Electronic stiffness of a superconducting niobium nitride single crystal under pressure

Xiao-Jia Chen, Viktor V. Struzhkin, Zhigang Wu, Ronald E. Cohen, Simon Kung,* Ho-kwang Mao, and Russell J. Hemley Geophysical Laboratory, Carnegie Institution of Washington, Washington, DC 20015, USA

> Axel Nørlund Christensen Højkolvej 7, DK-8210 Aarhus V, Denmark (Received 28 June 2005; published 23 September 2005)

We report a quantitative study of pressure effects on the superconducting transition temperature T_c and the electronic stiffness of niobium nitride. It is found that T_c increases initially with pressure and then saturates up to 42 GPa. Combining phonon and structural information on the samples obtained from the same single crystal, we derive a nonmonotonic pressure dependence of the electronic stiffness, rising moderately at low pressure while dropping slightly at high pressure. The theory of Gaspari and Gyorffy is found to reproduce the observed low-pressure results qualitatively but fails to predict the high-pressure data. The observed pressure effect on T_c is attributed to the pressure-induced interplay of the electronic stiffness and phonon frequencies.

DOI: 10.1103/PhysRevB.72.094514

PACS number(s): 74.62.Fj, 74.25.Jb, 74.70.Ad

I. INTRODUCTION

High-pressure studies have been crucial for tuning the superconducting transition temperatures T_c 's of materials, pursuing new classes of superconductors and shedding light on the theory of superconductivity. Over many decades, there has been no agreement regarding which of the quantitiesthe electronic stiffness $N(E_F)\langle I^2 \rangle$ or the lattice stiffness $M\langle\omega^2\rangle$ —determines the pressure dependence of T_c in a narrow-band superconductor to a greater degree $[N(E_F)]$ is the density of states per cell at the Fermi energy E_F , $\langle I^2 \rangle$ is the mean-square electron-ion interaction matrix element, M is the atomic mass, and $\langle \omega^2 \rangle$ is a weighted mean square of the phonon frequency].¹ The importance of $N(E_F)\langle I^2 \rangle$ was first addressed by Hopfield² to understand the pressure effect on T_c of transition metals. It was further emphasized³ in the theoretical study of the pressure effect in lanthanum, where the drastic increase in T_c under pressure is primarily attributed to the pressure-induced increase of $N(E_F)\langle I^2\rangle$. The determination of $N(E_F)\langle I^2 \rangle$ strongly depends on accurate measurements of both T_c and the phonon spectra. The theory of Gaspari and Gyorffy⁴ has been proven to be very powerful in predicting $N(E_F)\langle I^2\rangle$ for many transition metals and compounds.3-5 However, the absence of experimental determination of $N(E_F)\langle I^2\rangle$ at high pressure makes a comparison between the theory and experiments difficult.^{3,6} Experimental attempts in this direction are therefore highly desirable not only for understanding the pressure effect on T_c itself but also for examining whether the theory applies to highpressure conditions.

When investigating the pressure dependence of $N(E_F)\langle I^2\rangle$, one needs to determine the pressure-induced phonon frequency shift and the pressure dependence of T_c and assumes that the Coulomb pseudopotential μ^* depends weakly on pressure.² A direct experimental determination of phonon frequency can be made by measuring the electron-phonon coupling strength $\alpha^2(\omega)F(\omega)$ at different pressures by tunneling.⁷ Unfortunately, such measurements are exceedingly difficult for transition metals and their alloys and com-

pounds, even at atmospheric pressure,⁸ and they have not yet been attempted at high pressure. The measurement of the pressure dependence of the phonon density of states $F(\omega)$ by inelastic neutron scattering is also capable of providing good values of phonon frequencies. However, few neutron studies of the pressure dependence of phonon frequencies have been reported.⁹ On the other hand, experimental studies of some transition metals^{10,11} have shown that higher pressures sometimes give rise to a more complicated dependence of T_c than that at low pressure. The presence of such a nonmonotonic dependence of T_c calls for experiments in the region of high pressures to see how $N(E_F)\langle I^2 \rangle$ and/or $M\langle \omega^2 \rangle$ determine(s) the pressure behavior of T_c .

