Nonlinear self-phase-locking effect in an array of current-driven magnetic nanocontacts

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We show that in an array of magnetic nanocontacts driven by spin-polarized current a self-phase-locking effect can lead to a regime in which all the contacts are generating microwave oscillations with the same frequency and phase, and coherent addition of microwave power generated by individual nanocontacts is possible. The mechanism of this self-phase-locking effect is strongly nonlinear, while the frequency band of phase locking is an order of magnitude larger than in the usual coupled generators and depends on the direction of the bias magnetic field applied to the nanocontacts.

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The theoretical prediction[1–3] and experimental observation[4–9] of microwave generation in magnetic nanostructures driven by spin-polarized current open the possibility for the development of a new class of tunable (by both bias current and bias magnetic field) microwave generators for nanoworld. Recently, it has been demonstrated experimentally[10] that the microwave oscillation generated in a current-driven magnetic nanocontact can be phase locked to the frequency of a small external sinusoidal current added to the constant bias current passing through the nanocontact. This phenomenon of phase locking, besides being interesting fundamentally, is of substantial practical importance, especially in view of possible self-phase-locking of an array of magnetic nanocontacts by self-induced magnetic dipolar field or by common current passing through all the contacts. The use of self-phase-locked arrays of nanocontacts can lead to an increase in the resulting generated microwave power, and also to the reduction of the linewidth of the generated microwave signal, thus making this type of nanosized generator really practical.

In this paper we theoretically investigate the possibility of self-phase-locking of an array of magnetic nanocontacts by a self-induced dipolar magnetic field. First, we consider an isolated nanocontact driven by a constant bias current and determine the conditions of its phase locking to the frequency of a small external microwave magnetic field. We show that the mechanism of this phase-locking effect is strongly nonlinear in contrast with the case of the usual microwave oscillator.[11,12] We also show that the phase-locking band in a nanocontact (typically 100–300 MHz) is substantially larger than in the usual oscillator and larger than the typical linewidth (10–70 MHz) of the microwave generation in the nanocontact itself. Then we use the results obtained for an isolated nanocontact to determine the conditions for self-phase-locking of an array of nanocontacts, assuming that the coupling in the array takes place through the dipolar magnetic fields created by the individual generating nanocontacts.

The dynamics of the magnetization vector $\boldsymbol{M}$ in a "free" magnetic layer of a nanocontact is described by the modified Landau-Lifshitz equation[1,3]

$$d\boldsymbol{M}/dt = \gamma(\boldsymbol{H}_{\text{eff}} \times \boldsymbol{M}) + \boldsymbol{T}_I \tag{1}$$

with spin-transfer torque $\boldsymbol{T}_I$ that describes the effect caused by a spin-polarized current. In Eq. (1) $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio (g is the spectroscopic Landé factor, μ_B is the Bohr magneton, $\hbar = h/2\pi$, and h is the Planck constant), and $\boldsymbol{H}_{\text{eff}}$ is the effective magnetic field:

$$\boldsymbol{H}_{\text{eff}} = \boldsymbol{H}_0 - 4\pi(\boldsymbol{M}\cdot\hat{\boldsymbol{n}})\hat{\boldsymbol{n}} + \tilde{\boldsymbol{h}}(t). \tag{2}$$

Here $\boldsymbol{H}_0$ is the external static magnetic field, the second term describes the demagnetization field ($\hat{\boldsymbol{n}}$ is the unit vector in the direction of the normal to the free layer), and $\tilde{\boldsymbol{h}}(t) = \boldsymbol{h}(e^{-i\omega t} + \text{c.c.})$ is the external microwave magnetic field (ω is the external microwave frequency, and c.c. denotes a complex-conjugated term). For simplicity, we do not include in Eq. (2) contributions from the exchange energy and from the energy of crystallographic anisotropy. The spin-transfer torque $\boldsymbol{T}_I$ in Eq. (1) can be written in the form[1,3]

$$\boldsymbol{T}_I = \frac{\sigma_0 I}{M_0}[\boldsymbol{M} \times (\boldsymbol{M} \times \hat{\boldsymbol{p}})]. \tag{3}$$

Here I is the current, $\hat{\boldsymbol{p}}$ is the unit vector in the direction of the spin polarization of the current, and $\sigma_0 = \varepsilon g\mu_B/2eM_0LS$, where ε is the spin-polarization efficiency defined in Refs. 1 and 3, e is the modulus of the electron charge, L is the thickness of the free magnetic layer, and S is the cross-sectional area of the nanocontact.

