# Sound generation by the vortex flow in type-II superconductors

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Interaction of the magnetic flux flow with the crystal lattice in type-II superconductors is studied. Expression for the elastic stress is derived and previous works on the subject are critically analyzed. The power and spectrum of the acoustic waves generated by the moving flux lattice is computed. Our results, while qualitatively and quantitatively different from the results of Ivlev *et al.* [Phys. Rev. B **60**, 12419 (1999)], confirm their prediction that generation of ultrasound by the moving vortex lattice can be detected in experiment.

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## I. INTRODUCTION

The problem of ultrasound generation due to time-dependent effects in superconductors is rather old. Quantum generation of incoherent phonons at the gap frequency was reported by Eisenmenger and Dayem as early as in 1967.[1] In the same year, electromagnetic excitation of transverse microwave phonons in a superconducting thin film was reported by Abeles.[2] Generation of sound in Josephson junctions by Josephson oscillations due to phonon-assisted tunneling mechanism was studied theoretically by Ivanchenko and Medvedev[3] and subsequently observed by Kinder[4] (see also Refs. 5–7).

A more recent wave of activity in this field was inspired by experiments of Haneda and Ishiguro.[8] They demonstrated electromagnetic generation of acoustic waves by ac magnetic field in a mixed state of a high-temperature superconductor that occurs in the region of the phase diagram where pinning of vortices is important. Theoretically, the interaction of vortices with ultrasound and the generation of acoustic waves by vortices in the presence of pinning have been discussed by Domínguez *et al.*[9–11] and by Sonin.[12] There also exists another angle to sound generation in high-temperature superconductors. Very generally, an object moving through a substance at a supersonic speed can generate sound waves in that substance due to the possibility of fulfilling conditions for energy and momentum conservation. In high-temperature superconductors with large critical currents, such objects can be Abrikosov vortices driven by the Lorentz force of a large transport current.[13]

In this paper we consider electromagnetic generation of sound by a moving vortex lattice in the flux flow regime with negligible pinning. In the case of intrinsic Josephson junctions this mechanism was shown to be responsible for the generation of optical phonons by Josepshon oscillations in the absence of Josephson vortices.[14] Our approach to the generation of sound by vortices is based upon the following idea. A superconducting transport current perpendicular to the Abrikosov flux lines exerts the Lorentz force on the flux lattice. When this force exceeds the pinning force, the vortex lattice begins to drift in the direction perpendicular to the current and to the external magnetic field. The motion occurs at a speed at which the Lorentz force equals the friction force acting on normal electrons inside the vortex cores. For a static vortex lattice, the magnetic induction inside the superconductor is modulated in space due to the supercurrent flowing around vortex centers. When the vortex lattice moves, the magnetic induction becomes modulated both in space and in time. The time oscillations of the magnetic field at any given point in space are characterized by the "washboard" harmonics of frequency,

$$\omega_{wb}(\mathbf{g}) = \mathbf{v} \cdot \mathbf{g}, \tag{1}$$

where $\mathbf{v}$ is the velocity of the vortex lattice and $\mathbf{g}$ is a reciprocal vortex lattice vector.[15] In accordance with the Maxwell equation

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

the flow of a nonuniform magnetic flux through a superconductor generates spatially modulated time-dependent electric field. The action of this field on ions results in the time-dependent local elastic stress. For the generation of sound the frequency, $\omega$, and the wave vector, $\mathbf{k}$, of phonons should satisfy

$$\omega = sk = \omega_{wb}, \quad \mathbf{k} = \mathbf{g}, \tag{3}$$

which requires $v > s$, where $s$ is the speed of sound. Such a possibility exists in, e.g., clean copper oxides and borocarbides where the velocity of the vortex lattice can exceed the speed of sound for transport currents that are still small compared with the depairing critical current (see Sec. V).

The Cherenkov radiation of sound by a supersonic vortex lattice was first suggested by Ivlev *et al.*[13] They computed the electric field and the corresponding elastic strain produced by a single moving vortex without the use of the Maxwell equation (2). Individual effects of many vortices were then summed up over the vortex lattice. Deriving the electric field of a single vortex, the authors of Ref. 13 neglected screening of the electric field by the superconducting electrons. As a result, they missed the fact that the uniformity of the magnetic field, $\mathbf{B}(\mathbf{r})$, in a type-II superconductor goes up with the increasing magnitude of the field. Consequently, the electric field generated, through Eq. (2), by a moving vortex lattice goes down with the increasing density of vortices (as well as with increasing $g$). It becomes exactly zero at $B$

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$=H_{c2}$. Ignoring this effect leads to the overestimation of the radiated acoustic power. Contrary to the approach of Ivlev *et al.* we compute the electric field directly from the expression for the magnetic field of the moving vortex lattice. Our approach accounts for the screening and it gives the correct expression for the spectral power of the ultrasound. We demonstrate that the condition $v>s$ can be achieved in YBCO and borocarbides, where the power of the sound generated by a moving vortex lattice can be as high as 0.25 W/cm$^3$.

