Effect of spin-orbit coupling on zero-conductance resonances in asymmetrically coupled one-dimensional rings

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The influence of Rashba spin-orbit coupling on zero conductance resonances appearing in one-dimensional conducting rings asymmetrically coupled to two leads is investigated. For this purpose, the transmission function of the corresponding one-electron scattering problem is derived analytically and analyzed in the complex energy plane with focus on the zero-pole structure characteristic of transmission (anti)resonances. The lifting of real conductance zeros due to spin-orbit coupling in the asymmetric Aharonov-Casher ring is related to the breaking of spin reversal symmetry in analogy to the time-reversal symmetry breaking in the asymmetric Aharonov-Bohm ring.

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I. INTRODUCTION

An important feature of one-dimensional ring shaped conductors or electronic devices is the appearance of quantum interference effects under the influence of electromagnetic potentials, known as Aharonov-Bohm[1] (AB) and Aharonov-Casher[2] (AC) effect. In numerous investigations, the transmission properties of mesoscopic AB and AC rings coupled to current leads were studied under various aspects such as AB flux and coupling dependence of resonances,[3] geometric (Berry) phases[4–8] and spin flip, precession, and interference effects.[9–13] Most of the investigated models use symmetrically coupled rings. There are, however, mesoscopic systems such as nanographite ribbons showing conductance properties that are based on asymmetric configurations,[14] giving rise to a specific dip structure of antiresonances (zero-conductance resonances) in the model transmission. The effects of asymmetry on the transmission were considered mainly in quantum network models.[15–17] In quasi-one-dimensional (1D) systems, real conductance zeros appear under the condition of conserved time reversal symmetry[18,19] (TRS). The (anti)resonances in the transmission due to local quasibound states correspond to a specific zero-pole structure in the complex energy plane.[20–23] The application of an external magnetic field modifies this zero-pole structure, shifting the transmission zeros away from the real axis, with the shift as a function of the AB phase.[24] Thus, the lifting of conductance zeros is related to the breaking of TRS.

In this paper, the influence of spin-orbit coupling (SOC) on zero-conductance resonances in asymmetrically coupled rings is investigated by means of an AC ring, where an effective in-plane magnetic field results from the Rashba effect[25] of moving electrons in the presence of an electric field perpendicular to the ring plane, as considered in Refs. 12 and 13. This means that the role of time reversal symmetry is now transfered to inversion symmetry (parity). We will show that parity connected with the Rashba spin orbit coupling can be viewed in an analogous way as the case of time reversal symmetry for spinless particles.

This paper is organized as follows. In Sec. II, a single-particle description of the one-dimensional ring subject to Rashba-SOC in terms of Hamiltonian, eigenstates and eigenenergies is given, following Refs. 12 and 13. The section concludes with the results for the transmission of the asymmetric AC ring in the one-electron scattering picture which is derived in the Appendix. The analytic expression for the transmission function is analyzed in Sec. III with focus on geometry and SOC dependence of the transmission zeros. Section IV contains a symmetry argument which establishes an analogy between formation and lifting of the zeros due to Rashba SOC in the AC ring and the corresponding effects on spinless electrons due to the magnetic field in the AB ring. The main results are summarized in the conclusions of Sec. V.

II. AC RING IN SINGLE PARTICLE PICTURE

The coupling of electron spin and orbital degrees of freedom is due to the magnetic field generated in the reference frame of a moving electron by an electric field in the reference frame of the laboratory. In two-dimensional systems (e.g., due to the presence of a confinement potential along a specific direction), an important contribution of electric fields is the Rashba effect, a consequence of lack of inversion symmetry, that causes a spin band splitting proportional to the momentum. In the ring system under consideration, the Rashba field results from the asymmetric confinement along the direction perpendicular to the ring plane.

A. Hamiltonian

In the following investigation of one-dimensional rings, z is chosen as the direction of confinement, perpendicular to the plane of motion. The various SO-coupling mechanisms are accounted for using the following model Hamiltonian:

$$\hat{H}_{\rm SO} = \frac{\alpha}{\hbar}(\hat{\vec{\sigma}} \times \hat{\vec{p}})_z = i\alpha\left(\hat{\sigma}_y \frac{\partial}{\partial x} - \hat{\sigma}_x \frac{\partial}{\partial y}\right), \qquad (1)$$

where $(\hbar/2)\hat{\vec{\sigma}}$ is the spin operator in terms of the Pauli spin matrices, $\hat{\vec{\sigma}}=(\sigma_x,\sigma_y,\sigma_z)$, and α is the Rashba parameter characterizing the strength of the SOC corresponding to an electric field $\vec{E}_R=(0,0,E_z)$ in the z direction, arising from a potential $V(z)$ due to structural or confinement asymmetry. In polar coordinates $x=r\cos\varphi$ and $y=r\sin\varphi$ the total Hamiltonian in effective mass approximation reads[26]

$$\hat{H}(r,\varphi) = -\frac{\hbar^2}{2m^*}\left[\partial_r^2 + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\partial_\varphi^2\right] - \frac{i\alpha}{r}(\cos\varphi\sigma_x + \sin\varphi\sigma_y) \times \frac{\partial}{\partial\varphi} + i\alpha(\cos\varphi\sigma_y - \sin\varphi\sigma_x)\frac{\partial}{\partial r}, \qquad (2)$$

with the effective mass m^*. In the case of a one-dimensional ring, a confining potential $V(r)$ needs to be added in order to force the electron wave functions to be localized on the ring in the radial direction. It is shown in Ref. 26 that the exact form of the confining potential is not essential. A simple possibility is the harmonic potential centered around $r=\varrho$, $V(r)=\frac{1}{2}K(r-\varrho)^2$, where ϱ is the radius of the ring. Considering only the lowest radial mode, the resulting one-dimensional Hamiltonian for fixed radius ϱ is (see Ref. 26 for a complete derivation)

$$\begin{aligned}\hat{H}_{\rm 1D}(\varphi) &= \langle R_0(r)|\hat{H}(r,\varphi)|R_0(r)\rangle \\ &= -\frac{\hbar^2}{2m^*\varrho^2}\frac{\partial^2}{\partial\varphi^2} - \frac{i\alpha}{\varrho}(\cos\varphi\sigma_x + \sin\varphi\sigma_y)\frac{\partial}{\partial\varphi} \\ &\quad - \frac{i\alpha}{2\varrho}(\cos\varphi\sigma_y - \sin\varphi\sigma_x).\end{aligned} \qquad (3)$$