In the following we assume that the bias current I only slightly exceeds the threshold current I_c for microwave generation, so the transverse part of the magnetization $\boldsymbol{m}_\perp \equiv \boldsymbol{M} - M_z\hat{\boldsymbol{z}}$ (where $\hat{\boldsymbol{z}}$ is the direction of equilibrium magnetization) is small. We also assume that the angle γ_p between the equilibrium magnetization and the current polarization direction $\hat{\boldsymbol{p}}$ (i.e., $\hat{\boldsymbol{p}} = \cos\gamma_p\hat{\boldsymbol{z}} + \sin\gamma_p\hat{\boldsymbol{x}}$) is small, so we can approximately write the spin-transfer torque (3) in the form[13]

$$\boldsymbol{T}_I \simeq \sigma_0 I \cos\gamma_p \frac{M_z}{M_0}\boldsymbol{m}_\perp - \sigma_0 I \sin\gamma_p \hat{\boldsymbol{x}}, \tag{4}$$

The first term in Eq. (4) gives the current-induced negative damping and is essential to the phenomenon of microwave generation. The second term in Eq. (4) for a constant bias current I can be neglected, because it only describes small changes in the equilibrium orientation of $\boldsymbol{M}$. If, however, the bias current has a *microwave component* with the frequency close to the frequency of generated precession, the second term plays the role of a resonant external signal in exactly the same way as the external microwave magnetic field of a similar frequency.

Using the approximate perturbative analysis described in detail in Refs. 13 and 14, one can derive an equation for the dimensionless complex amplitude[15] $b(t) \simeq (m_x - im_y)/M_0$ of the magnetization precession in the same way as it was done in Ref. 13 [see also Eq. (11) in Ref. 14 and Eq. (25) in Ref. 13]:

$$\frac{db}{dt} = -i(\omega_0 + N|b|^2)b - \Gamma b + \sigma I(1 - |b|^2)b + \Lambda e^{-i\omega t}, \quad (5)$$

where $\sigma = \sigma_0 \cos \gamma_p$, $\omega_H = \gamma H$, $\omega_M = 4\pi\gamma M_0$,

$$\omega_0 = \sqrt{\omega_H(\omega_H + \omega_M \cos^2 \theta)}, \quad (6a)$$

$$N = \omega_M \omega_H (3\omega_H^2 \sin^2 \theta / \omega_0^2 - 1)/2\omega_0. \quad (6b)$$

Here H and θ are the magnitude and the out-of-plane angle of the *internal* bias magnetic field, which are connected with the magnitude H_0 and the out-of-plane angle θ_0 of the *external* bias magnetic field by the usual electrodynamic boundary conditions:

$$H_0 \cos \theta_0 = H \cos \theta, \quad H_0 \sin \theta_0 = (H + 4\pi M_z)\sin \theta. \quad (7)$$

In Eq. (5) we phenomenologically added a dissipation term $\sim\Gamma$ that accounts for both the internal damping in the layer and energy drift from the excitation region.[13]

The external signal amplitude Λ in Eq. (5) depends on the type of the external microwave signal. If the phase locking is achieved by addition of a microwave *magnetic field*, then

$$\Lambda = \gamma(h_y + ih_x)/\sqrt{2} \equiv \gamma h_0. \quad (8a)$$

If, on the other hand, the external signal is supplied in the form of a microwave *modulation of the bias current*, $I(t) \to I + \Delta I \cos \omega t$, then Λ arises from the second term in Eq. (4) and is given by

$$\Lambda = -\sigma \tan \gamma_p \Delta I / 2\sqrt{2}. \quad (8b)$$

It is clear that the phase locking by a microwave modulation of current is impossible for collinear orientations of equilibrium magnetization and spin polarization of the current, i.e., for $\gamma_p = 0$.

Note that Eq. (5) is the usual equation describing dynamics of an auto-oscillator with nonlinear frequency shift under the action of an external signal. Therefore, our results are valid for any such auto-oscillator.

