Low-temperature dephasing saturation from elastic magnetic spin disorder and interactions

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We treat the question of the low-temperature behavior of the dephasing rate of the electrons in the presence of elastic spin disorder scattering and interactions. In the frame of a self-consistent diagrammatic treatment, we obtain saturation of the dephasing rate in the limit of low-temperature for magnetic scattering, in agreement with the noninteracting case. The magnitude of the dephasing rate is set by the strength of the magnetic scattering rate. We discuss the agreement of our results with relevant experiments.

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An important quantity in disordered electronic systems is the dephasing rate τ_ϕ^{-1}. It provides a measure of the loss of coherence of the carriers, but in the two-particle channel, see Eq. (1) below. Decoherence arises from Coulombic interactions, scattering by phonons, magnetic fluctuations etc. The saturation of the dephasing rate at low temperature T seen in *numerous* experiments[1–13] has attracted a vigorous interest, especially given the long-standing theoretical prediction for a vanishing τ_ϕ^{-1} as the temperature $T\to 0$.[14–21]

Previous theoretical studies[14–25] have focused on the calculation of τ_ϕ^{-1} in the absence of spin-scattering disorder. The majority of these studies predict, correctly, a vanishing $\tau_\phi^{-1}(T\to 0)$. Here we determine and calculate the factors which contribute to dephasing in the presence of spin-scattering disorder. The saturation obtained allows for the consistent elucidation of this puzzle.

In the presence of spin-less disorder, the cooperon (particle-particle diffusion correlator, see Fig. 1) is given by

$$C_0(q,\omega)=\frac{1}{2\pi N_F\tau^2}\frac{1}{Dq^2-i\omega+\tau_\phi^{-1}}. \tag{1}$$

D is the diffusion coefficient, N_F is the density of states at the Fermi level, and τ^{-1} the total impurity scattering rate. We work in the diffusive regime $\epsilon_F\tau>1(\hbar=1)$, ϵ_F being the Fermi energy.

With spin-disorder present, the cooperon becomes spin dependent. The relevant terms C_i are shown in Fig. 1. We start by giving the explicit form of these $C^o_{0,1,2}$ without a dephasing rate. C^o_0 acquires a finite spin-dependent term in the denominator, which is crucial for the determination of the dephasing rate, see below.

For the case $\tau_S^{-1}>0$, $\tau_{SO}^{-1}=0$—with τ_S^{-1} the magnetic impurity scattering rate and τ_{SO}^{-1} the spin-orbit impurity scattering rate—the Cooperons are given by (see Refs. 26 and 27 and Appendix A)

$$C^o_0(q,\omega)=\frac{1}{2\pi N_F\tau^2}\frac{1}{Dq^2-i\omega+2/(3\tau_S)},$$

$$C^o_{1,2}(q,\omega)=b_{1,2}\left\{\frac{1}{Dq^2-i\omega+2/(3\tau_S)}-\frac{1}{Dq^2-i\omega+2/\tau_S}\right\}, \tag{2}$$

with $b_1=[3\tau_S/(2\tau)-2]/(2u)$, $b_2=1/u$, $u=4\pi N_F\tau^2$. Based on Eqs. (2), we expect a saturation of the dephasing rate. The simple diffusion pole is "cutoff" by the constant terms proportional to τ_S^{-1}. On symmetry grounds, the spin-conserving Coulomb interaction cannot eliminate these terms. We emphasize that the impurity scattering considered is elastic, bulk type. Interfacial impurity scattering, though similar to bulk type, is expected to differ in detail.

To calculate the dephasing rate, we write down and solve the appropriate coupled equations for *all* three renormalized Cooperons $C_i(q,\omega)$, $i=0,1,2$. We note that usually the terms containing the factors d_i and h_i below are completely omitted. The equations are shown schematically in Fig. 2:

$$C_0=C^o_0+C^o_0Y_0C_0, \tag{3}$$

$$C_1=C^o_1+C^o_1W_a+C^o_2W_b, \tag{4}$$

$$C_2=C^o_2+C^o_1W_b+C^o_2W_a, \tag{5}$$

with

$$W_a=Y_nC_1+Y_rC_2,\quad W_b=Y_nC_2+Y_rC_1 \tag{6}$$

and

$$Y_0=(1+h_0)\Sigma_0+d_0\Sigma_1,\quad Y_n=(1+h_1)\Sigma_1+d_1\Sigma_0,$$
$$Y_r=(1+h_2)\Sigma_2+d_2\Sigma_0. \tag{7}$$

