Field-induced chiral phase in isotropic frustrated spin chains

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It is argued that an external magnetic field applied to an isotropic zigzag spin chain with an arbitrary spin S and antiferromagnetic nearest-neighbor and next-nearest-neighbor exchange couplings J_1 and J_2 induces a phase with spontaneously broken parity, characterized by long-range ordering of vector chirality. To show that, we use a bosonization approach for $S=\frac{1}{2}$ and $S=1$, valid in the limit of a weak zigzag interaction $J_1/J_2 \ll 1$, as well as an effective large-S theory applicable in the vicinity of the saturation field. Relevance to real materials and the possibility of experimental observation of the chiral phase are discussed.

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I. INTRODUCTION

In recent years, phases with broken vector chirality in frustrated quantum spin chains have attracted considerable interest.[1–6] Such phases are characterized by nonzero long-range correlations of the vector product of two adjacent spins

$$\vec{\kappa}_n = \langle \vec{S}_n \times \vec{S}_{n+1} \rangle,$$

so that in a chirally ordered phase, spins tend to rotate in a certain preferred plane (e.g., chosen by anisotropic interactions) predominantly clockwise or counterclockwise. The vector chirality has to be distinguished from the so-called scalar chirality $\tilde{\kappa} \propto \mathbf{S}_{n-1}(\mathbf{S}_n \times \mathbf{S}_{n+1})$, which is often discussed in the context of isotropic spin chains.[7]

Classically, states with a broken chirality emerge only together with a helical long-range order and simply distinguish left and right spirals. In one-dimensional (1D) systems, however, existence of a true helical order is in most cases prohibited by the Mermin-Wagner theorem,[8] since strong quantum fluctuations prevent spontaneous breaking of the continuous rotation symmetry in the plane of the helix. In contrast to the helical spin order, the chiral order breaks only a discrete symmetry between left and right and, thus, can survive even in one dimension.[9,10] So, one can view the chiral order as a remnant of the classical helical order in a 1D spin system. At zero temperature, the long-range helical order will be reestablished at an arbitrarily small three-dimensional (3D) coupling. With increasing temperature, however, the helical order is suppressed faster than the chiral one, so that there is a finite temperature window where the helical order is already destroyed but the chiral order still persists.[9] Such a chiral ordering transition at finite temperature has possibly been observed experimentally[11] in the quasi-1D anisotropic organic magnet Gd(hfac)$_3$NITiPr.

Recently, chirally ordered phases were numerically found in frustrated spin chains with easy-plane anisotropy.[3,6] The aim of the present paper is to show that a chiral phase emerges in *isotropic* frustrated spin chains as well, if they are subject to a strong external magnetic field. We focus on the model of a zigzag chain defined by the Hamiltonian

$$\mathcal{H} = J_1 \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + J_2 \sum_j \vec{S}_j \cdot \vec{S}_{j+2} - h \sum_j S_j^z, \qquad (1)$$

where $\vec{S}_j$ are spin-S operators at the jth site and $J_{1,2}>0$. It is easy to analyze the *classical* counterpart of the above model, where spins are represented by vectors, $(S_n^{\pm}, S_n^z) \mapsto (S \sin \phi_n e^{\pm i\theta_n}, S \cos \phi_n)$. The applied field selects a preferred plane, reducing the symmetry of the Hamiltonian from SU(2) to U(1). Depending on the frustration strength

$$\alpha = J_2/J_1,$$

the in-plane ground-state configuration is given by $\theta_n = (\pi \pm \lambda)n$, where

$$\lambda = \begin{cases} 0, & \alpha < 1/4 \\ \arccos(1/4\alpha), & \alpha > 1/4. \end{cases} \qquad (2)$$

The spins are canted toward the field, $\cos \phi_n = h/h_s$, with

$$h_s = 4S\{J_1 \cos^2(\lambda/2) + J_2 \sin^2 \lambda\} \qquad (3)$$

being the saturation field. For $h>h_s$, the ground state is unique and corresponds to fully polarized spins. The classical ground state for $h<h_s$ is a canted antiferromagnet for α below the Lifshits point $\frac{1}{4}$ while for $\alpha>\frac{1}{4}$ one has two degenerate helical ground states, as reflected by the $\pm$ signs above, which correspond to the left and right chirality $\kappa = \pm S^2(1-h^2/h_s^2)\sin\lambda$. For $\alpha>\frac{1}{4}$ in the presence of a field, the initial SU(2) symmetry is thus reduced to U(1)$\times Z_2$. A schematic view of the classical phase diagram is shown in Fig. 1.

