

Electrostatic traps for dipolar excitons

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We consider the design of two-dimensional electrostatic traps for dipolar indirect excitons. We show that the exciton dipole-dipole interaction, combined with the in-plane electric fields that arise due to the trap geometry, constrains the maximal density and lifetime of trapped excitons. We derive an analytic estimate of these values and determine their dependence on the trap geometry, thus suggesting the optimal design for high density trapping as a route for observing excitonic Bose-Einstein condensation.

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For many years, excitons in semiconductors had been predicted to undergo a phase transition at high enough densities and low enough temperatures to form a Bose-Einstein condensate (BEC),¹⁻³ similar to the BEC of atomic gases, already observed a decade ago.⁸ This is expected to happen due to the predicted bosonic nature of excitons at densities that still disguise the fermionic nature of their constituents, i.e., the electron and the hole. A few major obstacles have, however, prevented a clear observation of an exciton BEC phase until this day, even though the typical predicted transition temperature is of the order of a Kelvin, much hotter than its atomic counterpart, and is available in many laboratories. Maybe the most crucial obstacle to excitonic BEC is the short exciton intrinsic radiative lifetime (of the order of hundred picoseconds) due to electron-hole recombination, which limits the time available for exciton thermalization. Since the initial state of the exciton after optical excitation is out of equilibrium, and full thermalization with the lattice becomes more difficult at low lattice temperatures, the thermalization time turns out to be longer than the intrinsic exciton lifetime. Thus the temperature of the exciton gas may not reach the required transition temperature to the condensed state, although the lattice temperature may in fact be well below that temperature. In recent years, a promising way to overcome the lifetime issue has emerged. The exciton lifetime can be considerably increased by spatially separating the electron and the hole. This is usually achieved by utilizing a double quantum well (DQW) system.^{4,5} The resulting excitons are constructed from electrons in one layer and holes in the other and are known as “spatially indirect excitons.” This trick can increase the exciton lifetime by many orders of magnitude (from less than a nanosecond to tens of microseconds)⁶ while only slightly reducing the exciton binding energy (due to the three-dimensional nature of the Coulomb interaction).

It seems that by utilizing these indirect excitons, the major obstacle to BEC has been removed. However, a new problem arises. The indirect excitons are dipolar in nature since they all carry a permanent dipole moment due to the charge separation of the electron and hole in the growth direction, perpendicular to the QW planes. All the dipoles are aligned in the same direction. As a result, there is a strong repulsive dipole-dipole interaction between all excitons. On one hand, this repulsive interaction has an additional advantage, since it prevents further binding of excitons into larger complexes,

such as biexcitons or electron-hole droplets. On the other hand, the strong repulsive interaction will also tend to spread the free exciton gas and to quickly reduce its density. Thus it is difficult to maintain a very dense gas of free dipolar excitons over long times due to this driven expansion. Recent experiments probing the dynamics of a dense excitonic gas indeed show a very fast expansion of the dense cloud over a short period of time followed by a much slower expansion of the dilute cloud.⁶ In a different work we show how this behavior can be quantitatively explained by an initial driven fast expansion that transforms into diffusive expansion when the density drops.⁷ One can show that at densities and temperatures required for observing excitonic BEC, the dipolar exciton gas expansion will always be initially driven outwards by the strong repulsive dipole-dipole interactions, quickly reducing its density. One possible solution for this problem is to create a homogeneous distribution of such excitons over the whole sample. This will indeed eliminate the fast expansion by eliminating the density gradients. Such a solution will, however, require a lot of excitation power and will end up heating the sample. One can think of a more elegant solution in which the dipolar excitons are trapped in an external potential, preventing them from expanding. Such methods have been extremely successful in trapping and cooling atoms, leading to their condensation.⁸ Since the excitons are already confined in one dimension (by quantum confinement of the quantum well), one needs to take care of only the in-plane confinement. One possibility of confining the excitons in the plane is by the use of applied localized stress to change the local band energies.⁹ In this paper we discuss an alternate scheme of circular, two-dimensional electrostatic traps which trap the dipolar excitons in a well-defined space. This trapping occurs via the interaction of the exciton’s permanent dipole with a nonuniform electric field. It was shown before that spatially indirect excitons can be transported¹⁰ and trapped in a one-dimensional periodic way¹¹ using spatially varying electric fields. The electrostatic trapping method allows trapping of dipolar excitons in a wide range of trap sizes and can also enable a fast, dynamic control of the trapped excitons by electrical modulation of the trap depth and shape, thus allowing, for example, evaporative cooling of the exciton gas.

