Edwards-Anderson spin glasses undergo simple cumulative aging

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We study and discuss rejuvenation and memory (numerical) experiments in Ising and Heisenberg three and four dimensional spin glasses. We introduce a quantitative procedure to analyze the results of temperature cycling experiments. We also run, compare, and discuss "twin" couples of experiments. We find that in our systems aging is always cumulative in nature, and rejuvenation and memory effects are also cumulative: they are very different from the ones observed in experiments on spin glass materials.

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Rejuvenation and memory effects (in the following we will denote by RME) are maybe the most striking features of spin glass materials, and their experimental evidence is very clear (see for example the discussion in Ref. 1 and references therein). The simplest experiment showing both rejuvenation and memory in a spin glass is the cycle in temperature, which is based on the following three steps:

(1) one lets a spin glass sample relaxing for a time t_1 at temperature T_1 in the low T phase;

(2) then one brings it to a temperature $T_2 < T_1$, where it relaxes for a time t_2;

(3) finally one heats it back to T_1 where relaxation continues.

The relevant experimental observations are mainly the following:
independently from the amount of time spent at T_1, when the sample is cooled to T_2 the relaxation process restarts completely *(rejuvenation)*;
when the sample is heated back to T_1 it seems to remember what happened during time t_1 and relaxation continues as if the second step was absent *(memory)*.

We have given here a very simplified description, but it is sufficient for our goal, that is to compare the situation to the results of numerical simulations. Actual experiments show a number of different and subtle effects; we address the interested reader to the experimental results of Ref. 1 (and references therein).

RME effects are poorly understood from the theoretical point of view: for example, it is still unclear which are the length scales that are relevant in such processes. Unfortunately length scales cannot be measured directly in experiments, and numerical simulations could be of great help in this context. It is not clear, if real RME (of the same nature of the ones observed in experiments) appear in numerical simulation of finite dimensional Edwards–Anderson (EA) models (with either Ising or Heisenberg spins). In this note we clarify this point. We use a phenomenological approach: rather than trying to interpret numerical data within a specific theory in order to validate it we focus on the comparison of numerical and experimental data. Our aim is to check whether RME, as observed in physical experiments, are also present in the EA model. In order to reach conclusions as general as possible, we consider EA models with different spin types (Ising and Heisenberg) and in $D=3$ and $D=4$.

Let us start with a brief review of RME as observed (or not) in numerical simulations of the EA model. A few years ago the first works on this issue[2,3] discussed numerical studies of such effects in the 3D Ising EA model with Gaussian couplings. Unfortunately the lack of a quantitative method for estimating RME brought the authors of the these two studies to give different interpretations of the outcome of a "cooling and stop" experiment. While Ref. 2 states that "the model exhibits the rejuvenation-like and memory effects within a time window of the present simulation," Ref. 3 says that "the model does not show, on the time scales we have access to, the strong RME real spin glasses show" (the time scales of the two numerical experiments are of the same order of magnitude, and the spatial volumes of the two systems are comparable).

A couple of years later, two further numerical works on this issue[4,5] reach again opposite conclusions. Berthier and Bouchaud[4] interpret their data for the 4D Ising EA model as showing strong RME. They also suggest that in 3D such effects are difficult to observe because the spatial correlation function does not change enough when varying the temperature. On the contrary Takayama and Hukushima[5] find the signature of a *cumulative aging* scenario for small $\Delta T \equiv T_1 - T_2$. The cumulative aging scenario assumes that, as long as the system is in a spin glass phase, temperature changes do not induce a restart of aging, so that effects of relaxations at different temperatures cumulate.

In a recent paper Jimenez *et al.*[6] find again RME in both 3D and 4D Ising EA models, although reduced with respect to previous studies.

Given such a confusing situation and such a number of different numerical results, we have decided to make very precise measurements with temperature cycle experiments in the EA model in order to try to answer the following three questions:

(1) Are true RME present in the EA model or is aging cumulative in nature?

(2) If we observe cumulative aging, can we try to understand if true RME can be recovered in the limit of very large (relaxing and probing) time scales, i.e., in the limit relevant for experiments?

(3) How much these effects depend on space dimension and spin type?