In this paper we address these issues by investigating pressure effects on niobium nitride. We report measurements of the pressure dependence of T_c in a single crystal up to 42 GPa. Combining the determined phonon and structural information on the samples that were separated from the same single crystal, we are able to obtain direct information on the pressure dependence of the electronic stiffness. We also calculate the electronic stiffness as a function of pressure up to 30 GPa based on the theory of Gaspari and Gyorffy.⁴ There is a qualitative agreement between the experiment and theory at low pressures but a large discrepancy for high-pressure data. We show that the pressure-induced interplay of the electronic stiffness and phonon frequencies is responsible for the pressure dependence of T_c in this material.

II. EXPERIMENT

Single crystals of NbN were grown from sintered rods using a zone-melting and zone-annealing technique, as described in detail elsewhere.^{12,13} Plate-shaped crystals were separated mechanically from the bar. X-ray diffraction confirmed that NbN has the sodium chloride (*B*1) structure with a lattice constant a_0 of 4.379 Å at ambient condition.¹⁴ The composition was determined to be NbN_{0.90(1)} from gravimetric chemical analysis.¹³ The measurements of T_c under pres-



FIG. 1. Magnetic susceptibility signal vs temperature for a $NbN_{0.90(1)}$ single crystal at various pressures. The superconducting transition temperature T_c is marked by an arrow.

sure were performed using a highly sensitive magnetic susceptibility technique.¹⁵ Hydrostatic pressure was generated in a Mao-Bell diamond anvil cell. Neon was loaded into the gasket to serve as the pressure medium. Pressure at low temperatures was determined by the R_1 fluorescence line of ruby.¹⁶

III. RESULTS AND ANALYSIS

Figure 1 shows representative temperature scans at different applied pressures for a $NbN_{0.90(1)}$ single crystal. The superconducting transition is identified as the temperature where the signal goes to zero on the high-temperature side which is the point at which magnetic flux completely enters the sample. The superconducting transition of 12.4 K is obtained at ambient pressure, which is in good agreement with previously reported values of samples with similar compositions.¹⁷ The transition temperature weakly depends on the applied pressure. At high pressures the shape of the signal does not change.

Figure 2 shows the experimental transition temperature as a function of pressure. There is a roughly linear dependence of the transition with applied pressure up to 4 GPa. The initial pressure derivative of T_c , dT_c/dP (~0.03 K/GPa) is in good agreement with the value of 0.04 K/GPa reported previously for NbN_{0.92} (Ref. 18). Measurements at higher pressures are of special interest as the transition temperature is observed to level off from 4 to 42 GPa. This behavior is very similar to observations of Nb, where T_c is nearly constant from 10 to 70 GPa.¹¹ This similarity suggests that d states dominate superconductivity at high pressure in such transition metals and their compounds. There are no reports of anomalies in compressibility and elastic constants of NbN over a wide pressure range.¹⁴ Until now, exact information about the phonon spectrum of niobium nitride under high pressure from neutron scattering or tunneling experiments has not been available. However, we have found¹⁹ that the pressure-induced phonon frequency shifts can be well determined from Raman scattering measurements because of a good agreement between the phonon density of states ob-



FIG. 2. Dependence of the superconducting transition temperature T_c on pressure in a NbN_{0.90(1)} single crystal. The error bars give the transition onset uncertainty. The solid curve is a fit to the experimental data.

tained from neutron scattering and the Raman response. The phonon frequency determined from Raman scattering for NbN was found to decrease with increasing pressure after passing through a maximum at around 20 GPa.¹⁹ It is indicated that the variation of $\langle \omega^2 \rangle$ with pressure alone is not adequate for a complete description of the observed pressure effect on T_c in this material. One should also note that the pressure behavior in NbN appears to agree with the chemical trends in structural properties proposed by Phillips.²⁰ Since pressure drives the crystal towards short-wavelength instabilities, T_c is expected to increase until reaching saturation, although $\langle \omega^2 \rangle$ may not necessarily increase under pressure.