Here we consider the limitations on the excitonic density and effective lifetime of such a trapping method. We show that there is a minimum required vertical applied electric

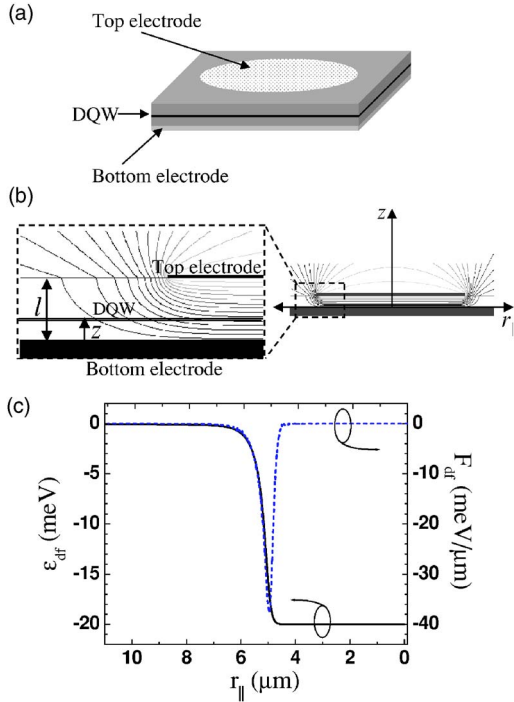


FIG. 1. (Color online) (a) An illustration of a suggested dipolar exciton trap design. (b) Equipotential lines of a circular trap with $R/l=10$. (c) Calculated confining energy (ϵ_{df} , solid line) and force (F_{df} , dashed line) for dipolar excitons for a circular trap with $R=5\ \mu\text{m}$, $l=0.5\ \mu\text{m}$, a potential difference $\Delta\phi_0$ of 1 V, and a dipole length $z_0=100\ \text{\AA}$. $r_{||}$ is measured from the center of the trap.

field to get the trapping energy larger than the dipole-dipole repulsion energy, in order to prevent the trapped excitons from escaping. However, in general, applying a vertical field will also result in a radial electric field depending on the geometry of the trap. This radial electric field will in turn cause exciton ionization at the trap boundaries and reduce the effective trapping lifetime. We then derive an analytic estimation of the maximal density and lifetime of trapped excitons. This analysis gives guidelines as well as constraints for optimal design of such dipolar traps.

The physical idea of an electrostatic trap is straightforward. Consider a geometry where a small circular, optically semitransparent metallic gate with a radius R is placed on top of the DQW sample. The sample substrate is made conductive to serve as the ground electrode, as illustrated in Fig. 1(a). The sample thickness, from the bottom to the top electrode, is denoted by l . The DQW plane is perpendicular to the growth direction (\hat{z}) and its vertical position in the sample is given by z , measured from the bottom electrode. The indirect excitons will tend to stay in the region under the gate contact when a sufficient voltage is applied between it and the substrate. This trapping effect is due to the fact that the indirect excitons are dipolar and thus they are high field seekers, gaining an additional negative energy term coming from the dipole-field interaction:

$$\epsilon_{df} = \vec{d}_X \cdot \vec{E}(r_{||}, z) = d_X E_z(r_{||}, z), \quad (1)$$

where $\vec{d}_X = -ez_0\hat{z}$ is the exciton dipole moment, $\vec{E} = -\nabla\phi$ is the applied electric field, and z_0 is the effective separation

between the electron and the hole and is equal to the separation between the centers of the two quantum wells to a good approximation. Figure 1(b) shows a cross section of a circular trap with equipotential lines illustrated, where $R/l=10$. Figure 1(c) depicts the confining energy, ϵ_{df} , as a function of the radial position of the excitons in the trap, $r_{||}$, for $R=5\ \mu\text{m}$, $l=0.5\ \mu\text{m}$, $z_0=100\ \text{\AA}$, and a potential difference $\Delta\phi_0$ of 1 V. The trapping energy at the center of the trap (where $E_z^{\text{center}} \equiv \Delta\phi_0/l$) is given by

$$\epsilon_{df}(r_{||}=0, z) = -ez_0\Delta\phi_0/l. \quad (2)$$

Since E_z varies significantly only over a range $\Delta r \approx l$ near $r_{||}=R$, then for $R \gg l$ the excitons experience the trapping dipole force just near the sharp boundaries, given by

$$F_{df}(r_{||}, z) = d_X \frac{\partial E_z}{\partial r_{||}}, \quad (3)$$

as is shown in Fig. 1(c).