The last term in the above expression for the 1D Hamiltonian encodes the correction due to the radial confinement. The Hamiltonian in Eq. (3) can be written in a dimensionless form[13]

$$H = \frac{2m^*\varrho^2}{\hbar^2}\hat{H}_{\rm 1D} = \left(-i\frac{\partial}{\partial\varphi} + \frac{\beta}{2}\sigma_r\right)^2, \qquad (4)$$

where $\beta=2\alpha m^*\varrho/\hbar^2$ is the dimensionless SOC constant, $\sigma_r=\cos\varphi\sigma_x+\sin\varphi\sigma_y$, and the additive constant $-\beta^2/4$ was neglected.[34]

B. Eigenstates and energy spectrum

The eigenstates of Hamiltonian (4) follow as the solution of the time-independent Schrödinger equation and have the general form[12,13]

$$\Psi_n^\sigma(\varphi) = e^{in\varphi}\chi^\sigma(\varphi), \qquad (5)$$

where n is the orbital quantum number and $\sigma=\uparrow,\downarrow \cong \pm 1$ labels the spin. For the isolated ring, $n\in\mathbb{Z}$, but when coupled to leads, n can adopt any real number allowed by energy, depending on spin and direction of motion. The spinors $\chi^\sigma(\varphi)$ are generally not aligned with the momentum dependent and spatially varying Rashba field $\vec{B}_R(r)=2\beta(\hat{z}\times\vec{p})/\varrho$, but make a tilt angle $\tilde{\theta}=\pi/2-\theta$ given by $\tan\theta=-\beta$ relative to the direction of the electric field $\vec{E}_R$ (see Fig. 1). The energy eigenvalues of the states in Eq. (5) are[13]

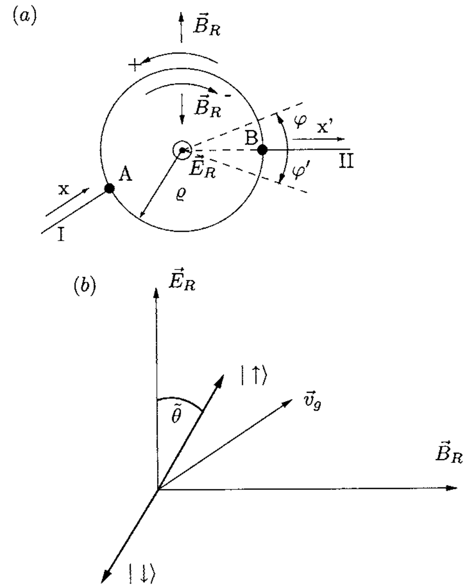

FIG. 1. (a) Momentum-dependent in-plane Rashba field $\vec{B}_R$, and (b) up and down spin eigenstates do not generally align with the Rashba field $\vec{B}_R$, but make a tilt angle θ with the electric field $\vec{E}_R$ perpendicular to the ring plane ($\vec{E}_R$, $\vec{B}_R$, and $\vec{v}_g$ form an orthogonal coordinate system).

$$E_n^\sigma = (n - \Phi_{\rm AC}^\sigma/2\pi)^2 \qquad (6)$$

with the Aharonov-Casher phase[12]

$$\Phi_{\rm AC}^\sigma = -\pi(1 - \sigma\sqrt{\beta^2+1}). \qquad (7)$$

At fixed energy E, the dispersion relation yields the quantum numbers $n_\lambda^\sigma(E)$ through

$$n_\lambda^\sigma(E) = \lambda\sqrt{E} + \Phi_{\rm AC}^\sigma/2\pi, \quad \lambda = \pm. \qquad (8)$$

For a plane wave arriving from lead I with wave vector k we get

$$n_\lambda^\sigma(k) = \lambda k\varrho + \Phi_{\rm AC}^\sigma/2\pi. \qquad (9)$$

The sense of propagation is determined by the sign of the group velocity, which in the latter case is given by

$$v_{g,\lambda}^\sigma = \frac{\hbar}{2m^*\varrho}\frac{dE_{n_\lambda^\sigma}^\sigma}{dn_\lambda^\sigma} = \frac{\hbar}{2m^*\varrho}(n_\lambda^\sigma - \Phi_{\rm AC}^\sigma/2\pi) = \lambda k\varrho, \qquad (10)$$

λ thus encoding the traveling direction. The quantum numbers for different spin and sense of propagation are related by

$$n_\lambda^\sigma = -(n_{-\lambda}^{-\sigma} + 1). \qquad (11)$$

The corresponding eigenstates of the closed ring are

$$\Psi_\lambda^\sigma(\varphi) = e^{in_\lambda^\sigma \varphi}\begin{pmatrix} \sin\left(\frac{\theta}{2}+\frac{\pi}{4}(1+\sigma)\right) \\ -\cos\left(\frac{\theta}{2}+\frac{\pi}{4}(1+\sigma)\right)e^{i\varphi}\end{pmatrix}\frac{1}{\sqrt{2\pi}},$$

$$\sigma = \pm 1, \quad \lambda = \pm. \tag{12}$$

These eigenstates differ from the solutions of the free system by the phase factors in the spin part.

