Sec. V.

V. EFFECTIVE THEORY FOR HALF-SKYRMION

In order to include antiferromagnetic spin fluctuation effects on the half-skyrmion, we shall derive the effective theory of the half-skyrmion. We make use of a duality mapping[25] for that purpose.

Before the application of the duality mapping, we point out that a half-skyrmion can be seen as a vortex in the CP[1] model. As stated in Sec. II, the topological charge of the half-skyrmion is represented by the gauge flux with respect to the CP[1] gauge field α_μ. In the CP[1] model, the vector $\mathbf{n}$ reads as

$$\mathbf{n} = (\bar{\zeta}_\uparrow \zeta_\downarrow + \bar{\zeta}_\downarrow \zeta_\uparrow, -i(\bar{\zeta}_\uparrow \zeta_\downarrow - \bar{\zeta}_\downarrow \zeta_\uparrow), \bar{\zeta}_\uparrow \zeta_\uparrow - \bar{\zeta}_\downarrow \zeta_\downarrow). \quad (50)$$

Since $\mathbf{n}$ is a unit vector, ζ and $\bar{\zeta}$ satisfy $\bar{\zeta}_\uparrow \zeta_\uparrow + \bar{\zeta}_\downarrow \zeta_\downarrow = 1$. In terms of ζ_σ and $\bar{\zeta}_\sigma$, the half-skyrmion solution, Eqs. (30) and (31), has the following form:

$$\zeta_\uparrow = \frac{r}{\sqrt{r^2 + \lambda^2}} \exp(\pm i\theta) \exp(i\chi), \quad (51)$$

$$\zeta_\downarrow = \frac{\lambda}{\sqrt{r^2 + \lambda^2}} \exp(i\chi), \quad (52)$$

where the minus sign is for the half-skyrmion and the plus sign is for the anti-half-skyrmion and χ is a constant. In general, the half-skyrmion solution is represented by

$$\begin{pmatrix} \zeta_\uparrow \\ \zeta_\downarrow \end{pmatrix} = \begin{pmatrix} u & -v^* \\ v & u^* \end{pmatrix} \begin{pmatrix} (\lambda/\sqrt{r^2 + \lambda^2}) \\ (r/\sqrt{r^2 + \lambda^2}) \exp(\pm i\theta) \end{pmatrix}, \quad (53)$$

where u and v are constant complex numbers and satisfy $|u|^2 + |v|^2 = 1$. The matrix

$$\begin{pmatrix} u & -v^* \\ v & u^* \end{pmatrix}$$

is a global SU(2) transformation. The boundary condition at infinity is transformed to $\mathbf{n} \rightarrow [-uv^* - vu^*, -i(uv^* - vu^*), -|u|^2 + |v|^2]$.

In order to see the relation between the half-skyrmion and a vortex, we take $u = v = 1/\sqrt{2}$. In this case, the half-skyrmions solution have the following form:

$$\zeta_\uparrow = \frac{\lambda - r \exp(\pm i\theta)}{\sqrt{2(r^2 + \lambda^2)}}, \quad (54)$$

$$\zeta_\downarrow = \frac{\lambda + r \exp(\pm i\theta)}{\sqrt{2(r^2 + \lambda^2)}}. \quad (55)$$

This has a vortex form at $r \gg \lambda$:

$$\begin{pmatrix} \zeta_\uparrow \\ \zeta_\downarrow \end{pmatrix} \sim \exp(\pm i\theta) \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}. \quad (56)$$

Thus, the half-skyrmion is seen as a vortex introduced in the system. Such a vortex is taken into account in the CP[1] model as follows:

$$\zeta_\sigma = \rho_\sigma^{1/2} \exp(i\phi) = \rho_\sigma^{1/2} \exp(i\phi_0 + i\phi_v). \quad (57)$$

Here ϕ_0 describes coherent motion of the bosons and ϕ_v describes the vortex. In the case of Eq. (56), ϕ_v is

$$\phi_v = \pm \tan^{-1} \frac{y - y_v}{x - x_v}, \quad (58)$$

with (x_v, y_v) representing the coordinate of the vortex.

