PHYSICAL REVIEW B **72**, 085347 (2005)

Nonlinear suppression of relaxation in dynamic localization phenomenon in a double quantum dot

Dmitry Solenov[1] and Vladimir A. Burdov[2]
[1]*Department of Physics, Clarkson University, Potsdam, New York 13699-5822, USA*
[2]*Department of Theoretical Physics, University of Nizhni Novgorod, Nizhni Novgorod 603950, Russia*

(Received 21 December 2004; revised manuscript received 16 May 2005; published 26 August 2005)

The role of decoherence in controllable electron dynamics in a double quantum dot, influenced by external ac and dc electric fields, has been analyzed. Using the quasienergy formalism we have obtained the reduced density matrix of the electron subsystem and found the time evolution of the occupation probabilities in each quantum dot. It was shown that the dissipation caused by the phonon environment disappears under certain relations between electric field parameters. In this case one may perform a dynamic localization and form stable electron states localized within one of the dots. The suppression of dissipation is essentially a nonlinear effect, which is possible only in strong electric fields.

DOI: 10.1103/PhysRevB.72.085347 PACS number(s): 73.21.La, 03.65.−w, 73.22.Dj

I. INTRODUCTION

Recent progress in nanotechnology has created the base for development of single-electron techniques. The principle of operation for single-electron devices implies the possibility of controlling the wave function of an individual electron using, for example, alternating external electromagnetic fields. Manipulation of charge or spin dynamics of an individual electron by means of external ac fields is often referred to as "dynamical control of electron states."

Suitable objects for investigation in this field include structures consisting of several tunnel-coupled quantum dots or wells. In particular, the possibility of creating electron states localized in some "site" and then transferring it to the neighboring site has been discussed for double quantum dots[1,2] and wells[3–5] as well as lattices of quantum wells and dots.[6]

Theoretical research in this field grew up from two opposite phenomena. One of them is acceleration of electron tunneling between two coupled quantum wells[7,8] in an ac field. The other one is the phenomenon of dynamic localization in quantum-well lattices,[9–11] double quantum wells,[12,13] and dots.[14,15] The name "dynamic localization," introduced by Dunlap and Kenkre,[9] means that the electron wave packet is "locked" inside one of the quantum wells (or dots) of a double- (multi)well (dot) structure by an alternating electric field. On the other hand, switching the ac field off allows the wave packet to tunnel through the potential barrier between the two adjacent wells (dots). For this reason, the localization of electron density within one of the wells in the double-well structure by an ac field has been called "coherent destruction of tunneling" by Grossmann *et al.*[13] Both terms are often mentioned since their first appearance and describe the same phenomenon as pointed out above. In what follows, we will use the term dynamic localization.

The authors of Refs. 9–15 have shown that in order to lock the electron density in one of the wells, one should apply a strong electric ac field with a certain relation between its amplitude and frequency. In connection with this, a strong ac field with either slowly varied amplitude[3,4] or adiabatically changed bias voltage[1,2,5,6] has been considered to be a candidate to drive the transfer of the electron wave packet from one of the dots (or wells) to the other.

It is important to note that all the theoretical results quoted above correspond to entirely coherent regime of wave packet evolution. The authors of all these works have ignored dissipation processes, which appear due to the interaction of electrons with emission fields. To a certain degree, the coherent mode may be realized only on a time scale restricted by the typical relaxation time of the system. Resent experimental observations of coherent oscillations and subsequent loss of coherence of electrons in a double quantum dot, carried out by Hayashi *et al.*,[16] have demonstrated the potential ability to manipulate electron dynamics in semiconductor nanostructures on this time scale. However, it is clear that several coherent oscillations are not enough to provide a solid base for dynamical control of quantum states. Therefore, it is extremely important to understand the impact of the interaction between a dynamically controlled electron system and the phonon or photon environment on times exceeding the characteristic relaxation time.

It is well known that dissipation can lead to decoherence processes in double quantum dots.[17–19] Moreover, some authors have shown that decoherence is able to destroy the regime of dynamical control.[20] In particular, relocation of the electron density in a double quantum dot under the action of an adiabatically changing external electric field turns out to be incomplete even at zero temperature. As a result, the final electron state after the transfer is not at all a pure state but a mixture. Does the electron-phonon interaction destroy the dynamic localization? If so, then how much can we influence this destructive action by choosing the system parameters, such as the magnitude and the frequency of electric field? In our paper we will try to obtain the answers to these two questions.

Attempts to answer these questions were made earlier for binary structures[13,21–23] and superlattices.[24] The simplest model of a double quantum dot connected to a single resonator mode described by a harmonic vibrator[25] has been studied previously.[21] It was shown that the dynamic localization effect and controllable transfer of electron charge density from one quantum dot to the other are quite possible despite the coupling to the resonator mode. However, the existence of a phonon environment with infinite size and continuously distributed frequencies leads to a different situation. The system dynamics becomes irreversible. The consequences of such irreversibility are not quite clear, since there is no stationary “final” state to which the system may decohere, when we deal with the essentially time-dependent Hamiltonian of a dynamical control or localization problem.

Theoretical studies of optical emission and absorption spectra in double quantum wells by Dakhnovskii, Bavli, and Metiu,[13] and in superlattices by Jonsen and Jauho,[24] have shown that dynamic localization remains possible, when amplitude-frequency conditions typical for this phenomenon occur. However, the authors of Refs. 13 and 24 did not focus on the time evolution of the charge distribution in tunneling-coupled quantum wells in the presence of dissipation, which is a key point for the problem of quantum control of electron states.

Nonstationary quasiclassical tunneling through chaotic electron states in a double-well potential was investigated numerically by Kohler *et al.*[22] The authors found that by choosing the system parameters in a certain way, one can reduce dissipative energy losses of the electron system. This effect has been called “chaos-induced coherence.”

Restoration of coherence for electron qubits by a high-frequency external field has been found by Fonseca-Romero *et al.*[23] They considered the situation when the ac field is very fast, such that its frequency is much larger than the frequency of the electron two-state system. The action of such a field results in the decrease of damping, which becomes exactly zero for some amplitude-frequency dependence. It is clear, however, that the most efficient influence on the electron system will be achieved under resonant ac field action. Therefore, below we will consider the last regime.