The electric field of the moving flux lattice will be considered in Sec. II. The power and the spectrum of sound generated by moving square and triangular lattices will be studied in Sec. III. The effect of the sound generation on the voltage-current characteristics of the superconductor will be considered in Sec. IV. The analysis of the results and consequences for experiment will be discussed in Sec. V.

## II. ELECTRIC FIELD OF A MOVING FLUX LATTICE

We consider the interval of the magnetic field $H_{c1} \ll B \ll H_{c2}$, with $H_{c1}$ and $H_{c2}$ being the first and second critical fields respectively. The static magnetic field in a mixed state of a type-II superconductor satisfies[16]

$$\nabla \times \nabla \times \mathbf{B} + \frac{1}{\lambda^2}\mathbf{B} = \frac{\Phi_0}{\lambda}\sum_i \delta(\mathbf{r}-\mathbf{r}_i), \tag{4}$$

where λ is the London penetration length in the plane perpendicular to the field, $\Phi_0$ is the flux quantum, and $\mathbf{r}_i$ is the position of the center of the $i$th vortex in the vortex lattice. We shall assume that the field is along the $z$ axis and that the vortex lattice is moving along the $x$ axis at a speed $\mathbf{v}=v\mathbf{e}_x$, so that

$$\mathbf{r}_i(t) = \mathbf{r}_i(0) + vt. \tag{5}$$

In the Fourier representation

$$B_z(\mathbf{k},\omega) = \sum_{\mathbf{g}} \frac{(2\pi)^4 B_0}{1+\lambda^2 g^2}\delta(\omega - v g_x)\delta(\mathbf{k}-\mathbf{g}), \tag{6}$$

where $\mathbf{g}$ are the reciprocal vortex lattice vectors and $B_0$ is the average field. For a square lattice

$$\mathbf{g} = g_0(m,n,0), \quad g_0 = 2\pi(B_0/\Phi_0)^{1/2}, \tag{7}$$

while for a triangular lattice

$$g_x = \pi(2B_0/\Phi_0)^{1/2}3^{1/4}n,$$

$$g_y = \pi(2B_0/\Phi_0)^{1/2}3^{-1/4}(2m-n), \tag{8}$$

with $m$ and $n$ being integers.[17,18]

A moving vortex lattice generates transverse electric field[19] satisfying $\nabla\mathbf{E}=0$. Together with Eq. (2), this gives for the Fourier transform of $\mathbf{E}$

$$k_x E_y - k_y E_x = -\sum_{\mathbf{g}} \frac{v\omega B_0 (2\pi)^4}{c(1+\lambda^2 g^2)}\delta(\omega - g_x v)\delta(\mathbf{k}-\mathbf{g})$$

$$k_x E_x + k_y E_y = 0. \tag{9}$$

From these equations one obtains

$$E_y = -\sum_{\mathbf{g}} \frac{g_x^2}{g^2}\frac{v}{c}\frac{B_0}{1+\lambda^2 g^2}(2\pi)^4 \delta(\omega - g_x v)\delta(\mathbf{k}-\mathbf{g}),$$

$$E_x = -\frac{g_y}{g_x}E_y. \tag{10}$$

## III. GENERATION OF SOUND

The phonon displacement field satisfies[20]

$$\frac{\partial^2 u}{\partial t^2} + \gamma\frac{\partial u}{\partial t} - s^2\nabla^2\mathbf{u} = \frac{\mathbf{F}}{\rho}. \tag{11}$$