In the *quantum* case, according to the Mermin-Wagner theorem,[8] the U(1) symmetry cannot be broken, but it is allowed to break the discrete Z_2 chiral symmetry. Such a scenario is indeed realized in frustrated *anisotropic* spin chains with easy-plane anisotropy, as predicted by Nersesyan *et al.*[2] and later confirmed numerically.[3,6,12] For a chain with easy-plane anisotropy the classical picture of the symmetry reduction from SU(2) to U(1)$\times Z_2$ is the same as in the case of applied magnetic field.

A natural question arises: Can an external magnetic field act similarly to the xy anisotropy,[13] favoring the chiral order in *isotropic* spin chains? At the first glance it seems that a

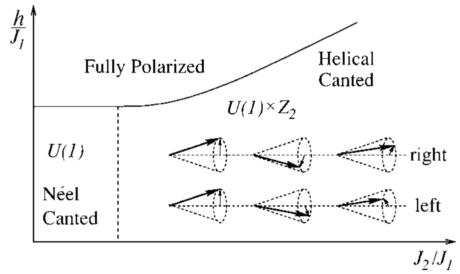

FIG. 1. A schematic phase diagram of the model (1) in the classical limit.

positive answer to this question is trivial. However, the connection between an isotropic antiferromagnetic chain in an external field and a chain with an easy-plane anisotropy is not so straightforward. For example, an integer-S antiferromagnetic chain undergoes a transition of the Berezinski-Kosterlitz-Thouless type from the Haldane phase to a gapless phase under the influence of an easy-plane anisotropy;[14] in an isotropic integer-S chain, an applied magnetic field can close the Haldane gap, also causing a transition to a gapless phase, but this latter transition belongs to the commensurate-incommensurate universality class.[15] For a frustrated $S=\frac{1}{2}$ chain in a magnetic field, recent numerical studies[16,17] propose a scenario which is different from that for an anisotropic chain, namely the scenario of a *two-component Luttinger liquid*, which does not involve any breaking of the Z_2 symmetry. The goal of the present study is to argue that the correct high-field physics of isotropic frustrated chains is indeed determined by the spontaneous breaking of the chiral symmetry. The paper is organized as follows: In Sec. II, we consider the model (1) for $S=\frac{1}{2}$ in the limit of strong frustration $\alpha \gg 1$ (the limit of two weakly coupled chains) and show the existence of the chirally ordered phase by means of bosonization and a mean-field decoupling in the spirit of Ref. 2. We also show that magnetization cusps similar to those observed in Refs. 16 and 17 naturally arise in our approach, and thus, the presense of cusps alone does not necessarily imply the scenario of the transition from a two-component to one-component Luttinger liquid. In Sec. III, the problem for an isotropic $S=1$ chain in an applied field is mapped onto that for a $S=\frac{1}{2}$ chain with easy-plane anisotropy, for which the existence of the chiral phase is established numerically;[3,6,12] in Sec. IV, we consider the general large-S case and show that the phase immediately below the saturation field is chirally ordered. Finally, Sec. V contains discussion and suggestions for possible experiments.

II. FIELD-INDUCED CHIRALITY IN $S=\frac{1}{2}$ CHAIN

We start with the "extreme quantum" spin-$\frac{1}{2}$ case which admits a field-theoretical description based on the bosonization approach. Consider the limit of strong frustration $\alpha \gg 1$ and strong magnetic fields $h \sim J_2$. The system may be viewed as two $S=\frac{1}{2}$ chains weakly coupled by the zigzag interaction J_1. A single spin-$\frac{1}{2}$ chain in a uniform magnetic field is known to be critical, its low-energy physics being effectively described by the standard Gaussian theory[18] known also as the Tomonaga-Luttinger liquid

$$\mathcal{H} = \frac{v}{2} \int dx \left\{ \frac{1}{K} (\partial_x \phi)^2 + K (\partial_x \theta)^2 \right\}. \tag{4}$$