The goal of the present paper is to study the dynamics of electron density distribution in a double quantum dot weakly coupled with a phonon environment and subject to a strong electric ac field. In this research we would also like to clarify the role that nonlinear phenomena, typical for the dynamic localization effect, play in electron-phonon interaction in a double quantum dot.

The paper is organized as follows. In Sec. II we discuss the theoretical model and formulate the basic equations of the problem. The solution and related techniques are presented in Sec. III, where we will derive the density matrix for the electron subsystem in a quasienergy basis. This will lead to the definition of the relaxation time for the double-dot system. In Sec. IV we analyze the dependence of the relaxation time on dc and ac field parameters, and the spatial characteristics of the probability distribution.

II. THEORETICAL MODEL

We consider a symmetric double quantum dot interacting with the phonon environment in the presence of constant, E, and variable harmonic, $F\cos\omega_0 t$, electric fields applied along the structure (see Fig. 1). The total Hamiltonian operator of the problem has the form

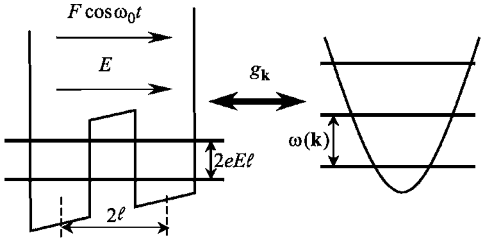

FIG. 1. Schematic representation of the system for an arbitrary fixed time. One of the phonon modes is shown as an oscillator with frequency $\omega(\mathbf{k})$.

$$H = H_e + H_B + V. \tag{1}$$

Here, H_e is the Hamiltonian operator of the single-electron system with the mentioned electric fields, and H_B represents the Hamiltonian of the phonon environment. It is common to describe the latter as a bosonic bath with an infinite number of modes[26–31]

$$H_B = \sum_{\mathbf{k}} \omega(\mathbf{k}) a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}, \tag{2}$$

where $a_{\mathbf{k}}^{\dagger}$ and $a_{\mathbf{k}}$ are the creation and annihilation operators of the phonon with wave vector $\mathbf{k}$. Here and throughout the paper we set $\hbar = 1$. The term V stands for the interaction between the electron subsystem in a double quantum dot and the phonon environment. It may often be written as a product of two operators $V = SX$, one of which, S, acts in the Hilbert space of the electron subsystem, while the other, X, acts in the space of the bath.

To proceed with the explicit formulation of H_e we shall make several key assumptions. First of all, we set the splitting energy Δ of symmetric, $|0\rangle$, and antisymmetric, $|1\rangle$, states of the double quantum dot to be significantly less than the size-quantization energy. For simplicity, we also move the energy reference line so that the energies of these two states are $\pm\Delta/2$. Second, we assume that the fields E and F are quite strong in the sense that the electron potential energy due to these fields in a double-dot structure of length 2ℓ is much higher than Δ, i.e., $eE\ell \gg \Delta$ and $eF\ell \gg \Delta$. At the same time, the values of $eE\ell$ and $eF\ell$ must be considerably smaller compared to the size-quantization energy or the height of the potential barrier. The last assumption allows neglecting electron excitation to higher energy levels. For the estimations we may accept the following values of the characteristic energies: ~ 1 meV for Δ; ~ 10 meV for $eE\ell$, $eF\ell$, and ω_0; and ~ 100 meV for the size-quantization energy. These values and the relationships are quite feasible in dc and ac fields with magnitudes of $10^3 - 10^4$ V/cm for silicon quantum dots with sizes of ~ 10 nm or less, embedded into a SiO_2 matrix. As a result, the model may be based on the two-level approximation.

Introducing the basis of orthonormal states $|L\rangle$ and $|R\rangle$ as $|L\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ and $|R\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, one may write H_e in the notation of the two-level pseudospin approximation:

$$H_e = -(\Delta/2)\sigma_X - e\ell(E + F\cos\omega_0 t)\sigma_Z. \tag{3}$$

Here, σ_X and σ_Z are Pauli matrices. The states $|L\rangle$ and $|R\rangle$ are almost completely localized in the left and right quantum dots. Such states are convenient in discussion of localization issues. The Hamiltonian operator (3) is typical for dynamic localization phenomenon studies.

An isolated electron system described by the Hamiltonian (3) may perform all the dynamical effects mentioned in the first section. In particular, the dynamic localization takes place when $2eF\ell/\omega_0$ coincides with any of the roots of the Bessel function $J_1(x)$ (see also Ref. 13). In addition, the regime of dynamical control is possible if the field E changes slowly and goes through the resonant value

$$2eE\ell = \omega_0. \tag{4}$$

The above relation states that the distance between the two lowest-energy levels, which equals approximately $2eE\ell$ in a strong field E, coincides with the quantum energy ω_0 of the ac field. As the system is pushed across the resonance (4), the electron wave packet, localized initially in one of the dots, relocates to the other and can be kept there as long as necessary.[2,5] This phenomenon is usually called dynamical control of electron states, since the strong ac field plays a major role in locking the electron density. It is important to note that the final state does not have to be the ground state of the biased double-dot system, and may well be a state polarized against the external dc field.

Now let us introduce the interaction of the electron subsystem (3) with the bath. In order to describe the electron-phonon coupling, one need to set the S and X operators defining the interaction term V in Hamiltonian (1). Let us choose the Pauli matrix σ_Z as S and the standard phonon coupling factor X as

$$X = \sum_{\mathbf{k}} (g_{\mathbf{k}} a^\dagger_{\mathbf{k}} + g^*_{\mathbf{k}} a_{\mathbf{k}}), \tag{5}$$

where $g_{\mathbf{k}}$ is the interaction constant between the **k**th phonon mode and electron subsystem. It is well known that the irreversibility in this description arises from the infinite number of modes in expressions (2) and (5). In this case, it is convenient to analyze the time evolution of the electron system by a reduced density matrix ρ_e, which is defined as the trace of the total density matrix over all the bath modes

$$\rho_e(t) = \mathrm{Tr}_B \rho(t). \tag{6}$$

The time dependence of the density matrix is given by the Liouville equation

$$i\dot{\rho}(t) = L\rho(t), \tag{7}$$

where L is the Liouville superoperator. Its action may be formulated by means of the relation

$$LO \equiv [H, O], \tag{8}$$

where O is any operator.

