Exact mappings between fermionic Ising spin-glass and classical spin-glass models

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(Received 21 June 2005; published 16 September 2005)

We present in this paper exact analytical expressions for the thermodynamical properties and Green's functions of a certain family of fermionic Ising spin-glass models with Hubbard interaction by noticing that their Hamiltonian is a function of the number operator only. The thermodynamical properties are mapped to the classical Ghatak-Sherrington spin-glass model, while the the density of states (DOS) is related to its joint spin-field distribution. We discuss the presence of the pseudogap in the DOS with the help of this mapping.

DOI: 10.1103/PhysRevB.72.104427 PACS number(s): 71.10.Fd, 71.23.An, 75.10.−b

I. INTRODUCTION

While ferromagnetism is theoretically grounded in models lying between two extreme pictures, that of localized spins and that of itinerant-electron theory, the theoretical description of spin-glass systems has been focused mainly on models of localized spins, whose paradigms are the Edwards-Anderson (EA) and Sherrington-Kirkpatrick (SK) models.[1,2] One reason for this is that, experimentally, most of the classic magnetic materials presenting spin-glass behavior correspond to this description of localized spins, another is that they encapsulate many aspects of theoretical challenge, while a third is that they relate to or emulate many problems of wider interest in the statistical physics of complex systems.

However, there do exist spin-glass materials that are more appropriately described in terms of itinerant electrons, requiring models that treat magnetic and conducting properties on the same footing. While early models can be found in Refs. 3 and 4 and the full problem remains to be tackled, our main goal here is to discuss a restricted class of models described by Oppermann *et al.*[5] and normally referred to as *fermionic Ising spin-glass models*.[6–13]

Even within an SK-like (infinite-ranged exchange) itinerant fermionic model, it is a very significant challenge to treat conducting and magnetic properties together. Hence, as a first step toward their understanding, simplified models have been studied in the so-called *insulating limit*.[6–13] This removes the essentially quantum complexity of the model and allows a classical treatment, albeit still with interesting consequences.

Independently of whether the insulating limit of fermionic Ising spin-glass models may or may not be useful to better understand real itinerant spin glasses, it is clear that, at least, we must fully understand the models arising from this limit. To this end, our goal is to point out that not only are the fermionic Ising spin-glass models completely mappable to classical spin-glass models at the level of the thermodynamics,[11] but also the densities of states (DOS), and hence the local (quantum) Green's functions, have a classical derivation and indeed are given by distributions of local fields of a corresponding classical model, without the need for sophisticated quantum treatment.

Hence we can exploit all the knowledge of classical spin glasses to shed light on the fermionic Ising spin-glass models in the insulating limit. In particular, the existence of a pseudogap in the DOS at half-filling and without a Hubbard term emerges as an immediate consequence of the mapping, when account is taken of the well-known fact that at zero temperature, the local field distribution of the SK spin glass has such a pseudogap.

In turn, this implies that the observed strong corrections to the DOS due to steps in the replica symmetry breaking (RSB)[13] do not have a fundamentally quantum origin.

We also notice, in passing, the strong temperature dependence of the DOS, mirroring that of the field distribution of the SK model.

This paper is organized as follows: In Sec. II the fermionic Ising spin-glass model is presented and some limits as a function of its parameters are discussed. In Sec. III we map this model to the Ghatak-Sherrington model and express the DOS as a function of its joint spin-field distribution. Then, in Sec. IV we discuss the mappings from a physical perspective and in Sec. V we discuss the existence of a DOS pseudogap in the light of the mapping. Section VI presents our conclusions.