C. Current

In order to investigate transport in our quantum mechanical system, an expression for the probability current density is requested. The probability current density j is determined by inserting the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi, \tag{13}$$

with H from Eq. (4), and its adjoint into the continuity equation imposed by probability conservation

$$\frac{\partial\rho}{\partial t} + \frac{\partial j}{\partial\varphi} = 0, \tag{14}$$

where $\rho = |\Psi|^2$ denotes the probability density. The probability current density can be expressed in terms of velocity operators

$$j = \frac{1}{2}[\Psi^\dagger(\hat{v}\Psi) + \Psi(\hat{v}\Psi)^\dagger]. \tag{15}$$

The velocity operators are derived from the Hamiltonian by[27]

$$\hat{v} = \frac{\partial\hat{H}}{\partial\hat{p}}, \tag{16}$$

where $\hat{p}$ is the momentum operator, whose explicit form depends on the coordinate system[35]

$$\hat{p}_\varphi = -i\frac{\partial}{\partial\varphi}(\text{ring}) \text{ and } \hat{p}_x = -i\varrho\frac{\partial}{\partial x}(\text{leads}). \tag{17}$$

In absence of SOC, only the kinetic energy term of the Hamiltonian contributes to the velocity operators, which in this case are

$$\hat{v}_0(\varphi) = -2i\frac{\partial}{\partial\varphi} \quad \text{and} \quad \hat{v}_0(x) = -2i\varrho\frac{\partial}{\partial x}. \tag{18}$$

For finite SOC, $H_{\rm SO}$ yields an additional term for the ring (assuming zero SOC in the leads)

$$\hat{v}_{\rm SO}(\varphi) = \beta\sigma_r(\varphi) \quad \text{and} \quad \hat{v}'_{\rm SO}(\varphi') = \beta\sigma'_r(\varphi'), \tag{19}$$

where $\sigma'_r(\varphi') \equiv \sigma_r(-\varphi') = \cos\varphi'\sigma_x - \sin\varphi'\sigma_y$.

The total velocity operator to consider in the expression of the probability current density given by Eq. (15) is

$$\hat{v} = \hat{v}_0 + \hat{v}_{\rm SO}. \tag{20}$$

The above results will be used when investigating the lead and ring currents in the Appendix.

D. Transmission amplitude from the one-electron scattering formalism

Conductance in mesoscopic structures can be expressed by means of the Landauer conductance formula,[28,29] which in our case reads

$$G = \frac{e^2}{h}\sum_{\sigma=\uparrow,\downarrow}|T_\sigma|^2, \tag{21}$$

where T_σ is the (spin dependent) transmission amplitude.[36] The previously obtained expressions for wave function and current are now used to calculate the transmission amplitude for the ring system from the proper requirements on wave function continuity and probability current conservation.[30] The calculation is performed in the Appendix and follows Refs. 13 and 12. It yields the transmission amplitude

$$T_\sigma(\phi,\beta,\gamma) = \frac{4i\left[e^{i(\Phi_{\rm AC}^\sigma/2)(1-\gamma)}\sin\left[\frac{\phi}{2}(1+\gamma)\right] + e^{-i(\Phi_{\rm AC}^\sigma/2)(1+\gamma)}\sin\left[\frac{\phi}{2}(1-\gamma)\right]\right]}{\cos\phi\gamma - 5\cos\phi + 4\cos\Phi_{\rm AC}^\sigma + 4i\sin\phi} \tag{22}$$

as a function of energy ($\phi = 2\pi k\varrho$), SOC $[\Phi_{\rm AC}^\sigma(\beta)]$ and asymmetry $[\gamma = (1-R)/(1+R)]$, where R stands for the ratio of lower and upper ring arm lengths (see Fig. 1). In the following discussion of transmission and conductance, spin index σ refers to the spinors in the ring eigenstates in Eq. (12), whereas the standard spinor basis (eigenvectors of σ_z) are labeled by s.

III. GEOMETRY AND SOC DEPENDENCE OF TRANSMISSION ZEROS

A. Free system (β=0)

The transmission function in Eq. (22) displays a peculiar resonant behavior characterized by a set of zeros and poles.

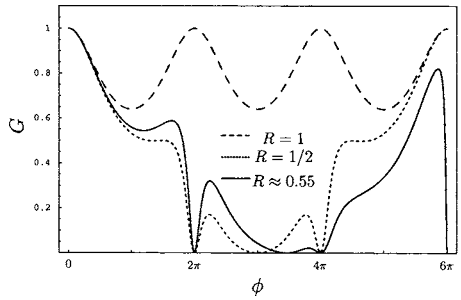

FIG. 2. Conductance for $\beta=0$ in symmetric ($R=1$) and asymmetric system ($R=1/2$ and $R=(2\sqrt{3}-1)/(2\sqrt{3}+1)\approx 0.55$. For the lead-ring coupling assumed in the present model ($\epsilon=4/9$), transmission is not perfect even in case of equal branch length. In the asymmetric system, periodical transmission zeros appear.

The transmission zeros are obtained from Eq. (22) as the solution of

$$\sin\left(\frac{\phi}{2}(\gamma-1)\right)=e^{-i\Phi_{\mathrm{AC}}^{\sigma}}\sin\left(\frac{\phi}{2}(\gamma+1)\right). \qquad (23)$$

For $\beta=0$, the phase factor equals unity, and Eq. (23) simplifies to

$$\cos\left(\frac{\phi}{2}\gamma\right)\sin\left(\frac{\phi}{2}\right)=0, \qquad (24)$$

which yields zeros at

$$\phi_{0,1}=2m\pi \text{ and } \phi_{0,2}=(2m+1)\pi/\gamma, \quad m\in\mathbb{Z}. \qquad (25)$$

Obviously, there are two types of zeros. The zeros of the first kind at $\phi_{0,1}$ correspond to the eigenstates of the closed ring, whereas the zeros of second type at $\phi_{0,2}$ are given by the geometry-dependent interference condition for nodes at the right junction[16] and appear only in an asymmetric configuration ($\gamma\neq 0$). The poles related to transmission resonances are determined by

$$\cos\phi\gamma-5\cos\phi+4\cos\Phi_{\mathrm{AC}}^{\sigma}+4i\sin\phi=0. \qquad (26)$$

Figure 2 shows the conductance in absence of SOC ($\beta=0$) for symmetry ($R=1$) and asymmetry parameters $R=1/2$ and $R=(2\sqrt{3}-1)/(2\sqrt{3}+1)\approx 0.55$. The oscillation in the conductance for the symmetric configuration is due to the coupling of lead and ring, which does not correspond to perfect transmission and therefore leads to resonances as a consequence of backscattering effects.[3] These resonances however do not give rise to conductance zeros: from Eq. (22) follows that zeros and poles of the conductance compensate each other and yield a finite value. In the asymmetric ring ($R=1/2, R\approx 0.55$), both types of zeros appear.