In Fig. 3 we show explicitly the components of the self-energy terms Y, see the figure legend for further details. Here, $h_0=h_1=h_2=-2/(\pi\epsilon_F\tau)$, $d_0=d_1=\{1/(\epsilon_F\tau)+4\pi\}\{\tau_{SO}^{-1}-\tau_S^{-1}\}/(2\pi^2\epsilon_F)$, and $d_2=-2\{1+2\pi\epsilon_F\tau\}\{\tau_{SO}^{-1}-\tau_S^{-1}\}/(\pi\epsilon_F)$. The terms containing the factors d_i—with a spin impurity line either looping around the Cooperon or crossing it—provide the coupling between the spin-independent and the spin-dependent Cooperons, as they produce spin flipping—see the spin configuration of C_2 in Fig. 1 and the figure legend. Also, note the minus relative sign between τ_{SO}^{-1} and τ_S^{-1} in d_i, coming from the respective spin flipping disorder vertices.[26] This

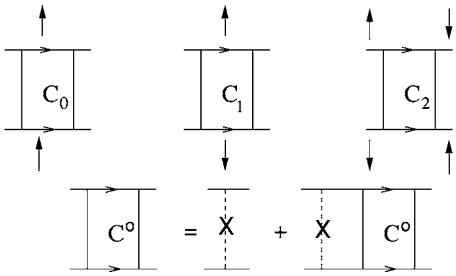

FIG. 1. The three Cooperons C_0, C_1, C_2. Note the spin indices. The Cooperons C_i^o do not contain a dephasing rate. The dashed line with the cross stands for impurity (disorder) scattering. The bare disorder vertices flipping spin, corresponding to the spin configuration of C_2, yield a coupling of all three Cooperons.

has the peculiar effect that for $\tau_{SO}=\tau_S$ the spin disorder signature disappears in $\tau_{\phi o}$—see Eq. (11) below.

In the spirit of Ref. 15 we obtain for the basic components Σ_i of the self-energy

$$\Sigma_i \equiv \Sigma_i(q=0,\omega=0) \simeq -c_o\sum_q \int_{-\infty}^{\infty} d\omega' \frac{C_i(q,\omega'+i0)\,\mathrm{Im}\,V(q,\omega'+i0)}{\sinh(\omega'/T)}, \quad (8)$$

where

$$V(q,\omega)=\frac{v_q}{1+v_q\Pi(q,\omega)}, \quad \Pi(q,\omega)=\frac{N_F D q^2}{Dq^2-i\omega}. \quad (9)$$

Here v_q is the bare Coulomb interaction and $c_o=8N_F^2\tau^4$.

In the foregoing we make the approximation[15]

$$\int_{-\infty}^{\infty} d\omega \frac{F(\omega)}{\sinh(\omega/T)} \simeq T\int_{-T}^{T} d\omega \frac{F(\omega)}{\omega}. \quad (10)$$

The bosonic modes with energy greater than T manifestly do *not* contribute to the self-energy and the dephasing process, as also emphasized in Refs. 17 and 21. The saturation obtained in Refs. 22 and 23 was due to an infinite upper limit for ω, which is incorrect.[17,21] We further discuss the approximation of Ref. 15 for the self-energy in the paragraph preceding Eq. (34) below.

In the limit $\epsilon_F\tau \gg 1$, we can decouple a 2×2 system of equations involving only $C_{1,2}$, to facilitate the solution of Eqs. (3)–(5). Then Eq. (3) yields directly for the dephasing rate $\tau_{\phi o}^{-1}$—note the index o—in the denominator of C_0

$$\tau_{\phi o}^{-1}(T) = -\frac{\Sigma_0 + d_0\Sigma_1}{2\pi N_F \tau^2}. \quad (11)$$

This equation means that the dephasing in the spin-dependent Cooperon channels also contributes to the dephasing rate in the spin-independent channel. We note that in the limit $\epsilon_F\tau \gg 1$ the factor d_0 in Eq. (11) is the only one remaining among the $d_{0,1,2}$ and $h_{0,1,2}$ appearing in Eqs. (3)–(5). But as shown below, the factor $d_0\Sigma_1$ is too small in comparison to the factors τ_S^{-1}, τ_{SO}^{-1} in the denominator of C_0.