We rewrite the CP[1] model by substituting Eq. (57) into Eq. (5):

$$S = \frac{1}{g} \int d^3x \left[\sum_\sigma \frac{1}{4\rho_\sigma} (\partial_\mu \rho_\sigma)^2 + \left(\sum_\sigma \rho_\sigma \right) (\partial_\mu \phi - \alpha_\mu)^2 \right]. \quad (59)$$

The amplitude fluctuations would be important only in the vicinity of the core. We focus on the outside of the core and assume a constant value for $\rho_\sigma : \rho_0 \equiv \Sigma_\sigma \langle \rho_\sigma \rangle$. After introducing a Stratonovich-Hubbard field J_μ, we obtain

$$S = \int d^3x \left[\frac{g}{4\rho_0} J_\mu^2 - iJ_\mu (\partial_\mu \phi_0 + \partial_\mu \phi_v - \alpha_\mu) \right]. \quad (60)$$

The field J_μ is associated with the spin current. Integrating out ϕ_0 leads to $\partial_\mu J_\mu = 0$. From this equation, we can represent J_μ in terms of a gauge field,

$$J_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\,\partial_\nu A_\lambda. \tag{61}$$

Substituting this equation into the action, and integrating out α_μ, we obtain

$$S_d = S_A + S_{\rm int}, \tag{62}$$

where

$$S_A = \frac{1}{4e_A^2}\int d^3x(\partial_\mu A_\nu - \partial_\nu A_\mu)^2, \tag{63}$$

and

$$S_{\rm int} = -i\int d^3x\, A_\mu j_\mu^v, \tag{64}$$

with

$$j_\mu^v = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu\,\partial_\lambda\phi_v. \tag{65}$$

The value of the gauge charge e_A depends on the action of the gauge field α_μ, which would be massive because of Bose-Einstein condensation of the Schwinger bosons. Here we treat e_A as a parameter of the theory.

Since J_μ describes the spin current and A_μ is related to J_μ through Eq. (61), the gauge field A_μ is associated with spin excitations, such as antiferromagnetic spin waves. Indeed, A_μ is massless in the Néel-ordered phase and the velocity of the mode is $c_{\rm sw}$.

By taking into account the fact that the half-skyrmion is given by Eq. (46), we obtain the following action for the half-skyrmion:

$$S = \int d^3x\left[\sum_\sigma \bar{\psi}_\sigma[\gamma_\mu(\partial_\mu - iq_\sigma A_\mu) + m]\psi_\sigma + \frac{1}{4e_A^2}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2\right], \tag{66}$$

where q_σ is an index for the sign of the topological charge. Hereafter, we set $c_{\rm sw}=1$.

In order to study the effect of spin fluctuations on the half-skyrmion, we formulate the theory on the square lattice. The lattice form of (66) is given by

$$\mathcal{L} = \sum_a\sum_j\sum_\mu\left(\frac{1}{2}(\bar{\psi}_{ja}\gamma_\mu e^{-iq_\sigma\phi_\mu(j)}\psi_{j+\hat{\mu},a} - \bar{\psi}_{j+\hat{\mu},a}\gamma_\mu e^{iq_\sigma\phi_\mu(j)}\psi_{ja}) + m\bar{\psi}_{ja}\psi_{ja}\right) + \frac{1}{2e_A^2}\sum_j\sum_{\mu\nu}\{1-\cos[\phi_\nu(j+\hat{\mu}) - \phi_\nu(j) - \phi_\mu(j+\hat{\nu}) + \phi_\mu(j)]\}. \tag{67}$$

We expand $\exp[\pm iq_\sigma\phi_\mu(j)]$ with respect to $\phi_\mu(j)$. The first-order term $\mathcal{L}^{(1)}$ is

$$\mathcal{L}^{(1)} = -i\sum_a\sum_\mu\sum_{k,q} q_\sigma\bar{\psi}_{k+q,a}\gamma_\mu\phi_\mu(q)\psi_{k,a}e^{-iq_\mu/2}\cos\left(k_\mu + \frac{q_\mu}{2}\right). \tag{68}$$

The second-order term is

$$\mathcal{L}^{(2)} = -\frac{i}{2}\sum_{k,q,q'}\sum_\mu\sum_a\bar{\psi}_{k+q+q',a}\gamma_\mu\phi_\mu(q)\phi_\mu(q')\times\psi_{k,a}e^{-i(q_\mu+q'_\mu)/2}\sin\left(k_\mu + \frac{q_\mu+q'_\mu}{2}\right). \tag{69}$$

We evaluate the self-energy $\Sigma_k^{(I)}$ that comes from the second-order term of $\mathcal{L}^{(1)}$ and $\Sigma_k^{(II)}$ that comes from the first-order term of $\mathcal{L}^{(2)}$. The self-energy $\Sigma_k^{(II)}$ is given by

$$\sum_k^{(II)} = \frac{i}{2\Omega}\sum_q{}' D_q\gamma_\mu\sin k_\mu, \tag{70}$$

with $D_q=1/[(i\omega_n)^2-\omega_q^2]$. The mass renormalization due to this self-energy is

$$\frac{m}{1-\frac{1}{2\Omega}\sum_q{}' D_q} = \frac{m}{1-0.123e_A^2}. \tag{71}$$