By examination of the transmission amplitude in the complex energy plane we find a certain connection between the conductance zeros and transmission resonances. Zeros on the real axis are accompanied by nearby poles in complex plane (Figs. 3 and 4). Figure 4 shows zeros (a) and poles (b) at $R=1/2$ separately. A similar feature is known from the quantum waveguide systems with an attached resonator.[21] In the present case a pair of poles is associated with each zero of

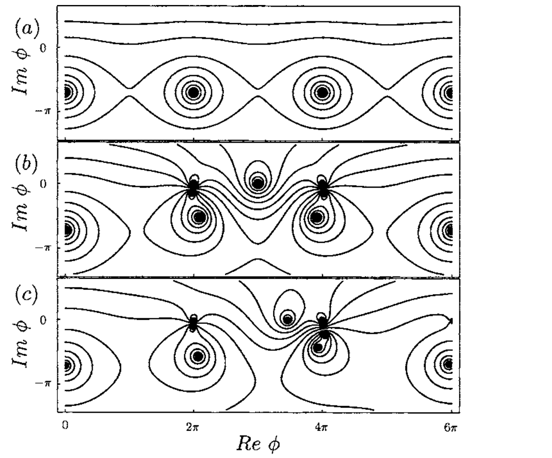

FIG. 3. Zero-pole structure of G in the complex plane for $\beta=0$ and (a) $R=1$, (b) $R=1/2$, (c) $R\approx 0.55$. The zeros lie on the real axis, whereas the poles have a finite imaginary part.

the first kind at $\phi_{0,1}$. The real part of the energies of the zeros and poles are not exactly identical, which results in an asymmetric shape of the resonance (Fano type).[31] These characteristic features can be clearly observed in Fig. 2, e.g., at $\phi_{0,1}=2\pi$. Note that at $\phi=0$ and $\phi=6\pi$ both numerator and denominator of the transmission amplitude vanish simultaneously for $R=1/2$, such that they annihilate at these places, as can be easily observed in Fig. 2.

B. Finite Rashba SOC ($\beta\neq 0$)

There are two remarkable features in the transmission characteristics arising as effects of SOC. The first is the finite transmission probability in the spin channel opposite to the incident spin orientation. This is the result of spin precession along the ring branches due to SOC as considered in Ref. 32. The conductance zeros in the opposite channel correspond to a frequency of precession which reproduces the incident spin orientation at the right junction. The second aspect, and the one on which we will concentrate in the following, is the lifting of certain conductance zeros in the incident channel. These features can be observed in Fig. 5 where the transmission for finite SOC is displayed. It is instructive to analyze the modification of the transmission amplitude in the complex energy plane. Figure 6 shows the lifting of the zeros of

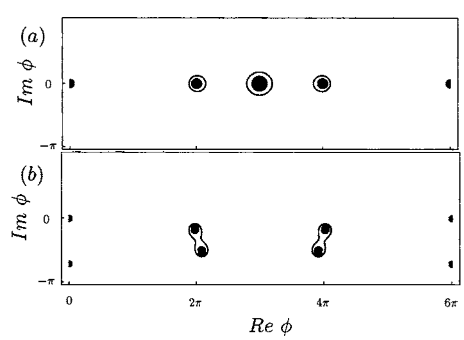

FIG. 4. (a) Zeros and (b) poles of G in the asymmetric case ($R=1/2$) for $\beta=0$.

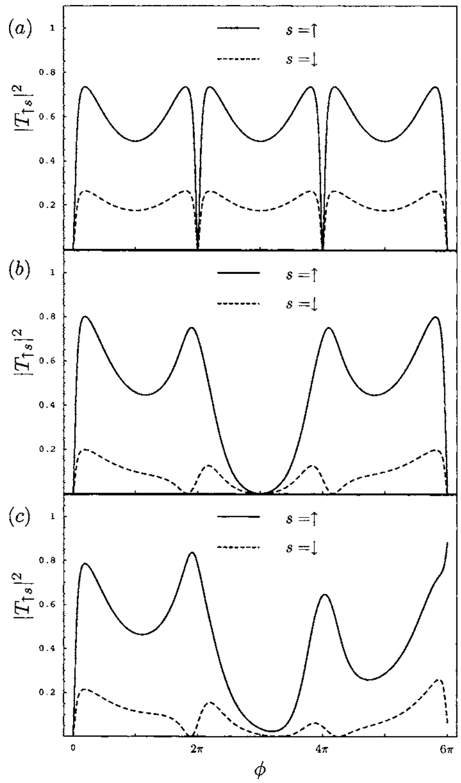

FIG. 5. Transmission probability for nonzero SOC β=0.6 from s=↑ into s=↑ and s=↓ spin channels for symmetric [(a) R=1] and asymmetric system [(b) R=1/2, (c) $R \approx 0.55$]. In the symmetric AC ring, spin-orbit interaction causes the appearance of transmission zeros, in the asymmetric configuration however, the latter are partially lifted. For particular asymmetry ratios R, the geometry dependent zeros persist.

the first kind as well as the emergence of zeros that were canceled by poles in the free system. The shifting of zeros and poles away from the real axis is displayed in Fig. 7. In the conductance, the zeros of the first kind appear no longer. They are still present in the up- and down-transmission amplitudes, but different spin components are shifted in opposite directions, as it is shown in Fig. 8.

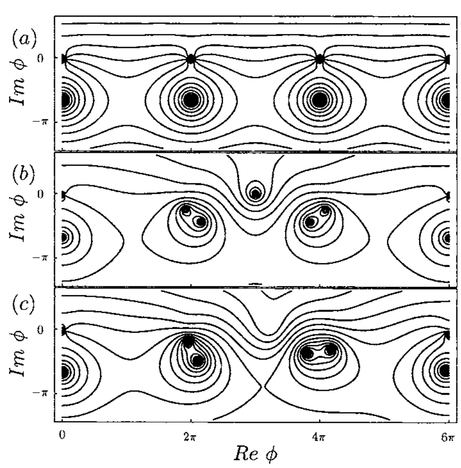

FIG. 6. Zero-pole structure of G in the complex plane for β =0.6 and (a) R=1, (b) R=1/2, (c) $R \approx 0.55$.