We consider the EA model with Gaussian couplings and both Ising spins (in 3D, I3D, and 4D, I4D) and Heisenberg spins (in 3D, H3D). Typical sizes used are $L=40$ for I3D, $L=20$ for I4D and $L=60$ for H3D. We have checked that our lattices are large enough to avoid any detectable finite size effect; in particular, the choice of a large size for H3D samples is due to the very large length scales involved in the dynamics of Heisenberg spin glasses.[7] We have computed the disorder averages by using 16 samples for H3D, 176 samples for I4D and 256 samples for I3D. The dynamics of Ising spins models has been based on the popular single spin-flip Metropolis algorithm. For Heisenberg spins we use again Metropolis updates but when changing temperature we fix the acceptance ratio, so the amplitude of the trial updates depends on the temperature: in this way we reproduce at best the physical dynamics. Most of the numerical simulations of I3D were performed on the APEmille parallel computer,[8] while I4D and H3D were simulated on a PC cluster.

The choice for T-cycle experiments, which are in principle more complicated than T-shift experiments, is dictated by two main reasons. First, T-cycle experiments allow for the study of both rejuvenation and memory at the same time. Second, the first part of any relaxation process may be affected by large finite time corrections. Thus it is important that, when extracting the effective age of the system (see below), one compares relaxation processes where the initial steps are performed at the same temperature. In other words the effective age must be measured deep into the aging regime and not in the initial part of the relaxation process.

In order to simplify the analysis, especially when taking the large time limit, we introduce in our experimental procedure only one single relevant time scale t_p, that corresponds to the period of the measuring field used in real experiments. t_p is the number of MC steps on which we average data: in other words we divide the total number of MC steps in groups of t_p steps over which we compute expectation values. We use t_p also as the time distance for computing time dependent correlations over spin configurations. We perform different runs for each fixed value of $t_p=200$, 500, 1000, 2000, 5000 (I3D), $t_p=100$, 1000 (I4D, H3D). All the other time scales will be proportional to t_p. In particular, if t_i is the time spent in the phase i of the experiment, we fix $t_1=t_2=t_3=20t_p$.

Our first aim is to define properly an effective time t_{eff}, such that after a T-cycle (i.e., t_1 steps at T_1, t_2, at T_2 and t_3 at temperature T_1 again) the system is in the same state as if it was let relaxing isothermally at T_1 during the time $t_1+t_{\text{eff}}+t_3$. Because of possible transient effects (just after restoring temperature T_1) one should avoid to use small values of t_3.

Checking that two systems are statistically equivalent is not easy. We have done that by comparing a number of observable quantities and checking whether their values coincide in our statistical accuracy. We have considered both one time and two time quantities. As one time quantities we look at the Edwards-Anderson overlap order parameter q_{EA} and the spatial correlation function $G(x,t)$:

$$q_{\text{EA}}(t) \equiv \frac{1}{N}\sum_{i=1}^{N} \overline{m_i(t) \cdot m_i(t)}, \tag{1}$$

$$G(x,t) \equiv \frac{1}{zN}\sum_{\|i-j\|=x} \overline{[S_i^a(t) \cdot S_j^a(t)][S_i^b(t) \cdot S_j^b(t)]}, \tag{2}$$

where $\overline{\cdot}$ denotes the average over the quenched disordered couplings and thermal histories, z is the coordination number of the simple cubic D-dimensional lattice, $N=L^{\text{D}}$, and a, b are real replica indexes. The basic fields of the theory take values $S_i=\pm 1$ variables for the Ising spin glasses (I3D, I4D), while are vectors on a sphere of unitary radius in the case of the Heisenberg spin glass (H3D). We define the corresponding time-integrated magnetization (over the time t_p) as

$$m_i(t) \equiv \frac{1}{t_p}\sum_{\tau=t-t_p+1}^{t} S_i(\tau). \tag{3}$$

We have also measured and used some two time quantities: the in-phase and out-of-phase susceptibilities. Provided that we are in the quasiequilibrium regime (so that FDT holds) they can be estimated via the spin autocorrelation function:

$$C(t,t') \equiv \frac{1}{N}\sum_{i=1}^{N} \overline{S_i(t) \cdot S_i(t')}. \tag{4}$$

In order to improve the signal-to-noise ratio we have integrated the above autocorrelation function over short times, $\tau < t_p$, and defined

$$\tilde{C}(t,t') \equiv \frac{1}{N}\sum_{i=1}^{N} \overline{m_i(t) \cdot m_i(t')}. \tag{5}$$