We look in detail at the case of pure magnetic scattering, i.e., $\tau_S^{-1}>0$ and $\tau_{SO}^{-1}=0$. In Appendix C we discuss the case of pure spin-orbit scattering. Diagonalizing the system formed

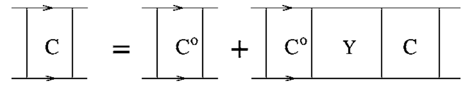

FIG. 2. Schematic form of the Eqs. (3)–(5) involving the Cooperons and the self-energies Y.

by Eqs. (4) and (5) yields for the Cooperons $C_{1,2}$

$$C_{1,2}(q,\omega) = S_{1,2}\left\{\frac{1}{Dq^2-i\omega+R_-} - \frac{1}{Dq^2-i\omega+R_+}\right\}. \quad (12)$$

Here $M_1=[b_1m_1m_2+(b_2^2-b_1^2)m_0\Sigma_1]/m$, $M_2=[b_2m_1m_2-(b_2^2-b_1^2)m_0\Sigma_2]/m$, $R_\pm=[m_1m_2\pm m_0(b_2\mp b_1)(\Sigma_1\mp\Sigma_2)]/m$, $X=2(b_2\Sigma_1+b_1\Sigma_2)$, $m_1=2/(3\tau_S)$, $m_2=2/\tau_S$, $m_0=m_2-m_1$, $m=m_1+m_2$. The derivation of Eqs. (12) can be found in Appendix B. We note that it is important to keep the term $b_j(Dq^2-i\omega)$ in S_j, to arrive at the correct solution. Further, Eqs. (12) are valid in any dimension.

In two dimensions 2D, $v_q=2\pi e^2/q$ and, following Refs. 15 and 18, we take $V(q,\omega)=(2\pi e^2/q)[(Dq^2-i\omega)/(D\kappa q-i\omega)]$ with $\kappa=4\pi N_F e^2$.

To calculate the self-energies, we first evaluate the integral in Eq. (10)

$$I_j(q) = \int_{-T}^{T} d\omega \frac{b_j(Dq^2-i\omega)+M_j}{(D\kappa q)^2+\omega^2}\left\{\frac{1}{Dq^2-i\omega+R_-} - \frac{1}{Dq^2-i\omega+R_+}\right\}. \quad (13)$$

Since we are interested in the low-T limit, we take

$$Dq^2 > T, \quad (14)$$

obtaining

$$I_j(q) \simeq 2T\frac{b_jDq^2+M_j}{(D\kappa q)^2}\left\{\frac{1}{Dq^2+R_-} - \frac{1}{Dq^2+R_+}\right\}. \quad (15)$$

Subsequently, we evaluate

$$\int_{\sqrt{T/D}}^{\sqrt{z/D}} I_j(q)(\kappa-q)q\,dq, \quad (16)$$

with

$$z = \max Dq^2 = \tau^{-1}. \quad (17)$$

Finally we obtain the following equations for the self-energies:

$$\Sigma_j = \frac{T^2 a_{2\mathrm{D}}}{X}\left\{\frac{M_j - b_j R_+}{R_+}\left[\kappa \ln\left(\frac{R_+ + z}{R_+ + T}\right) - 2\sqrt{\frac{R_+}{D}}\left(\arctan\sqrt{\frac{z}{R_+}} - \arctan\sqrt{\frac{T}{R_+}}\right)\right] - \frac{M_j - b_j R_-}{R_-}\left[\kappa \ln\left(\frac{R_- + z}{R_- + T}\right) - 2\sqrt{\frac{R_-}{D}}\left(\arctan\sqrt{\frac{z}{R_-}} - \arctan\sqrt{\frac{T}{R_-}}\right)\right] + \kappa M_j\left(\frac{1}{R_-} - \frac{1}{R_+}\right)\ln\left(\frac{z}{T}\right)\right\}, \quad (18)$$

with $j=1,2$ and $a_{2\mathrm{D}}=2c_o e^2/(D\kappa^2)$.

The solution is

$$\Sigma_j = s_j T^2,$$
$$s_j = b_j a_{2\mathrm{D}} \tau_S \left\{\sqrt{\frac{2}{D\tau_S}}\arctan\sqrt{2z\tau_S} - \kappa \ln(1+2z\tau_S)\right\}, \quad (19)$$

which is valid for

$$\Sigma_j \ll 1/(2\tau_S). \quad (20)$$

This condition accompanies the solutions for Σ_j in 1D and 3D as well.