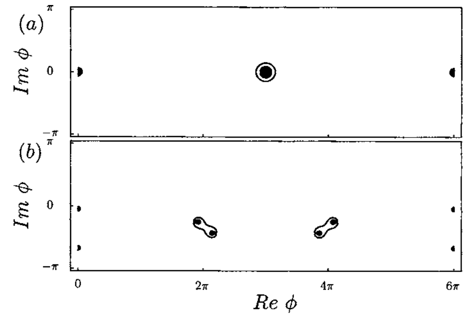

FIG. 7. (a) Zeros and (b) poles of G in the asymmetric case (R=1/2) for β=0.6.

To study the behavior of the transmission zeros under the influence of SOC, an expansion around the zeros in the AC phase $\Phi_{\mathrm{AC}}^{\sigma}$ (mod2π) of the transmission probability $\mathcal{J}_\sigma = |T_\sigma|^2$ is performed:

$$\mathcal{J}_\sigma(\Phi_{\mathrm{AC}}^{\sigma}) = 8\csc(\pi m\gamma)(\Phi_{\mathrm{AC}}^{\sigma})^2 + O[(\Phi_{\mathrm{AC}}^{\sigma})^3] \quad \text{at } \phi_{0,1} = 2m\pi, \tag{27}$$

$$\mathcal{J}_\sigma(\Phi_{\mathrm{AC}}^{\sigma}) = \frac{16\{1+\cos[(2m+1)\pi/\gamma]\}}{\{5-3\cos[(2m+1)\pi/\gamma]\}^2}(\Phi_{\mathrm{AC}}^{\sigma})^2 + O[(\Phi_{\mathrm{AC}}^{\sigma})^3] \quad \text{at } \phi_{0,2} = (2m+1)\pi/\gamma. \tag{28}$$

Equation (27) shows that the zeros of the first type are removed by the action of SOC for all values of γ. For the zeros of the second type however there are geometries where the zeros persist even in presence of the interaction. From Eq. (28) follows the geometry condition for persistent zeros:

$$\gamma_{\mathrm{per}} = \frac{2m+1}{2n+1} \Leftrightarrow R_{\mathrm{per}} = \frac{m+n+1}{n-m}, \quad n,m \in \mathbb{Z}, \quad n > m. \tag{29}$$

IV. ANALOGY TO AB RING AND SYMMETRY ARGUMENT

It was shown[14,33] for the AB ring that zero conductance energies belong to states of vanishing vorticity, i.e., the circular currents in the loop system change sign at these energies. The zero conductance resonances can therefore be regarded as the signatures of destructive interference resulting from the superposition of circular currents of opposite chirality corresponding to degenerate resonant states of the loop system. The possibility of superposition is due to the degen-

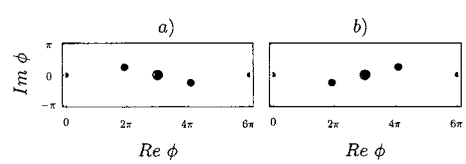

FIG. 8. Zeros of (a) $T_\uparrow$ and (b) $T_\downarrow$ for β=0.6 and R=1/2. SOC shifts the zeros away from the real axis, the direction of the shift depending on the spin.

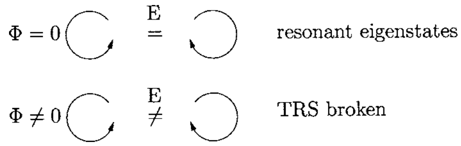

FIG. 9. Resonant states and broken symmetry for the AB ring (E=energy).

eracy of the two chiral states as a consequence of time reversal symmetry in absence of external fields. A magnetic field, respectively the resulting flux Φ through the loop, destroys this degeneracy as a consequence of broken TRS (Fig. 9).

In the present case of a one-dimensional ring subject to Rashba SOC, the role of the magnetic flux Φ is played by the Rashba term depending on the coupling β. In fact, the transmission function in Eq. (22) equals the expression obtained in Ref. 14 for the asymmetric AB ring, except that the AB phase $\Phi_{\rm AB}=2\pi\Phi/\Phi_0$ is replaced by the (spin dependent) AC phase $\Phi_{\rm AC}^{\sigma}$. In analogy to the AB ring, there are conductance zeros due to resonant states of different chirality for the free system at $\beta=0$. At finite SOC, configurations of opposite spin *and* chirality are still degenerate as a consequence of time reversal symmetry: with the time reversal operator given by $\hat{T}=-i\sigma_y\hat{K}$, where $\hat{K}$ is the operator for complex conjugation, and using the relations in Eq. (11), we find

$$\hat{T}\Psi_{n,\lambda}^{\sigma} = -\sigma\Psi_{n,-\lambda}^{-\sigma}. \tag{30}$$

For a fixed spin orientation, however, states of opposite chirality are no longer degenerate as parity is broken for $\beta \neq 0$. The situation with SOC is illustrated in Fig. 10. The vanishing of circular currents corresponding to time reversed degenerate states is easily derived: from Eqs. (18) and (19) follow the currents

$$j_{\lambda}^{\sigma} = 2\left[n_{\lambda}^{\sigma} + \sin^2\left(\frac{\theta}{2} + \frac{\pi}{4}(1-\sigma)\right)\right] + \sigma\beta\sin\theta,$$
$$\sigma = \pm 1, \quad \lambda = \pm. \tag{31}$$

The total circular current of time reversed states has to vanish such that

$$j_{tot}(\Psi_{\lambda}^{\sigma},\Psi_{-\lambda}^{-\sigma}) = 2(n_{\lambda}^{\sigma} + 1 + n_{-\lambda}^{-\sigma}) \equiv 0 \quad \forall\, \beta, \tag{32}$$

whereas the total circular current for states of a equal spin disappear only for $\beta \to 0$,

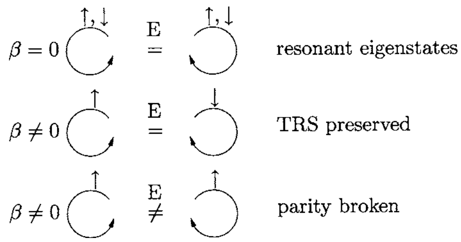

FIG. 10. Resonant states and broken parity for ring subject to Rashba SOC.