Here ϕ is a compactified scalar bosonic field and θ is its dual, $\partial_t \phi = v \partial_x \theta$, with the commutation relations $[\phi(x), \theta(y)] = i\Theta(y-x)$, where $\Theta(x)$ is the Heaviside function and the regularization $[\phi(x), \theta(x)] = i/2$ is assumed. Integrability of the $S=\frac{1}{2}$ chain model makes it possible to relate explicitly the coupling constants of the theory, the spin wave velocity v, and the Luttinger liquid (LL) parameter K to the microscopic parameters J_2, h. The exact functional dependences $v(h)$ and $K(h)$ are known (see Ref. 19 and references therein) from the numerical solution of the Bethe ansatz integral equations.[20] Particularly, K increases with the magnetic field from $K(h=0)=\frac{1}{2}$ to $K=1$ for h approaching the saturation value $2J_2$.

In the infrared limit, the following representation of the lattice spin operators holds:[18]

$$S_n^z = \frac{1}{\sqrt{\pi}} \partial_x \phi + \frac{a}{\pi} \sin\{2k_F x + \sqrt{4\pi}\phi\} + m,$$

$$S_n^- = (-1)^n e^{-i\theta\sqrt{\pi}} \{c + b \sin(2k_F x + \sqrt{4\pi}\phi)\}. \tag{5}$$

Here $m(h)$ is the ground-state magnetization per spin which determines the Fermi wave vector $k_F = \left(\frac{1}{2} - m\right)\pi$ and is known exactly from the Bethe ansatz results.[20] Nonuniversal constants a, b, and c for general h have been extracted numerically from the density matrix renormalization group (DMRG) calculations.[21]

We treat the J_1 interchain coupling term perturbatively, representing two decoupled chains in terms of Gaussian models of the form (4). It is convenient to pass to the symmetric and antisymmetric combinations of the fields describing the individual chains,

$$\phi_\pm = (\phi_1 \pm \phi_2)/\sqrt{2K}, \quad \theta_\pm = (\theta_1 \pm \theta_2)\sqrt{K/2}.$$

The effective Hamiltonian describing low-energy properties of the model (1) takes the following form:

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_0^+ + \mathcal{H}_0^- + \mathcal{H}_{\text{int}}, \quad \mathcal{H}_0^\pm = \frac{v}{2}[(\partial_x \theta_\pm)^2 + (\partial_x \phi_\pm)^2],$$

$$\mathcal{H}_{\text{int}} = g_1 \cos k_F \cos(k_F + \sqrt{8\pi K_-}\phi_-) - g_2 \partial_x \theta_+ \sin(\sqrt{2\pi/K_-}\theta_-). \tag{6}$$

Here the Fermi velocity $v \propto J_2$, while the couplings $g_{0,1,2} \propto J_1 \ll v$. The renormalized LL parameter is given by

$$K_- = K(h)\{1 + J_1 K(h)/[\pi v(h)]\}. \tag{7}$$

Note that to the first order in J_1/J_2 the correction to K_- for the zigzag type of interchain coupling is twice larger compared to that for the ladder type of coupling.

Only the relevant terms are shown in Eq. (6), including the "twist operator" with nonzero conformal spin[2] (the second term in $\mathcal{H}_{\text{int}}$) which is responsible for the existence of chiral phase in chains with easy-plane anisotropy.[2,5] All the other terms, omitted in (6), are made either irrelevant or incommensurate by the external magnetic field. The intersector part of (6) contains a term which can be identified as an infrared limit of the product of in-chain and interchain chiralities: one can show that

$$\partial_x \theta_+ \sin \sqrt{\frac{2\pi}{K_-}}\theta_- \propto (\kappa^z_{2i-1,2i+1} + \kappa^z_{2i,2i+2})\kappa^z_{2i,2i+1}, \tag{8}$$

where $\kappa^z_{i,j} \equiv (\vec{S}_i \times \vec{S}_j)^z$. Among the irrelevant terms omitted in (6) there is one which is nevertheless important for the discussion of magnetization cusps below, namely

$$\mathcal{H}_{\rm int} \mapsto \mathcal{H}_{\rm int} + g_3 \partial_x \phi_+ \sin(\sqrt{8\pi K_-}\phi_-), \tag{9}$$

where $g_3 \propto J_1$.