Here $\gamma=\alpha\omega$ is the coefficient of the attenuation of sound of frequency $\omega, \rho$ is the mass density of the crystal, $s$ is the speed of the transverse sound, and

$$\mathbf{F} = ZeN\mathbf{E} \tag{12}$$

is the electric force density acting on the ions with the electric charge $Ze$ and concentration $N=\rho/M_i$, with $M_i$ being the ion mass. For the Fourier transform of $\mathbf{u}$ one obtains

$$\mathbf{u}(\mathbf{k},\omega) = \rho^{-1}\frac{\mathbf{F}(\mathbf{k},\omega)}{s^2 g^2 - \omega^2 + i\gamma\omega}, \tag{13}$$

with

$$F_y = -\sum_{\mathbf{g}} \frac{g_x^2}{g^2}\frac{v}{c}\frac{ZeB_0 N(2\pi)^4}{g^2(1+\lambda^2 g^2)}\delta(\omega - g_x v)\delta(\mathbf{k}-\mathbf{g}),$$

$$F_x = -\frac{g_y}{g_x}F_y. \tag{14}$$

Note that besides the electric force given by Eq. (12), the ions are also subject to the Lorentz force, $\mathbf{F}_L = (\dot{\mathbf{u}}/c)\times\mathbf{B}$, and to the force coming from the interaction of the vortex lattice with the defects of the crystal structure (residual pinning). The latter force makes every defect a source of an additional sound. Due to the destructive interference of sound coming from randomly distributed defects, the net effect of such generation should be small compared to the coherent radiation of sound at $\mathbf{k}=\mathbf{g}$ by the supersonic vortex lattice. As to the Lorentz magnetic force, it is small compared to the electric force as long as the velocities of ions, $\dot{u}$, are small compared to the speed of the vortex lattice $v$, which is always true at $v \sim s$ that we are concerned with in this paper.

The power of the sound generated per cubic centimeter of the superconductor is given by

$$W = \langle \mathrm{Re}[\mathbf{F}(\mathbf{r},t)\dot{\mathbf{u}}(\mathbf{r},t)]\rangle, \tag{15}$$

where averaging is over a large volume and long period of time. With the help of the Eqs. (13) and (14), and relations

$$\langle \exp[-i(g_x + g_x')vt]\rangle = \delta_{g_x,-g_x'},$$

$$\langle \exp[i(\mathbf{g}+\mathbf{g}')\cdot\mathbf{r}]\rangle = \delta_{\mathbf{g},-\mathbf{g}'}, \tag{16}$$

we obtain

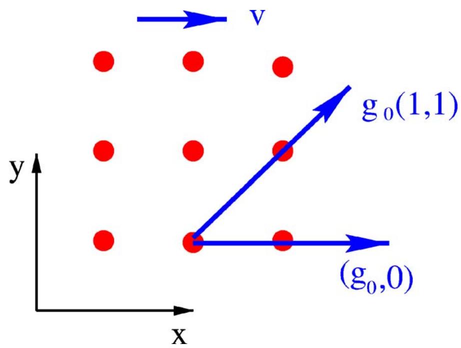


FIG. 1. (Color online) Rectangular vortex lattice, moving at a speed $\mathbf{v}$ close to the sound velocity $s$, generates sound with a frequency $\omega_r$ in the direction of $v$, shown by the arrow $(g_0,0)$. When the speed of the vortex lattice is close to $\sqrt{2}s$, the sound at a frequency $\sqrt{2}\omega_r$, propagating at 45° with $v$ [shown by the arrow $g_0(1,1)$], is generated.

$$W(v) = \frac{(ZeB_0Nv)^2}{2\rho c^2} \times \sum_{\mathbf{g}} \frac{g_x^2}{g^2(1+\lambda^2 g^2)^2} \frac{\gamma/2}{(sg - vg_x)^2 + (\gamma/2)^2}. \tag{17}$$

Due to the condition $\alpha \equiv \gamma/\omega \ll 1$ the main contribution to the acoustic power comes from those $\mathbf{g}$ in Eq. (17) that give the smallest values of $(sg - vg_x)^2$. Hence, the sound radiation occurs at $v \approx sg/g_x$, that is, with the wave vectors $\mathbf{k}=\mathbf{g}$ that make an angle with the direction of $\mathbf{v}$ satisfying

$$\cos\theta \equiv \frac{g_x}{g} \approx \frac{s}{v}. \tag{18}$$

The vortex lattice moving at a speed $\mathbf{v}$ is oriented in such a way that one of its sides coincides with the direction of $v$ (see Refs. 21–23). This rule allows us to determine the orientation of the phonon wave vectors $\mathbf{k}=\mathbf{g}$.

