Quantum corrections to the conductivity and Hall coefficient of a two-dimensional electron gas in a dirty $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$ quantum well: From the diffusive to the ballistic regime

V. T. Renard,[1,2,*] I. V. Gornyi,[3,4] O. A. Tkachenko,[1,5] V. A. Tkachenko,[5] Z. D. Kvon,[1,5] E. B. Olshanetsky,[1,5] A. I. Toropov,[5] and J.-C. Portal[1,2,6]

[1]*GHMFL, MPI-FKF/CNRS, BP-166, F-38042, Grenoble Cedex 9, France*
[2]*INSA-Toulouse, 31077, Cedex 4, France*
[3]*Forschungszentrum Karlsruhe, Institut für Nanotechnologie, 76021 Karlsruhe, Germany*
[4]*A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia*
[5]*Institute of Semiconductor Physics, Novosibirsk 630090, Russia*
[6]*Institut Universitaire de France, Toulouse, France*

(Received 17 December 2004; revised manuscript received 19 May 2005; published 3 August 2005)

We report an experimental study of quantum conductivity corrections in a low mobility, high density two-dimensional electron gas in an AlGaAs/GaAs/AlGaAs quantum well in a wide temperature range (1.5–110 K). This temperature range covers both the diffusive and the ballistic interaction regimes for our samples. It has been therefore possible to study the crossover between these regimes for both the longitudinal conductivity and the Hall effect. We perform a parameter-free comparison of our experimental data for the longitudinal conductivity at zero magnetic field, the Hall coefficient, and the magnetoresistivity to the recent theories of interaction-induced corrections to the transport coefficients. A quantitative agreement between these theories and our experimental results has been found.

DOI: 10.1103/PhysRevB.72.075313 PACS number(s): 73.21.Fg, 73.20.Fz

I. INTRODUCTION

At low temperatures the conductivity of a degenerated two-dimensional electron gas (2DEG) is governed by quantum corrections to the Drude conductivity σ_D. In general, these corrections have two principal origins: the weak localization (WL) and the electron-electron (e-e) interaction.[1] Until recently our understanding of the interaction corrections to the conductivity of a 2DEG was based on the seemingly unrelated theories developed for two opposite regimes: the diffusive regime[2] $k_BT\tau/\hbar \ll 1$, and the ballistic regime[3] $k_BT\tau/\hbar \gg 1$. In the diffusive regime the quasiparticle interaction time $\hbar/k_BT$ is larger than the momentum relaxation time τ and two interacting electrons experience multiple impurity scattering. In the ballistic regime the e-e interaction is mediated by a single impurity.

Recently, Zala, Narozhny, and Aleiner (ZNA) have developed a new theory of the interaction related corrections to the conductivity[4,5] that bridges the gap between the two theories known previously.[2,3] One of the important conclusions of the new theory is that the interaction corrections to the conductivity in both regimes have a common origin: the coherent scattering of electrons by Friedel oscillations. This can be also reformulated in terms of returns (diffusive and ballistic) of an electron to a spatial region which it has already visited. Conformably to the previous results,[2,3] the new theory predicts a logarithmic temperature dependence of the longitudinal conductivity and the Hall coefficient in the diffusive regime, whereas in the ballistic regime the temperature dependence of these parameters becomes linear and T^{-1}, respectively. Finally a further step in the generalization of the interaction theory was realized in Refs. 6 and 7 that considered application of strong perpendicular magnetic fields for arbitrary type of disorder potential and influence of electron-phonon impurity scattering, respectively.

Despite a surge of experimental activity[8–11,13–16] following the publication of the theory,[4,5] to our knowledge, no experiment has been reported where the crossover between the two regimes would have been clearly observed. One of the reasons is that the temperature at which the crossover is expected to occur is given by $k_BT\tau/\hbar \approx 0.1$ (see Refs. 4–6). In the relatively high-mobility 2D systems that are commonly studied the crossover temperature is by far too low to be accessed experimentally ($T < 100$ mK for $\tau > 10^{-11}$ s). Thus, the ZNA theory has to our knowledge been verified only in the intermediate and ballistic regimes[17] ($k_BT\tau/\hbar = 0.1–10$).

To shift the crossover to higher temperatures one should use low mobility samples (small τ). At the same time high carrier densities N_s are necessary in order to maintain high conductivity and avoid strong localization. Note that such samples were already grown and studied[18–20] in the diffusive regime, but the crossover between the ballistic and diffusive regimes was not considered. In high density 2D systems the characteristic parameter $r_s = E_C/E_F \propto 1/N_s^{1/2}$, the ratio between Coulomb energy and Fermi energy is small ($r_s < 1$) and hence the effect of e-e interaction is relatively weak. In this case the ZNA theory[4] predicts insulatinglike behavior of conductivity $d\sigma_{xx}/dT > 0$ at all temperatures, whereas the "screening" theory[3] predicts metalliclike behavior $d\sigma_{xx}/dT < 0$ in the high-temperature ballistic regime. Moreover, for such small r_s the Fermi liquid interaction constant F_0^σ, the only parameter in the expressions for the quantum corrections to the conductivity in the theory,[4] can be calculated explicitly.

In this respect low-mobility high-density systems appear to offer certain advantages for testing the theory,[4,5] as compared to high-mobility low-density systems. Indeed not only do they provide an opportunity for studying an experimentally accessible temperature crossover between the diffusive and the ballistic interaction regimes but also the comparison between the theory and experiment requires no fitting parameters. Also, in such systems the disorder potential is mostly due to the short-range impurities which yields almost isotropic scattering on impurities as assumed in Refs. 4 and 5. As shown in Ref. 6, the nature of disorder becomes crucially important in the ballistic regime. Finally, the interaction-induced longitudinal magnetoresistance (MR) $\rho_{xx}(B,T)$ in the ballistic regime has been already studied on systems with long-range[12] and mixed[13] disorder, where the theoretical results of Ref. 6 have been confirmed. However, no experimental results for $\rho_{xx}(B,T)$ have been reported so far for low-mobility systems in the ballistic regime.

The aim of the present work is to experimentally study the interaction related corrections to the conductivity, magnetoresistivity, and the Hall coefficient in a broad temperature range covering both the diffusive and ballistic interaction regimes and the crossover between them. The experimental results obtained in the weak interaction limit are expected to allow for a parameter-free comparison with the ZNA theory for both σ_{xx} and ρ_{xy}. We also compare our results on the MR for short-range disorder with the predictions of Ref. 6.

II. EXPERIMENTAL SETUP

The experimental samples had a 2DEG formed in a narrow (5 nm) AlGaAs/GaAs/AlGaAs quantum well δ-doped in the middle. Such doping results in a low mobility and a high carrier density. Also impurities located in the layer give rise to a large-angle scattering of electrons. A detailed description of the structure can be found in Ref. 21. Two samples from the same wafer have been studied for which similar results were obtained. Here we present the data obtained for one of the samples with the following parameters at $T=1.4$ K depending on prior illumination: the electron density $N_s=(2.54-3.41)\times 10^{12}$ cm^{-2} and the mobility $\mu=(380-560)$ cm^2/V s. The Hall bar shaped samples were studied between 1.4 and 110 K in magnetic fields up to 15 T using a superconducting magnet and a VTI cryostat and also a flow cryostat ($T>5$ K) placed in a 20 T resistive magnet. The data was acquired via a standard four-terminal lock-in technique with the current 10 nA.

Figure 1 shows the longitudinal and Hall resistances of the sample as a function of magnetic field at temperatures up to 110 K. As can be seen both are strongly temperature dependent. Before analyzing the role of the quantum corrections in the behavior of the transport coefficients shown in Fig. 1, let us estimate the possible contribution from other unrelated temperature dependent factors.