The in-phase and out-of-phase susceptibilities are then expressed as a function of this time-integrated correlation function

$$\tilde{\chi}'(t,t+t_p) \equiv \frac{\tilde{C}(t,t) - \tilde{C}(t,t+t_p)}{T}, \tag{6}$$

$$\tilde{\chi}''(t,t+t_p) \equiv \frac{1}{T}[\tilde{C}(t,t) + \tilde{C}(t-t_p,t+t_p) - \tilde{C}(t-t_p,t) - \tilde{C}(t,t+t_p)]. \tag{7}$$

Since $\tilde{\chi}''$ showed too small excursions upon T changes, we preferred to use the out-of-phase susceptibility defined via the spin–spin autocorrelation (4)

$$\chi''(t,t+t_p) \equiv \frac{1}{T}[1 + C(t-t_p,t+t_p) - C(t-t_p,t) - C(t,t+t_p)]. \tag{8}$$

Notice that relations (7) and (8) hold apart from an overall multiplicative factor.[2]

For each period t_p we measure one-time quantities only at the end of period, and we integrate measurements of two-time quantities over the period, in close analogy with real experiments. This observation can be relevant since the only data which have been interpreted as a rejuvenation effect in Ref. 4 have been measured before the end of the first period after the T shift: this is not done in real experiments. A possibility that we consider plausible is indeed that the RME showed in (Ref. 4) are not related to the experimental RME effects, but to the fact that right after the temperature change the system is (for a very short time) strongly out of equilibrium. In such a situation the response measured in (Ref. 4) with the expression $\chi(t) \equiv [1-C(t,t+t_p)]/T$ may overesti-

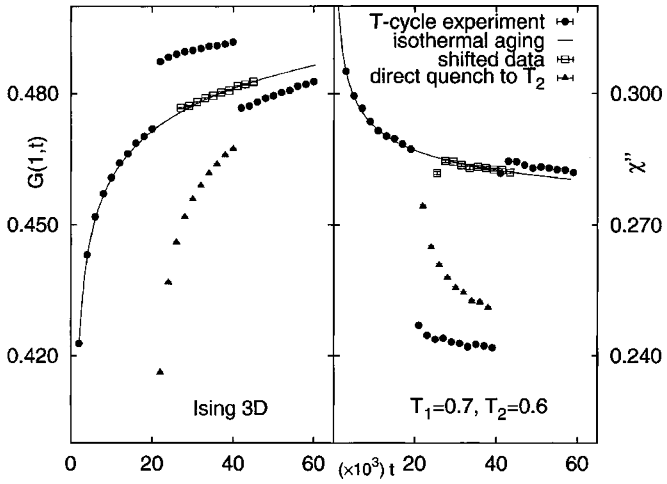

FIG. 1. Spatial correlation function at distance one (that coincides with the link overlap) and out-of-phase susceptibility in a temperature cycle of I3D. The time evolution of $G(1,t)$ and χ'' are compared with the evolution coming from a direct quench at T_2. The continuous line is a fit on data from a long isothermal run at T_1.

mate the true susceptibility, giving rise to a signal which looks like a stronger rejuvenation. A deeper analysis of this artifact has been done recently in (Ref. 6).

Figures 1 and 2 show how our method for estimating t_{eff} works. In Fig. 1 we show $G(1)$ and χ'' for I3D in a $T_1=0.7$, $T_2=0.6$ cycle (remember that here $T_c\simeq 0.95$), while in Fig. 2 we show $G(1)$ and χ' for I4D in a $T_1=1.3$, $T_2=0.9$ cycle (here $T_c\simeq 1.8$). For clarity only half of the data points are presented in the figures. In each plot we show raw data measured during the T-cycle (●). Isothermal aging data, from very long $(300t_p)$ simulations at fixed $T=T_1$, are fitted on a simple smooth function $f(t)$ (that fits perfectly the data and is only used as a book keeping device for the matching procedure) that we represent with a solid line. t_{eff} is calculated by shifting horizontally $f(t)$ to fit the data from the third stage of the T cycle, and adding the needed time shift t_s to the time t_2. t_s enters the procedure as a fitting parameter, allowing a fully automatized estimation of $t_{\text{eff}}=20t_p+t_s$, so we do not introduce any systematic error due to human perception of collapsing goodness.