TABLE I. Symmetry breaking analogy between AB and Rashba rings.

	AB ring	Ring with Rashba SOC
ext field	$\vec{B}=(0,0,B_z)$	$\vec{E}=(0,0,E_z)$
Hamiltonian	$\hat{H}=\frac{1}{2m\varrho^2}\left(\frac{\hbar}{i}\frac{\partial}{\partial\varphi}+\frac{\Phi}{\Phi_0}\right)^2$ $\Phi_0=\frac{hc}{e}$	$\hat{H}=\frac{1}{2m\varrho^2}\left(\frac{\hbar}{i}\frac{\partial}{\partial\varphi}+A(\varphi)\right)^2$ $A(\varphi)=\frac{\beta\hbar^2}{2}\sigma_r(\varphi)$
broken symm.	time reversal $\hat{T}$	spin parity $\hat{P}_s$
	$[\hat{H},\hat{T}]=\frac{2\hbar k}{m}\frac{\Phi}{\Phi_0}$ $=0 \Leftrightarrow \Phi=0$	$[\hat{H},\hat{P}_s]=-i\beta\sin\varphi\sigma_z$ $=0 \Leftrightarrow \beta=0$

$$j_{\rm tot}(\Psi_{+}^{\uparrow},\Psi_{-}^{\uparrow}) = 2\left(n_{+}^{\uparrow} + n_{-}^{\uparrow} + 2\sin^2\frac{\theta}{2} + \beta\sin\theta\right), \tag{33}$$

$$j_{\rm tot}(\Psi_{+}^{\downarrow},\Psi_{-}^{\downarrow}) = 2\left(n_{+}^{\downarrow} + n_{-}^{\downarrow} + 2\cos^2\frac{\theta}{2} - \beta\sin\theta\right). \tag{34}$$

The symmetry breaking analogy between AB rings and rings subject to Rashba-SOC appears already in the corresponding Hamiltonians and their symmetries. For Rashba SOC, the normalized magnetic flux Φ/Φ_0 breaking time reversal symmetry in the AB ring is replaced by the spin-dependent vector potential $A(\varphi)$ which respects the TRS of $\hat{H}$, i.e.,

$$[\hat{H},\hat{T}]_{\Psi_{n,\lambda}^{\sigma}} = (n+1+\Phi_{\rm AC}^{-\sigma})^2 - (n-\Phi_{\rm AC}^{\sigma})^2 \equiv 0 \quad \forall\, \beta, \tag{35}$$

but changes under spin reversal, and which is related to the Aharonov-Casher phase by Eq. (6) for the eigenenergies. The main results of this analysis are summarized in Table I.

In Ref. 24, a relation was established between the breaking of TRS by a magnetic field in an AB ring and the location of the transmission zeros in the complex plane. It was shown that real zeros appear if the flux is an integer or half integer multiple of the flux quantum Φ_0, and are shifted off the real axis for other flux values. Due to the analogy to the AB ring, the behavior of the transmission zeros of the ring subject to Rashba SOC follows the same rules, now depending on the value of the AC phase. This implies the periodical dependence of transmission properties on the value of the SOC constant β: real transmission zeros demand a (half) integer AC phase, $\Phi_{\rm AC}^{\sigma}/2\pi=(2m+1)/2$, $m\in\mathbb{Z}$, which is satisfied by[12] $\beta=\sqrt{4(m+1)^2-1}$.

It was shown by Lee and co-workers that conductance zeros occur generically in quasi-1D systems if time reversal is a symmetry.[18,19] In the proof, they used the constraints laid upon the elements of the scattering matrix describing the system of spinless particles by the symmetry and unitarity requirement. In the case of particles with spin, these conditions are reproduced only in the presence of time reversal symmetry *and* parity, apart from special situations (geometries).

As in Ref. 13, it is possible to combine the AB and AC effects by the inclusion of a finite magnetic flux in the Hamiltonian (4),

$$H = \left(-i\frac{\partial}{\partial\varphi} + \frac{\beta}{2}\sigma_r - \frac{\Phi}{\Phi_0}\right)^2. \tag{36}$$

Equation (9) becomes[13]

$$n_\lambda^\sigma(k) = \lambda k \varrho + (\Phi_{AC}^\sigma + \Phi_{AB})/2\pi. \tag{37}$$

Thus, the AB effect contributes just a spin-independent additive phase, i.e., in Eq. (22), the AC phase has to be replaced by the sum of AB and AC phases.

V. CONCLUSIONS

In summary, we have shown that zero conductance resonances appearing as a signature of interfering resonant states of the loop system, and as a consequence of its asymmetry, behave in a similar way under the influence of a magnetic flux through the loop as in presence of a perpendicular electric field generating Rashba spin-orbit coupling. Real conductance zeros are lifted by the influence of these external fields, being shifted into the complex plane depending on the value of the AB(AC) phase. In the case of the magnetic flux, it is the breaking of time reversal symmetry which destroys the energetic degeneracy of states with opposite chirality, preventing the destructive interference leading to the zeros. For Rashba SOC, time reversal symmetry is respected, but not spin reversal symmetry, which again leads to a chiral dependence in the energy of the loop wave function and eventually to the lifting of the conductance zeros.

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APPENDIX: DERIVATION OF THE TRANSMISSION AMPLITUDE IN THE SINGLE-ELECTRON SCATTERING PICTURE

This derivation follows Refs. 13 and 12. System geometry and coordinates are shown in Fig. 1. Asymmetry is introduced by choosing different lengths l_{up} and l_{low} for upper and lower branches of the ring in the figure, and is expressed by means of the asymmetry factor $R = l_{low}/l_{up}$. This leads to different phases at the left junction (A):

$$\varphi(A) = \frac{2\pi}{R+1} \equiv \varphi_A, \quad \varphi'(A) = \frac{2\pi R}{R+1} \equiv \varphi_A'. \tag{A1}$$

The connection of leads and ring is described by the application of spin-dependent Griffith's boundary conditions,[30] which demand (a) continuity of the wave function and (b) probability current conservation at the junctions (A) and (B).[37]