For a square lattice, $g_x = g_0 n$, $g_y = g_0 m$, with $n,m = 0, \pm 1, \ldots$. At $v \approx s$ the main contribution comes from $g_x = g_0$ and $g_y = 0$. It describes the generation of sound at the fundamental frequency,

$$\omega_r = sg_0, \tag{19}$$

propagating along the $x$ axis. The power of this sound is given by

$$W(\omega_r) = \frac{\hbar^2 N^2 s^2}{16\pi^2 \rho \lambda^4 \alpha \omega_r} = \frac{\hbar^2 N^2 s \Phi_0^{1/2}}{32\pi^3 \rho \lambda^4 \alpha B^{1/2}}. \tag{20}$$

For $v$ farther away from $s$, the term with $g_y = g_0$ becomes important when $v \approx \sqrt{2}s$. This term in Eq. (17) leads to the generation of sound with frequency $\omega = \sqrt{2}\omega_r$ and the wave vector $\mathbf{k} = (g_0, g_0)$. This sound is emitted at the angle $\theta = \pi/4$ with respect to the direction of $\mathbf{v}$. Its power is by the factor $2\sqrt{2}$ smaller than for the sound emitted at the fundamental frequency $\omega_r$. The directions of sound propagation for the two lowest frequencies is shown in Fig. 1.

Each term in Eq. (17) with $g_x = ng_0$ and $g_y = mg_0$ permits generation of sound with frequency

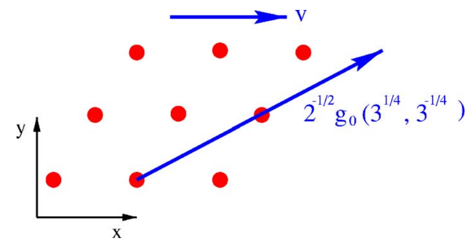


FIG. 2. (Color online) Triangular vortex lattice moving at a speed $v$ close to $2s/\sqrt{3}$ generates sound at a frequency $\omega_r(4/3)^{1/4}$ at 30° with $\mathbf{v}$, shown by the arrow $2^{1/2}g_0(3^{1/4}, 3^{-1/4})$.

$$\omega = \sqrt{n^2 + m^2}\,\omega_r \tag{21}$$

and power

$$W_{mn} = \frac{W(\omega_r)}{(n^2 + m^2)^{5/2}}. \tag{22}$$

It occurs at the vortex lattice speed

$$v_{mn} = s\sqrt{n^2 + m^2}. \tag{23}$$

This sound is emitted at an angle with the direction of $\mathbf{v}$ satisfying

$$\cos\theta_{mn} = n/\sqrt{n^2 + m^2}. \tag{24}$$

A triangular vortex lattice is described by Eq. (8). The fundamental acoustic frequency of the moving triangular lattice corresponds to $n=1$ and $m=0,1$, and is given by

$$\omega_t = \omega_r 2^{1/2} 3^{-1/4}. \tag{25}$$

The emission of this sound requires

$$v \approx 2s/\sqrt{3}. \tag{26}$$

The fundamental phonons have the wave vector $\mathbf{k} = g_0 2^{1/2} 3^{-1/4}$ and are emitted at an angle $\theta = \pi/6$, see Fig. 2. In contrast with the square lattice there is no sound generation along the direction of the motion of the vortex lattice. Due to the contribution of two terms, $m=0$ and $m=1$, the fundamental harmonic is generated by the triangular lattice with a power that is by factor $3/2$ greater than that for a square lattice.

## IV. SOUND GENERATION AND THE *I-V* CURVE

The velocity of the vortex lattice, $v$, is given by

$$v = \frac{cE_0}{B}, \tag{27}$$

where $E_0 = j/\sigma_{ff}$ is the average dc electric field in the superconductor. Here $j$ is the density of the superconducting transport current perpendicular to $\mathbf{B}$ and $\sigma_{ff}$ is the flux flow conductivity. The latter is given by

$$\sigma_{ff} = \frac{B\sigma_n}{H_{c2}} \tag{28}$$

in the Bardeen-Stephen approximation,[24] with $\sigma_n$ being the normal state conductivity. Consequently, the transport current needed to achieve $v=s$ is

$$j_s = \frac{sB^2\sigma_n}{cH_{c2}}. \tag{29}$$

Generation of sound leads to the additional energy losses by the moving vortex lattice. This should result in the enhancement of the current density seen in the *I-V* characteristic at certain voltages $V$, corresponding to the electric field $E_0$, see Ref. 13. The equation describing the energy balance is

$$jE_0(v) = \sigma_{ff}E_0^2(v) + W(v). \tag{30}$$

Generation of sound occurs at discrete values of $v$ computed in the previous section. At these values of $v$ the *I-V* curve must exhibit peaks, $\Delta j$, above the background curve determined by the equation $j_0=\sigma_{ff}E_0$. The relative height of these peaks is given by

$$\frac{\Delta j(n,m)}{j_0} = \frac{\hbar^2 N^2 c^2 \Phi_0 H_{c2}}{32\pi^3\rho\lambda^4\alpha\sigma_n B^{7/2} s n^4 (n^2+m^2)^{3/2}}. \tag{31}$$