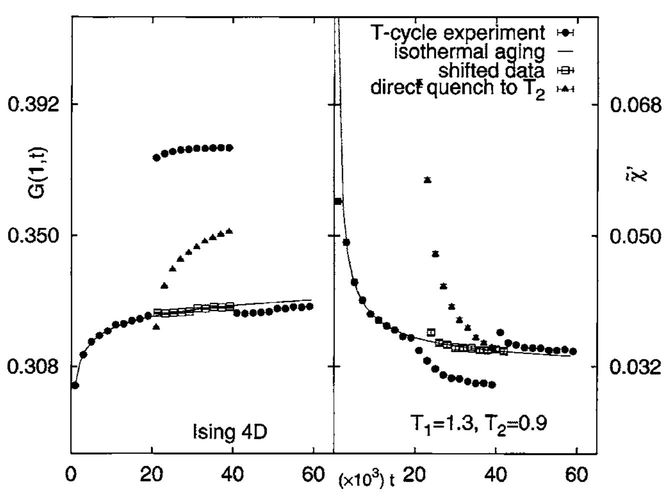

FIG. 2. As in Fig. 1 for I4D, but $\tilde{\chi}'$ instead of χ''.

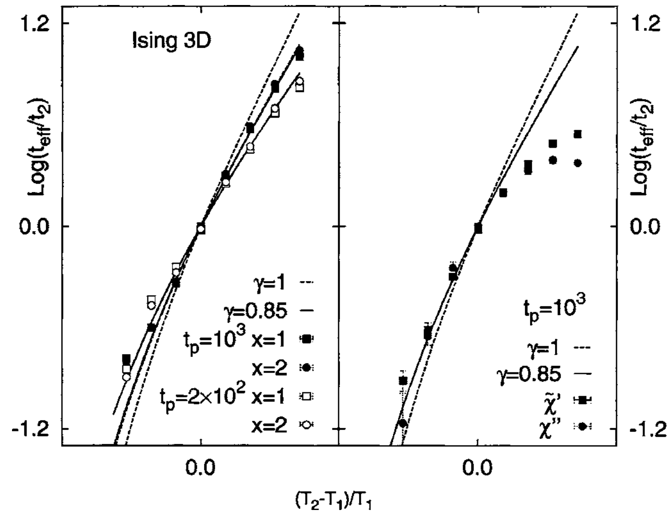

FIG. 3. Comparison between the ratio $t_{\text{eff}}/(t_2)$ and the cumulative hypothesis, for I3D. Measurements of $t_{\text{eff}}/(t_2)$ are from cycle measurements of $G(x,t)$ with $x=1$, 2 (left frame) and of the susceptibilities (right frame). In the left frame filled symbols are from measurements in cycles with $t_p=1000$, while empty symbols come from cycles with $t_p=200$. These plots are representative of the behavior of one-time and two-times quantities. Full and dashed lines are the theoretical prediction in the cumulative hypothesis with $\gamma=0.85$ and $\gamma=1$, respectively, (on the right part of the plot higher lines have a larger t_p value).

In these plots we do not see a real and complete rejuvenation as in experiments, where the susceptibility decays in the second stage as if the first stage was absent (at least for a large ΔT as the one we are using here). This is clear especially if we compare the second part of the T cycle with a direct quench at T_2 (▲). The authors of Ref. 4 suggested that in real experiments even the fastest quench always corresponds to a cooling, so that the starting configuration of any relaxation process is never completely random. In order to check how much this fact could affect our hypothesis that the relaxation at T_2 strongly depends on the time spent at T_1, we have computed a new direct quench curve, starting this time not from a completely random configuration $(T=\infty)$, but from a configuration thermalized at temperature $T=2T_c$. Again we find substantial differences in the observables decays between these *softer* direct quenches and the relaxation in the second stages of the T cycles: the two decays are not the same, and the discrepancy is not too different than the one from a direct quench (see Figs. 1 and 2). This shows that even starting from a slightly correlated spin configuration, that is what could be happening in real experiments, we do not recover the behavior of the temperature cycle.