To be able to apply the boundary conditions, we need the wave functions of leads and branches. The wave functions Ψ_I and Ψ_{II} for incoming and outgoing leads respectively, can be expanded in terms of the spinors χ^σ at the junctions

$$\Psi_I(x) = \sum_{\sigma=\uparrow,\downarrow} \Psi_I^\sigma(x)\chi^\sigma(\varphi_A), \quad x \in [-\infty, 0], \tag{A2}$$

$$\Psi_{II}(x') = \sum_{\sigma=\uparrow,\downarrow} \Psi_{II}^\sigma(x')\chi^\sigma(0), \quad x' \in [0, \infty]. \tag{A3}$$

The expansion coefficients are the orbital wave functions

$$\Psi_I^\sigma(x) = i_\sigma e^{ikx} + r_\sigma e^{-ikx}, \tag{A4}$$

$$\Psi_{II}^\sigma(x') = t_\sigma e^{ikx'}, \tag{A5}$$

where we assume an incident plane wave from the left with wave number k. The coefficients i_σ of the incoming wave are chosen such that $\Sigma_\sigma |i_\sigma|^2 = 1$. r_σ and t_σ are the spin-dependent reflection and transmission coefficients, respectively. A similar expansion in terms of the ring eigenstates in Eq. (12) yields the wave functions Ψ_{up} and Ψ_{low} of upper and lower branches, respectively,

$$\Psi_{up}(\varphi) = \sum_{\sigma=\uparrow,\downarrow} \Psi_{up}^\sigma(\varphi)\chi^\sigma(\varphi), \quad \varphi \in [0, \varphi_A], \tag{A6}$$

$$\Psi_{low}(\varphi') = \sum_{\sigma=\uparrow,\downarrow} \Psi_{low}^\sigma(\varphi')\chi^\sigma(-\varphi'), \quad \varphi' \in [0, \varphi_A'], \tag{A7}$$

with the corresponding orbital components

$$\Psi_{up}^\sigma(\varphi) = \sum_{\lambda=+,-} a_\lambda^\sigma e^{in_\lambda^\sigma \varphi}, \tag{A8}$$

$$\Psi_{low}^\sigma(\varphi') = \sum_{\lambda=+,-} b_\lambda^\sigma e^{-in_\lambda^\sigma \varphi'}, \tag{A9}$$

where n_λ^σ is given by Eq. (9).

Imposing the boundary conditions mentioned above, it is now possible to relate the transmission and reflection coefficients r_σ and t_σ to the input parameters i_σ. The continuity conditions for the wave function demand $\Psi_{II}^\sigma(0) = \Psi_{up}^\sigma(0) = \Psi_{low}^\sigma(0)$ and $\Psi_I^\sigma(0) = \Psi_{up}^\sigma(\varphi_A) = \Psi_{low}^\sigma(\varphi_A')$, yielding the equations

$$\sum_{\lambda=+,-} a_\lambda^\sigma = \sum_{\lambda=+,-} b_\lambda^\sigma = t_\sigma, \tag{A10}$$

$$\sum_{\lambda=+,-} a_\lambda^\sigma e^{in_\lambda^\sigma \varphi_A} = \sum_{\lambda=+,-} b_\lambda^\sigma e^{-in_\lambda^\sigma \varphi_A'} = r_\sigma + i_\sigma. \tag{A11}$$

Probability current density conservation requires $j_{up}^\sigma + j_{low}^\sigma + j_{I(II)}^\sigma = 0$ at the junctions. The current densities follow evaluating the expressions derived in Sec. II for the wave functions above. The (dimensionless) ring currents read

$$j_{up}^\sigma(\varphi) = \frac{1}{2}[(\Psi_{up}^\sigma \chi^\sigma)^\dagger (\hat{v}\Psi_{up}^\sigma \chi^\sigma) + \Psi_{up}^\sigma \chi^\sigma (\hat{v}\Psi_{up}^\sigma \chi^\sigma)^\dagger](\varphi), \tag{A12}$$

$$j_{\mathrm{low}}^{\sigma}(\varphi') = \frac{1}{2}[(\Psi_{\mathrm{low}}^{\sigma}\chi_{-}^{\sigma})^{\dagger}(\hat{v}'\Psi_{\mathrm{low}}^{\sigma}\chi_{-}^{\sigma}) + \Psi_{\mathrm{low}}^{\sigma}\chi_{-}^{\sigma}(\hat{v}'\Psi_{\mathrm{low}}^{\sigma}\chi_{-}^{\sigma})^{\dagger}] \times(\varphi'), \quad \text{(A13)}$$

where $\hat{v}(\varphi)=\hat{v}_0(\varphi)+\hat{v}_{\mathrm{SO}}(\varphi)$, $\hat{v}'(\varphi)=\hat{v}_0(\varphi)-\hat{v}_{\mathrm{SO}}(\varphi)$, and $\chi_{-}^{\sigma}(\varphi')=\chi^{\sigma}(-\varphi')$. The currents in the leads are given by

$$j_{\mathrm{I}}^{\sigma}(x) = \frac{1}{2}[(\Psi_{\mathrm{I}}^{\sigma}\chi_{A}^{\sigma})^{\dagger}(\hat{v}_0\Psi_{\mathrm{I}}^{\sigma}\chi_{A}^{\sigma}) + \Psi_{\mathrm{I}}^{\sigma}\chi_{A}^{\sigma}(\hat{v}_0\Psi_{\mathrm{I}}^{\sigma}\chi_{A}^{\sigma})^{\dagger}](x), \quad \text{(A14)}$$

$$j_{\mathrm{II}}^{\sigma}(x') = \frac{1}{2}[(\Psi_{\mathrm{II}}^{\sigma}\chi_{B}^{\sigma})^{\dagger}(\hat{v}_0\Psi_{\mathrm{II}}^{\sigma}\chi_{B}^{\sigma}) + \Psi_{\mathrm{II}}^{\sigma}\chi_{B}^{\sigma}(\hat{v}_0\Psi_{\mathrm{II}}^{\sigma}\chi_{B}^{\sigma})^{\dagger}](x'), \quad \text{(A15)}$$

where $\chi_{A(B)}^{\sigma}=\chi_{\sigma}\{\varphi[A(B)]\}$. Using the equality of the wave function at the junctions and noting that $\hat{v}_{\mathrm{SO}}(\varphi)\chi^{\sigma}(\varphi) =-\hat{v}_{\mathrm{SO}}(\varphi)\chi_{-}^{\sigma}(\varphi')$, the probability current density conservation condition simplifies to

$$\hat{v}_0\Psi_{\mathrm{up}}^{\sigma}|_{\varphi=0(\varphi_A)} + \hat{v}_0\Psi_{\mathrm{low}}^{\sigma}|_{\varphi'=0(\varphi_A')} + \hat{v}_0\Psi_{\mathrm{I(II)}}^{\sigma}|_{x(x')=0} = 0. \quad \text{(A16)}$$