The self-energy $\Sigma_k^{(I)}$ is given by

$$\Sigma_k^{(I)} = -\frac{2}{\Omega}\sum_q{}'\gamma_\mu D_q G_{k+q}\gamma_\mu\cos^2\left(k_\mu + \frac{q_\mu}{2}\right). \tag{72}$$

Substituting the explicit form of G_k,

$$G_k = \frac{1}{\sin^2 k_\mu + m^2}(-i\gamma_\mu\sin k_\mu + m), \tag{73}$$

we obtain

$$\Sigma_k^{(I)} = -\frac{2e_A^2}{\Omega}\sum_q{}'\frac{[i\gamma_\rho\sin(k_\rho+q_\rho) + mc^2]\cos^2\left(k_\mu+\frac{q_\mu}{2}\right) - 2i\gamma_\mu\sin(k_\mu+q_\mu)\cos^2\left(k_\mu+\frac{q_\mu}{2}\right)}{\sin^2 q_\nu[\sin^2(k_\nu+q_\nu)+m^2]} = -me_A^2\int\frac{d^3q}{(2\pi)^3}\frac{\sum_\mu\cos^2\left(k_\mu+\frac{q_\mu}{2}\right)}{\left(\sum_\nu\sin^2 q_\nu\right)\left[\sum_\rho\sin^2(k_\rho+q_\rho)+m^2\right]}. \tag{74}$$

Since the dominant contribution comes from the region of $q_\mu \sim 0$ and $q_\mu \sim (\pi, \pi)$, we make an approximation for evaluating the integral,

$$\Sigma_k^{(I)} \simeq -2me_A^2 \int \frac{d^3q}{(2\pi)^3}\frac{1}{q^2}\frac{\sum_\nu \cos^2 k_\nu}{\sum_\rho \sin^2 k_\rho + m^2}$$

$$= -\frac{me_A^2}{\pi}\frac{\sum_\mu \cos^2 k_\mu}{\sum_\nu \sin^2 k_\nu + m^2}. \quad (75)$$

At the zone centers, the energy is shifted by $-2e_A^2/(\pi m)$.

VI. DISCUSSION

On the description of the half-skyrmion spin texture formation, it is essential that the Zhang-Rice singlet state has the form of superposition of the *d*-orbital spin-up state and the *d*-orbital spin-down state. However, each of the *d*-orbital spin states accompanies the oxygen *p*-orbital hole state. As argued by Aharony *et al.*,[26] there is the possibility of replacing the antiferromagnetic superexchange interaction with a ferromagnetic interaction on the bond where the doped hole occupies the oxygen *p*-orbital state. In the picture of the Zhang-Rice singlet where a copper site hole spin forms a singlet state with the Wannier state of the symmetric combination of the four oxygen *p*-orbital hole states surrounding the copper site, the interactions between the nearest neighbor spins could be replaced with ferromagnetic interactions. If this is the case, the skyrmion spin texture would be formed for the opposite spin state compared with the antiferromagnetic interaction case. However, obviously this does not affect the conclusion on the half-skyrmion formation. Another possibility on the effect of the oxygen hole state would be related to the core structure of the half-skyrmion. The discussion on the core structure is beyond the description of the effective theories. For the investigation of this effect, one would require a microscopic model, such as the d-p model.

As argued in Sec. V, the half-skyrmion spin texture can be seen as a vortex introduced in the Bose-Einstein condensate of the Schwinger bosons that describes the Néel ordering. Generally, vortices disturb the phase coherence of condensate. Rapid suppression of Néel ordering by hole doping could be understood by this picture.[27] We expect that this effective Néel order suppression is absent in the electron doping case. Our picture for the half-skyrmion formation is based on the fact that the Zhang-Rice singlet has a form of superposition of the spin-up and spin-down states. This suggests a picture for the difference between the hole doping and the electron doping. In the electron doping case, the doped electrons occupy copper sites. The occupied copper site has spin zero. However, the electronic state at this site does not have a form of superposition of different spin states but is simply given by a doubly occupied site. Therefore, we do not expect the half-skyrmion formation for the electron-doped systems. Similar arguments can be applied to Zn doping. If we replace the copper with Zn, the electronic state at that site does not have the superposition of different spin states. Since vortices are not induced by either the electron doping or the Zn doping, those dopings are not so effective to destroy the Néel ordering, as observed experimentally. By contrast, if we replace a copper with Li, then a hole is introduced. That hole is believed to occupy an oxygen *p*-orbital state. If we assume the Zhang-Rice singlet formation between the hole spin and a copper site spin, the half-skyrmion would be created. This is consistent with the experiments that report that the critical Li doping concentration for the destruction of the Néel ordering is almost the same as that of the hole doping concentration.[28] A skyrmionlike spin texture[29] formation for the Li-doping case was discussed by Haas *et al.*[30]