Having estimated t_{eff} for different values of T_2 we summarize our results in Fig. 3. As already noticed for example in Ref. 3 (see also Refs. 4 and 5), positive ΔT cycles do not reinitialize aging as in real experiments. On the contrary the time spent at $T_2>T_1$ strongly increases the relaxation rate: for $\Delta T>0$, t_{eff} is larger than t_2. Full and dashed lines in Fig. 3 correspond to predictions obtained in a fully cumulative

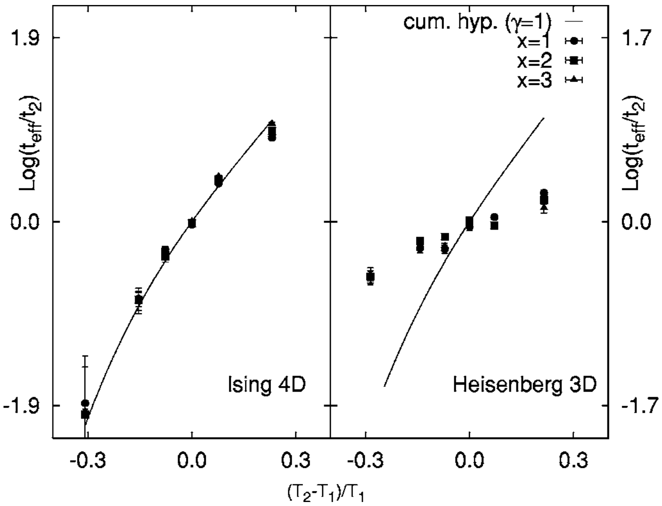

FIG. 4. As in Fig. 3, but for I4D and H3D. Values are extracted from measurements of $G(x,t)$, $x=1$, 2, 3, $t_p=1000$.

aging scenario (see below). t_{eff} values are very far from experimental observations, which predict $t_{\text{eff}}=0$ for $\Delta T<0$ and $t_{\text{eff}}=-t_1$ for $\Delta T>0$. Moreover the experimental behavior does not seem to be approached when we let the simulation time scales grow. We show in the left frame of Fig. 3 the data for t_{eff} obtained with $t_p=1000$ and $t_p=200$. Both of them are well described by the cumulative hypothesis: this suggests that the cumulative aging scenario remains valid for very long ages of the system. In other words this "cumulative" behavior does not change when changing the total duration of the experiment by modifying the value of t_p. In the right frame of the same figure we show the same measurements obtained from two-times observables. They do not appear to be in good agreement with the cumulative hypothesis for $T_2>T_1$ where our data points flatten: relaxation of these two time observables is much flatter in the third stage of the cycle (especially for large positive T shifts) and the fitting procedure to estimate t_{eff} is affected by a very large incertitude.

Let us discuss the way we have obtained the analytical predictions shown in Fig. 3. We assume that the off-equilibrium correlation length grows as $\xi_T(t)$ (a T dependent functional dependence over time). In this case the cumulative aging prediction for t_{eff} is that:

$$\xi_{T_1}(t_1+t_{\text{eff}})=\xi_{T_2}\{\xi_{T_2}^{-1}[\xi_{T_1}(t_1)]+t_2\}. \tag{9}$$

The correlation length in Ising EA models is believed to grow as[9] $\xi_T(t)\propto t^A$, with $A=aT$ (with a $\sim$0.17 in 3D). Using this functional dependence we obtain the dashed line in Fig. 3. We also explore the possibility of a more general dependence[10] by assuming that $A=aT^{\gamma}$ (the usual dependence assumes $\gamma=1$). The best fit to new high-precision data[10] gives $\gamma=0.85\pm0.04$. With $\gamma=0.85$ the prediction for t_{eff} becomes the one plotted with a full line in Fig. 3, which is a much better interpolation of the numerical data.

In Fig. 4 we present the analogous data for I4D and H3D, and we compare them with a cumulative hypothesis based on $\gamma=1$ (this is only to allow to compare to a scenario where ξ grows as a power of the time: at least for the Heisenberg case we have no precise hints about a given rate of growth, so that the fact that the solid curve does not fall on the numerical data cannot be seen as a "discrepancy"). Data for H3D show a very weak dependence of t_{eff} [and of $\xi(t)$] on T, which deserves (and is undergoing) deeper investigations.[10]

The authors of Ref. 5 find cumulative aging only for small ΔT, while for $\Delta T\geq 0.3$ their data are incompatible with the cumulative aging scenario. This incompatibility shows up as an asymmetry in the laws for transforming times from T_1 to T_2 and that from T_2 to T_1. In the cumulative aging scenario these two functions should be one the inverse of the other, while in Ref. 5 they are shown not to be so. This discrepancy could be due to the fact that T-shift experiments of Ref. 5 also take into account the very first part of the relaxation after the initial quench, which is typically plagued by finite time effects. We believe that in order to avoid this kind of problems any measurement should be taken late enough after the initial quench, in such a way that the system has already entered the asymptotic aging regime (and in any case all the region of very large ΔT is bound to be affected by non-universal effects, very resilient to a clean theoretical analysis).