The primary determinant of T_c is the electron-phonon coupling constant λ . The direct dependence of λ on the electronic characteristics can be taken into account by the electronic stiffness $N(E_F)\langle I^2\rangle$. We have performed first-principles calculations using the all-electron linearized augmented plane-wave (LAPW) method²¹ within the local density approximation (LDA). The k-point mesh is of $16 \times 16 \times 16$, and the convergence parameter RK_{max} = 8.0. The calculated zero-pressure equilibrium lattice constant a_0 is 4.360 Å. The theoretical pressure is thus rescaled to fit the experimental value of a_0 . The density of states N(E) derived from the tetrahedral method at three selected pressures for stoichiometric NbN is shown in Fig. 3. The general shape and relative magnitude of the peaks agree quite well with previous theoretical calculations.²² The Fermi energy E_F is about 4 eV above Γ_{15} and occurs within the t_{2g} manifold of the niobium 4d bands. The peak of the density of states for the higher bands is at 2.0 eV above E_F . The dominant effect of pressure on N(E) is a broadening of the bands and the resulting decrease of N(E) in most energy regions. The density of states at E_F , $N(E_F)$, decreases from 0.85 eV⁻¹ cell⁻¹ at ambient pressure to $0.72 \text{ eV}^{-1} \text{ cell}^{-1}$ at 50 GPa (inset of Fig. 3). We have estimated the contributions of states of different symmetry to the density of states at the Fermi level. It turns out that for NbN the contribution of d-type states to $N(E_F)$ substantially exceeds those of the s and p states. This shows that mainly d states are responsible for the superconductivity



FIG. 3. (Color online) Plot of the calculated density of states vs energy of NbN at different pressures. Inset: density of states at the Fermi level $N(E_F)$ as a function of pressure in NbN. The solid line is a fit to the calculated data.

over the pressure range studied. Similar behavior has also been reported previously by Palanivel *et al.*²³ from band structure calculations. Their predicted T_c behavior under pressure is in contrast with our measurements.

IV. DISCUSSION

We now determine the electronic stiffness under pressure. Based on McMillan's decomposed expression for λ (Ref. 24), we have $N(E_F)\langle I^2\rangle(P)/N(E_F)\langle I^2\rangle(0) = [\lambda(P)/\lambda(0)] \times [\langle \omega^2(P) \rangle / \langle \omega^2(0) \rangle]$. The pressure dependence of λ can be expressed through McMillan's formula

$$\lambda(P) = \frac{1.04 + \mu^*(P)\chi(P)}{\chi(P) - 1.04 - 0.62\mu^*(P)\chi(P)},$$
(1)

with $\chi(P) = \ln[\Theta_D(P)/1.45T_c(P)]$ using the experimentally measured Debye temperature Θ_D and T_c and the Coulomb pseudopotential μ^* with a value of 0.13 at ambient pressure for transition metals and their compounds. The pressure dependence of μ^* is generally believed to be very small and therefore neglected.² However, an analysis of magnetostriction data²⁵ reveals that μ^* does change with volume (pressure), usually in the sense of decreasing μ^* with decreasing volume. The pressure enters μ^* as $\mu^*(P)$ $= \mu(P)/[1 + \mu(P) \ln \beta(P)]$ through the screened Coulomb interaction $\mu = 0.5 \ln[(1+a^2)/a^2]$ and $\beta = E_F/\omega_{ph}$ with a^2 $=\pi e^2 N(E_F)/k_F^2$ and ω_{ph} a characteristic phonon frequency.²⁶ Considering that $k_F = [3\pi^2 Z/\Omega]^{1/3}$ and $E_F = \hbar^2 k_F^2/2M$ with Ω the atomic volume and Z the valence, one can write $\beta(P) = \beta(0) [\omega_{ph}(0) / \omega_{ph}(P)] [\Omega(0) / \Omega(P)]^{2/3}$ and $a^2(P)$ $=a^{2}(0)[N(E_{F})(P)/N(E_{F})(0)][\Omega(0)/\Omega(P)]^{2/3}.$