First, since the measurements were performed up to relatively high temperatures, the question of the role of phonon scattering becomes important. In this connection we believe that the following argument can be used. It is well known that in ultraclean GaAs samples sufficiently high values of

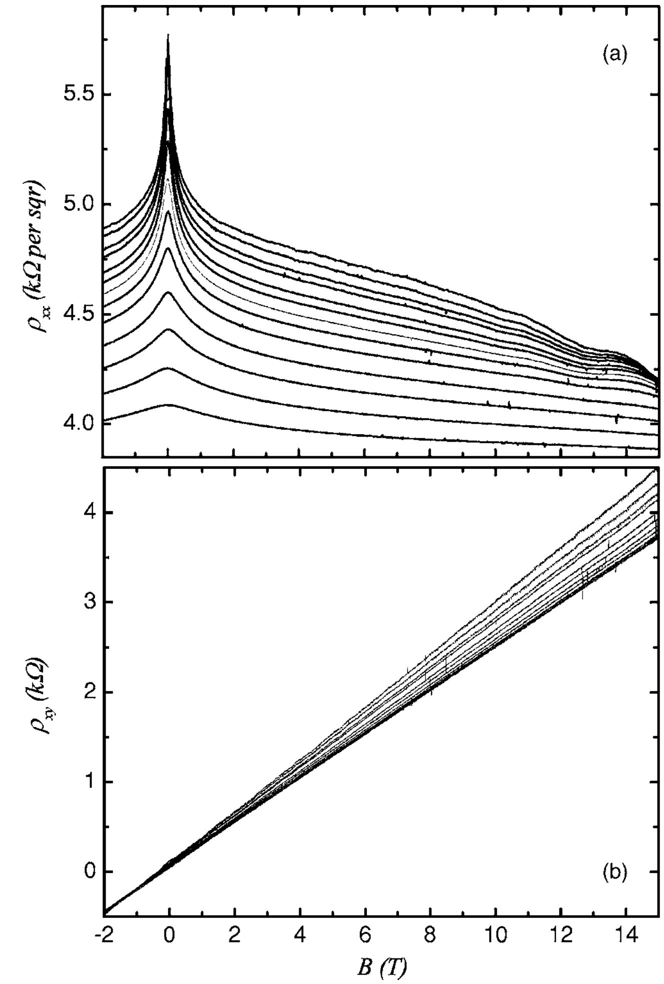

FIG. 1. (a) Longitudinal resistivity of the sample at $N_s=2.56\times 10^{12}$ cm^{-2} for temperature=1.4, 1.9, 3.1, 4, 7.2, 10.25, 15.45, 21.5, 31, 46.2, 62.8, 84.5, and 110 K from top to bottom. (b) Hall resistance at the same temperatures (from top to bottom).

mobility are reported even at liquid nitrogen temperatures (see, for example, Refs. 22 and 23, where $\mu=4\times 10^5$ cm^2/V s at $T=77$ K). At these temperatures the phonon scattering is the dominant scattering mechanism in these samples and yet the mobilities are still a thousand times larger than in our sample. In our experiment, the pure electron-phonon contribution to the conductivity is thus negligible compared to impurity scattering.

Recently a theory of the interplay between electron-phonon and impurity scattering was developed.[7] It was argued that these interference effects might play a significant role at intermediate temperatures. However, we have evaluated the phonon contribution to be a few percent of the Drude conductivity at 100 K. Also our estimates show that this contribution is still smaller than the *e*-*e* interaction one. For these reasons the effect of phonons can be neglected in the entire experimental temperature range in these samples.

Now, as can be seen in Fig. 1, the Hall coefficient varies with T at low temperatures but remains practically constant for $T>20$ K. One might argue that the behavior at low temperatures could be due to a variation of the electron density with temperature. However, we believe that this is not the case. Indeed, from the measurements carried out up to 20 T where the Shubnikov–de Haas (SdH) oscillations are better resolved, we find that the density remains constant at $T < 30$ K. Also we find that the density given by the SdH oscillations is the same as we get from the slope of the Hall resistance at $T > 20$ K where it is T-independent. We conclude therefore that the electron density remains constant in the entire experimental temperature range and all the data presented in Fig. 1 correspond to $N_s = 2.56 \times 10^{12}$ cm^{-2}.

Having excluded the phonon scattering and the density variation as possible causes of the behavior shown in Fig. 1 we associate the observed temperature dependencies with the quantum corrections to the transport coefficients. Our data will be analyzed in the framework of the recent theories[4,5] valid for a degenerated 2DEG ($k_B T \ll E_F$). According to Ref. 21 only one subband is occupied in our quantum wells at $N_s = 2.56 \times 10^{12}$ cm^{-2}. Also $E_F \approx 1000$ K and so the theory[4,5] should apply under our experimental conditions.

III. QUANTUM CORRECTIONS TO THE CONDUCTIVITY TENSOR: BACKGROUND

The longitudinal conductivity is a sum of three components: the Drude conductivity, the WL contribution, and the e-e interaction correction. For the correct evaluation of the interaction related correction at $B=0$ T, the knowledge of the first two contributions to the conductivity is essential. Unfortunately, in our case there is no direct means of knowing the value of the Drude conductivity σ_0^D because of a considerable (up to 20%) variation of the zero field conductivity with temperature. On the other hand, to single out the e-e interaction correction we have to eliminate the weak localization contribution, which might be stronger than the interaction correction at $B=0$.

The WL correction to the conductivity at low temperatures and magnetic fields is described by a well-known expression[24] involving digamma functions. However, at high enough temperatures and/or magnetic fields (when the contribution of nondiffusive paths becomes more and more pronounced) the WL correction is given by a rather complicated analytical expression.[25] Nevertheless, there exists a method (see the next section) that can be used for the evaluation of all three contributions to the conductivity at zero magnetic field based on the knowledge of the high-B behavior of the magnetoconductivity. This method has the advantage that one can forgo the usual procedure of fitting the low field data with the theoretical expressions for the WL magnetoresistance,[24,25] thus eliminating a possible source of error at this stage.

A general formula for the conductivity tensor in a magnetic field can be derived using the quantum kinetic equation of Ref. 4. The longitudinal and the Hall conductivities can be written for $k_B T \ll E_F$ in the following form:[26]

$$\sigma_{xx}(T,B) = \frac{\sigma_D(T)}{1+\omega_c^2\tau^2(T)} + \Delta\sigma_{ee}^{\mathrm{Diff}}(T) + \Delta\sigma_{xx}^{\mathrm{WL}}(T,B), \quad (1)$$

$$\sigma_{xy}(T,B) = \frac{\omega_c\tau(T)\sigma_D(T)}{1+\omega_c^2\tau^2(T)} + \omega_c\tau(T)\Delta\sigma_{ee}^{\mathrm{H}}(T) + \Delta\sigma_{xy}^{\mathrm{WL}}(T,B). \quad (2)$$

Generally, the zero-B Drude conductivity $\sigma_D(T)$ depends on T due to the interaction-induced renormalization of both the transport scattering time $\tau(T)$ and the Fermi velocity $v_F(T)$. Strictly speaking, the cyclotron frequency ω_c also depends on T via the renormalization of the effective mass $m^*(T)$; however, it appears in Eqs. (1) and (2) only in combination $\omega_c\tau(T)$ so that one can absorb its renormalization into the T-dependence of the effective scattering time. While the first terms in Eqs. (1) and (2) have the structure of the classical Drude conductivity in a finite B, the terms $\Delta\sigma_{ee}^{\mathrm{Diff}}(T)$ and $\omega_c\tau(T)\Delta\sigma_{ee}^{\mathrm{H}}(T)$ appear as quantum corrections to the Drude terms.