Ideally, for $R \gg l$, such a trap behaves like a “pool” of free moving excitons, subject to perfectly reflecting boundary conditions at the edges (one can get a nonflat potential well for $R \sim l$). One has to compare the trapping energy ϵ_{df} to the dipole-dipole repulsion energy ϵ_{dd} and to kT . As long as $\epsilon_{df} > \epsilon_{dd}$, kT , the excitons will be confined within the boundaries of the trap, being reflected from the walls by the trap’s dipole force. If, however, the opposite condition arises, the excitons will “spill over” the trap due to the internal repulsive force. The dipole-dipole repulsion is given by

$$\epsilon_{dd} = \frac{4\pi e^2 z_0}{\epsilon} n_X, \quad (4)$$

where ϵ is the background dielectric constant. The condition for trapping,

$$|\epsilon_{df}/\epsilon_{dd}| > \alpha, \quad (5)$$

where α is a parameter of order 1, and is determined by how much “residual” trapping energy is required in the experiment. This condition then yields the maximal trapped exciton density:

$$n_X^{\text{max}} = \frac{\epsilon E_z^{\text{center}}}{4\pi e \alpha}. \quad (6)$$

Here, we have neglected the thermal energy kT since it is always much smaller than ϵ_{dd} at temperatures and densities relevant for excitonic BEC. This inequality actually leads to some interesting consequences and is discussed elsewhere.⁷

Equation (6) seems to suggest that an arbitrary density of excitons can be trapped, depending only on E_z^{center} and thus on the external potential difference. Unfortunately, the picture given above is too simplified. While the electric field inside the trap and away from the edge is always aligned with the dipoles (i.e., $\vec{E} \equiv E_z \hat{z}$), at the boundary of the trap, the radial component of the electric field, E_r , can be appreciable, depending on z . As z increases (moving the quantum well away from the bottom electrode), the magnitude of E_r increases compared to E_z , as is shown in Fig. 2(a) for two different exemplary z/l positions. One can extract the ratio of the maximal radial component of the electric field, E_r^{max} to

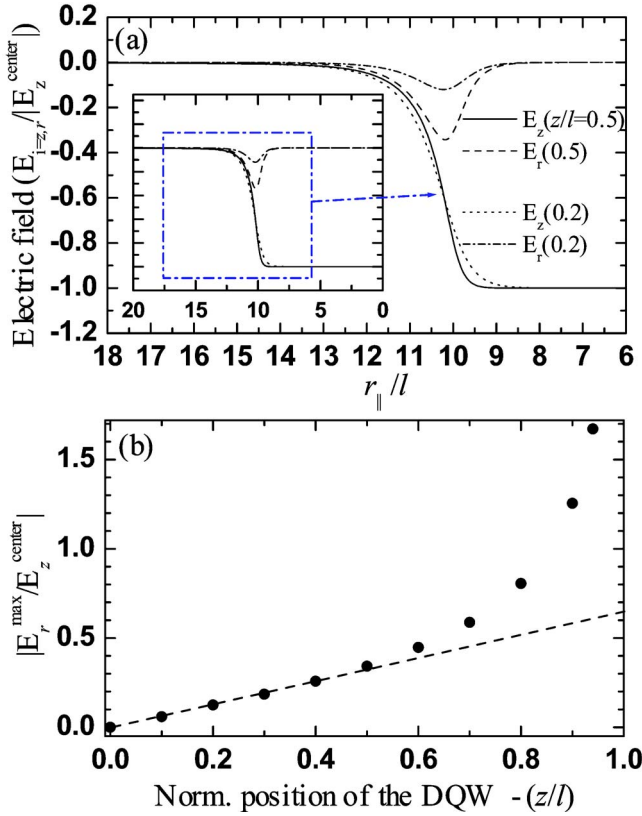


FIG. 2. (Color online) (a) Vertical (E_z) and radial (E_r) components of the electric field in a circular trap as a function of $r_{||}$ for two different vertical positions ($z/l=0.2, 0.5$) of the DQW structure. The inset shows the whole range of radial positions from the center of the trap. (b) The ratio of the maximal value of the radial component of the electric field, E_r^{\max} , to the vertical component of the field at the trap center, E_z^{center} , for various vertical position z/l values (circles). The dashed line is a linear fit to the range $0 \leq z/l < 0.5$.

the vertical component at the center of the trap, E_z^{center} . This is plotted in Fig. 2(b). For $z/l < 1/2$, this ratio can be well approximated by a linear dependence,

$$\left| \frac{E_r^{\max}}{E_z^{\text{center}}} \right| \cong \beta z/l, \quad (7)$$

with $\beta=0.625$.