The Hamiltonian (6) gives the minimal effective field theory describing the low-energy dynamics of a strongly frustrated ($\alpha \gg 1$) spin-$\frac{1}{2}$ zigzag chain for a nonzero magnetization m. For small m, the LL parameter $K_- \simeq \frac{1}{2}$, and the intersector g_2 term has a higher scaling dimension than the strongly relevant g_1 term in the antisymmetric sector. In this case, the system is in a phase with relevant coupling in antisymmetric sector, as is discussed in Ref. 22 (this phase was later dubbed "even-odd (EO) phase" in Ref. 16). In contrast to that, at $h=0$ all terms generated by the zigzag coupling are only marginal.

When the magnetic field h increases, the chirality product operator (8) can become more relevant than the g_1 term controlling the field ϕ_-; the latter term becomes less relevant with the increase of h as well as with the increase of the zigzag *antiferromagnetic* coupling J_1. To study this situation, one can apply a mean-field decoupling procedure to the intersector (twist) term in the spirit of Ref. 2. Strictly speaking, mean-field arguments are potentially dangerous in one dimension; however, since the mean-field-based predictions of Nersesyan *et al.*[2] were later confirmed numerically,[6] one may hope that this procedure is able to capture the essential physics of the system. At the mean-field level, the interaction $\mathcal{H}_{\rm int}$ takes the form

$$\mathcal{H}_{MF} = g_1 \cos k_F \cos(k_F + \sqrt{8\pi K_-}\phi_-) - g_2 \partial_x \theta_+ \left\langle \sin \sqrt{\frac{2\pi}{K_-}}\theta_- \right\rangle - g_2 \langle \partial_x \theta_+ \rangle \sin \sqrt{\frac{2\pi}{K_-}}\theta_-. \tag{10}$$

Remarkably, the mean-field Hamiltonian reveals a competition between the basic and dual field terms of the form $\sin(\gamma\phi_-)$ and $\sin(\delta\theta_-)$ with $\gamma\delta = 4\pi$; under this latter "self-duality" condition, one can show that the competition of $\sin(\gamma\phi_-)$ and $\sin(\delta\theta_-)$ leads to the Ising quantum phase transition.[23,24] To find the critical magnetic field h_{cr}, which corresponds to this transition, we equate the masses produced by the two competing operators

$$\left(\frac{g_1}{v}\right)^{1/(2-d_1)} \sim \left(\frac{g_2}{v}\langle \partial_x \theta_+ \rangle\right)^{1/(2-d_2)}, \tag{11}$$

where

$$d_1 = 2K_-, \quad d_2 = 1/(2K_-)$$

are the scaling dimensions of the corresponding operators. The averages in (10) can now be found from the mean-field equations (self-consistency conditions)

$$\langle \partial_x \theta_+ \rangle \sim \frac{g_2}{v} \langle \sin \sqrt{2\pi/K_-}\theta_- \rangle,$$

$$\langle \sin \sqrt{2\pi/K_-}\theta_- \rangle \sim \left(\frac{g_2}{v}\langle \partial_x \theta_+ \rangle\right)^{d_2/(2-d_2)}, \tag{12}$$

which yields

$$\langle \partial_x \theta_+ \rangle \sim (g_2/v)^{1/(1-d_2)}. \tag{13}$$

Substituting (13) back into (11) and taking into account that both g_1 and g_2 are of the order of J_1, one can obtain the transition condition by equating the exponents of g_1 and g_2 on the right-hand and on the left-hand sides. In this way, one obtains the simple equation $4K_-^2 - 2K_- - 1 = 0$ for the renormalized LL parameter K_- at the transition, which together with (7) leads to the following equation for the critical field h_{cr}:

$$K(h_{cr}) = \frac{q}{2}\left\{1 - \frac{J_1 K(h_{cr})}{\pi v(h_{cr})}\right\}, \tag{14}$$

where $q \equiv (\sqrt{5}+1)/2$ is the celebrated "golden mean," $2K_-(h_{cr}) = q$.