The expressions (1) and (2) are justified for overlapping Landau levels under the condition

$$\omega_c \ll \pi/\tau + 2\pi^2 k_B T/\hbar \quad (3)$$

which governs the strength of the influence of magnetic field on the collision integral in the kinetic equation[4] and allows one to neglect cyclotron returns to the same impurity. Under this condition, the bending of relevant electron trajectories by the magnetic field is weak. It is taken into account by a proper definition of the B-independent quantities $\sigma_D(T)$, $\tau(T)$, $\Delta\sigma_{ee}^{\mathrm{Diff}}(T)$, and $\Delta\sigma_{ee}^{\mathrm{H}}(T)$. This makes it possible[26] to extract the interaction-induced corrections to the conductivity at $B=0$ from the magnetoconductivity obtained in relatively strong magnetic fields, see Sec. IV. The condition (3) is fulfilled in the whole range of relevant T and B we address in this work.

The term $\Delta\sigma_{ee}^{\mathrm{Diff}}$ in Eq. (1) corresponds to the "diffusive" contribution of e-e interactions, which is due to the coherent processes involving multiple impurity scattering events. In the diffusive regime, $\Delta\sigma_{ee}^{\mathrm{Diff}}$ diverges logarithmically[2] with decreasing T, $\Delta\sigma_{ee}^{\mathrm{Diff}}(T) \propto \ln(k_B T\tau/\hbar)$ [we note in passing that, in contrast to Eqs. (9) and (10) below, this logarithmic contribution is cut off by $\hbar/\tau$ rather than by E_F]. At high temperatures the contribution of diffusive paths is expected to vanish, since the probability of "diffusive" returns involving more than one impurity scattering is suppressed in the ballistic regime (each additional impurity scattering yields an extra factor $\hbar/k_B T\tau$). In effect, the term $\Delta\sigma_{ee}^{\mathrm{Diff}}$ in Eq. (1) also takes into account the influence of the magnetic field on the return probability determining the correction to the T-dependent part of the effective transport time, see discussion in Ref. 6. This contribution to $\Delta\sigma_{ee}^{\mathrm{Diff}}$ dominates in the ballistic regime. As for the term $\Delta\sigma_{ee}^{\mathrm{H}}$ in Eq. (2), the contribution of diffusive paths to it is exactly zero,[2], so that this term is completely determined by the influence of the magnetic field on the collision integral. Therefore in the diffusive regime $\Delta\sigma_{ee}^{\mathrm{H}}$ has no logarithmic divergency,[2] unlike $\Delta\sigma_{ee}^{\mathrm{Diff}}$.

Taking into account the e-e interaction effects related to the scattering on a single impurity results in the T-dependent renormalization[3,4,26] of $\tau(T)$. The T-dependence of $\sigma_D(T)$ in the ballistic limit is dominated by the T-dependence of $\tau(T)$ since the interaction-induced correction to the Fermi velocity yields a weaker T-dependence. Thus in the ballistic limit the linear-in-T interaction correction to the zero-B conductivity[4] enters Eq. (1) only via the renormalized transport scattering time $\tau(T)$ in the first term (both in the numerator and the denominator).

The terms $\Delta\sigma_{xx}^{\mathrm{WL}}$ and $\Delta\sigma_{xy}^{\mathrm{WL}}$ in Eqs. (1) and (2) are the WL corrections to the longitudinal and Hall conductivities, respectively. Actually, the WL corrections arise solely from the renormalization of the transport scattering time[27] and hence they can be completely absorbed into the first "classical" terms in Eqs. (1) and (2) via the B-dependent correction to $\tau(T)$.

A general method for the analysis of the magnetotransport data is based on Eqs. (1) and (2). For a given temperature one can treat the B-independent quantities $\sigma_D(T)$, $\tau(T)$, $\Delta\sigma_{ee}^{\mathrm{Diff}}(T)$, and $\Delta\sigma_{ee}^{\mathrm{H}}(T)$ as four fitting parameters to fit the two experimental curves: $\sigma_{xx}(T,B)$ vs B and $\sigma_{xy}(T,B)$ vs B. Under the assumption that the WL corrections are suppressed it follows from Eqs. (1) and (2) that

$$\sigma_{xx}(T,B) = \frac{\sigma_{xy}(T,B)}{\omega_c \tau(T)} + \Delta\sigma_{ee}^{\mathrm{Diff}}(T) - \Delta\sigma_{ee}^{\mathrm{H}}(T).$$

This equation allows one to find the values of $\tau(T)$ from the slope of $\sigma_{xx}(T,B)$ vs $\sigma_{xy}(T,B)/\omega_c$ dependence. Then $\sigma_D(T)$ and $\Delta\sigma_{ee}^{\mathrm{Diff}}(T)$ can be found from the $\sigma_{xx}(T,B)$ vs $[1+\omega_c^2\tau^2(T)]^{-1}$ plot. A detailed analysis of the magnetoconductivity based on this procedure will be published elsewhere.[28] In this paper we will concentrate on the zero-B interaction-induced correction to σ_{xx}. For this purpose a simpler fitting procedure described in Sec. IV is sufficient.

IV. LONGITUDINAL CONDUCTIVITY AT $B=0$: EXPERIMENTAL METHOD

Let us describe how the experimental quantum corrections were extracted from the row data and then turn to the analysis of the obtained corrections. The main idea of our method is to use the MR and Hall data obtained in a relatively *strong magnetic field*, where the weak localization is suppressed, to find the value of interaction-induced corrections in the limit of *zero magnetic field*.

With the magnetic field increasing, the MR in Fig. 1(a) goes through two distinct types of behavior. An abrupt drop of resistance at low fields and then a much weaker magnetic field dependence at higher B. As is well known the weak localization is suppressed at magnetic fields larger than $B_{tr} = \hbar/(2el^2)$, where l is the mean free path. In our samples $B_{tr} \approx 1.5$ T that roughly coincides with the field at which the crossover from the one type of MR to the other takes place. We conclude therefore that the strong MR observed at low fields can be associated with the WL suppression in our samples and that the MR observed at higher fields must be attributed entirely to the e-e interaction effects.[2]

As a first step of our procedure, the experimental values of the longitudinal and Hall conductivities are obtained by inverting the resistivity tensor using the data shown in Fig. 1. The result for the longitudinal conductivity is shown in Fig. 2. The weak localization correction dominates at low fields

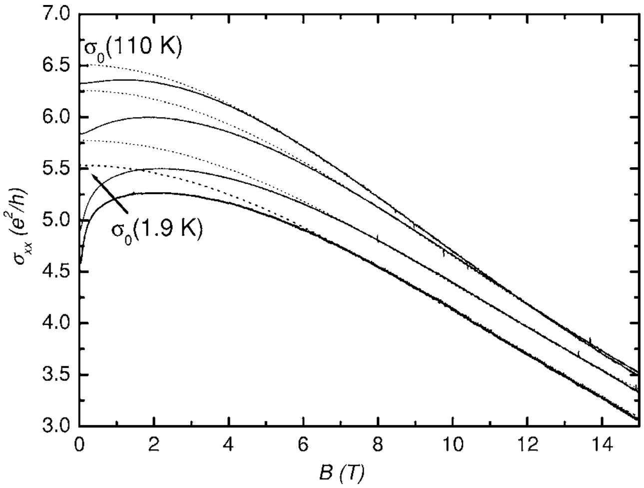

FIG. 2. Experimental longitudinal conductivity at $T=1.9$, 10.25, 62.8, and 110 K from bottom to top (solid line) and the fit to Eq. (1) according to the description in the text. The result is then extrapolated to $B=0$ T (dotted line) for the same temperatures.