The radial field will tend to pull the electron and the hole into opposite directions. Classically, if the electrostatic energy due to the in-plane external field is larger than the exciton binding energy, the exciton will be ionized. This happens when $e|E_r|a_X \approx \varepsilon_X$, where ε_X is the dipolar exciton binding energy and a_X is its in-plane radius. Quantum mechanically, however, one expects that ionization of excitons becomes significant at much smaller values of in-plane field, due to tunneling of the electron and hole through the ‘‘hill’’ in the Coulomb potential that binds the exciton.^{12,13} Such tunneling will break some of the excitons on the edge of the trap, thus decreasing the total density of excitons with time and giving rise to an effective trap lifetime. In other words,

the quantum ionization process effectively makes the perfectly reflecting boundaries of the trap partly absorbing.

The ionization rate of a two-dimensional exciton in its ground state subject to a one-dimensional electric field has been calculated in Ref. 15. We define the ratio between the exciton binding energy and the typical electrostatic energy the exciton experiences due to the in-plane field as

$$\gamma = \varepsilon_X / (e|E_r|a_X), \quad (8)$$

where the local field correction due to the induced in-plane polarization of the excitons can be neglected.¹⁴ Since E_r changes over a length scale $\sim l \gg a_X$, we follow Ref. 15 to get the ionization rate of the exciton at the trap boundary:

$$\Gamma_{\text{ion}} = \frac{32\varepsilon_X}{\sqrt{\pi\hbar}} \sqrt{\gamma} e^{-(8\gamma/3)}. \quad (9)$$

Assuming a homogenous distribution of excitons in the trap ($N_X = n_X \pi R^2$), the number of excitons within a distance l from the edge is given by $n_X 2\pi R \Delta R \approx n_X 2\pi R l$. The depletion of the exciton density in the trap can be described by a simple rate equation:

$$\frac{dn_X}{dt} = -\frac{2l}{R} \Gamma_{\text{ion}} n_X = -\frac{n_X}{\tau_{\text{trap}}}, \quad (10)$$

where we have defined an effective trap lifetime, $1/\tau_{\text{trap}} = (2l/R)\Gamma_{\text{ion}}$ and assumed $\tau_{\text{trap}} \ll \tau_X$ (τ_X being the exciton intrinsic lifetime).

The constraints on the trap performance can already be seen: due to the dipole-dipole interaction ε_{dd} , there is a minimum required vertical field in order to get the trapping energy ε_{df} larger than ε_{dd} for a given required trap density [Eq. (6)]. This requirement will introduce a radial field due to the geometrical relationship between E_z and E_r given by Eq. (7). The radial field will in turn increase the ionization rate at the trap boundaries and will reduce the effective trap lifetime [Eqs. (10) and (9)].

By combining Eqs. (6)–(10) one can get the effective trap lifetime as a function of the trap energy (and hence the maximal trapped exciton density):

$$\frac{1}{\tau_{\text{trap}}} = \frac{32}{\pi} \sqrt{\frac{\varepsilon^2(l/z)\varepsilon_X^3}{\hbar^2 R^2 \alpha \beta e^2 a_X n_X}} \exp\left(\frac{-2(l/z)\varepsilon\varepsilon_X}{3\pi\alpha\beta e^2 a_X n_X}\right). \quad (11)$$

This is plotted in Fig. 3(a) for various z/l values, for a trap with $R/l=50$, $\varepsilon_X=5$ meV, $a_X=140$ Å, $z_0=100$ Å, and setting α to be 1.2. As the exciton density increases, there is a strong reduction of τ_{trap} . This will dramatically reduce the time available for exciton thermalization even if their intrinsic lifetime is very long.

Inverting the previous argument yields a bound on the maximal E_r^{\max} (through the unitless parameter γ), for a minimum desired effective trap lifetime τ_{trap} which we should choose to be long enough for efficient exciton thermalization for BEC. With such a requirement, Eqs. (9) and (10) give