The "unforced" (i.e., for $\Lambda = 0$) equation (5) admits a stationary solution that describes a free autogeneration and has the form[13,14] $b(t) = B_0 e^{-i\tilde{\omega}_0 t}$, where

$$|B_0|^2 = (\zeta - 1)/\zeta, \quad \tilde{\omega}_0 = \omega_0 + N|B_0|^2, \quad (9)$$

and the supercriticality ζ is defined as $\zeta \equiv I/I_c \equiv \sigma I/\Gamma$, where I_c is the critical bias current at which microwave generation starts.

The forced Eq. (5) has a stationary phase-locked solution in the form $b(t) = Be^{-i\omega t}$, where the amplitude B is implicitly determined from the equation

$$B = -\frac{\Lambda}{\Gamma(\zeta - 1 - \zeta|B|^2) + i(\omega - \omega_0 - N|B|^2)}. \quad (10)$$

One can significantly simplify this equation, using the definitions

$$P_0 \equiv |B_0|^2, \quad P \equiv |B|^2, \quad \Omega \equiv \omega - \omega_0 - NP_0, \quad (11a)$$

and introducing the following dimensionless variables:

$$x \equiv \frac{P}{P_0}, \quad \lambda^2 \equiv \left(\frac{|\Lambda|}{\zeta\Gamma}\frac{1}{P_0}\right)^2 \frac{1}{P_0}, \quad (11b)$$

$$\eta \equiv \frac{N}{\zeta\Gamma}, \quad \xi \equiv \frac{\Omega}{\zeta\Gamma}\frac{1}{P_0}. \quad (11c)$$

Here x is the normalized power of forced generation, λ is the normalized amplitude of the external signal, and η and ξ are, respectively, the normalized nonlinear frequency shift and frequency detuning of the external signal.

In these dimensionless variables one can rewrite (10) as a third-order equation for x:

$$(x-1)^2 x + [\xi - \eta(x-1)]^2 x = \lambda^2. \quad (12)$$

This equation always has at least one positive root, i.e., for any amplitude λ and frequency ξ of the external microwave field there exists a forced phase-locked solution. This solution, however, is not always stable. At the same time, only the stable solutions correspond to a real phase locking of the autogenerator.

Performing the standard stability analysis of Eq. (5), one can derive the critera for the stability of the phase-locked solution. These criteria can be written as two simple conditions for the normalized power of generation x. The first condition, arising from the requirement of stability of the solution with respect to small perturbations having *the same frequency* ω ("internal" stability) can be written as

$$\frac{dx}{d\lambda^2} > 0, \quad (13a)$$

i.e., it requires the increase of the power of the phase-locked oscillation with the increase of the external signal power. The condition of "external" stability (i.e., stability with respect to small perturbations having *different frequencies*) for the same solution has the following form:

$$x > \frac{1}{2}. \quad (13b)$$

Thus, we are interested in large-amplitude solutions of Eq. (12). We note that the large amplitude of the normalized power x does not necessarily mean the large absolute power P of the forced oscillation. The condition (13b) simply means that to be stable the forced oscillation should have power that is comparable to the power of the free-running oscillation in the same nanocontact.

It is clear from Eq. (12) that for small external signal λ

and, respectively, small frequency deviation ξ, the amplitude of such a solution only slightly deviates from the amplitude of the free-running oscillation, $|x-1| \ll 1$. Using the smallness of $|x-1|$, λ, and ξ, we can reduce Eq. (12) to a second-order equation for $(x-1)$, neglecting all third-order terms in small quantities:

$$(1+\eta^2)(x-1)^2 - 2\eta\xi(x-1) + \xi^2 - \lambda^2 = 0. \tag{14}$$

The solution of this equation, which satisfies both stability criteria (13) and represents a real experimentally observable phase-locked solution, has the following form:

$$x = 1 + \frac{\eta\xi + \sqrt{(1+\eta^2)\lambda^2 - \xi^2}}{1+\eta^2}. \tag{15}$$

This solution exists only when the expression under the square root in Eq. (15) is positive, which immediately gives the frequency interval of phase locking in the form $|\xi| < \tilde{\Delta} \equiv \sqrt{1+\eta^2}\lambda$. In dimensional units the phase-locking interval can be written as

$$\Delta = \frac{|\Lambda|}{\sqrt{P_0}}\sqrt{1+\left(\frac{N}{\zeta\Gamma}\right)^2}. \tag{16}$$