but is suppressed at $B > B_{tr}$. Therefore at $B \gg B_{tr}$ the shape of the σ_{xx} vs B dependence should be determined by the first term in Eq. (1). The term $\Delta\sigma_{ee}^{\mathrm{Diff}}$, which is B-independent, should only result in a vertical shift of this contribution. At low temperatures we experimentally find that with the WL suppressed at higher magnetic fields the MC corresponding to different temperatures forms parallel vertically shifted traces (see Fig. 2) whose shape is given by the first term in Eq. (1) with a T-independent τ. At temperatures larger than 30 K the shape of the curves begins to deviate slightly from that of the low temperature traces. This change is attributed to the renormalization of the scattering time by e-e interactions in the ballistic limit.[4]

To interpolate between all the relevant regimes (diffusive vs ballistic, classically weak B vs strong B) we use a simplified version of Eq. (1). Within this procedure, we attribute the T-dependence of $\sigma_D(T)$ solely to the T-dependence of $\tau(T)$, using

$$\sigma_D(T) = e^2 n \tau(T)/m^*. \tag{4}$$

This amounts to treating all the interaction-induced corrections to the collision integral related to $\sigma_D(T)$ as the renormalization of the effective transport scattering time. Further, we assume that the T-dependence of the product $\omega_c\tau$ in the classical terms in Eqs. (1) and (2) is the same as the T-dependence of $\sigma_D(T)$. This approximation [used earlier together with Eq. (5) in Refs. 20 and 29] yields the proper asymptotics of the conductivity correction that are governed by $\Delta\sigma_{ee}^{\mathrm{Diff}}$ and $\tau(T)$ in the diffusive and the ballistic regimes, respectively.

It is possible to determine the scattering time by fitting the curves for $B > 6$ T using Eq. (1) at a given value of T with $\tau(T)$ and $\Delta\sigma_{ee}^{\mathrm{Diff}}(T)$ as fitting parameters (see Fig. 2). This was done for all the temperatures and the results for both $\Delta\sigma_{ee}^{\mathrm{Diff}}$ and $\tau(T)$ are presented in Fig. 3. The momentum relaxation time is observed to increase linearly with temperature at $T > 20$ K. This linear behavior is expected in the bal-

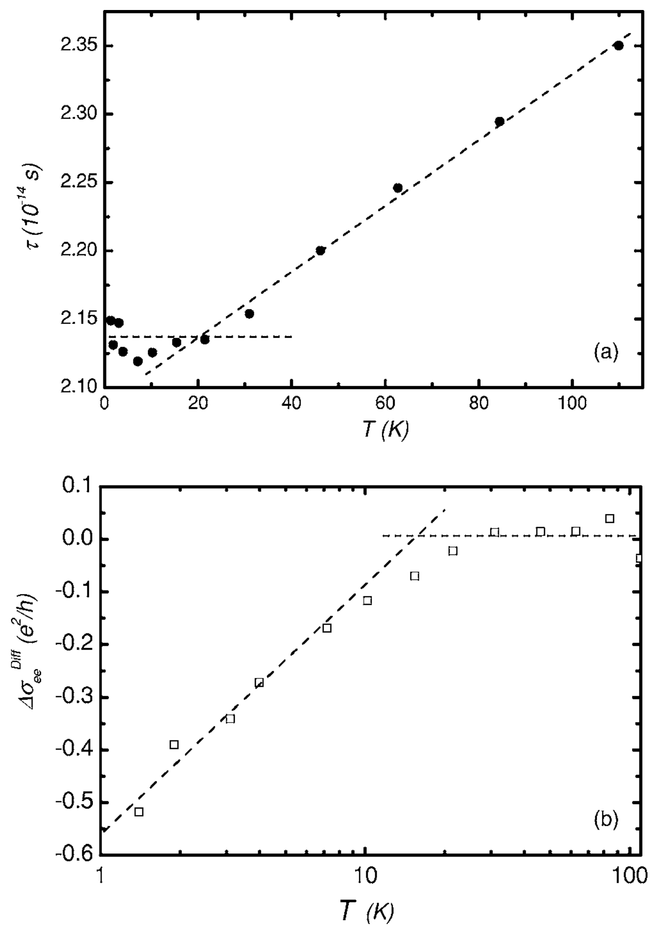

FIG. 3. Obtained values of the scattering time (a) and of the term $\Delta\sigma_{ee}^{\mathrm{Diff}}$ (b). A clear change in their behavior is observed from constant (logarithmic) to linear (constant) at $T\sim 20$ K. The lines are a guide for the eyes.

listic limit.[3,4] As for the term $\Delta\sigma_{ee}^{\mathrm{Diff}}$, it is observed to decrease in amplitude with temperature increasing and to vanish at $T>20$ K. It is important to stress that a significant change in the behavior of these two parameters occurs at $T=20$ K.

Once fitted for $B>6$ T, the term

$$\tilde{\sigma}_{xx}(T,B)=\frac{e^2n}{m^*}\frac{\tau(T)}{1+\omega_c^2\tau^2(T)}+\Delta\sigma_{ee}^{\mathrm{Diff}}(T) \tag{5}$$

was then extrapolated for each of the curves down to $B=0$ (see Fig. 2). We believe that the value

$$\sigma_0(T)=\tilde{\sigma}_{xx}(T,B=0)=\sigma_D(T)+\Delta\sigma_{ee}^{\mathrm{Diff}}(T) \tag{6}$$

obtained at $B=0$ is free of the T-dependent WL contribution.[30]

Finally, the temperature independent term σ_0^D was subtracted from σ_0 at all temperatures. This was made to obtain the value of the e-e interaction correction to the conductivity

$$\Delta\sigma_{xx}^{ee}(T,B=0)=\sigma_0(T)-\sigma_0^D \tag{7}$$

which is presented in Fig. 4. The value $\sigma_0^D=(6.3\pm 0.1)\times e^2/h$ was found from the analysis of the MR data as the

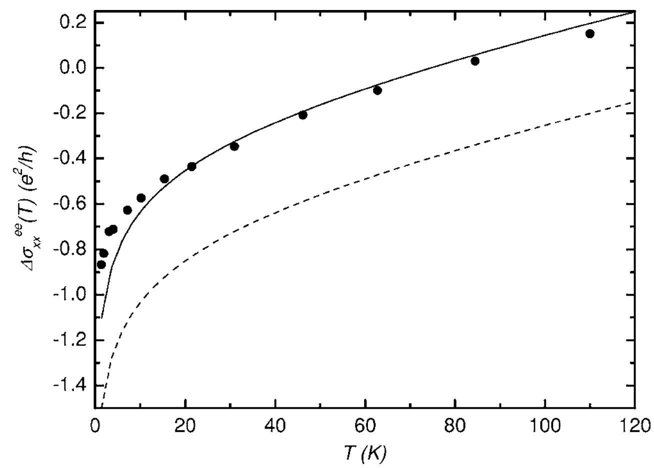

FIG. 4. Experimental temperature dependence of the e-e correction to conductivity (black dots). The dashed line corresponds to the first evaluation of the model of Ref. 4. The solid line corresponds to the theory taking into account the temperature independent contribution (see the text).

value of the conductivity at the point $\omega_c\tau=1$, where the MR curves corresponding to the diffusive range of T intersect, see Sec. VII and Eq. (19) there.