At $h = h_{cr}$ a second-order phase transition belonging to the Ising universality class takes place. On the $h < h_{cr}$ side of this transition, one has the EO phase which enjoys all microscopic symmetries of the Hamiltonian and, to our knowledge, is not characterized by any conventional order parameter. The other side of this transition (at $h > h_{cr}$) corresponds to the chiral phase with spontaneously broken parity. Indeed, using the spin operator representation (5), it is easy to see that the leading term of the z component of the vector chirality is in a simple way connected to the antisymmetric dual field θ_-

$$\kappa^z_{2n} \mapsto \frac{1}{2} i \{S^+_{1,n} S^-_{2,n+1} - S^-_{1,n} S^+_{2,n+1}\} \mapsto \sin\left(\sqrt{\frac{2\pi}{K_-}}\theta_-\right),$$

where the indices 1 and 2 label two weakly coupled chains. The transition at $h = h_{cr}$ is accompanied by the emergence of a nonzero value of $\langle \sin(\sqrt{2\pi/K_-}\theta_-)\rangle$, and thus, it is a transition into the chiral phase. The scalar chirality also gets long-range ordered in the chiral phase, due to the explicitly broken time-reversal symmetry by external magnetic field (or, more simply, due to the presense of a finite magnetization). The average scalar chirality $\tilde{\kappa}$ is trivially obtained as a product of the vector chirality κ^z and the average magnetization m.

The fact that $K(h)$ is a monotonically increasing function[19,25] implies that the critical field decreases with increasing the *antiferromagnetic* zigzag coupling J_1

$$(\partial h_{cr}/\partial J_1) < 0 \quad \text{for } J_1 > 0. \tag{15}$$

Numerically solving Eq. (14), one obtains that the maximal

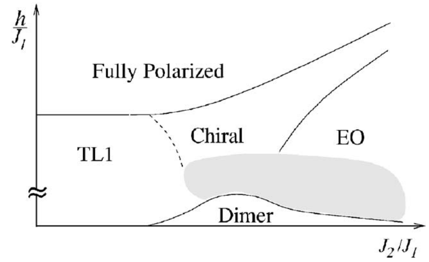

FIG. 2. Schematic view of the high-field part of the phase diagram of a $S=\frac{1}{2}$ frustrated chain. TL1 and EO are nonchiral gapless phases described by one-component Tomonaga-Luttinger liquids. The transition line between EO and chiral phases belongs to the Ising univerality class. The transition between TL1 and chiral phase (marked with a dashed line), as well as the intermediate field region (marked gray) are not captured within the present approach.

value of h_{cr}, achieved at $J_1 \to 0$, is approximately $h_{cr} \simeq 1.7 J_2$, and the spin wave velocity in this limit is still of the order of the bandwidth, $v(h_{cr}) \simeq 0.6 J_2$, which justifies the applicability of bosonization formalism close to h_{cr}. Within this approach, there is no indication that the chiral phase would be destabilized by a further increase of the magnetic field, so one may conclude that it extends from h_{cr} up to the saturation field h_s.

Recently, influence of strong magnetic fields on a spin-$\frac{1}{2}$ zigzag chain was studied numerically by means of the DMRG technique.[17,16] The authors of Refs. 16 and 17 in analogy with the tight-binding *t-t′* model of free spinless fermions (with nearest-neighbor and next-nearest-neighbor hopping) identify the phase below the saturation field for large $\alpha = J_2/J_1$, in terms of the *two-component* Luttinger liquid phase. Our findings suggest an alternative scenario, according to which this phase (the high-field phase denoted TL2 in Fig. 1 of Ref. 16) is still described by a *one-component Luttinger liquid*, albeit with a spontaneously broken left-right symmetry. The conjectured high-field phase diagram is shown in Fig. 2.

It should be remarked that phase boundaries were determined in Ref. 16 by means of analyzing the magnetization curves; the point of transition into the TL2 phase was identified with the point where a *magnetization cusp*[16,17] occurred. The authors of Ref. 17 explained the origin of this cusp, again in the analogy with the *t-t′* model, as the point where two Fermi seas (Luttinger liquids) coalesce into one. We would like to note that a similar magnetization cusp emerges in our description as well, although our approach excludes the existence of a two-component Luttinger liquid. Indeed, the presence of the irrelevant term shown in Eq. (9) implies that in the EO phase, where $\langle \sin(\sqrt{8\pi K_-}\phi_-) \rangle$ acquires a finite value, the Hamiltonian receives an additional contribution of the form const$\times \partial_x \phi_+$. This, in turn, leads to renormalization of the equilibrium value $\langle \partial_x \phi_+ \rangle \mapsto \langle \partial_x \phi_+ \rangle + \text{const} \times \langle \sin(\sqrt{8\pi K_-}\phi_-) \rangle$, so that the magnetization gets renormalized on the EO phase side ($h < h_{cr}$) as

$$m(h) \mapsto m(h) + \text{const} \times (h_{cr} - h)^{2K_-/(2-2K_-)}.$$

Thus, existence of a magnetization cusp does not necessarily imply a transition from the one-component to two-component Luttinger liquid. Comparing our results with the DMRG phase diagram,[16] we suggest that the TL2 phase in Fig. 1 of Ref. 16 should be identified as a chirally ordered phase. For high fields, in the limit $J_1 \ll J_2$ the stability region of this phase expands with increasing J_1, in agreement with our result (15).