In Sec. IV, we have argued that the dispersion (49) is qualitatively in good agreement with the ARPES result on the parent compounds. A similarity between the dispersion (49) without the mass term and the ARPES result was first pointed out by Laughlin.[31] However, in the absence of the mass term, cusp structures appear around the $(\pm\pi/2, \pm\pi/2)$ points, which does not agree with the experiment. This discrepancy is corrected by including the mass term. A recent estimation of the mass term[32] reports a value that is close to the theoretical prediction, as discussed in Sec. IV. Another point of view has been suggested based on a self-consistent Born approximation[3] of the t-J model. The analysis of the t-J model predicts a relatively flat dispersion along the $(\pi, 0)$ point to the $(0, \pi)$ point, whose bandwidth is much smaller than the ARPES result. This discrepancy is improved by taking into account next and third nearest neighbor hopping terms. In contrast, the dispersion (49) predicts the same dispersion along the $(0, 0)$ point to the (π, π) point and the $(\pi, 0)$ point to the $(0, \pi)$ point. This seems consistent with the experiment by Ronning *et al.*[32]

The quite broad peaks observed by ARPES would be associated with the coupling to some bosonic modes. The question is what boson mode plays the major role for this broadening. A scenario based on an electron-phonon coupling has been proposed by Mischenko and Nagaosa.[33] It is argued that the electron-phonon coupling is in the strong coupling regime for the quasiparticles in the t-J model. The quasiparticle dispersion in the t-J model is associated with the antiferromagnetic spin wave effects, whereas in the half-skyrmion picture, the dispersion is given by the soliton character of the half-skyrmion in the spin system. The antiferromagnetic spin fluctuations can play some role for the broadening of the spectrum. This issue will be examined in the future publication. This effect comes from the coupling between the half-skyrmion and the gauge field that is associated with the antiferromagnetic spin fluctuations, as discussed in Sec. V. Of course, this just suggests another possibility for the broadening of the spectrum.

Ng argued[34] that the vortex excitations in the Schwinger boson mean field theory correspond to the quasiparticles in the π-flux phase. Similarities between the skyrmionlike spin texture[29] and the quasiparticle of the π-flux phase was pointed out by Gooding,[35] based on a numerical simulation. These seem to be consistent with the half-skyrmion picture. In the context of the spin-charge separation,[36] Baskaran argued[37] that the half-skyrmion can be seen as a deconfined spinon from the analysis of the n-skyrmion solution of the NLσM.

The half-skyrmion picture may be extended to the slightly doped regime. If we increase the number of holes, then the interaction between the half-skyrmions becomes important. A half-skyrmion lattice state might be possible in an appropriate doping concentration. We expect that a skyrmionlike spin texture is formed, even in the magnetically disordered phase. Since the antiferromagnetic correlation length ξ_{AF} is finite, the topological charge that characterizes the spin texture is given by

$$Q = \frac{1}{2\pi}\int_{\lambda}^{\xi_{AF}} d^2\mathbf{r}(\partial_x\alpha_y - \partial_y\alpha_x). \tag{76}$$

Probably the term "topological" is not appropriate in the disordered regime because the index Q would be no longer quantized. If doped holes accompany a spin texture with nonzero Q, then we can apply a mechanism of d-wave superconductivity based on a skyrmion like spin texture.[38]

VII. SUMMARY

To summarize, it has been argued that the half-skyrmion spin texture is created by doping a hole into the CuO_2 plane. The picture is based on the Zhang-Rice singlet. The Zhang-Rice singlet wave function has the form of superposition of different d-orbital spin states. The half-skyrmion formation is the result of superposition of the skyrmion spin texture and the trivial uniform state associated with each of d-orbital spin states.

The excitation spectrum of the half-skyrmion is qualitatively in good agreement with ARPES experiments on the parent compounds. Although the theory is based on the effective theories, such as the NLσM and the CP_1 model, an estimation of the parameters in the dispersion gives us the values close to experimentally obtained values.

Since the half-skyrmion picture is based on the superposition nature of the Zhang-Rice singlet wave function, the picture is not applicable to the electron-doped systems, where the description of the doped electrons is not expected to have that character.

An interesting extension of the half-skyrmion picture would be to examine whether the picture can be applicable to a slightly doped regime, where the interaction between the half-skyrmions is not negligible. It would also be interesting to examine whether the skyrmionlike spin texture is formed in the disordered spin state. In the absence of the Néel ordering, the topological charge could be arbitrary. However, we expect that strong antiferromagnetic correlations can stabilize a skyrmionlike spin texture created by a doped hole with nonvanishing topological charge.

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