V. LONGITUDINAL CONDUCTIVITY AT $B=0$: EXPERIMENT VS THEORY

According to Ref. 4, the e-e interaction correction to the conductivity is given by the following expressions:

$$\Delta\sigma_{xx}^{ee}=\delta\sigma_C+3\delta\sigma_T, \tag{8}$$

where

$$\delta\sigma_C=\frac{e^2}{\pi\hbar}\frac{k_BT\tau}{\hbar}\left[1-\frac{3}{8}f(k_BT\tau/\hbar)\right]-\frac{e^2}{2\pi^2\hbar}\ln\left[\frac{E_F}{k_BT}\right] \tag{9}$$

is the charge channel correction and

$$\delta\sigma_T=\frac{F_0^\sigma}{[1+F_0^\sigma]}\frac{e^2}{\pi\hbar}\frac{k_BT\tau}{\hbar}\left[1-\frac{3}{8}t(k_BT\tau/\hbar;F_0^\sigma)\right]-\left[1-\frac{\ln(1+F_0^\sigma)}{F_0^\sigma}\right]\frac{e^2}{2\pi^2\hbar}\ln\left[\frac{E_F}{k_BT}\right] \tag{10}$$

is the correction in the triplet channel. The detailed expression of $f(x)$ and $t(x;F_0^\sigma)$ can be found in Ref. 4.

In these expressions the linear-in-T term is due to the renormalization of $\tau(T)$ by Friedel oscillation. This contribution comes from $\sigma_D(T)$ in Eq. (6) and dominates in the ballistic limit $k_BT\tau/\hbar\gg 1$. In the diffusive limit, the conductivity correction is determined by the logarithmic terms, which can be roughly split in two parts as follows: $\ln(E_F/k_BT)=\ln(\hbar/k_BT\tau)+\ln(E_F\tau/\hbar)$. Here the first (singular) term comes from $\Delta\sigma_{ee}^{\mathrm{Diff}}$ in Eqs. (1) and (6). The second (constant) term is the contribution of $\sigma_D(T)$. In the ballistic regime $\Delta\sigma_{ee}^{\mathrm{Diff}}$ gets suppressed, so that the whole logarithmic term $\ln(E_F/k_BT)$ comes from $\sigma_D(T)$.

It is worth mentioning that for small r_s the interaction constant F_0^σ as function of r_s can be calculated explicitly. As suggested by ZNA[4] we used

$$F_0^\sigma \to -\frac{1}{2}\frac{r_s}{r_s+\sqrt{2}} = -0.1 \tag{11}$$

in the first line of $\delta\sigma_T$ (this form reflects the backscattering character of e-e interaction related to Friedel oscillations) and

$$F_0^\sigma \to -\frac{1}{2\pi}\frac{r_s}{\sqrt{2-r_s^2}}\ln\left(\frac{\sqrt{2}+\sqrt{2-r_s^2}}{\sqrt{2}-\sqrt{2-r_s^2}}\right) = -0.17 \tag{12}$$

in the second line so that no additional fitting parameter has been introduced. The calculations were done for $r_s=0.35$ corresponding to the electron density in our sample.

In Fig. 4 we plot the theoretical curve (dashed line) calculated for our system parameters using Eqs. (8)–(10), as well as the experimental data points. As can be seen, there is a systematic shift of the experimental points with respect to the theoretical curve. This shift can be explained by the fact that ZNA theory describes only the temperature dependence of the conductivity but not the magnitude of the total interaction-induced contribution.

First, in addition to the correction $\Delta\sigma_{xx}^{ee}$ given by Eqs. (8)–(10), there is a large T-independent interaction-induced contribution to conductivity which is due to the T-independent part of the renormalization (screening) of impurities by Friedel oscillations [see Eq. (3.33) of Ref. 4]. For $r_s \gtrsim 1$, this contribution is of the same order in magnitude as the value of the Drude conductivity of a noninteracting electron gas, while for $r_s \ll 1$ it contains an additional factor $\sim r_s$. However, in the presence of interactions this contribution cannot be experimentally separated from the noninteracting part of the Drude conductivity. Therefore the value of σ_0^D used in Eq. (7) already takes into account this screening-induced term, so that the observed shift cannot be explained in this way.

Second, the logarithmic terms in Eqs. (9) and (10) yield a T-independent contribution which depends on the ultraviolet cutoff. It is worth noting that E_F appears in Eqs. (9) and (10) only due to the contribution of $\sigma_D(T)$ (this fact becomes important in a finite magnetic field). It follows that, similarly to the linear-in-E_F term discussed above, the T-independent term $\sim\ln(E_F\tau/\hbar)$ is also already absorbed in σ_0^D when the latter is found from the analysis of the MR data. Thus it is not surprising to observe a vertical shift between the predictions of Ref. 4 written in the form of Eqs. (9) and (10) with $\ln(E_F/k_BT)$ and the experimental data obtained using a specific choice of the value of the Drude conductivity.

In Fig. 4 we shifted the theoretical curve given by Eqs. (9) and (10) upward by replacing E_F by a quantity of order of $\hbar/\tau$ in the logarithmic terms (solid line).

A reasonably good quantitative agreement between the model of Ref. 4 and the data is found for the entire temperature range. Note that contrary to the previous works[8–11,13,14] we have used no fitting parameter. Moreover, we find that using the interaction constant F_0^σ as a fitting parameter does not result in a better agreement between theory and experiment.

Let us now return to the analysis of Fig. 3 which we believe to reveal important information. Indeed, the total correction to the conductivity is the sum of $\Delta\sigma_{ee}^{\mathrm{Diff}}(T)$ and a ballistic contribution which is proportional to $\tau(T)$. As can be seen the logarithmic diffusive part vanishes at $T>20$ K when the ballistic part starts to vary linearly with temperature. Therefore we believe that Fig. 3 establishes a crossover from the diffusive to the ballistic limit in the behavior of the interaction-induced correction to the zero-B conductivity. This change of behavior is observed at $T\sim 20$ K. This is in a qualitative agreement with the ZNA theory, predicting the crossover to occur at $k_BT\tau/\hbar \sim 0.1$ which corresponds in our case to $T\sim 30$ K.

Finally, not only does Fig. 3 show that the scattering time effectively varies linearly with temperature at high temperature[3,4] but it also shows that the sign of the variation is positive (i.e., insulatinglike). It is due to the fact that at small r_s the exchange (singlet) contribution is more important than Hartree (triplet) contribution.[4] While predicted by ZNA theory at low interaction this behavior is not allowed by the screening theory[3] which does not take into account the exchange part in the calculation of the corrections.

VI. HALL EFFECT

We now turn to the analysis of the Hall data presented in Fig. 1(b). According to Ref. 5 the Hall resistivity may be written as

$$\rho_{xy} = \rho_H^D + \delta\rho_{xy}^C + \delta\rho_{xy}^T, \tag{13}$$

where ρ_H^D is the classical Hall resistivity and $\delta\rho_{xy}^C, \delta\rho_{xy}^T$ are the corrections in the charge and triplet channel. These corrections are given as follows:

$$\frac{\delta\rho_{xy}^C}{\rho_H^D} = \frac{1}{\pi E_F\tau}\ln\left(1+\lambda\frac{\hbar}{k_BT\tau}\right), \tag{14}$$

$$\frac{\delta\rho_{xy}^T}{\rho_H^D} = \frac{3h(k_BT\tau/\hbar;F_0^\sigma)}{\pi E_F\tau}\ln\left(1+\lambda\frac{\hbar}{k_BT\tau}\right). \tag{15}$$

The detailed expression for $h(x;F_0^\sigma)$ can be found in Ref. 5, $\lambda=11\pi/192$ and the value of ρ_H^D is obtained from the high temperature curves for which $\delta\rho_{xy}\to 0$. Therefore according to the theory of the e-e interaction[5] one should observe a logarithmic temperature dependence of ρ_{xy}/ρ_H^D-1 in the diffusive regime replaced by a hyperbolic decrease $1/T$ at higher temperatures.