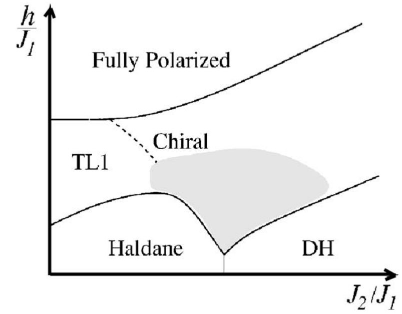

FIG. 3. The conjectured view of the phase diagram for a $S=1$ zigzag chain. DH denotes the "double Haldane" phase separated from the Haldane phase by a first-order transition (Ref. 30). The TL1-chiral transition (dashed line) as well as the intermediate field region (marked gray) are not captured in our approach.

III. $S=1$ FRUSTRATED CHAIN IN A HIGH FIELD

One can obtain a bosonized description of a single (unfrustrated) spin-1 chain in a magnetic field exceeding the Haldane gap Δ by accessing the Luttinger liquid parameters K and v either directly from numerical DMRG studies[26,27] or, in a more exotic way, from the exact solution of the integrable $O(3)$ nonlinear σ model (NLSM), which itself is believed to provide a proper effective field-theoretical description.[28] Then, for fields above the gap, one can use the same bosonized expressions (5) for spin operators, and a zigzag $S=1$ chain in the regime of strong frustration $J_1 \ll J_2$ *and high fields* $h > \Delta \simeq 0.4 J_2$ can be studied along the same lines as was done above for the spin-$\frac{1}{2}$ case.

Treating the zigzag interaction as a perturbation coupling two LLs yields the same effective field theory (6). However, in contrast to the $S=\frac{1}{2}$ case, the LL parameter of a $S=1$ chain turns out to be *increasing* from the free fermion value $K=1$ at $h=\Delta$ with the further increase of the field, so that generally for $h > \Delta$ one has $K > 1$,[26–28] which resembles a 1D Bose gas.[29] This fact leads to a considerable simplification: since $K > 1$, the term proportional to g_1 in (6) is irrelevant, and the only important term is the product of chiralities (8).

In this way, one gets an effective field theory for the isotropic frustrated spin-1 chain in a strong magnetic field which is essentially the same as the one obtained by Nersesyan *et al.*[2] for the *XY*-anisotropic spin-$\frac{1}{2}$ zigzag chain. For the latter model, the mean-field scenario predicting the emergence of a chiral phase was confirmed by extensive numerical work.[3,6,12]

Thus, in contrast to the $S=\frac{1}{2}$ case, in a strongly frustrated spin-1 chain, the EO phase is absent, and the chiral phase probably extends all the way down to the first critical field $h=\Delta$ (see Fig. 3). The crucial property $K(h) \geq 1$, derived by Konik and Fendley,[28] is an intrinsic feature of the $O(3)$

NLSM and, in fact, does not depend on the spin value, so one can expect the same scenario to apply to any integer-S zigzag chain.

IV. LARGE-S FRUSTRATED CHAIN IN THE VICINITY OF THE SATURATION FIELD

In the vicinity of the saturation field h_s the emergence of chirality can be analyzed for an arbitrary spin value S and for a general frustration strength J_2/J_1. In the coherent-state path integral representation, the effective Lagrangian is given by $\mathcal{L}=-\hbar S\Sigma_n(1-\cos\phi_n)\partial_t\theta_n-\langle\mathcal{H}\rangle$. One can introduce the variables

$$z_n=(-1)^n\sin(\phi_n/2)e^{i\theta_n}, \qquad (16)$$

then the dynamical part of the Lagrangian can be rewritten as $i\hbar S\Sigma_n(z_n^*\dot{z}_n-\dot{z}_n^*z_n)$, where the dot denotes differentiation with respect to time. It is also easy to show that the scalar spin product $\vec{S}_n\cdot\vec{S}_{n+d}$, in terms of the new variables, takes the form

$$\vec{S}_n\cdot\vec{S}_{n+d}=(1-2|z_n|^2)(1-2|z_{n+d}|^2)+2(-1)^d(z_n^*z_{n+d}+\text{c.c.})\times\sqrt{1-|z_n|^2}\sqrt{1-|z_{n+d}|^2}. \qquad (17)$$