Geballe *et al.*²⁷ reported a value of Θ_D =331 K from lowtemperature heat capacity measurements of a NbN_{0.84}. This value is similar to the reported Θ_D =320(25) K for a NbN_{0.90(1)} single crystal.¹³ Therefore, we believe that a choice of 331 K for Θ_D is reasonable for the present investigation. Substituting this value along with the experimentally determined $T_c(0)$ into Eq. (1), we obtain $\lambda(0)$ =0.87.



FIG. 4. Pressure dependence of the phonon frequency of the acoustic branch $\langle \omega_{ac}^2 \rangle^{1/2}$ in a NbN_{0.90(1)} single crystal deduced from high-pressure Raman data (Ref. 19). The solid curve is a fit to the data points.

Taking a typical value of 0.4 for $a^2(0)$ (Ref. 26), one then obtains $\beta(0)$ of 448 for the material studied. These parameters determined at ambient condition will be used to determine the high-pressure behavior of $N(E_F)\langle I^2 \rangle$.

In NbN, the mass of the metal $M_{\rm Nb}$ is apparently larger than that of the nitrogen $M_{\rm N}$. The ratio between them is equal to the ratio between the average square of phonon frequencies in the optical branch $\langle \omega_{op}^2 \rangle$ and in the acoustic branch $\langle \omega_{ac}^2 \rangle$ expected from nearest-neighbor forces.²⁸ Thus, the information on the change in $\langle \omega^2 \rangle$ can be obtained from the shift in phonon frequency ω_{ac} or ω_{op} due to the relation $\langle \omega^2 \rangle^{1/2} \approx (2M_{\rm Nb}/M)^{1/2} \langle \omega_{ac}^2 \rangle^{1/2} \approx (2M_{\rm N}/M)^{1/2} \langle \omega_{op}^2 \rangle^{1/2}$ (Refs. 19, 28, and 29). Our Raman studies of $NbN_{0.90(1)}$ single crystal¹⁹ showed that the low-frequency acoustic branch is more pronounced and broadened compared to the optical branch. Such a feature leads us to conclude that the frequency of the acoustic phonon branch would provide more accurate information than that of the optical branch. Using the approximation calculation of $\langle \omega^2 \rangle$ = $\int \omega F(\omega) d\omega / \int \omega^{-1} F(\omega) d\omega$ (Ref. 29), we deduce the values of $\langle \omega_{ac}^2 \rangle^{1/2}$ under various pressures based on the Raman scattering data corrected for background contribution in the wave number range of $50-400 \text{ cm}^{-1}$ (Ref. 19). The pressure behavior of $\langle \omega_{ac}^2 \rangle^{1/2}$ is shown in Fig. 4.

Assuming that both Θ_D and ω_{ph} are proportional to $\langle \omega^2 \rangle^{1/2}$ and that $\langle \omega^2 \rangle^{1/2} \sim \langle \omega_{ac}^2 \rangle^{1/2}$, we obtain the pressure dependence of these quantities through $\langle \omega_{ac}^2(P) \rangle^{1/2}$. Previously, we determined the pressure dependence of the lattice constant in a NbN_{0.90(1)} single crystal.¹⁴ Using these high-pressure relations along with the determined $T_c(P)$ and $N(E_F)(P)$ data taken to 30 GPa, we can evaluate the pressure dependence of $N(E_F)\langle I^2 \rangle$ based on the equations developed above (Fig. 5). As can be seen, $N(E_F)\langle I^2 \rangle$ increases as expected to a maximum value at ~20 GPa where it is about 7% greater than its zero-pressure value and then begins to decrease slightly at higher pressure behavior of $N(E_F)\langle I^2 \rangle$ is very similar to that of $\langle \omega_{ac}^2 \rangle^{1/2}$, with a maximum value at almost same pres-