From that follows an additional pair of equations for the coefficients:

$$\sum_{\lambda=+,-} a_{\lambda}^{\sigma}\frac{n_{\lambda}^{\sigma}}{k\varrho} - \sum_{\lambda=+,-} b_{\lambda}^{\sigma}\frac{n_{\lambda}^{\sigma}}{k\varrho} + t_{\sigma} = 0, \quad \text{(A17)}$$

$$\sum_{\lambda=+,-} a_{\lambda}^{\sigma}e^{in_{\lambda}^{\sigma}\varphi_A}\frac{n_{\lambda}^{\sigma}}{k\varrho} - \sum_{\lambda=+,-} b_{\lambda}^{\sigma}e^{-in_{\lambda}^{\sigma}\varphi_A'}\frac{n_{\lambda}^{\sigma}}{k\varrho} + i_{\sigma} - r_{\sigma} = 0. \quad \text{(A18)}$$

Together with Eqs. (A10) and (A11), we now have enough equations to determine the coefficient set $\{r_{\sigma},t_{\sigma},a_{\lambda}^{\sigma},b_{\lambda}^{\sigma}\}$, $\lambda =\pm$, for both spin polarizations $\sigma=\uparrow,\downarrow$ as a function of the input coefficients i_{σ}, the incident wave number k, ring radius ϱ and SOC constant β. For an incident current from the right, an analogous calculation is performed with $\{i_{\sigma},r_{\sigma}\}$ (left lead) and $\{t_{\sigma},0\}$ (right lead) replaced by $\{0,t_{\sigma}'\}$ and $\{r_{\sigma}',i_{\sigma}'\}$, respectively. This enables us to formulate the scattering matrix of the ring system $\vec{o}=\underline{S}\vec{i}$, where $\vec{o}$ stands for outgoing and $\vec{i}$ for incoming wave coefficients. The relations can be written as $t_{\sigma}^{(\prime)}=\Sigma_{\sigma'}T_{\sigma\sigma'}^{(\prime)}i_{\sigma'}^{(\prime)}$, $r_{\sigma}^{(\prime)}=\Sigma_{\sigma'}R_{\sigma\sigma'}^{(\prime)}i_{\sigma'}^{(\prime)}$. A careful examination shows that no spin flip amplitudes for transmission or reflection in this spinor basis are present, and a possible modification of the spinor is only due to a difference between propagating channels. Thus, the scattering matrix reads

$$\underline{S} = \begin{pmatrix} R_{\uparrow} & 0 & T_{\uparrow}' & 0 \\ 0 & R_{\downarrow} & 0 & T_{\downarrow}' \\ T_{\uparrow} & 0 & R_{\uparrow}' & 0 \\ 0 & T_{\downarrow} & 0 & R_{\downarrow}' \end{pmatrix}. \quad \text{(A19)}$$

The overall conductance then follows from the entries of the scattering matrix by means of the Landauer conductance formula[28,29] shown in Eq. (21), with the spin-dependent transmission amplitude given by Eq. (22). The corresponding expression for the reflection amplitude is

$$R_{\sigma}(\phi,\beta,\gamma) = \frac{\cos\phi\gamma + 3\cos\phi - 4\cos\Phi_{\mathrm{AC}}^{\sigma}}{\cos\phi\gamma - 5\cos\phi + 4\cos\Phi_{\mathrm{AC}}^{\sigma} + 4i\sin\phi}. \quad \text{(A20)}$$

The (time reversed) functions for incident wave in the right lead are related to those above by $T_{\sigma}'(\Phi_{\mathrm{AC}}^{\sigma})=T_{\sigma}(-\Phi_{\mathrm{AC}}^{\sigma})$ and $R_{\sigma}'=R_{\sigma}$.

The transmission and reflection coefficients with respect to the standard σ_z basis $\{|s\rangle\}$ are obtained by the corresponding spin rotation $\Lambda(\{|s\rangle\}\to\{|\sigma\rangle\})$ of the diagonal transmission and reflection blocks in the scattering matrix (A19), e.g., for the transmission

$$T_{ss'} = \langle s'|\Lambda^{-1}(0)\circ[T_{\sigma\sigma'}]\circ\Lambda(\varphi_A)|s\rangle, \quad \text{(A21)}$$

where

$$\Lambda(\varphi) = \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & -e^{-i\varphi}\cos\frac{\theta}{2} \end{pmatrix} \quad \text{(A22)}$$

and

$$[T_{\sigma\sigma'}] = \begin{pmatrix} T_{\uparrow} & 0 \\ 0 & T_{\downarrow} \end{pmatrix}, \quad \text{(A23)}$$

and analogously for the reflection coefficients.

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[34] The neglected term in Eq. (4) produces a spin-independent modification of the spectrum, which should not influence the interference effects.
[35] Note that using H, we are still working with dimensionless quantities.
[36] Generally, $G=(e^2/h)\Sigma_{\sigma,\sigma'=\uparrow,\downarrow}|T_{\sigma,\sigma'}|^2$, where $T_{\sigma,\sigma'}$, $\sigma\neq\sigma'$ is the spin flip amplitude. The conductance does not depend on the choice of spinor basis, i.e., it is invariant under spin rotation, we can therefore make use of the ring-spinor basis where the spin flip amplitudes vanish, and define $T_{\sigma\sigma}\equiv T_{\sigma}$.
[37] In the scattering matrix approach of Ref. 3, these conditions correspond to a coupling parameter $\epsilon=4/9$, which is below the value of $\epsilon=1/2$ for perfect transmission.