In order to investigate this potential problem we have repeated the "twin-experiments" of Refs. 5 and 11. They are based on four stages: the first stage (t_1 steps at T_1) is the same in both twin experiments, and it is only used to bring the system in the asymptotic aging regime (in this stage there are no measurements). In the following two stages the two experiments are complementary: one consists of t_2 steps at T_2 and then t_3 steps at $T_3=T_1$, while the other goes first with t_3' steps at $T_3=T_1$ and then t_2' steps at T_2. In the fourth and last stage both experiments are run at the same temperature $T_4=T_1$.

Assuming the validity of the cumulative aging hypothesis, it is not difficult to choose times t_1, t_2, t_2', t_3, and t_3' at fixed temperatures T_1 and T_2 such that the correlation length takes the same value at the end of the complementary stages (the second and the third ones). If the cumulative aging hypothesis is correct, one should observe that in the fourth stage measurements from the twin experiments coincide. We show in Fig. 5 the results of measurements of $G(1)$ from twin experiments of I3D with $T_1=0.7$, $T_2=0.4$: stage durations are marked by dotted vertical lines. In the fourth stage measurements of $G(x)$ turn out to coincide (in our statistical accuracy) even for large values of x: the structures built by the two system undergoing different histories are, as far as we can check, equivalent.

We believe that we have been able to give *quantitative* evidence that aging in finite dimensional spin glasses is *cumulative* in nature. It is clear that, as always in numerical simulations, our statements are valid in the limit of, among others, the time scales we are able to investigate (that are far shorter than the experimental ones). Still, our search for some potential asymptotic restoration of true, experimental-like RME has failed. Our findings concern both Ising and Heisenberg systems, both in 3D and in 4D: in our time windows the behavior is not substantially affected under a

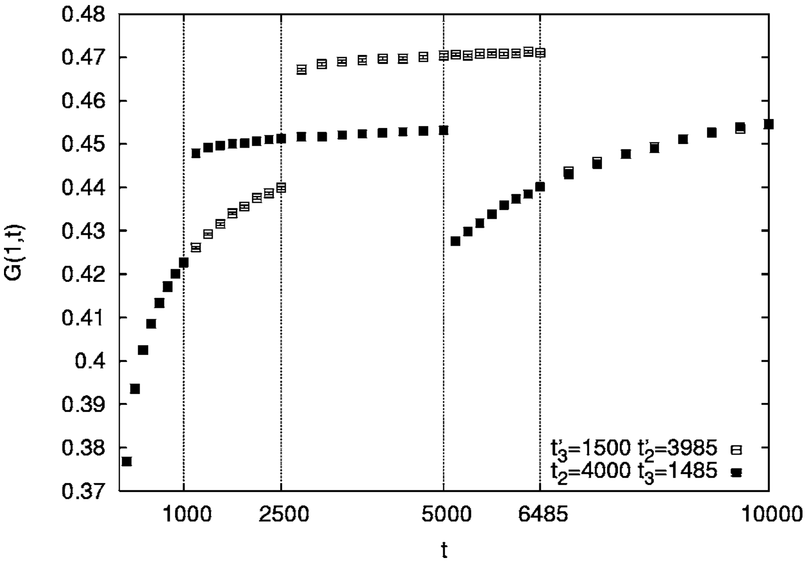

FIG. 5. Twin $T-$cycle experiment ($T_1=0.7$, $T_2=0.4$) of I3D, based on measurements of $G(1,t)$. Stage durations were predicted using a power law cumulative hypothesis and imposing the equivalence of the correlation length at the end of the third stage.

sizable change of time window, even if we should not forget that we are still very far from the experimental time scales.

It is clear that further studies are required. There is a clear mismatch with experimental data, where a nontrivial aging is observed.

ACKNOWLEDGMENTS

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