FIG. 5. Pressure dependence of the normalized electronic stiffness $N(E_F)\langle I^2 \rangle$ of niobium nitride up to 30 GPa.

sure level. Below this level, although pressure-induced enhancement of $N(E_F)\langle I^2 \rangle$ tends to increase T_c , the hardening of phonon under pressure weakens this increase. After passing through this level, the decrease in $N(E_F)\langle I^2 \rangle$ with pressure would result in a reduction of T_c . However, pressureinduced phonon softening beginning from this point obviously suppresses such a reduction. The present studies suggest that the pressure effect on T_c is a result of a pressureinduced interplay of the electronic stiffness and phonon frequencies. It is worth emphasizing that a similar monotonic pressure dependence of $N(E_F)\langle I^2\rangle$ was also reported previously for aluminum³⁰ where T_c was reduced to 0.075 K at 6.2 GPa from its zero-pressure value of 1.18 K. This similarity indicates that such a behavior may be common for a narrow-band superconductor no matter how T_c varies with pressure.

The electronic stiffness $N(E_F)\langle I^2 \rangle$ has been calculated based on the rigid muffin-tin approximation (RMTA) in the theory of Gaspari and Gyorffy:⁴

$$N(E_F)\langle I^2 \rangle_{\alpha} = \sum_{l} \frac{2(l+1)E_F N_l^{\alpha} N_{l+1}^{\alpha} \sin^2(\delta_{l+1}^{\alpha} - \delta_l^{\alpha})}{\pi^2 N(E_F) N_l^{(1)\alpha} N_{l+1}^{(1)\alpha}}.$$
 (2)

In this expression δ_l^{α} are the scattering phase shifts at E_F for atom α and angular momentum l, N_l^{α} are the lth angular momentum components of the densities of states at E_F , and $N_l^{(1)\alpha}$ are the *l*th components of the single-scatterer densities of states evaluated at E_F . At P=0 GPa, we obtained $\langle I^2 \rangle_{\rm Nb}$ =13.1 eV²/Å² and $\langle I^2 \rangle_{\rm N}$ =5.3 eV²/Å², which are consistent with the previous calculations.⁵ The minor discrepancy is possibly due to different schemes of the LDA. Under high pressure, the muffin-tin radii decrease proportionally to the lattice constant. The theoretical results are also plotted in Fig. 5 for comparison. Over the pressure range studied, the metal component $N(E_F)\langle I^2\rangle_{\rm Nb}$, which has its main contribution from the metal d states at E_F , is much larger than the nitrogen component $N(E_F)\langle I^2 \rangle_N$, which is dominated by the nitrogen p states at E_F . At low pressures below 13 GPa, theoretical calculations for $N(E_F)\langle I^2\rangle$ agree qualitatively with our experimental determination. However, the theoretical value of $N(E_F)\langle I^2 \rangle$ always increases with increasing pressure.

This monotonic pressure dependence of $N(E_F)\langle I^2 \rangle$ was also theoretically observed in lanthanum.³ The RMTA assumes that the *d*-wave function of the transition metal is strongly peaked and confined inside the muffin tin, so that the change in potential of the interstitial region has small effect on the *d* electrons. With increasing pressure, the *d*-wave function extends more outside the muffin tin region, and therefore the RMTA becomes less accurate at high pressures. More accurate theoretical studies are necessary to bring theory into agreement with experiment at high pressure.