Figure 5 shows how this prediction works in our case. A simple calculation (carried out without any attempt at fitting the experiment) results in the dashed curve ($F_0^\sigma=-0.17$). This prediction is compared with the experimental correction (black dots). At each temperature the Hall coefficient was obtained by linearly fitting the experimental curves shown in Fig. 1(b). The corresponding range of magnetic field satisfies $\omega_c\tau<0.6-0.8$, which allowed us to neglect the finite-B cor-

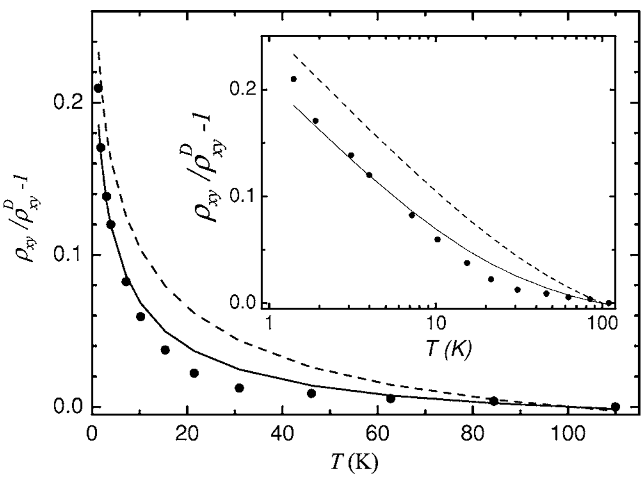

FIG. 5. Temperature dependence of the Hall coefficient (dots) compared to Eq. (13) (dashed line) and to Eq. (13) with $\lambda=0.065$ (solid line). The same data are plotted in a logarithmic scale.

rections to Eqs. (13)–(15) in our analysis. As shown in Ref. 6, such corrections are small even at $\omega_c\tau\sim 1$ because of small numerical factors, so that one can safely use the results of Ref. 5 obtained in the limit $B\rightarrow 0$ in a rather wide range of B.

On the whole, there is a qualitative agreement between theory and experiment but the quantitative agreement is lacking. Using F_0^{σ} as a fitting parameter does not improve the agreement. Nevertheless we have found that if the coefficient $\lambda=11\pi/192\simeq 0.18$ is replaced by $\lambda=4\pi/192\simeq 0.065$, then the theoretical curve (the solid line) fits the experimental dependence quite well.

This result might be related to an anisotropy of electron scattering in the sample which reduces the electron return probability and so weakens the correction at low fields ($\omega_c\tau\ll 1$). The reduction of the prefactor λ could just be the way in which this anisotropy reveals itself since the correction is proportional to λ in the ballistic limit. It is worth noting that in the ballistic regime the correction to the Hall coefficient is more sensitive to the anisotropy of impurity scattering than the leading correction to the longitudinal conductivity. This is because the relevant processes giving rise to $\delta\rho_{xy}$ involve at least three impurity scattering events,[5] while those leading to the linear-in-T correction to σ_{xx} involve a single backscattering. Clearly, each large-angle scattering event yields a reduction factor even for the weak anisotropy of scattering.

Finally, in Fig. 6 we show the experimental data points for the transverse conductivity tensor as a function of magnetic field [Fig. 6(a)] and as a function of temperature for two different values of magnetic field [Fig. 6(b)]. The conductivity is observed to be temperature independent at low temperatures and varies linearly with temperature at high temperatures. While conforming to the theoretical prediction in the diffusive regime ($\Delta\sigma_{ee}^{\mathrm{H}}=0$ and $\Delta\rho_{xy}^{\mathrm{WL}}=0$, according to Ref. 2), the behavior at high temperatures is less obvious. However, this behavior follows from Eq. (2) which takes into account the ballistic renormalization of the scattering time. The measured values of the Hall conductivity are indeed well

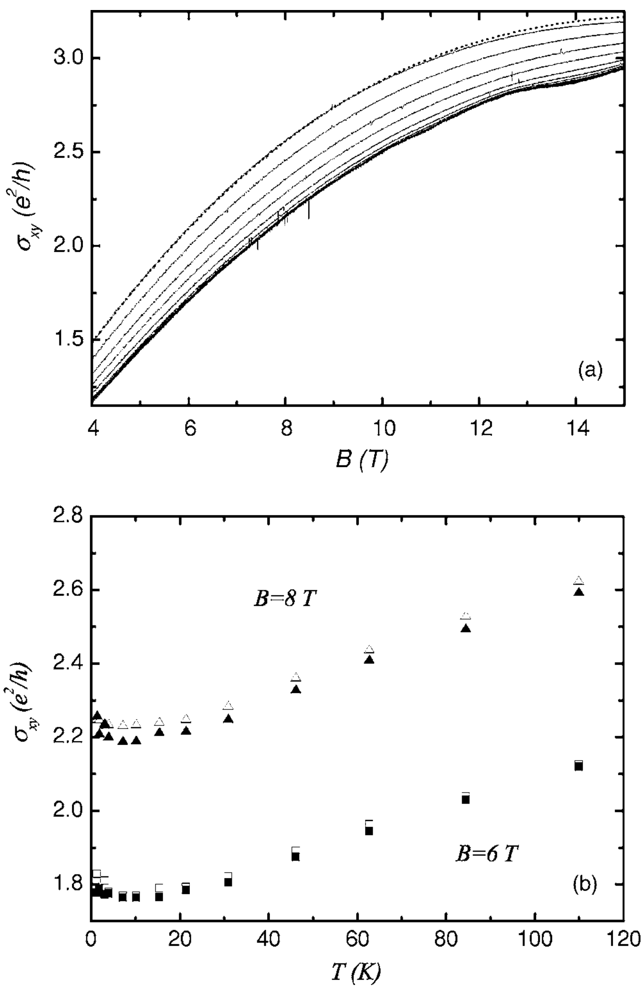

FIG. 6. (a) Transverse conductivity as a function of magnetic field (shown in the range of B relevant to the interaction-induced corrections) for the temperatures listed in the caption of Fig. 1. The dotted line corresponds to Eq. (16) taken at $T=110$ K. (b) Transverse conductivity as a function of T for two different values of magnetic fields (black symbols). It is compared to the value calculated using Eq. (16) (open symbols).

compared to values of $\sigma_{xy}(T)$ calculated using the simple Drude-like formula:

$$\sigma_{xy}=\frac{e^2n}{m^*}\frac{\omega_c\tau^2(T)}{1+\omega_c^2\tau^2(T)}. \qquad (16)$$

In this formula we neglected terms $\omega_c\tau(T)\Delta\sigma_{ee}^{\mathrm{H}}(T)$ and $\Delta\sigma_{xy}^{\mathrm{WL}}(T,B)$ from Eq. (2) and used Eq. (4) for $\sigma_D(T)$. To evaluate σ_{xy} we used the values of the scattering time shown in Fig. 3. Again the data are well described by the model which includes no fitting parameter. Note that we also calculated the expected field dependence at $T=110$ K [see dotted curve in Fig. 6(a)] which also reproduced well the experimental data. A more detailed analysis of the Hall conductivity within the general method outlined in Sec. III [taking into account all the terms in Eq. (2)] will be presented elsewhere.[28]

VII. LONGITUDINAL MAGNETORESISTANCE

Let us return to the analysis of the longitudinal resistivity $\rho_{xx}(B)$ shown in Fig. 1. This analysis is aimed to obtain a consistent description including all the transport coefficients, $\rho_{xx}(B)$, $\sigma_{xx}(B)$, $\rho_{xy}(B)$, and $\sigma_{xy}(B)$. Furthermore, the behavior of $\rho_{xx}(B)$ in the ballistic regime is determined by more subtle effects as compared to the behavior of the conductivity tensor. It turns out that in the longitudinal MR, unlike in the conductivity components, the leading B-independent e-e correction to τ cancels out. In fact, the T dependence of $\rho_{xx}(B)$ reflects the weak influence of magnetic field on the collision integral in the quantum kinetic equation of Ref. 4.