Near the saturation field, one can expect that the deviations from the fully polarized state are small, $|z_n|\ll 1$, so one can expand the square roots in (17), keeping terms up to the quartic ones.

For $h>h_s$, the fully polarized state is an exact ground state of the Hamiltonian. One-magnon excited states are exact eigenstates as well, and the dispersion of a single magnon has the form

$$\varepsilon(k)=h+J_1\cos k+J_2\cos(2k),$$

with two minima at $k=\pi\pm\lambda$ at $J_2>J_1/4$, where λ is the classical helix pitch given by (2). When h is decreased below h_s, magnons with momenta around $\pi\pm\lambda$ start to "condense" into the ground state. This leads us to the following ansatz for passing to the continuum:

$$z_n=\psi_{R,n}e^{i\lambda n}+\psi_{L,n}e^{-i\lambda n}. \qquad (18)$$

Treating $\psi_{R,L}$ as smooth fields and keeping only nonoscillating terms, one obtains

$$\langle\mathcal{H}\rangle=\int dx\{2S(h-h_s)(|\psi_R|^2+|\psi_L|^2)+2Sh_s(|\psi_R|^2+|\psi_L|^2)^2 + 4S[h_s-4SJ_1(1+J_1^2/J_2^2)\sin^2\lambda]|\psi_R|^2|\psi_L|^2 + 8J_2S^2\sin^2\lambda(|\partial_x\psi_R|^2+|\partial_x\psi_L|^2)\}, \qquad (19)$$

where we have set the lattice constant to 1. It is convenient to rescale the bosonic fields

$$(2S)^{1/2}\psi_{R,L}\to\psi_{1/2},$$

and set the Planck constant to 1; then one finally arrives at the Lagrangian of the form

$$\mathcal{L}=\int dx\sum_{\sigma=1,2}\left\{i\psi_\sigma^*\partial_t\psi_\sigma-\frac{1}{2m}|\partial_x\psi_\sigma|^2+\mu|\psi_\sigma|^2\right\} - \frac{1}{2}\int dx\{u(|\psi_1|^2+|\psi_2|^2)^2+w|\psi_1|^2|\psi_2|^2\}, \qquad (20)$$

recently discussed in the context of 1D, two-component Bose condensates.[31] The Lagrangian parameters are, in our case, given by

$$\mu=h_s-h,\quad m^{-1}=8J_2S\sin^2\lambda,$$

$$u=h_s/S,\quad w=2\{u-4J_1(1+J_1^2/J_2^2)\sin^2\lambda\}. \qquad (21)$$

In the harmonic fluid approach,[32] the field operators and densities can be expressed through scalar bosonic fields ϑ, φ as

$$|\psi_\sigma|^2=\{\rho_\sigma+\partial_x\varphi_\sigma/\pi\}\sum_m e^{2im(\pi\rho_\sigma x+\varphi_\sigma)},$$

$$\psi_\sigma=\{\rho_\sigma+\partial_x\varphi_\sigma/\pi\}^{1/2}e^{i\vartheta_\sigma}\sum_m e^{2im(\pi\rho_\sigma x+\varphi_\sigma)}, \qquad (22)$$

and for $\mu>0$ the Lagrangian (20) describes two LLs of the form (4), with a density-density interaction. In contrast to Ref. 31, in our case, the total particle numbers of the components $n_{1,2}=\int dx|\psi_{1,2}|^2$ (which are separately conserved) are not fixed, but are chosen by the system so as to minimize the energy at $\mu>0$. It is easy to show that for $w>0$ the system is unstable against any perturbation making $\rho_1\neq\rho_2$: indeed, e.g., for $\rho_1>\rho_2$ the interaction term leads to renormalization $\rho_\sigma\mapsto\rho_\sigma+\langle\partial_x\varphi_\sigma\rangle/\pi$ with $\langle\partial_x\varphi_1\rangle>\langle\partial_x\varphi_2\rangle$. In terms of the two-component Bose condensates this corresponds to the "demixing" instability.[31] As a result, the chiral Z_2 symmetry breaks spontaneously and one of the bands σ gets fully depleted. The effective theory is a *single* Luttinger liquid with the parameter $K>1$ depending on the dimensionless coupling constant