Our experimental data for NbN provide an estimate of the electronic stiffness, which agrees with the theory of Gaspari and Gyorffy over roughly half the pressure range studied. It should be emphasized that the experimental curve plotted in Fig. 5 does not include errors in measurements. Considering the errors of $T_c(P)$ within 2% and $\langle \omega^2 \rangle^{1/2}$ within 0.5%, we may have the $N(E_F)\langle I^2 \rangle$ errors less than 1.6%. Note that there exist systematic errors in Raman measurements. These errors may contribute to the pressure dependence of the phonon density of states. Since the same calculation procedure was used to derive $\langle \omega^2 \rangle$ after correcting for the background contribution over a wide wave number range, we believe that the pressure dependence of $\langle \omega^2 \rangle$ should not be altered due to systematic errors. An estimate of 0.5% scatter of $\langle \omega^2 \rangle^{1/2}$ would fully cover errors in Raman measurements. Although there exists a deviation of $N(E_F)\langle I^2 \rangle$ at individual pressure points, the deviatoric $N(E_F)\langle I^2 \rangle$ does not change the tendency of its pressure dependence. Good agreement between the experiment and theory still remains up to 15 GPa, even if the experimental errors would be taken into account.

Unlike previous theoretical treatments,^{2,30} we have already considered the pressure-dependent electron-electron interaction through the standard Coulomb pseudopotential approximation for μ^* . For a narrow-band superconductor, μ^* may play an important role in determining T_c only if the applied pressure is sufficiently high.³¹ NbN is generally considered to be a standard narrow-band superconductor. Thus, all assumptions made for the parameters entering the standard expression for μ^* are believed to be sufficient for providing a reliable estimate of $N(E_F)\langle I^2 \rangle$ over the pressure range studied. Richardson and Ashcroft³² have determined the effective electron-electron interaction for monovalent and low-density alkali metals such as hydrogen and lithium by a method which treats electrons and phonons on an equivalent footing. This treatment is probably essential for providing a more accurate estimate of transition temperatures at high pressure in such lower-density systems, which waits for further experimental examination. The fact that T_c depends weakly on $N(E_F)$ in NbN over a wide pressure range indicates that the standard Coulomb pseudopotential approximation for μ^* is physically plausible. The inclusion of the effective Coulomb interaction on a completely equal footing with the phonon-mediated interaction may not greatly affect the behavior of $N(E_F)\langle I^2 \rangle$ for NbN under pressure.

V. CONCLUSION

We have measured the superconducting transition temperature of niobium nitride under pressure up to 42 GPa. We observe that T_c increases initially with pressure and then saturates at around 4 GPa. Combining the phonon and structural information determined previously by Raman scattering and x-ray diffraction, we obtain the pressure dependence of the electronic stiffness in terms of the McMillan theory. We find that the electronic stiffness rises moderately at low pressure but drops slightly at high pressure. Our low-pressure results are in qualitative agreement with predictions from the theory of Gaspari and Gyorffy, but the high-pressure data differ markedly from theory. Our results follow that the interplay of the counteracting changes in the electronic stiff-

- *Present address: California Institute of Technology, MSC 593, Pasadena, CA 91126, USA.
- ¹J. W. Garland and K. H. Bennemann, in *Superconductivity in d-and f-band Metals*, edited by D. H. Douglass (American Institute of Physics, New York, 1972), p. 255.
- ²J. J. Hopfield, Physica (Amsterdam) 55, 41 (1971).
- ³W. E. Pickett, A. J. Freeman, and D. D. Koelling, Phys. Rev. B **22**, 2695 (1980).
- ⁴G. D. Gaspari and B. L. Gyorffy, Phys. Rev. Lett. 28, 801 (1972).
- ⁵D. A. Papaconstantopoulos, W. E. Pickett, B. M. Klein, and L. L. Boyer, Phys. Rev. B **31**, 752 (1985).
- ⁶R. Evans, V. K. Ratti, and B. L. Gyorffy, J. Phys. F: Met. Phys. **3**, L199 (1973).
- ⁷J. P. Franck and W. J. Keeler, Phys. Lett. **25A**, 624 (1967).
- ⁸J. Geerk, G. Linker, and R. Smithey, Phys. Rev. Lett. **57**, 3284 (1986).
- ⁹S. Klotz, J. M. Besson, M. Braden, K. Karch, P. Pavone, D. Strauch, and W. G. Marshall, Phys. Rev. Lett. **79**, 1313 (1997).
- ¹⁰Y. Akahama, M. Kobayashi, and H. Kawamura, J. Phys. Soc. Jpn. 59, 3843 (1990).
- ¹¹ V. V. Struzhkin, Yu. A. Timofeev, R. J. Hemley, and H. K. Mao, Phys. Rev. Lett. **79**, 4262 (1997).
- ¹²A. N. Christensen, O. W. Dietrich, W. Kress, W. D. Teuchert, and R. Currat, Solid State Commun. **31**, 795 (1979).
- ¹³A. N. Christensen, Acta Chem. Scand., Ser. A **32**, 89 (1978).
- ¹⁴X. J. Chen, V. V. Struzhkin, Z. G. Wu, M. Somayazulu, J. Qian, S. Kung, A. N. Christensen, Y. S. Zhao, R. E. Cohen, H. K. Mao, and R. J. Hemley, Proc. Natl. Acad. Sci. U.S.A. **102**, 3198 (2005).