Even though the interaction we have chosen is not complicated, an exact solution of Eq. (7) is impossible. In our case it is useful to utilize projector superoperators and obtain an approximation for the reduced density matrix similar to the one used in Refs. 32 and 33 (see the Appendix),

$$i\dot{\rho}_e(t) = L_e\rho_e(t) - i\int_0^t \mathrm{Tr}_B L_V U(t) U^\dagger(t') \times [e^{-i(t-t')L_B} L_V \rho_B \widetilde{\rho}_e(t')] U(t') U^\dagger(t) dt'. \tag{9}$$

Here $L_e O \equiv [H_e, O]$, $L_B O \equiv [H_B, O]$, $L_V O \equiv [V, O]$, and ρ_B represents the initial density matrix of the bath. It is considered to be thermalized, i.e., $\rho_B = e^{-H_B/kT}/[\mathrm{Tr}_B e^{-H_B/kT}]$. The electron subsystem is initially unentangled with the bath. In Eq. (9) we have used the evolution operator $U(t)$ which is the solution to

$$i\dot{U}(t) = H_e U(t). \tag{10}$$

It should be noted that the approximation (9) is only valid for a weak electron-to-bath coupling. All the calculations below are restricted to these conditions.

III. THE SOLUTION

As follows from Eq. (3), the electron Hamiltonian and, hence, the total Hamiltonian operator (1) are periodic in time with the period $2\pi/\omega_0$. This circumstance allows us to simplify the solution significantly. One of the techniques for solving nonstationary quantum problems with periodic Hamiltonian, proposed first by Shirley, Zel'dovich, and Ritus,[34–36] is called the "quasienergy formalism." It has been successfully applied to problems of dynamic localization and dynamical control in coherent modes,[1–6,10,12,13] as well as in some other areas.[37] The quasienergy approach may be also very useful in studies of the dissipation effects in time-dependent electron systems.

Single-electron quasienergy states for the two-level Hamiltonian (3) were obtained earlier (see, e.g., Ref. 5) and have the form

$$|\pm\rangle = e^{\mp i\varepsilon t}\left[A^{(\pm)} \exp\left(i\frac{\omega_0}{2}t + i\frac{\lambda}{2}\sin\omega_0 t\right)|L\rangle \mp A^{(\mp)} \exp\left(-i\frac{\omega_0}{2}t - i\frac{\lambda}{2}\sin\omega_0 t\right)|R\rangle\right]. \tag{11}$$

Here $\pm\varepsilon$ are the quasienergies, where

$$\varepsilon = \sqrt{\delta^2 + \Delta_{-1}^2}/2. \tag{12}$$

The parameters δ and Δ_{-1}, respectively, represent the deviation from the exact resonance (4) and Rabi frequency in a strong ac field. They are defined by the expressions $\delta = 2eE\ell - \omega_0$, $\Delta_{-1} = J_{-1}(\lambda)\Delta$, where J_{-1} is the Bessel function and $\lambda = 2eF\ell/\omega_0$. The coefficients $A^{(\pm)}$ are defined as follows:

$$A^{(\pm)} = \sqrt{\frac{1}{2}\left(1 \mp \frac{\delta}{2\varepsilon}\right)}. \tag{13}$$

The advantage of the quasienergy basis (11) here based on the fact that representation of the wave function in it does not depend on time,[35] i.e., the expansion coefficients of the wave function in the $|\pm\rangle$ basis are constant. Note that the evolution operator defined by Eq. (10) can be written in terms of (11).

Note also that the quasienergy wave functions (11) are built on states $|L\rangle, |R\rangle$ localized within the left or the right dot, respectively. The degree of localization of the quasienergy states is given by the coefficients $A^{(\pm)}$, which are smooth steps as functions of δ. For example, $A^{(+)}$ decreases from unity to zero when δ goes from $-\infty$ to $+\infty$. At the same time $A^{(-)}$ goes backward, i.e., increases from zero to unity.

Therefore, when δ is far enough from zero and negative, the state $|+\rangle$ becomes localized in the left quantum dot, while the state $|-\rangle$ occupies the right dot. On increasing the field E we move the system across the resonance and make δ positive. As a result, the states $|+\rangle$ and $|-\rangle$ turn into each other, changing their localization to the opposite one. Taking into account the time independence of the density-of-states distribution in the quasienergy basis for the coherent mode, one may conclude that the electron that is in a certain quasienergy state (11) will stay there, provided the system is far from the resonance.

The situation is different when we consider decoherence. As has been mentioned in the previous section, it is convenient to use the density matrix formalism for the description of dissipative processes in the system. The natural basis for the density matrix in the case of Hamiltonian (1) with H_e written in the form (3) is the quasienergy basis

$$\rho_e(t) = \sum_{\mu\nu} C_{\mu\nu}(t)\rho_{\mu\nu}(t). \tag{14}$$

Here μ and ν run over the values $\pm$, $\rho_{\mu\nu} = |\mu\rangle\langle\nu|$, and $C_{\mu\nu}(t)$ are expansion coefficients. Note that $C_{\mu\nu}(t)$ are now time dependent, in contrast to the coherent evolution mode. Substituting Eq. (14) into Eq. (9) we obtain the time-evolution equations in the form

$$\dot{C}_{\mu\nu}(t) = -\sum_{\mathrm{s}\xi}\int_0^t C_{\mathrm{s}\xi}(t')I^{\mathrm{s}\xi}_{\mu\nu}(t,t')dt' \tag{15}$$

with the kernel

$$\begin{aligned} I^{\mathrm{s}\xi}_{\mu\nu}(t,t') = {} & \mathrm{Tr}_{QE}\rho_{\nu\mu}(t)\mathrm{Tr}_B L_V U(t)U^\dagger(t') \\ & \times e^{-i(t-t')L_B} L_V \rho_B \rho_{\mathrm{s}\xi}(t')U(t')U^\dagger(t), \end{aligned} \tag{16}$$

where $\mathrm{Tr}_{QE}O = \Sigma_\nu \langle\nu|O|\nu\rangle$. One may notice that the kernel (16) is of the second order in V and, thus, is small. This means that $C_{\mu\nu}(t)$ change slowly compared to the functions (11). At the limit $V \to 0$ the kernel $I^{\mathrm{s}\xi}_{\mu\nu}(t,t')$ vanishes, which corresponds to the coherent mode and $C_{\mu\nu} = \mathrm{const}$.