As discussed in Sec. IV, the low-B part of the curves is dominated by the WL-induced MR, while the MR for $B > B_{tr}$ is governed by the interaction effect. In the diffusive regime $k_B T\tau/\hbar \ll 1$, the interaction-induced resistivity correction,

$$\frac{\delta\rho_{xx}(B)}{\rho^D} = \frac{1-(\omega_c\tau)^2}{2\pi E_F\tau}\left(1+3\left[1-\frac{\ln(1+F_0^\sigma)}{F_0^\sigma}\right]\right)\ln\left(\frac{\hbar}{k_B T\tau}\right) \tag{17}$$

(ρ^D is the classical Drude resistivity), gives rise to a parabolic MR $\Delta\rho_{xx}=\delta\rho_{xx}(B)-\delta\rho_{xx}(B=0)$ in arbitrary magnetic field.[31,32]

In the ballistic regime, as shown in Ref. 6, the interaction-induced MR remains quadratic in magnetic field, while the T behavior of the proportionality coefficient depends on the type of disorder. In the present case of short-ranged impurities, Ref. 6 predicts the following ballistic ($k_B T\tau/\hbar \gg 1$) asymptotic behavior of the MR for $\hbar\omega_c \ll 2\pi^2 k_B T$:

$$\frac{\Delta\rho_{xx}}{\rho^D} = -(\omega_c\tau)^2\frac{1+3g(F_0^\sigma)}{2\pi E_F\tau}\frac{17\pi\hbar}{192 k_B T\tau}, \tag{18}$$

where the function $g(F_0^\sigma)$ describes the contribution of the triplet channel. It is worth stressing that in high-density systems with $r_s \ll 1$ (i.e., for $|F_0^\sigma| \ll 1$), the parabolic MR is dominated by the contribution of the singlet channel and hence is negative.

Equations (17) and (18) can be obtained by inverting the conductivity tensor given by Eqs. (5) and (16). One can see that the classical part of the conductivity tensor [i.e., Eq. (16) and the first term in Eqs. (1) and (5)] does not yield B dependence of the resistivity, even when the interaction effects are taken into account through the T dependence of $\tau(T)$. Indeed, neglecting the term $\Delta\sigma_{ee}^{\mathrm{Diff}}(T)$ one gets $\rho_{xx}(T) = m^*/e^2 n\tau(T)$ which is independent of B. Thus the MR is solely generated by the term $\Delta\sigma_{ee}^{\mathrm{Diff}}$. We remind the reader that in the ballistic regime $\Delta\sigma_{ee}^{\mathrm{Diff}}$ appears to be dominated by the effect of magnetic field on the collision integral, see Sec. III. Thus we conclude that the main source of the MR for $T > 20-30$ K is the weak B dependence of the transport scattering time.

Let us now compare our experimental data with the above theoretical predictions. It is worth mentioning that the comparison is again parameter free. Figure 7 presents the longitudinal resistivity as a function of $(\omega_c\tau)^2$. It shows that the MR is indeed parabolic and negative. The slope of the curves ρ_{xx} vs $(\omega_c\tau)^2$ was obtained in the relevant ranges $(\omega_c\tau)^2 = 0.1-0.4$ for the curves corresponding to $T < 20$ K and for $(\omega_c\tau)^2 > 0.2$ for $T > 20$ K. This has allowed us to reduce the influence of WL in the high T data and the SdH oscillations in the low T data. The slope of these lines is presented in Fig. 8. The error in the determination of the slope due to the choice of the evaluation range was estimated from the deviations obtained using the interval 0.2–0.35 of $(\omega_c\tau)^2$ for the linear fit. In Fig. 7 we have extrapolated the MR lines to the region of higher magnetic fields $\omega_c\tau \sim 1$. From this plot we again clearly see the crossover between the diffusive and the ballistic regimes which occurs at $T \sim 20$ K. Indeed the lines for $T < 20$ K intersect each other at a single point close to $(\omega_c\tau)^2 = 1$, as predicted by the diffusive expression (17). As follows from Eq. (17), at the intersection point the quantum

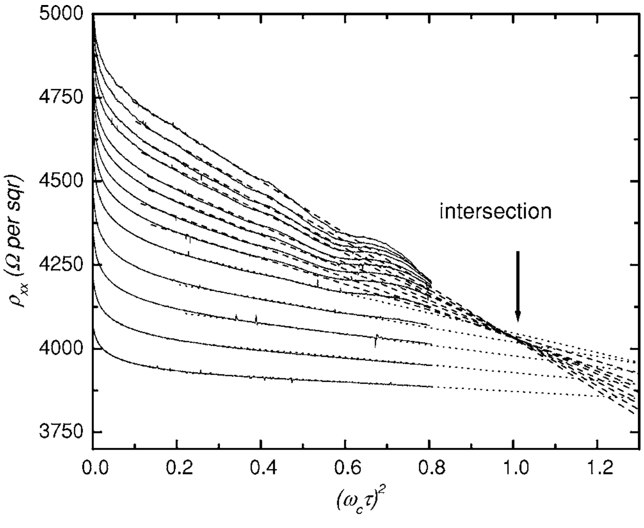

FIG. 7. ρ_{xx} plotted as a function of $(\omega_c\tau)^2$ for the temperatures listed in the caption of Fig. 1. The dashed lines are the extrapolation of the linear behavior of the curves corresponding to the diffusive regime. They cross each other at $\omega_c\tau=1$. The dotted lines represent the extrapolation of the curves in the ballistic regime.

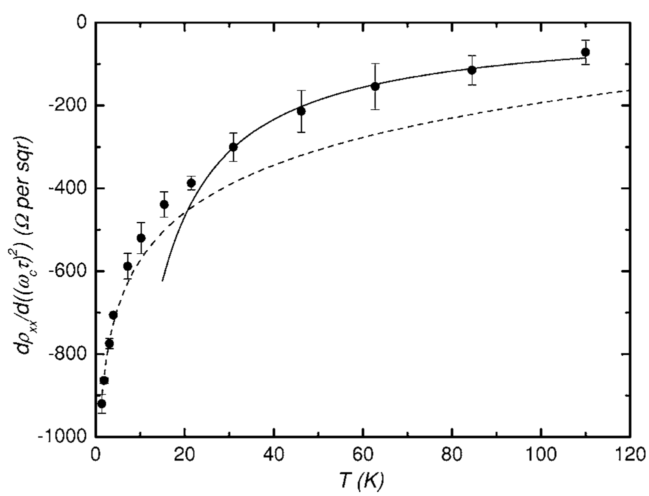

FIG. 8. Slope of the curves shown in Fig. 7 compared to the theoretical predictions. The dashed line corresponds to the diffusive regime [Eq. (17)] and the solid line to the ballistic limit [Eq. (18)].

correction to the longitudinal resistivity is zero, so that the value of ρ_{xx} at this point corresponds to the classical Drude value of the resistivity:

$$\rho_{xx}(\omega_c\tau = 1) = \rho^D = 1/\sigma_0^D. \quad (19)$$

This value of σ_0^D was used in Sec. IV to find the magnitude of the interaction-induced conductivity correction at $B=0$. For higher temperatures ($T > 20$ K), the MR lines in Fig. 7 no longer intersect each other at a single point. At this point the system enters the crossover region, where the T-dependence of $\sigma_D(T)$ starts to become important.