II. MODEL DEFINITIONS

Our starting point is the following model for itinerant electrons involving frustrated magnetic order:

$$\hat{\mathcal{H}} = U\sum_{i=1}^{N} \hat{n}_{i\uparrow}\hat{n}_{i\downarrow} - \sum_{i<j=1}^{N} \mathcal{J}_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z - \mu\sum_{i=1}^{N}\sum_{s\in\{\uparrow,\downarrow\}} \hat{n}_{is} + \hat{\mathcal{H}}_{\text{rest}}, \tag{1}$$

where the couplings $\mathcal{J}_{ij}$ are drawn randomly and independently from a distribution

$$P(\mathcal{J}_{ij}) = \frac{1}{\sqrt{2\pi\mathcal{J}^2/N}} \exp\left[-\frac{N}{2\mathcal{J}^2}\left(\mathcal{J}_{ij} - \frac{\mathcal{J}_0}{N}\right)^2\right]. \tag{2}$$

The spin and charge operators are defined by

$$\hat{\sigma}_i^z = \hat{n}_{i\uparrow} - \hat{n}_{i\downarrow}, \quad \hat{n}_{is} = \hat{a}_{is}^\dagger \hat{a}_{is}, \tag{3}$$

where $\hat{a}_{is}^\dagger$ and $\hat{a}_{is}$ are, respectively, the fermion creation and annihilation operators. The label $s \in \{\downarrow, \uparrow\} \equiv \{-1, 1\}$ indicates the spin state. The Hamiltonian $\hat{\mathcal{H}}_{\text{rest}}$ contains those terms that cannot be expressed as a function of the number operator only, as, for example, the hopping and the pair-hopping terms

$$\hat{\mathcal{H}}_{\text{hopping}} = \sum_{(i,j)} \sum_{s \in \{\uparrow,\downarrow\}} t_{ij} \hat{a}_{is}^{\dagger} \hat{a}_{js},$$

$$\hat{\mathcal{H}}_{\text{hopping}}^{\text{pair}} = \sum_{(i,j)} t_{ij}^{\text{pair}} \hat{a}_{i\downarrow}^{\dagger} \hat{a}_{i\uparrow}^{\dagger} \hat{a}_{j\uparrow} \hat{a}_{j\downarrow}, \tag{4}$$

as well as transverse spin-exchange terms.

The class of models described by (1) with $\hat{\mathcal{H}}_{\text{rest}}=0$ have been called fermionic Ising spin-glass (FISG) models[14] and may be considered as the insulating limit of the larger class of itinerant models described by the Hamiltonian (1). Henceforth, we consider this limit.

In the past, these models have been studied using techniques of coherent fermionic states (see, for instance, Ref. 13). Our purpose here is to point out that quantum techniques are unnecessary and a classical treatment suffices.

III. MAPPING TO CLASSICAL SPIN-GLASS MODELS

For a general fermionic problem, the coherent-states representation is a powerful technique and is likely to be useful for a treatment of the full quantum Hamiltonian (1). However, for the unfamiliar, its use is likely to obscure a simplicity of the insulating case. Hence, here we proceed differently, in what we consider to be a much simpler way, for the Hamiltonian

$$\hat{\mathcal{H}} = U \sum_{i=1}^{N} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \sum_{i<j=1}^{N} \mathcal{J}_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z - \mu \sum_{i=1}^{N} \sum_{s \in \{\uparrow,\downarrow\}} \hat{n}_{is}. \tag{5}$$

First, we note that the Hamiltonian is a function of the number operator $\hat{n}_{is}$ only. We are therefore in the ideal position of knowing the eigenstates of the Hamiltonian exactly. If we define

$$|\mathbf{n}\rangle = \prod_{i=1}^{N} |n_{i\uparrow} n_{i\downarrow}\rangle_i, \tag{6}$$

then the partition function expressed in this set of states becomes

$$\mathcal{Z}(\beta) = \sum_{\mathbf{n}} e^{-\beta \mathcal{H}(\mathbf{n})}, \tag{7}$$

where $\mathcal{H}(\mathbf{n})$ is the Hamiltonian (5) with the operators now just numbers. This Hamiltonian is quite similar to such a three-state spin-glass model. In order to make this similarity more apparent, we express the partition function and the Hamiltonian as a function of the two new variables $S_i = n_{i\uparrow} - n_{i\downarrow}$ and $\tau_i = n_{i\uparrow} + n_{i\downarrow}$. After doing the trace with respect to the variables τ_i, we end up with the partition function