Equations (15) are still impossible to solve exactly unless some further approximations are made. Let us introduce correlation function of the bath operators

$$C(t) = \mathrm{Tr}_B X X(t)\rho_B. \tag{17}$$

To clarify the role of the exponent in the kernel (16) we need an identity, which follows from the definition of Liouville superoperators:

$$e^{itL_B}O = e^{itH_B}Oe^{-itH_B}. \tag{18}$$

Using Eq. (18) and performing some algebra we write the kernel (16) as a sum, each term of which splits into $\mathrm{Tr}_{QE}(\cdots)\mathrm{Tr}_B(\cdots)$, where the second factor is the trace of various combinations of X, $X(t) \equiv \exp(iH_B t)X\exp(-iH_B t)$, and ρ_B. Moreover, one may notice, that the trace of all these combinations turns out to be $C(t-t')$ or $C(t'-t)$.

The correlation function (17) is well known (see, e.g., Ref. 26). Since we are interested more in the result of the dissipation effect, it is possible to use the Markov approximation.[38,39] By doing this, we neglect short-time fluctuations of the bath states and restrict ourselves to a much longer time scale. In other words, we assume that the characteristic time of the bath correlation function (17) is small compared to the system evolution time. Naturally, if one wants to obtain the correct short-time behavior this approximation should not be made.

In order to find $C(t)$, we may write the expression (17) explicitly as an integral over phonon frequencies

$$C(t) = \int_0^\infty J(\omega)\left(\cos\omega t \coth\frac{\omega}{2kT} + i\sin\omega t\right)d\omega. \tag{19}$$

Here, k is the Boltzmann constant, T is the temperature, and $J(\omega)$ stands for the phonon spectral function. Considering the coupling of the electron subsystem to a single acoustic phonon branch only, we write $J(\omega)$ in the form

$$J(\omega) = g^2\omega\left(1 + \frac{ab}{g}\omega + \frac{b^2}{g^2}\omega^2\right)\exp\left(-\frac{\omega}{\omega_c}\right), \tag{20}$$

where g is a dimensionless interaction constant that is responsible for the intensity of umklapp processes in the electron-phonon scattering, b is the similar constant for direct processes, a is some parameter, and ω_c is the cutoff frequency. The limiting cutoff frequency, as usual (see, e.g., Ref. 26), is assumed to be much greater than all other frequencies of the problem, such as Δ, kT, and the reciprocal relaxation time. In other words, ω_c^{-1} is the smallest temporal parameter of the problem.

Expression (20) may be transformed into a more simple form $b^2\omega^3\exp(-\omega/\omega_c)$, provided we neglect the umklapp processes. This situation corresponds to the "super-Ohmic" case of the spectral function. It is often considered[26] when studying electron tunneling in solids. However, as it be shown below, consideration of the umklapp processes is crucial in determining the correct form of the correlation function. Indeed, substitution of Eq. (20) into Eq. (19) yields $C(t) = \gamma\delta(t) + \tilde{C}(t)$, where the first term is due to the umklapp processes only, while the second term is a result of the direct electron-phonon scattering. Here, $\gamma = 2\pi g^2 kT$ and $\delta(t)$ is the Dirac delta function. It appears due to the fact that the system evolution time is much greater than ω_c^{-1}, as mentioned earlier. $\tilde{C}(t)$ is some quickly oscillating function, which has zero mean value, and varies in time on the scale $\sim\omega_c^{-1}$. Thus, integration of $\tilde{C}(t)$ with a slow function, varying on time scales essentially exceeding ω_c^{-1}, gives zero. With this reasoning, one may see that $\tilde{C}(t)$ does not contribute to the right-hand part of Eq. (15), and direct electron-phonon scattering has a negligible effect in system evolution. On the contrary, umklapp processes indeed influence the time evolution of $C_{\mu\nu}$ in Eq. (15), and, therefore, they have to be taken into account. Note, however, that umklapp processes are efficient only for nonzero temperatures, since the parameter γ is linear in T.

Considering the above arguments we may neglect $\widetilde{C}(t)$ and write the correlation function as

$$C(t) = \gamma\delta(t). \tag{21}$$

This corresponds, in fact, to the "Ohmic" spectral function,[26] which is formally derived from the general expression (20) by setting $b=0$.

Using expression (21), Eqs. (15) may be simplified to the form

$$\begin{aligned}\dot{C}_{++}(t) = &-\gamma[C_{++}(t)T_{++}^{++} + C_{--}(t)T_{++}^{--} + C_{+-}(t)T_{++}^{+-}e^{-2i\varepsilon t}\\ &+ C_{-+}(t)T_{++}^{-+}e^{2i\varepsilon t}],\\ \dot{C}_{--}(t) = &-\gamma[C_{++}(t)T_{--}^{++} + C_{--}(t)T_{--}^{--} + C_{+-}(t)T_{--}^{+-}e^{-2i\varepsilon t}\\ &+ C_{-+}(t)T_{--}^{-+}e^{2i\varepsilon t}],\\ \dot{C}_{+-}(t) = &-\gamma[C_{++}(t)T_{-+}^{++}e^{-2i\varepsilon t} + C_{--}(t)T_{-+}^{--}e^{-2i\varepsilon t}\\ &+ C_{+-}(t)T_{-+}^{+-}e^{-4i\varepsilon t} + C_{-+}(t)T_{-+}^{-+}],\\ \dot{C}_{-+}(t) = &-\gamma[C_{++}(t)T_{+-}^{++}e^{2i\varepsilon t} + C_{--}(t)T_{+-}^{--}e^{2i\varepsilon t} + C_{+-}(t)T_{+-}^{+-}\\ &+ C_{-+}(t)T_{+-}^{-+}e^{4i\varepsilon t}].\end{aligned} \tag{22}$$

Here

$$T_{\mu\nu}^{s\xi} \equiv 4(A^{(\mu)}A^{(\xi)}B^{(s)}B^{(\nu)} + A^{(s)}A^{(\nu)}B^{(\mu)}B^{(\xi)}), \tag{23}$$

where $B^{(\pm)} = \mp A^{(\mp)}$. As one can see, in the case $\gamma=0$, corresponding to the absence of an electron-phonon interaction, all the coefficients $C_{\mu\nu}$ become constant, as was already pointed earlier.