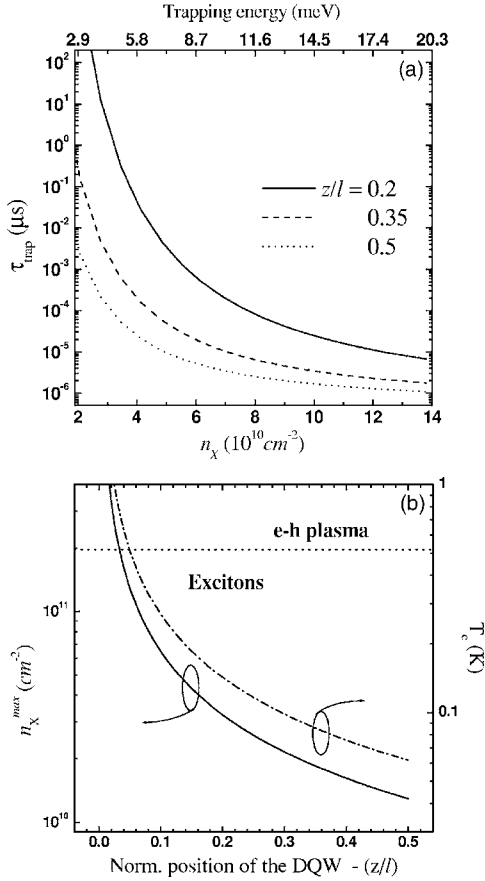


FIG. 3. (a) Effective trap lifetime as a function of the trapped density n_X and the trap energy ε_{df} for various vertical positions of the DQW structure in the trap, z/l . Here $R/l=50$, $\varepsilon_X=5$ meV, $a_X=140$ Å, $z_0=100$ Å, and $\alpha=1.2$. (b) The calculated maximal density (solid line) of trapped dipolar excitons [with the same parameters as in (a)] and the expected BEC transition temperature (dashed-dot line) as a function of z/l . The horizontal line marks the approximated Mott density.

$$\gamma - \frac{3}{16} \log_e \gamma = \frac{3}{8} \log_e \left(\frac{64l}{\sqrt{\pi R}} \frac{\tau_{\text{trap}}}{\tau_{gs}} \right), \quad (12)$$

where we have defined $\tau_{gs} = \hbar / \varepsilon_X$. This bound¹⁶ yields an upper limit on E_z^{center} through the geometrical relation of Eq. (7). The upper limit on E_z^{center} then sets a limit on n_X^{max} , due to the competition of ε_{df} and ε_{dd} , as reflected in Eq. (6).

Combining Eqs. (7) and (6) results in a compact expression for estimating the maximal density of excitons that can be trapped, depending on the various requirements and trap design:

$$n_X^{\text{max}} = \frac{(\varepsilon \varepsilon_X)^2 (l/z)}{4\pi e^4 \alpha \beta \gamma}. \quad (13)$$

Figure 3(b) shows the maximal trapped density as a function of (z/l) , for $\tau_{\text{trap}}=1$ μs and with the same trap parameters as before. Note that γ depends logarithmically on τ_{trap} , hence relaxing the trapping time constraint by orders of magnitude will result only in a small increase of n_X^{max} . The predicted BEC transition temperature T_c for the exciton gas in a circular trap with $R=25$ μm , given by¹⁷ $T_c = \pi \hbar^2 n_X^{\text{max}} / [2k_B M_X \log_e(n_X^{\text{max}} \pi R^2 / 4)]$, is also plotted. As the DQW gets further away from the bottom electrode, there is a strong reduction of n_X^{max} due to the increased in-plane electric field. Thus this is an important issue in a design of a trap. The guideline for designing a trap for high density dipolar exciton gas is then to minimize z/l as much as possible (z/l should be smaller than 0.25 for an exciton density larger than 10^{10} cm^{-2}).

There are a few other possible ways of getting around this complication: (a) one can design a trap with symmetric top and bottom electrodes. This will eliminate in-plane fields exactly at $z/l=1/2$. However, constructing such a trap is much more difficult from a fabrication point of view. (b) It is possible to use a deep trap and continuously pump the trap to achieve a higher steady-state density to compensate for the fast tunneling time. This will, unfortunately, tend to heat up the trap while the time available for excitons to thermalize will be significantly shorter due to the fast ionization, even for intrinsically long lifetime excitons, as can be seen from Fig. 3(a). (c) It may be possible to further stabilize the excitons by applying a perpendicular magnetic field, as was suggested in Ref. 13. (d) Finally, in order to keep the DQW structure close to the center ($z/l \approx 1/2$), but minimize the in-plane field ionization problem, a trap can be designed where a doped QW layer is inserted just below the DQW structure. This will greatly reduce the E_r component at the DQW due to its vicinity to a charged, metalliclike layer.

In summary, we have analyzed the constraints on the design of electrostatic traps for dipolar excitons and derived expressions relating the trapping lifetime and the trap design parameters to the maximal density of excitons that can be trapped and to their corresponding expected BEC transition temperature. We show that it is feasible to construct dipolar traps that will trap excitons with high enough densities and for long times for a possible observation of excitonic BEC. Strong experimental evidence for high density exciton trapping has already been observed in our laboratory, utilizing similar trap designs as discussed above, and will be presented in a separate paper.

Note added. After the completion of this paper, we were informed of a related ongoing work by L. V. Butov and co-workers.

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