$$\mathcal{Z}(\beta) = \sum_{\mathbf{S}} e^{-\beta \mathcal{H}_{\text{GS}}(\mathbf{S})},$$

$$\mathcal{H}_{\text{GS}}(\mathbf{S}) = -\sum_{i<j=1}^{N} \mathcal{J}_{ij} S_i S_j - D \sum_{i=1}^{N} S_i^2 - \frac{N}{\beta} \ln(1 + e^{-\beta U + 2\beta\mu}), \tag{8}$$

with

$$D = \mu - T \ln(1 + e^{-\beta U + 2\beta\mu}), \tag{9}$$

and with notation $\mathbf{S} = (S_1, \ldots, S_N)$, $S_i \in \{0, \pm 1\}$. The Hamiltonian (8) is known in classical spin-glass literature as the Ghatak-Sherrington (GS) model,[15] a particular case of the Blume-Emery-Griffiths-Capel spin-glass model[16] without biquadratic interaction.

In the GS formulation, the new spin variables $\mathbf{S}$ do not reflect the difference between unoccupied and doubly occupied sites. It is, therefore, useful to consider the calculation of the expectation value of the fermion number operator in this formulation. By defining

$$n = \frac{1}{N} \sum_{i=1}^{N} \sum_{s=\pm 1} \langle \hat{n}_{is} \rangle_{\mathcal{H}_{\text{FISG}}}, \quad \rho = \frac{1}{N} \sum_{i=1}^{N} \langle S_i^2 \rangle_{\mathcal{H}_{\text{GS}}}, \tag{10}$$

we can then write the following relation:

$$n = \frac{2(1-\rho)}{e^{\beta(U-2\mu)} + 1} + \rho, \tag{11}$$

with $\langle \cdots \rangle_{\mathcal{H}}$ the thermal average with respect a Hamiltonian $\mathcal{H}$

$$\langle \cdots \rangle = \mathcal{Z}^{-1}(\beta) \text{Tr}\, e^{-\beta \hat{\mathcal{H}}} (\cdots), \tag{12}$$

where Tr denotes the trace. Thus, half-filling $(n=1)$ corresponds to $\mu = U/2$.

This mapping between the fermionic Ising spin-glass model (1) and the classical SG model (8) was noticed and used fruitfully in Ref. 11, but unfortunately these authors appear not to have noticed that the Green's function, and the DOS that can be derived from it, can also be obtained simply from the classical model.[21]

Instead of using the fermionic path integral definition for the DOS, let us start with the standard definition for the retarded Green's function

$$\mathcal{G}_{ij}^{ss'}(t-t') = -i\theta(t-t') \langle [\hat{a}_{is}(t), \hat{a}_{js'}^{\dagger}(t')] \rangle, \tag{13}$$

with $\{\hat{A}, \hat{B}\} \equiv \hat{A}\hat{B} + \hat{B}\hat{A}$ and the creation and annihilation operators given in the Heisenberg representation

$$\hat{a}_{is}(t) = e^{(i/\hbar)\hat{\mathcal{H}}t} \hat{a}_{is} e^{-(i/\hbar)\hat{\mathcal{H}}t}. \tag{14}$$

Using the set of states $|\mathbf{n}\rangle$, and after some standard manipulations, the retarded Green's function takes the form

$$\mathcal{G}_{ij}^{ss'}(t-t') = \delta_{i,j}\delta_{s,s'}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\mathcal{G}_i^s(\omega), \tag{15}$$

with

$$\mathcal{G}_i^s(\omega) = \mathcal{Z}^{-1}(\beta)\sum_{\mathbf{n},\mathbf{m}}|\langle\mathbf{n}|\hat{a}_{is}|\mathbf{m}\rangle|^2 \times \frac{e^{-\beta\mathcal{H}(\mathbf{n})}+e^{-\beta\mathcal{H}(\mathbf{m})}}{\omega-[\mathcal{H}(\mathbf{m})-\mathcal{H}(\mathbf{n})]/\hbar+i\delta}. \tag{16}$$