The solution of Eqs. (22) with the assumption $\gamma \ll \varepsilon$ to the first order in γ is

$$\begin{aligned}C_{++}(t) &= C_{--}(0)\frac{1-\exp(-t/\tau)}{2} + C_{++}(0)\frac{1+\exp(-t/\tau)}{2},\\ C_{--}(t) &= C_{--}(0)\frac{1+\exp(-t/\tau)}{2} + C_{++}(0)\frac{1-\exp(-t/\tau)}{2},\\ C_{+-}(t) &= C_{+-}(0) + i[C_{++}(0) - C_{--}(0)]\frac{\gamma\Delta_{-1}\delta}{4\varepsilon^3}\\ &\quad\times[\exp(-t/\tau)\exp(-2i\varepsilon t) - 1],\\ C_{-+}(t) &= C_{-+}(0) + i[C_{--}(0) - C_{++}(0)]\frac{\gamma\Delta_{-1}\delta}{4\varepsilon^3}\\ &\quad\times[\exp(-t/\tau)\exp(2i\varepsilon t) - 1],\end{aligned} \tag{24}$$

where the decrement τ^{-1} is defined as

$$\tau^{-1} = \gamma\Delta_{-1}^2/\varepsilon^2. \tag{25}$$

A detailed discussion of this decrement will be carried out in the next section.

Higher-order corrections in the small parameter γ to the solution (24) would be artificial since in the approximation (9) we kept only the terms of the first order of magnitude. The coefficients $C_{\mu\nu}$, in fact, represent the distribution of the density matrix over quasienergy states. Therefore, the solution (24) shows that the system tends to have equal probabilities for both quasienergy states at $t\to\infty$ irrespective of the initial conditions.

IV. RESULTS AND DISCUSSION

To gain insight into the electron density distribution we need a relation that explicitly connects the solution (24) to the reduced density matrix in the basis of states $|L\rangle$ and $|R\rangle$. Such a relation is obtained by substituting Eq. (11) into Eq. (14):

$$\begin{aligned}\rho_{LL}(t) = &C_{++}(t)(A^{(+)})^2 + C_{--}(t)(A^{(-)})^2 + C_{+-}(t)e^{-2i\varepsilon t}A^{(+)}A^{(-)}\\ &+ C_{-+}(t)e^{2i\varepsilon t}A^{(+)}A^{(-)},\\ \rho_{LR}(t) = &[C_{+-}(t)e^{-2i\varepsilon t}(A^{(+)})^2 + C_{-+}(t)e^{2i\varepsilon t}(A^{(-)})^2\\ &+ A^{(+)}A^{(-)}]e^{i\omega_0 t + i\lambda\sin\omega_0 t}.\end{aligned} \tag{26}$$

The density matrix is Hermitian and positively defined; therefore $\rho_{RR}(t) = 1-\rho_{LL}(t)$, $\rho_{RL}(t) = \rho_{LR}^*(t)$. As usual, $\rho_{LL}(t) = \langle L|\rho(t)|L\rangle$ and $\rho_{RR}(t) = \langle R|\rho(t)|R\rangle$ are the probabilities of the electron charge to be in the left or the right dot, respectively. On the other hand the expansion coefficients $C_{\mu\nu}(t)$ may be obtained from Eq. (14) in the form

$$C_{\mu\nu}(t) = \langle\mu|\rho_e(t)|\nu\rangle, \tag{27}$$

which gives a simple "recipe" for constructing $C_{\mu\nu}(0)$ in the relations (24):

$$\begin{aligned}C_{\mu\nu}(0) = &A^{(\mu)}A^{(\nu)}\rho_{LL}(0) + B^{(\mu)}B^{(\nu)}\rho_{RR}(0) + A^{(\mu)}B^{(\nu)}\rho_{LR}(0)\\ &+ B^{(\mu)}A^{(\nu)}\rho_{RL}(0).\end{aligned} \tag{28}$$

Expressions (26), (28), and (24) provide a complete "toolkit" for analysis of electron density distribution inside the dots.

Let us examine now the choice of initial distribution that is often discussed for the phenomena of dynamic localization and dynamical control. In particular, the electron density distribution in the double dot will be considered fully polarized, i.e., the whole electron charge is initially placed in one of the dots. Such a state is natural, from the energy point of view, for an electron situated in a double-dot structure in an electric dc field. For the sake of definiteness, the left dot may be chosen occupied, i.e., $\rho_{LL}(0)=1$. In this case, the expression (28) gives $C_{\mu\nu}(0) = A^{(\mu)}A^{(\nu)}$.

The occupation probability for the left quantum dot with the above initial settings can be written as

$$\begin{aligned}\rho_{LL}(t) = &\frac{\Delta_{-1}^2}{4\varepsilon^2}\cos^2(\varepsilon t) + \frac{\delta^2}{8\varepsilon^2}[1+\exp(-t/\tau)] + \frac{\gamma}{\varepsilon}\frac{\delta^2}{4\varepsilon^2}\frac{\Delta_{-1}^2}{4\varepsilon^2}\\ &\times[\exp(-t/\tau)\sin 4\varepsilon t - \sin 2\varepsilon t].\end{aligned} \tag{29}$$

The time dependence of ρ_{LL} for several values of the ratio Δ_{-1}/δ, where $\delta = 2eE\ell - \omega_0$, is shown in Fig. 2. In the limit-