## V. DISCUSSION

The most crucial question for the possibility of observation of the effects studied in this paper is whether the superconducting transport current can be sufficiently large so that the condition $v \gtrsim s$ is satisfied. The required current is given by Eq. (29). Taking for borocarbides $B=0.5$ T, $\sigma_n = 10^4(\Omega\ \mathrm{cm})^{-1}$, and $H_{c2}=10$ T we get $j_s \approx 2.5\ \mathrm{kA/cm^2}$. For YBCO, where $H_{c2}$ is higher by one order of magnitude, $j_s$ must be one order of magnitude lower. Such values of the transport current are achievable in both compounds when the pinning is low.

We shall now estimate the power of the sound generation. At the fundamental frequency, the estimates for square and triangular lattices differ by a factor of order unity. According to Eq. (20) the estimate depends crucially on the dimensionless parameter of the attenuation of sound $\alpha$. The problem of sound dissipation in superconductors has been studied over the last forty years by a number of authors, both theoretically and experimentally; see, e.g., Refs. 25–29. At low temperature the attenuation of sound due to its interaction with electrons in $s$-wave superconductors is very weak because of the gap. The main source of the attenuation is then scattering of ultrasound by the imperfections of the crystal lattice, which should be weak in good crystals as well. The generation of acoustic power must be then limited not so much by the imperfections of the crystal lattice but by the imperfections of the moving vortex lattice itself. In, e.g., good crystals of $NbSe_2$ the correlation length of the translational order in a moving vortex lattice in the direction of the magnetic field has been found to exceed 400 intervortex spacings, while the correlation length in the plane perpendicular to the magnetic field remains unknown.[30] The correlation length greater than 100 intervortex spacings would result in the effective $\alpha < 10^{-2}$. Taking $\lambda=10^{-5}$ cm, $B_0=0.1$ T, $\alpha<10^{-2}$, $s=10^5$ cm/s, $N=10^{21}\ \mathrm{cm^{-3}}$, and $\rho=5\ \mathrm{g/cm^3}$, we get from Eqs. (19) and (20) the frequency $\omega_r=4.5\times10^{10}\ \mathrm{s^{-1}}$ and the power $W_r>0.25\ \mathrm{W/cm^3}$. This is a rather high phonon power that can be easily detectable by the existing methods.

Finally, one should estimate the magnitudes of the peaks in the *I-V* curve of the superconductor due to the emission of ultrasound by the moving vortex lattice at discrete speeds. From Eq. (31) we find that at $n=1$, $m=0$, and $B=0.1$ T, in borocarbides $\Delta j(1,0)/j>0.01$, while for YBCO we get $\Delta j(1,0)/j>0.1$. Such peaks, even though they are significantly lower than predicted in Ref. 13, must be possible to see in experiment. Higher peaks are, probably, too weak to be observed.

We shall conclude this paper by noting that in magnetic superconductors, like borocarbides, the power of sound generation by the moving vortex lattice may be enhanced due to the magnetic order. We have seen from Eq. (2) that the ac electric field is determined by the magnetic induction. In the presence of the magnetic order, the latter changes by a factor that equals the magnetic permeability, $\mu(\mathbf{k},\omega)$. In borocarbides $\mu(\mathbf{k},\omega)$ can be large, see Ref. 31. Correspondingly, the sound power can be enhanced by a factor $\mu^2(\mathbf{k},\omega)$. The permeability is given by $\mu(\mathbf{k},\omega)=1+4\pi\chi(\mathbf{k},\omega)$, where

$$\chi(\mathbf{k},\omega) = \frac{\omega_M\omega_m(\mathbf{k})}{\omega_m^2(\mathbf{k}) - \omega^2 - i\omega\nu_m} \tag{32}$$

is the magnetic susceptibility of a magnetic superconductor. Here $\hbar\omega_M=\mu_M^2 n_M$, $\mu_M$ and $n_M$ are the magnetic moment and concentration of the magnetic ions, $\omega_m(\mathbf{k})$ and $\nu_m$ are the dispersion and the relaxation rate of magnetic excitations. The sound power can be enhanced if the washboard frequency, $\omega_{wb}$, matches both the frequency of the sound and the frequency of the magnetic excitations. This requires $\omega_{wb}$ above the gap for the magnetic excitations, which at this time is unknown for borocarbides.

## ACKNOWLEDGMENTS

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