ness and phonon frequencies due to pressure is responsible for the observed pressure effect on T_c in this material.

ACKNOWLEDGMENTS

We thank W. E. Pickett for sharing the RMTA code and H. Krakauer and D. J. Singh for useful discussions. This work was supported by U.S. Department of Energy Grant No. DEFG02-02ER4595, Carnegie/Department of Energy Alliance Center Grant No. DEFC03-03NA00144, and Office of Naval Research Grant No. N000140210506.

- ¹⁵Yu. A. Timofeev, V. V. Struzhkin, R. J. Hemley, H. K. Mao, and E. A. Gregoryanz, Rev. Sci. Instrum. **73**, 371 (2002).
- ¹⁶H. K. Mao, J. Xu, and P. M. Bell, J. Geophys. Res. **91**, 4673 (1986).
- ¹⁷A. N. Christensen, S. E. Rasmussen, and G. Thirup, J. Solid State Chem. **34**, 45 (1980).
- ¹⁸H. Neubauer, Z. Phys. **226**, 211 (1969).
- ¹⁹X. J. Chen, V. V. Struzhkin, S. Kung, H. K. Mao, R. J. Hemley, and A. N. Christensen, Phys. Rev. B **70**, 014501 (2004).
- ²⁰J. C. Phillips, Phys. Rev. Lett. **26**, 543 (1971).
- ²¹D. J. Singh, *Planewaves*, *Pseudopotentials*, and the LAPW Method (Kluwer Academic, Boston, 1994).
- ²²L. F. Mattheiss, Phys. Rev. B 5, 315 (1972).
- ²³B. Palanivel, G. Kalpana, and M. Rajagopalan, Phys. Status Solidi B **176**, 195 (1993).
- ²⁴W. L. McMillan, Phys. Rev. **167**, 331 (1968).
- ²⁵E. Fawcett and G. K. White, J. Appl. Phys. **39**, 576 (1968).
- ²⁶ P. Morel and P. W. Anderson, Phys. Rev. **125**, 1263 (1962).
- ²⁷T. H. Geballe, B. T. Matthias, J. P. Remeika, A. M. Clogston, V. B. Compton, J. P. Maita, and H. J. Williams, Physics (Long Island City, N.Y.) **2**, 293 (1966).
- ²⁸J. C. Phillips, J. Appl. Phys. **43**, 3560 (1972).
- ²⁹W. Weber, Phys. Rev. B **8**, 5093 (1973).
- ³⁰D. U. Gubser and A. W. Webb, Phys. Rev. Lett. **35**, 104 (1975).
- ³¹X. J. Chen, H. Zhang, and H.-U. Habermeier, Phys. Rev. B 65, 144514 (2002).
- ³²C. F. Richardson and N. W. Ashcroft, Phys. Rev. Lett. **78**, 118 (1997); Phys. Rev. B **55**, 15130 (1997).