The proportionality coefficient of ρ_{xx} vs $(\omega_c\tau)^2$ dependence is compared in Fig. 8 to the theoretical asymptotics given by Eqs. (17) and (18). In Eq. (17) we used the "diffusive" value $F_0^\sigma = -0.17$ given by Eq. (12). In Eq. (18) we used $g(F_0^\sigma) = F_0^\sigma/(1+F_0^\sigma)$ with $F_0^\sigma = -0.1$ given by Eq. (11). This is consistent with the above observation that the "ballistic" MR is mostly due to the B-dependent corrections to the collision integral. An almost perfect quantitative agreement between the predictions of Refs. 31, 32, and 6 and the experimental data is found for both diffusive and ballistic temperature regimes.

VIII. CONCLUSION

In conclusion, we have presented a study aiming at observing the crossover from the diffusive to the ballistic regime in the weak interaction limit. We find strong evidence of such crossover in the obtained measurements. We realized a parameter-free comparison of our experimental data for the longitudinal conductivity and Hall coefficient to the recent ZNA theory as well as the longitudinal resistivity to the theory of Ref. 6. We find these theories to be in a good qualitative agreement with our experimental results.

ACKNOWLEDGMENTS

We are very grateful to A. Dmitriev, A. Germanenko, G. Minkov, A. Mirlin, and B. Narozhny for useful discussions. This work was supported by PICS-RFBR (Grant No. 1577), RFBR (Grants No. 02-02-16516, 05-02-17800, and 05-02-17802), NATO, INTAS (Grant No. 01-0014), DFG-Schwerpunktprogramm "Quanten-Hall-Systeme," programs "Physics and Technology of Nanostructures" of the Russian ministry of Industry and Science, "Low dimensional quantum structures" of RAS, and "Russian Scientific School" (Grant No. 2192.2003.2).

*Electronic address: renard@grenoble.cnrs.fr

[1] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1984).

[2] B. L. Altshuler and A. G. Aronov, *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).

[3] A. Gold and V. T. Dolgopolov, Phys. Rev. B **33**, 1076 (1986).

[4] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001).

[5] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 201201(R) (2001).

[6] I. V. Gornyi and A. D. Mirlin, Phys. Rev. Lett. **90**, 076801 (2003); Phys. Rev. B **69**, 045313 (2004).

[7] A. Sergeev, M. Yu. Reizer, and V. Mitin, Phys. Rev. B **69**, 075310 (2004).

[8] P. T. Coleridge, A. S. Sachrajda, and P. Zawadzki, Phys. Rev. B **65**, 125328 (2002).

[9] A. A. Shashkin, S. V. Kravchenko, V. T. Dolgopolov, and T. M. Klapwijk, Phys. Rev. B **66**, 073303 (2002).

[10] Y. Y. Proskuryakov, A. K. Savchenko, S. S. Safonov, M. Pepper, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. Lett. **89**, 076406 (2002).

[11] Z. D. Kvon, O. Estibals, G. M. Gusev, and J. C. Portal, Phys. Rev. B **65**, 161304(R) (2002).

[12] L. Li, Y. Y. Proskuryakov, A. K. Savchenko, E. H. Linfield, and D. A. Ritchie, Phys. Rev. Lett. **90**, 076802 (2003).

[13] E. B. Olshanetsky, V. Renard, Z. D. Kvon, J. C. Portal, N. J. Woods, J. Zhang, and J. J. Harris, Phys. Rev. B **68**, 085304 (2003).

[14] C. E. Yasin, T. L. Sobey, A. P. Micolich, A. R. Hamilton, M. Y. Simmons, L. N. Pfeiffer, K. W. West, E. H. Linfield, M. Pepper, and D. A. Ritchie, cond-mat/0403411 (unpublished).

[15] S. A. Vitkalov, K. James, B. N. Narozhny, M. P. Sarachik, and T. M. Klapwijk, Phys. Rev. B **67**, 113310 (2003).

[16] V. M. Pudalov, M. E. Gershenson, and K. Kojima, in *Fundamental Problems of Mesoscopic Physics: Interactions and Decoherence*, edited by I. V. Lerner, B. L. Altshuler, and Y. Gefen (Kluwer Academic Publishers, Dordrecht, 2004), p. 309; cond-mat/0401396 (unpublished); V. M. Pudalov, M. E. Gershenson, H. Kojima, G. Brunthaler, A. Prinz and G. Bauer, Phys. Rev. Lett. **91**, 126403 (2003).

[17] E. A. Galaktionov, A. K. Savchenko, S. S. Safonov, Y. Y. Proskuryakov, L. Li, M. Pepper, M. Y. Simmons, D. A. Ritchie, E. H. Linfield, and Z. D. Kvon, in *Fundamental Problems of Mesoscopic Physics: Interactions and Decoherence*, edited by I. V. Lerner, B. L. Altshuler, and Y. Gefen (Kluwer Academic Publishers, Dordrecht, 2004), p. 349; cond-mat/0402139 (unpublished).

[18] V. A. Kulbachinskii, V. G. Kytin, V. I. Kadushkin, E. L. Shangina, and A. de Visser, J. Appl. Phys. **75**, 2081 (1994).

[19] G. M. Minkov, O. E. Rut, A. V. Germanenko, A. A. Sherstobitov, V. I. Shashkin, O. I. Khrykin, and V. M. Daniltsev, Phys. Rev. B **64**, 235327 (2001); G. M. Minkov, O. E. Rut, A. V. Germanenko, A. A. Sherstobitov, B. N. Zvonkov, E. A. Uskova, and A. A. Birukov, *ibid.* **65** 235322 (2002).

[20] G. M. Minkov, O. E. Rut, A. V. Germanenko, A. A. Sherstobitov, V. I. Shashkin, O. I. Khrykin, and B. N. Zvonkov, Phys. Rev. B **67**, 205306 (2003).

[21] Z. D. Kvon, V. A. Tkachenko, O. A. Tkachenko, A. I. Toropov, A. K. Bakarov, V. Renard, and J.-C. Portal, Physica E (Amsterdam) **21**, 742 (2004).

[22] L. Pfeiffer, K. W. West, and K. W. Baldwin, Appl. Phys. Lett. **55**, 1888 (1989).

[23] B. J. F. Lin, D. C. Tsui, M. A. Paalanen, and A. C. Gossard, Appl.

Phys. Lett. **45**, 695 (1984).
[24] S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
[25] See, e.g., S. McPhail, C. E. Yasin, A. R. Hamilton, M. Y. Simmons, E. H. Linfield, M. Pepper, and D. A. Ritchie, Phys. Rev. B **70**, 245311 (2004), and references therein.
[26] I. V. Gornyi (unpublished).
[27] A. P. Dmitriev, V. Yu. Kachorovskii, and I. V. Gornyi, Phys. Rev. B **56**, 9910 (1997).
[28] V. Renard *et al.* (unpublished).
[29] G. M. Minkov, A. A. Sherstobitov, A. V. Germanenko, O. E. Rut, V. A. Larionova, and B. N. Zvonkov, cond-mat/0503567 (unpublished).
[30] A subtle point is that the residual T-independent "ballistic" WL contribution decreases rather slowly ($\propto B^{-1/2}$) at $B > B_{tr}$. As discussed in Refs. 19 and 20, in relatively low-mobility structures this WL tail may be still important for the analysis of magnetotransport data, when the fitting is performed in the range $B < 10B_{tr}$. However, this T-independent WL term drops out in Eq. (7) and therefore the interaction correction $\Delta\sigma_{xx}^{ee}(T,B=0)$ obtained from Eq. (7) is almost unaffected by the WL tail.
[31] A. Houghton, J. R. Senna, and S. C. Ying, Phys. Rev. B **25**, 2196 (1982).
[32] S. M. Girvin, M. Jonson, and P. A. Lee, Phys. Rev. B **26**, 1651 (1982).