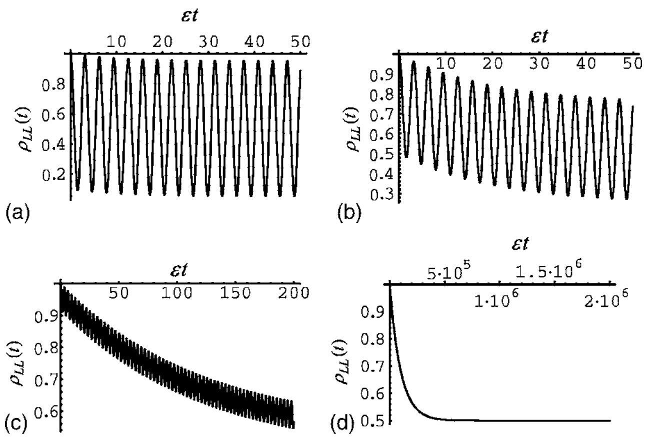

FIG. 2. Occupation probability for the left quantum dot for various values of the $|\Delta_{-1}/\delta|$ ratio: (a) 3.0, (b) 1.0, (c) 0.3, (d) 0.01. For all the cases γ/ε has been kept constant and equal to 0.1. One can see that, independently of $|\Delta_{-1}/\delta|$, the mean value of ρ_{LL} tends to 0.5.

ing case $|\Delta_{-1}/\delta| \gg 1$ the second and the third terms in expression (29) vanish, and $\rho_{LL}(t) \approx \cos^2(\varepsilon t)$, which corresponds, in fact, to Rabi oscillations of the electron density in strong electric fields. In contrast to the well-known Rabi oscillations in a weak ac field, which demonstrate periodic transitions of the electron wave function between two stationary states, here, Rabi oscillations indicate periodic transfer of electron charge from one quantum dot to the other. We should note that such an evolution mode corresponds to equal initial occupation of the + and − quasienergy states, which remain equally occupied for the whole time: $C_{++}(t) = C_{--}(t) = C_{++}(0) = C_{--}(0) = 1/2$.

Another limiting case, $|\Delta_{-1}/\delta| \ll 1$, allows us to omit the first and the last terms in Eq. (29). Choosing, e.g., the field E so that δ is negative, we define the initial density matrix in the quasienergy basis with sufficient accuracy [see Eqs. (13) and (28)], assuming

$$C_{\mu\nu}(0) = \delta_{\mu,+}\delta_{\nu,+}. \tag{30}$$

Equation (30), in fact, states that the system is in the pure quasienergy state $|+\rangle$. Substitution of the initial condition (30) into the general solution (24) yields

$$C_{++}(t) = 1 - C_{--}(t) = \frac{1 + \exp(-t/\tau)}{2},$$

$$C_{+-}(t) = C^{*}_{-+}(t) = i\gamma\frac{\Delta_{-1}\delta}{4\varepsilon^3}[\exp(-t/\tau)\exp(-i2\varepsilon t) - 1]. \tag{31}$$

The effect of dissipation, as we see from Eq. (31), is manifested by the decay of the pure quasienergy states, i.e., $C_{++}(t)$ and $C_{--}(t)$ converge to 1/2 at $t \to \infty$. Furthermore, the off-diagonal elements rise gradually up to some limited value, which may be considered as an additional trait of a quantum mixture of $|+\rangle$ and $|-\rangle$ quasienergy states.

Utilizing Eq. (27) we obtain explicit expressions for the density matrix in the "left-right" representation,

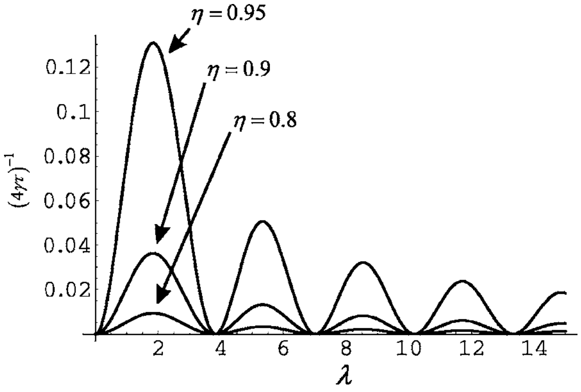

FIG. 3. Reciprocal relaxation time in units of 4γ for various dimensionless resonance detunings η as a function of parameter λ, which is proportional to the ac field magnitude F (see the text). $\omega_0/\Delta = 30$.

$$\rho_{LL}(t) = \frac{1 + \exp(-t/\tau)}{2},$$

$$\rho_{LR}(t) = i\gamma\frac{\Delta_{-1}\delta}{4\varepsilon^3}[\exp(-t/\tau - 2i\varepsilon t) - 1] \times \exp(-2i\varepsilon t + i\omega_0 t + i\lambda \sin\omega_0 t). \tag{32}$$

Off-diagonal elements $\rho_{RL}(t) = \rho^{*}_{LR}(t)$ are nonzero, but always remain small. They are directly proportional to the constant of the electron-phonon interaction. The diagonal elements $\rho_{LL}(t)$ and $\rho_{RR}(t)$ change from the values of 1 and 0 to 1/2 at $t \to \infty$, respectively. This means that an extremely polarized double quantum dot at $t = 0$ will be almost equally populated and lose its polarization when $t \gg \tau$.

Thus, we have answered the first question stated in the Introduction. Indeed, even a weak connection between the double quantum dot and the phonon environment leads to relaxation processes, which give equal population of both quasienergy states, leading to equal population of both quantum dots in the case $|\Delta_{-1}/\delta| \ll 1$. In a certain sense such behavior of the electron density under the action of strong dc and ac electric fields is similar to what takes place for static and symmetric double dots without any external fields. Nevertheless, there is one essential difference between these two cases. This difference is caused by the characteristic decay time τ, which appears in solutions (31) and (32).