Notice that the preceding expression is fully general for any Hamiltonian system depending on the number operator only. Even though we could continue with the general calculation quite easily, at this stage we believe it to be helpful first to analyze the family of fermionic Ising spin-glass models without Hubbard interaction, which have been studied extensively with coherent-state methods.[9–12]

A. Fermion Ising spin-glass model without Hubbard interaction

Due to the presence of $|\langle\mathbf{n}|\hat{a}_{is}|\mathbf{m}\rangle|^2$ in Eq. (16), the only states that contribute have

$$\mathcal{H}(\mathbf{m})-\mathcal{H}(\mathbf{n}) = -sh_i(\mathbf{S})-\mu, \tag{17}$$

with $h_i(\mathbf{S})=\sum_{j(\neq i)}^N \mathcal{J}_{ij}S_j$, the local field at site i. Hence, after some algebra, we can write

$$\mathcal{G}_i^s(\omega) = \int\frac{dh}{\omega+(\mu+sh)/\hbar+i\delta} \times\mathcal{Z}^{-1}(\beta)\sum_{\mathbf{n}}e^{-\beta\mathcal{H}(\mathbf{n})}\delta[h-h_i(\mathbf{S})]. \tag{18}$$

Notice that all the spin dependence is in the last term. We proceed as before, changing variables $S_i=n_{i\uparrow}-n_{i\downarrow}$ and $\tau_i=n_{i\uparrow}+n_{i\downarrow}$ and tracing out the dependence on the τ's. We can then rewrite the preceding expression as

$$\mathcal{G}_i^s(\omega) = \int dh\frac{p_i^{\mathrm{GS}}(h)}{\omega+(\mu+sh)/\hbar+i\delta}, \tag{19}$$

with $p_i^{\mathrm{GS}}(h)$ the density of local fields at site i

$$p_i^{\mathrm{GS}}(h) = \langle\delta[h-h_i(\mathbf{S})]\rangle_{\mathcal{H}_{\mathrm{GS}}}. \tag{20}$$

Defining the DOS as the imaginary part of the spectral density function averaged over all sites, over spin orientation, and over the disorder, denoting the latter by an overline and using the identity $1/(x+i\delta)=\mathbf{P}(1/x)-i\pi\delta(x)$ and shifting the energy levels $\epsilon=\hbar\omega+\mu$, we obtain finally

$$\rho^{\mathrm{DOS}}(\epsilon) = -\frac{1}{2\pi N}\sum_{s=\pm1}\sum_{i=1}^N\overline{\mathrm{Im}\,\mathcal{G}_i^s(\omega)} = \frac{1}{2}\sum_{s=\pm1}p^{\mathrm{GS}}(s\epsilon), \tag{21}$$

with the definition

$$p^{\mathrm{GS}}(h) \equiv \frac{1}{N}\sum_{i=1}^N\overline{p_i^{\mathrm{GS}}(h)} \tag{22}$$

If $\mathcal{J}_0=0$, then we have that the distribution of fields is an even function, i.e., $p^{\mathrm{GS}}(\epsilon)=p^{\mathrm{GS}}(-\epsilon)$, and therefore the expression (21) reveals that the DOS in the fermionic Ising spin-glass model without Hubbard interaction is exactly the distribution of local fields in the corresponding classical Ghatak-Sherrington model for any value of the chemical potential and temperature.

B. Fermion Ising spin-glass model with Hubbard interaction

The fermionic Ising spin-glass model with Hubbard interaction was studied in Ref. 13. Again, due to the term $|\langle\mathbf{n}|\hat{a}_{is}|\mathbf{m}\rangle|^2$, in Eq. (16), we can replace

$$\mathcal{H}(\mathbf{m})-\mathcal{H}(\mathbf{n}) = Un_{i\bar{s}}-sh_i(\mathbf{S})-\mu, \tag{23}$$

to yield

$$\mathcal{G}_i^s(\omega) = \sum_{\gamma=0,1}\int\frac{dh}{\omega+(\mu+sh-\gamma U)/\hbar+i\delta} \times\mathcal{Z}^{-1}(\beta)\sum_{\mathbf{n}}\delta_{n_{i\bar{s}},\gamma}e^{-\beta\mathcal{H}(\mathbf{n})}\delta[h-h_i(\boldsymbol{\sigma})], \tag{24}$$