$$\gamma=\frac{mu}{\rho_0}\simeq\frac{\pi}{2S\sin\lambda}\left(\frac{h_s}{4J_2S(1-h/h_s)}\right)^{1/2}, \qquad (23)$$

where $\rho_0=(2\mu m)^{1/2}/\pi$ is the equilibrium density for small μ (i.e., in the vicinity of the saturation field). For $h\to h_s$, when $\gamma\gg 1$, the LL parameter tends to 1 and is given by $K\simeq 1+4/\gamma$, and for $\gamma\ll 1$ (which, despite the condition $\rho_0\ll 1$, is formally possible for large S) one has $K\simeq\pi/\sqrt{\gamma}$.[29]

The chirality order parameter is directly related to the density difference, $\kappa\simeq\langle|\psi_1|^2-|\psi_2|^2\rangle\sin\lambda$. Neglecting the depleted field and using a known expression for the density correlator,[29] one obtains the leading asymptotics of the chirality correlation function

$$\langle\kappa(x)\kappa(0)\rangle\simeq\frac{S^2}{\pi^2}\left\{\frac{h_s-h}{J_2S}-\frac{2K\sin^2\lambda}{x^2}\right\}. \qquad (24)$$

The longitudinal spin correlator $\langle S^z(x)S^z(0)\rangle$ is also related to the density and behaves similarly to (24). The leading part of the transversal spin correlator can be expressed through $\langle\psi^\dagger(x)\psi(0)\rangle$ and is given by

$$\langle S^{+}(0)S^{-}(x)\rangle \simeq 2S\rho_0\left(\frac{K}{\pi\rho_0 x}\right)^{1/(2K)} e^{i\lambda x}. \qquad (25)$$

V. DISCUSSION

In summary, we have shown that a sufficiently strong magnetic field applied to a spin-S isotropic J_1-J_2 zigzag chain induces a phase with spontaneously broken Z_2 symmetry, which is characterized by the long-range vector chirality order and emerges immediately below the saturation field if the frustration strength J_2/J_1 exceeds the classical Lifshits point value $\frac{1}{4}$. This chiral phase is *gapless* and its low-energy physics is effectively described by a *one-component* Luttinger liquid. Our results disagree with the two-component Luttinger liquid scenario proposed in Refs. 16 and 17 and, in fact, may necessitate reconsidering the phase diagrams of other frustrated spin models, particularly of a biquadratic-bilinear spin-1 chain in a magnetic field.[33] To clarify this issue, one could calculate numerically the chirality correlator $\langle\kappa_0^z\kappa_n^z\rangle$ in the limit $n\to\infty$ above the cusp singularity. For spin-$\frac{1}{2}$ chain, such a correlator was calculated only for very short distances[34] and indicated emergence of at least short-range chirality correlations for h directly below the saturation field h_s.

The chiral phase should be able to survive finite temperature effects since it involves breaking of the discrete Z_2 symmetry. Less trivially, it has also a chance to survive the three-dimensional interaction without tranforming into a usual helical long-range order: As noted by Villain,[9] at finite temperatures the chirality correlation length is much larger than the spin correlation length, so with decreasing temperature the chiral order should set in before the helical spin order does.

Several materials are known which realize zigzag spin-$\frac{1}{2}$ chains (see Table 1 of Ref. 35). A promising candidate substance for detecting the field-induced chirality would be $(N_2H_5)CuCl_3$, since its small exchange constants $J_1\simeq 4$ K and $J_2\simeq 16$ K make feasible the task of attaining magnetic fields comparable to J_2. Experimentally, the projection of vector chirality $\vec{\kappa}$ on the applied field direction could be detected by comparing inelastic scattering intensities for oppositely polarized neutrons, as it was done for the triangular lattice antiferromagnet $CsMnBr_3$;[36] a similar route can be employed with polarized light. We hope that our results will stimulate further experimental work in this direction.

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