While the decay time in a static two-level system depends only on the constant of electron-phonon interaction, the time of decay in a two-level system with strong dc and ac electric fields has a nontrivial dependence on the harmonic field amplitude F and the magnitude of the constant field E, as clearly follows from the definition of τ. It is instructive to write it explicitly as

$$\tau^{-1} = 4\gamma\frac{\Delta^2 J^2_{-1}(\lambda)}{\Delta^2 J^2_{-1}(\lambda) + (2eE\ell - \omega_0)^2}. \tag{33}$$

The decrement τ^{-1} as a function of the dimensionless parameter $\lambda = 2eF\ell/\omega_0$ for several values of $\eta = 2eE\ell/\omega_0$ is plotted in Fig. 3. When $\eta = 1$ the decrement has a maximum 4γ, which corresponds to the resonant condition (4). A remarkable feature is that τ^{-1} vanishes at an infinite number of points on the λ axis, independently of η. These points are the roots λ_j of the Bessel function $J_{-1}(\lambda)$. This yields the following values for the ac field amplitude:

$$F_j = \frac{\omega_0 \lambda_j}{2e\ell}. \tag{34}$$

Thus, by choosing the amplitude of the ac field close or equal to any value of F_j, we can significantly reduce the decrement τ^{-1} or even make it zero to the first order in γ. In this case, as follows from Eq. (32), relaxation of the electron subsystem inside the dots will be suppressed, and the density of charge initially localized within one of the dots will stay there. Note that solution (32) was obtained assuming $|\delta| \gg |\Delta_{-1}|$. This assumption is automatically satisfied for any nonzero δ when Δ_{-1} is an exact zero.

Evidently, the choice of F corresponding to one of the values in set (34) maintains the dynamic localization of the electron wave packet even when the energy exchange between the electron subsystem and the phonon environment is possible. This effect may be called dynamic suppression of relaxation. It is important to notice that the phenomenon of dynamic suppression of relaxation is an essentially nonlinear effect, which would never appear in a weak ac field where $\lambda \ll 1$. The smallest value of the amplitude F suitable for dynamical control over the dissipation equals $3.832\omega_0/2e\ell$, where the numerical coefficient 3.832 is the first root of the Bessel function $J_{-1}(\lambda)$. Thus, an answer to the second question stated in the Introduction has also been obtained.

Finally, we would like to discuss the nondissipative electron-subsystem evolution from the "stability" point of view. In order to cancel the dissipation, the parameter λ must be precisely equal to any of λ_j, as was noted above. Even the smallest deviation from λ_j will return the system to the dissipative regime. It is quite clear, however, that such accuracy cannot be achieved experimentally because of, e.g., random fluctuations existing in any real system. Thus, the problem is actually reduced to the problem of possible minimization of the energy losses in some controllable way. The solution of the problem highlights the role the dc field E plays in the reduction of dissipation. Comparing the above situation with the nonresonant case $E=0$, considered in Ref. 23, we can conclude that the former is preferable in terms of the controllable reduction of dissipation processes. Evidently, it is possible to choose the constant electric field E, such that $|\delta| \gg \Delta$. The decrement τ^{-1}, then, behaves in the vicinity of the jth zero of the Bessel function as $\tau^{-1} \sim (\Delta/\delta)^2(\lambda-\lambda_j)^2$. To compare, the corresponding expression for τ^{-1} in the case $E=0$ has the form $\tau^{-1} \sim |\lambda-\lambda_j|$, since it is proportional[23] to the first order of the Bessel function $J_0(\lambda)$. As one can see, for any small, but finite, $\lambda-\lambda_j$, the dissipation arising will be essentially weaker in the resonant case considered here. This is due to the fact that $\lambda-\lambda_j$ enters quadratically in τ^{-1}, and, which is also important, due to the presence of the controlling factor $(\Delta/\delta)^2$ that is determined by the dc field magnitude. In fact, this factor can be decreased down to sufficiently small values, which will also help to suppress the dissipation.

ACKNOWLEDGMENTS

D.S. is thankful to Vladimir Privman for useful discussions. The authors acknowledge support of this research by the National Security Agency and Advanced Research and Development Activity under Army Research Office Contract No. DAAD-19-02-1-0035, by the National Science Foundation, Grant No. DMR-0121146, and by Russian Foundation for Basic Research, Grants No. 04-02-16493 and No. 05-02-16762.

APPENDIX

Let us obtain Eq. (9) using Eqs. (7), (3), and (1). The approach used below is often called the Zwanzig projection technique.[40] First of all (see Refs. 33, 40, and 41) we need to introduce "relevant" and "irrelevant" parts of the density matrix using projector superoperators P and $Q \equiv 1-P$,

$$\rho_R(t) \equiv P\rho(t), \quad \rho_I(t) \equiv Q\rho(t). \tag{A1}$$

The projector superoperator P should be chosen so that $\rho_R(t)$ is proportional to the reduced density matrix $\rho_e(t)$. One may notice that if we set $P=\rho_B \,\mathrm{Tr}_B$, then the relevant part of the density matrix at $t=0$ is simply $\rho_B\rho_e(0)=\rho(0)$, and $\rho_I(0)=0$. At all other times $\rho_R(t)$ is $\rho_B\rho_S(t)$.

Applying the projectors (A1) to Eq. (7), and using the identity $\rho(t)=\rho_R(t)+\rho_I(t)$, we obtain

$$i\dot{\rho}_R(t) = P[L_e(t)+L_B+L_V]\rho_R(t) + P[L_e(t)+L_B+L_V]\rho_I(t), \tag{A2}$$

$$i\dot{\rho}_I(t) = Q[L_e(t)+L_B+L_V]\rho_R(t) + Q[L_e(t)+L_B+L_V]\rho_I(t). \tag{A3}$$

It is usual[31] to assume that $\mathrm{Tr}_B X\rho_B=0$. This assumption cancels the term $PL_V\rho_R(t)$ in Eq. (A2). One may notice, that due to Eq. (A1) and properties of P and Q, the second term of the first parentheses in Eq. (A2), as well as the first two terms in the second, vanish. The same way, in Eq. (A3) only four terms survive, producing the system

$$i\dot{\rho}_R(t) = L_e(t)\rho_R(t) + PL_V\rho_I(t), \tag{A4}$$

$$i\dot{\rho}_I(t) = QL_V\rho_R(t) + QL_B\rho_I(t) + L_e(t)\rho_I(t) + QL_V\rho_I(t). \tag{A5}$$