Equation (16) in the limiting case $|N| \ll \zeta\Gamma$ is reduced to the standard equation that determines the phase-locking bandwidth for an autogenerator *without nonlinear frequency shift*. The phase-locking bandwidth Δ_0 in this case can be written as

$$\Delta_0 \equiv \frac{|\Lambda|}{\sqrt{P_0}} = \omega_M \left|\frac{h_0}{4\pi m_\perp}\right|, \tag{17a}$$

where $m_\perp = M_0\sqrt{P_0}$ is the characteristic amplitude of the free-running oscillation. The locking interval written in this form (characteristic frequency ω_M of the system times the ratio of the amplitude h_0 of the external signal to the amplitude $4\pi m_\perp$ of the internal free-running oscillation) literally coincides with the well-known classical result [see, e.g., Eq. (19c) in Ref. 12]. If the external signal is supplied in the form of the microwave modulation of the bias current, the locking bandwidth can be written in a similar form:

$$\Delta_0 \equiv \frac{|\Lambda|}{\sqrt{P_0}} = \omega_I \left|\frac{\delta I}{I_\perp}\right|, \tag{17b}$$

where $\omega_I = \sigma I$ is the characteristic frequency associated with the bias current, $\delta I = \Delta I \tan \gamma_p / 2\sqrt{2}$ is the *effective microwave signal current*, and $I_\perp = I\sqrt{P_0}$ is the characteristic microwave current of the free-running oscillation.

We would like to stress that the classic expression (17) obtained in the limit $|N| \ll \zeta\Gamma$ gives a *qualitatively* incorrect result in the case of phase locking of an autogeneration in magnetic nanocontacts: the nonlinear frequency shift in such systems is of the order of the generated frequency, $|N| \sim \omega_0$ [see Eq. (6b) and Ref. 13], while the relaxation frequency can be estimated as $\Gamma \sim \alpha_G \omega_0$, where $\alpha_G \simeq 0.01$ is the Gilbert damping parameter.[16] Thus, for $\zeta \sim 1$ one gets $|N|/\zeta\Gamma \sim 1/\alpha_G \simeq 100 \gg 1$, and the phase-locking interval Eq. (16) is dominated by the second term under the square root. In this case the locking interval Δ is much larger ($\sim$100 times) than the classical result (17). The physical reason for such a significant increase of the phase-locking interval is clear: in the case of a strongly nonlinear oscillator even small changes in the oscillation amplitude can result in matching of the nonlinear eigenfrequency of the auto-oscillator to the frequency of the external sinusoidal signal. Thus, in this case the primary phase-locking mechanism is a *nonlinear frequency matching* with the external signal, and this mechanism is very different from the classical phase-locking mechanism.[12]

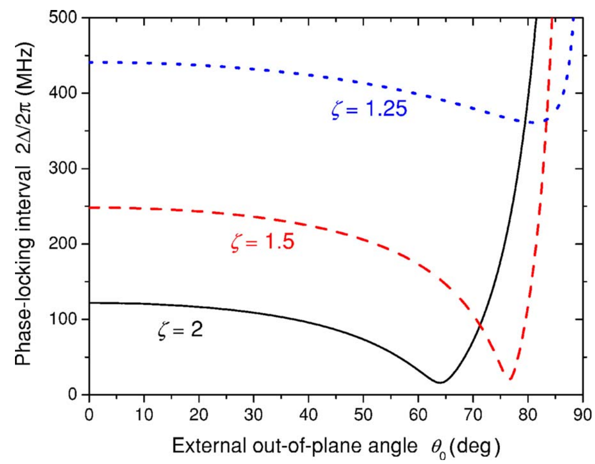

FIG. 1. (Color online) Dependence (16) of the phase-locking bandwidth Δ on the direction of the external bias magnetic field θ_0 for different values of the supercriticality ζ: solid line, $\zeta=2$; dashed line, $\zeta=1.5$; dotted line, $\zeta=1.25$. Saturation magnetization $4\pi M_0 = 9$ kOe, external magnetic field $H_0 = 7$ kOe, Gilbert damping parameter $\alpha_G = 0.01$, amplitude of the external microwave magnetic field $|h_0| = 2$ Oe (equivalent to a microwave current $\Delta I = 1$ mA for $\gamma_p = 5°$).