where $\bar{s}\equiv -s$. We proceed as before and map to the GS model. This calculation is a bit more involved but fairly straightforward and, after some algebra, we arrive at

$$\mathcal{G}_i^s(\omega) = \sum_{\gamma=0,1}\sum_{\tau=0,\pm1}a_\tau^\gamma(s)\int dh\frac{p_i^{\mathrm{GS}}(\tau,h)}{\omega+(\mu+sh-\gamma U)/\hbar+i\delta}, \tag{25}$$

where we have introduced the joint spin-field distribution at site i of the GS spin glass

$$p_i^{\mathrm{GS}}(\tau,h) = \langle\delta_{S_i,\tau}\delta[\epsilon-h_i(\mathbf{S})]\rangle_{\mathcal{H}_{\mathrm{GS}}}, \tag{26}$$

with

$$a_\tau^\gamma(s) = \delta_{\tau,0}\frac{\delta_{0,\gamma}+\delta_{1,\gamma}e^{-\beta U+2\beta\mu}}{1+e^{-\beta U+2\beta\mu}}+\delta_{0,\gamma}\delta_{\tau,s}+\delta_{1,\gamma}\delta_{\tau,\bar{s}}. \tag{27}$$

From here, we have that the DOS is given by

$$\rho^{\mathrm{DOS}}(\epsilon) = \frac{1}{2}\sum_{s=\pm1}\sum_{\gamma=0,1}\sum_{\tau=0,\pm1}a_\tau^\gamma(s)p^{\mathrm{GS}}[\tau,s(\gamma U-\epsilon)], \tag{28}$$

with

$$p^{\mathrm{GS}}(\tau,h) \equiv \frac{1}{N}\sum_{i=1}^N\overline{p_i^{\mathrm{GS}}(\tau,h)} \tag{29}$$

In this case, we have again an intimate relationship between the DOS in the fermonic Ising spin-glass model and the joint spin-field distribution of the classical GS spin-glass model.

IV. PHYSICAL MAPPING OF DOS TO FIELD DISTRIBUTIONS

Complementary to the formal mathematical mappings discussed earlier, in this section we describe how the preceding connection between the classical and quantum systems also appears naturally, based solely on physical arguments. For the sake of simplicity, we restrict the discussion to zero temperature and $\mathcal{J}_0=0$.

Let us consider initially that $U=0$ and $\mu=0$, i.e., half-filling. In this case, the number of fermions N_{f} is equal to the number of sites N_{sites}. It is a reasonable ansatz, which we shall later show to be true, that in the ground state every site will carry a single fermion, whose spin can be either up or down. It is immediately clear that this is nothing but the usual classical SK model. Consequently, the ground state of the fermionic Ising spin-glass model is the same as that of the classical SK model, and the DOS of the former system is simply

$$\rho^{\mathrm{DOS}}(\epsilon) = p^{\mathrm{SK}}(-|\epsilon|), \tag{30}$$

where

$$p^{\mathrm{SK}}(h) = \frac{1}{N}\sum_{i=1}^{N}\left\langle \delta\left(h - \left|\sum_j \mathcal{J}_{ij}\sigma_j\right|\right)\right\rangle_{\mathcal{H}_{\mathrm{SK}}}. \tag{31}$$

Next, let us consider further the ansatz that each site is singly occupied. Were a site to be unoccupied, then clearly it would contribute no energy to the ground state. Neither would a doubly occupied site, since the two spins would both see the same effective field due to the other spins and they would contribute cancelling energies. However, we also know that in the ground state of the SK model all local fields are finite and spins are oriented to yield negative energies. It follows that removing a fermion from one singly occupied site and depositing it on another already favorably singly occupied site incurs two energetic penalties. Furthermore, since the distribution of local fields in the SK model goes to zero at zero field and any single-spin-coupling strength scales as N^{-1}, the loss cannot be compensated by further readjustments on other sites.