At this point Eq. (A5) may be formally integrated and substituted in Eq. (A4). We neglect the last term of the right-hand side of Eq. (A5), which produces higher-order corrections, when integrating, as was done in Refs. 32 and 33. We also assume that the solution to Eq. (10) is known, which allow us to express $L_e(t)$ in terms of $U(t)$ in Eq. (A5). As a result, the equation for $\rho_R(t)$ is correct to the second order of electron-to-bath coupling and has the form

$$i\dot{\rho}_R(t) = L_e(t)\rho_R(t) - iPL_V\int_0^t U(t)U^\dagger(t') \times[e^{-i(t-t')L_B}QL_V\rho_R(t')]U(t')U^\dagger(t)dt', \quad (A6)$$

where we have used the fact that $\rho_I(0)=0$ and $\exp(iQL_Bt)Q=\exp(iL_Bt)Q$. Introducing P explicitly and recalling that $\mathrm{Tr}_B X\rho_B=0$, so that $QL_VP=L_VP$, we obtain

$$i\rho_B\dot{\rho}_e(t) = L_e(t)\rho_B\rho_e(t) - i\rho_B\int_0^t \mathrm{Tr}_B L_V U(t)U^\dagger(t') \times[e^{-i(t-t')L_B}L_V\rho_B\rho_e(t')]U(t')U^\dagger(t)dt'. \quad (A7)$$

As we see, Eq. (A7) easily turns into Eq. (9) by canceling out ρ_B.

[1] V. A. Burdov, Pis'ma Zh. Eksp. Teor. Fiz. **76**, 25 (2002) [JETP Lett. **76**, 21 (2002)].

[2] V. A. Burdov and D. S. Solenov, Physica E (Amsterdam) **24**, 217 (2004).

[3] R. Bavli and H. Metiu, Phys. Rev. A **47**, 3299 (1993).

[4] A. A. Gorbatsevich, V. V. Kapaev, and Yu. V. Kopaev, Zh. Eksp. Teor. Fiz. **107**, 1320 (1995) [JETP **80**, 734 (1995)].

[5] V. A. Burdov, Zh. Eksp. Teor. Fiz. **116**, 217 (1999) [JETP **89**, 119 (1999)].

[6] V. A. Burdov and D. S. Solenov, Phys. Lett. A **305**, 427 (2002).

[7] W. A. Lin and L. E. Ballentine, Phys. Rev. Lett. **65**, 2927 (1990).

[8] M. Holthaus, Phys. Rev. Lett. **69**, 1596 (1992).

[9] D. H. Dunlap and V. M. Kenkre, Phys. Rev. B **34**, 3625 (1986).

[10] M. Holthaus and D. Hone, Phys. Rev. B **47**, 6499 (1993).

[11] S. Raghavan, V. M. Kenkre, D. H. Dunlap, A. R. Bishop, and M. I. Salkola, Phys. Rev. A **54**, R1781 (1996).

[12] F. Grossmann, T. Dittrich, P. Jung, and P. Hanggi, Phys. Rev. Lett. **67**, 516 (1991); F. Grossmann and P. Hanggi, Europhys. Lett. **18**, 571 (1992).

[13] Y. Dakhnovskii and R. Bavli, Phys. Rev. B **48**, 11010 (1993); **48**, 11020 (1993); Y. Dakhnovskii and H. Metiu, Phys. Rev. A **48**, 2342 (1993).

[14] C. A. Stafford and N. S. Wingreen, Phys. Rev. Lett. **76**, 1916 (1996).

[15] R. H. Blick, D. W. van der Weide, R. J. Haug, and K. Eberl, Phys. Rev. Lett. **81**, 689 (1998).

[16] T. Hayashi, T. Fujisawa, H. D. Cheong, Y. H. Jeong, and Y. Hirayama, Phys. Rev. Lett. **91**, 226804 (2003).

[17] S. D. Barrett and G. J. Milburn, Phys. Rev. B **68**, 155307 (2003).

[18] L. Fedichkin and A. Fedorov, Phys. Rev. A **69**, 032311 (2004).

[19] G. Levine and V. N. Muthukumar, Phys. Rev. B **69**, 113203 (2004).

[20] T. Brandes and T. Vorrath, Phys. Rev. B **66**, 075341 (2002).

[21] V. A. Burdov and D. S. Solenov, Zh. Eksp. Teor. Fiz. **125**, 684 (2004) [JETP **98**, 605 (2004)].

[22] S. Kohler, R. Utermann, P. Hanggi, and T. Dittrich, Phys. Rev. E **58**, 7219 (1998).

[23] K. M. Fonseca-Romero, S. Kohler, and P. Hanggi, Chem. Phys. **296**, 307 (2004).

[24] K. Johnsen and A.-P. Jauho, Phys. Rev. B **57**, 8860 (1998).

[25] A. B. Pippard, *Physics of Vibration* (Cambridge University Press, Cambridge, UK, 1978).

[26] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987) [Erratum: **67**, 725 (1995)].

[27] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[28] G. W. Ford, M. Kac, and P. Mazur, J. Math. Phys. **6**, 504 (1965).

[29] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981).

[30] A. O. Caldeira and A. J. Leggett, Physica A **121**, 587 (1983).

[31] S. Chakravarty and A. J. Leggett, Phys. Rev. Lett. **52**, 5 (1984).

[32] D. Loss and D. P. DiVincenzo, cond-mat/0304118 (unpublished).

[33] D. Solenov and V. Privman, Proc. SPIE **5436**, 172 (2004); cond-mat/0403510 (unpublished).

[34] J. H. Shirley, Phys. Rev. **138**, B979 (1965).

[35] Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **51**, 1492 (1966) [Sov. Phys. JETP **24**, 1006 (1967)].

[36] V. I. Ritus, Zh. Eksp. Teor. Fiz. **51**, 1544 (1966) [Sov. Phys. JETP **24**, 1041 (1967)].

[37] M. L. Glasser, L. M. Nieto, and B. F. Samsonov, J. Phys. A **36**, L585 (2003); F. Casas, J. A. Oteo, and J. Ros, *ibid.* **34**, 3379 (2001).

[38] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).

[39] K. Blum, *Density Matrix Theory and Applications* (Plenum Press, New York, 1996).

[40] R. Zwanzig, J. Chem. Phys. **33**, 1338 (1960).

[41] V. F. Los, J. Phys. A **34**, 6389 (2001); D. Zubarev *et al.*, *Statistical Mechanics of Nonequilibrium Processes* (Wiley, New York, 1996).