In Fig. 1 we show the phase-locking band (16) as a function of the *external* magnetization angle θ_0 calculated from Eq. (16) using Eqs. (7) and (6b). Following the angular dependence of the nonlinear frequency shift coefficient N, the phase-locking bandwidth has a pronounced minimum at some critical angle for which $N=0$. At this minimum point the bandwidth coincides exactly with the classical value given by Eq. (17).

One can see that for reasonable values of the parameters the frequency range of phase locking can be around 100–300 MHz. Taking into account that the linewidth of the microwave generation in a nanocontact is of the same order of magnitude or smaller,[7] it is possible to predict with a large degree of confidence that the effect of self-phase-locking in an array of magnetic contacts coupled either by means of the dipole-dipole interaction or by means of the common bias current can be observed experimentally.

Let us make an approximate estimation of the maximum distance between the nanocontacts in an array that still allows the possibility of self-phase-locking of the array. We shall assume that individual nanocontacts are coupled by the dipole-dipole interaction, while the bias currents passing through the contacts are independent. In this approximate estimation we shall ignore the vectorial properties of the dipolar field and details of the geometry of the considered array of nanocontacts.

In a real array of magnetic nanocontacts a distribution of contact parameters (such as contact radius R_c) will lead to the situation when with the increase of bias current I one of the contacts will start to generate first at a certain frequency $\omega^* \approx \omega_0$. The variable magnetization of this contact precessing with the frequency ω^* will create a variable dipolar field with amplitude that can be approximately estimated as $h_{\text{dip}} \simeq m_\perp V_c / a^3$, where $V_c = \pi R_c^2 L$ is the volume of the nanocontact (R_c is the nanocontact radius, and L is the thickness of the free magnetic layer). This field will play the role of an external forcing signal $\Lambda \simeq \gamma h_{\text{dip}}$ for all the neighboring nanocontacts, and if this signal has a sufficient amplitude to guarantee that the phase-locking band Eq. (16) larger than the frequency mismatch between different nanocontacts (i.e., inhomogeneous broadening of the generation linewidth) $\delta\omega$, other nanocontacts (which start to generate with the increasing bias current) will be phase locked to the first one. Thus, due to this mutual dipole-dipole interaction a single generation frequency will be imposed on all the generating nanocontacts.

With the help of Eq. (16), the condition $\Delta > \delta\omega$ of such self-phase-locking can be written in the form

$$\omega_M \left| \frac{h_{\text{dip}}}{4\pi m_\perp} \right| \frac{|N|}{\zeta\Gamma} > \delta\omega. \tag{18}$$

Using the estimation for the dipolar field h_{dip}, one can find the restriction for the distance a between the neighboring contacts:

$$a^3 < a_{\max}^3 \equiv \frac{\omega_M}{4\pi\delta\omega} \frac{|N|}{\zeta\Gamma} V_c. \tag{19}$$

In the particular case of the in-plane ($\theta_0 = 0$) magnetization by a moderate bias field ($H_0 \leqslant 4\pi M$) the expression for the maximum distance between the contacts can be written in an especially simple form:

$$a_{\max} = \left(G \frac{\omega_0}{\delta\omega} \frac{1}{\alpha_{\text{G}}} V_c \right)^{1/3}, \tag{20}$$

where G is the coefficient of the order of unity that accounts for the vectorial properties of the dipolar magnetic field and the details of the array geometry. We note that since the dependence of $a_{\max}$ in Eq. (20) on both the coefficient G and the bias magnetic field is rather weak (cubic root), Eq. (20) with $G \sim 1$ can be used for approximate estimations of the maximum distance between the contacts in most cases. In particular, Eq. (20) shows that for typical values of parameters ($\alpha_{\text{G}} = 0.01$, $R_c = 15$ nm, $L = 5$ nm), even for a relatively large inhomogeneous broadening $\delta\omega / \omega_0 = 0.1$, the maximum distance between the contacts at which self-phase-locking is still possible is quite large: $a_{\max} \simeq 150$ nm$=10R_c$.

In conclusion, we have shown that the mechanism of phase locking in generating current-driven magnetic nanocontacts is strongly nonlinear and the frequency band of phase locking is much larger than the natural linewidth of generation. We also demonstrated that in an array of generating nanocontacts, even in the case when the inhomogeneous distribution of generation frequencies is about 10%, the self-phase-locking of all the contacts is possible if the spatial separation between them significantly exceeds the contact radius ($a \sim 10R_c$).

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