For $\mu\neq 0$, account must be taken of the fact that in the fermionic model, some sites must be unoccupied for $\mu<0$ or doubly occupied for $\mu>0$. In both cases, the system behaves energetically as though it were a diluted classical SK model with spins absent on the sites of either zero or double occupancy in the fermionic model. Furthermore, the location of these holes is chosen so as to minimize the total ground-state energy, i.e., the system behaves as though one has an effective Hamiltonian

$$\mathcal{H}(\boldsymbol{\sigma},\mathbf{n}) = -\sum_{i<j=1}^{N}\mathcal{J}_{ij}\sigma_i\sigma_j n_i n_j - \tilde{\mu}\sum_{i=1}^{N} n_i,$$

$$\sigma = \pm 1, \quad n = 0,1 \tag{32}$$

with two types of annealed variables, Ising spins (characterised by the σ) and "quasiparticles" (characterized by the n_i and not to be confused with the real fermions of number operator $\hat{n}_{is}$). We shall refer to this system as the *anneal-diluted SK model* (ADSK). The two chemical potentials, of Eqs. (32) and (1), are related by

$$\tilde{\mu} = -|\mu|. \tag{33}$$

The DOS is given by

$$\rho^{\mathrm{DOS}}(\epsilon) = p^{\mathrm{ADSK}}(-|\epsilon|), \quad |\epsilon| > |\tilde{\mu}|, \tag{34}$$

where p^{ADSK} is defined analogously to p^{SK} but with the sum over only the singly occupied sites and averaged over the ADSK Hamiltonian (32).

It is tempting to think that the truncation of site occupation might modify the DOS of the $\mu=0$ case by simply moving the Fermi level of the $\rho^{\mathrm{DOS}}(\epsilon)$ corresponding to the pure SK model so as to occupy only the lowest states up to $\tilde{\mu}$, but this does not take account of the loss of contribution to the fields of the unoccupied sites of (32). In fact, computer studies of the Thouless-Anderson-Palmer (TAP) equations have shown that $\rho^{\mathrm{DOS}}(\epsilon)$ now goes to zero at $\epsilon=\tilde{\mu}$, in a manner at least qualitatively similar to what happens at $\epsilon=0$ for the case of $\mu=0$ (Ref. 10). A replica symmetric analysis of ρ close to $\epsilon=\tilde{\mu}$ also behaves analogously to the corresponding replica symmetric study for the undiluted SK model near $\epsilon=0$; the full replica symmetry breaking calculation has not yet been done explicitly.

For $\rho^{\mathrm{DOS}}(\epsilon)$ with $|\epsilon|<|\tilde{\mu}|$, it is necessary to calculate the local field distribution $h_i=\Sigma_j\mathcal{J}_{ij}\sigma_j$ at sites of (32) where there is no quasiparticle so that such sites do not contribute to the total energy or the field or spin orientation at other sites. With this extension, (34) applies for all ϵ.

For $U>0$, the mapping of (32) continues to apply with $\tilde{\mu}$ appropriately chosen. Again, if $\mu<U/2$, there are $(N_{\mathrm{site}}-N_{\mathrm{f}})$ unoccupied sites, and for $\mu>U/2$, there are $(N_{\mathrm{f}}-N_{\mathrm{site}})$ doubly occupied fermion sites. Consequently, in both cases, there are $|(N_{\mathrm{f}}-N_{\mathrm{site}})|$ sites without quasiparticles. $\tilde{\mu}$ is given by (Ref. 22)

$$\tilde{\mu} = \begin{cases} \mu, & \mu < U/2 \\ U-\mu, & \mu > U/2 \end{cases}, \tag{35}$$

and $\rho^{\mathrm{DOS}}(\epsilon)$ is given by

$$\rho^{\mathrm{DOS}}(\epsilon) = \begin{cases} p^{\mathrm{ADSK}}(-|\epsilon|), & \epsilon<0 \\ 0, & 0<\epsilon<U \\ p^{\mathrm{ADSK}}(-|\epsilon-U|), & \epsilon>0. \end{cases} \tag{36}$$

These mappings may be related to those of the preceding section by noting that Hamiltonian (32) is also another way of writing the GS model of (8) with $\tilde{\mu}=D-T\ln 2$.[16]

V. EXISTENCE OF A PSEUDOGAP IN FERMIONIC ISING SPIN GLASSES

Having demonstrated the mapping between DOS and the distribution of fields, we can draw some conclusions and speculate about the nature of the pseudogap in the DOS at the Fermi energy.[13] First, let us notice that the nature of the pseudogap and of the strong corrections of different steps of replica symmetry breaking can only be of classical origin. The strong corrections to the DOS from RSB corrections found in fermionic Ising spin-glass models are common in classical spin glasses.

In particular, it has been shown for some time from ∞–RSB calculations[17,18] and Thouless-Anderson-Palmer equations[19] that the field distribution at zero temperature in the SK model vanishes with $h \to 0$ as $p^{SK}(h)=a|h|$ (See also Ref. 20). This, therefore, predicts the existence of a pseudogap for the fermionic Ising-spin glass model without a Hubbard term and at half-filling.[14]

The distribution of fields for fermionic SK models without Hubbard interaction for $\mu \neq 0$ was studied numerically using a TAP approach in Ref. 10 for $\mu > 0$, indicating that it seems to vanish, again quasilinearly, at $h = \pm\mu$, implying that the DOS also presents pseudogaps.

We might note also that the field distribution is very temperature dependent and the pseudogap becomes filled in as the temperature rises. Therefore (and unusually), the density of fermionic states will mirror this strong temperature dependence.

VI. CONCLUSIONS

In this paper, we have shown that the fermionic Ising spin-glass model (with SK-like interactions) is mappable to the classical GS spin-glass model not only at the level of the free energy but also at the DOS, the latter being given exactly by the local field distribution of the GS model in the case without Hubbard interaction and obtainable from it when a Hubbard term is present. By using known results from spin-glass models, we can show the existence of a pseudogap, from full RSB and TAP approaches. It should be noted that the pseudogap and strong corrections in the different steps of RSB are purely classical effects and not due to quantum fluctuations. It would be interesting to see how this picture changes when the Hamiltonian $\hat{\mathcal{H}}_{\mathrm{rest}}$ is switched on and also when one passes to a more realistic (but also more difficult to solve and controversial) model with short-range interactions.

We have concentrated on single-fermion Green's functions and their averages. A similar procedure to that outlined in Sec. III can be applied to higher-order Green's functions, mapping into higher-order field distributions, and for averages of products of Green's functions.

Finally, we note that the GS model has a first-order phase transition at a critical negative-valued $D_c(T)$ and beneath a tricritical temperature T_3, between a magnetic state ($|\langle S_i\rangle| \neq 0$) and a nonmagnetic ($|\langle S_i\rangle|=0$) solution. This reflects in the FISG to a critical μ_c for magnetic breakdown.[14,23]

ACKNOWLEDGMENTS

The authors thank R. Oppermann for informative discussions. I.P.C. also acknowledges R. Jack, B. Doyon, J. Chalker, and F. Essler for discussions. This work was supported by EPSRC under Grant No. GR/R83712/01.

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[20] R. Oppermann and D. Sherrington (unpublished).
[21] In later work (Ref. 13), one of these authors, together with one of the present authors, has demonstrated that within a replica treatment of the coherent-state formulation, the FISG requires only static terms, and examination shows their consequence to be the same as would be obtained from a replica treatment of the classical model, but this point was not made explicitly, nor was the correspondence even before replication that we demonstrate in the present paper.
[22] Note that $\tilde{\mu}$ varies continuously with N_f, always nonpositive and with $|\tilde{\mu}|$ monotonically increasing with $|(N_f - N_{\mathrm{site}})|$, whereas μ jumps discontinuously as $(N_f - N_{\mathrm{site}})$ increases past 0 for $U \neq 0$.
[23] More precisely, it leads to two critical μ_c, one positive and one negative.