We consider the so-called weak localization contribution to the conductivity, given by the sum of[26]

$$\delta\sigma_o = -\frac{e^2 Du}{\pi}\sum_q \{C_0(q,0) + C_2(q,0)\}, \quad (21)$$

which in a magnetic field H perpendicular to the 2D system becomes

$$\delta\sigma_o = -\frac{e^2 Du}{\pi}\frac{eH}{\pi}\sum_{n=0}^{N_H} \times\left\{\frac{1}{2\pi N_F \tau^2}\frac{1}{4DeH(n+1/2) + 2/(3\tau_S) + \tau_{\phi o}^{-1}} + \frac{4b_2 eDH(n+1/2) + M_2}{X} \times\left(\frac{1}{4DeH(n+1/2)+R_-} - \frac{1}{4DeH(n+1/2)+R_+}\right)\right\}, \quad (22)$$

with $N_H = 1/(4De\tau H)$.

If this formula were fit to the 2D formula without magnetic (or spin-orbit) scattering

$$\delta\sigma_o = -\frac{2e^3 DH}{\pi^2}\sum_{n=0}^{N_H}\frac{1}{4DeH(n+1/2) + \tau_\phi^{-1}}, \quad (23)$$

saturation of the dephasing is obtained due (mostly) to the factor $2/(3\tau_S)$ in the denominator of C_0. The factors $R_\pm$ in Eq. (22) satisfy $R_\pm = 1/(2\tau_S) + O(T^2)$. As a result, the contribution of the C_2 term is small, because $R_+ - R_- = O(T^2)$. The same applies to 1D and 3D, with the power law being $T^{3/2}$ in 1D, as shown below.

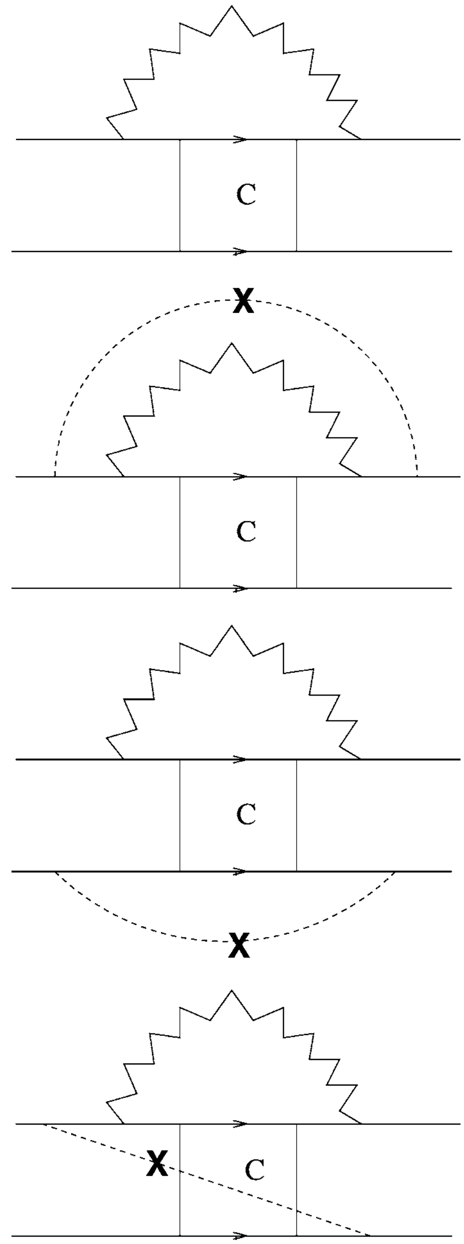

FIG. 3. The various components of the self-energy Y. The wiggly line represents the screened Coulomb interaction of Eq. (9). The diagrams with the extra spin-disorder impurity line are responsible for the contribution $d_0\Sigma_1$ in $\tau_{\phi o}^{-1}$ [see Eq. (11)]. We consider *all* possible variations of the diagrams shown here. That is, including terms in which the positions of the interaction and impurity vertices are interchanged along the particle lines. For example, in the second diagram above, suppose we label the Coulomb and disorder vertices along the upper electron line by the numbers (1,2,3,4). For this diagram we also consider the permutations (2,1,3,4), (1,2,4,3), and (2,1,4,3).

In 1D, $v_q = 2e^2 \ln(q_m/q)$, q_m being the inverse of the largest transverse dimension (width) of the system. Here,

$$\mathrm{Im}\, V(q,\omega) = \frac{-4e^4 N_F \omega D q^2 \ln^2(q_m/q)}{\omega^2 + (Dq^2)^2[1 + 2e^2 N_F \ln(q_m/q)]^2}.$$

To calculate the self-energies in Eq. (8), we first evaluate the integrals

$$L_j(\omega) = \int_0^z dx \frac{\sqrt{x}(b_j(x-i\omega)+M_j)}{B^2x^2+\omega^2}\left\{\frac{1}{x-i\omega+R_-} - \frac{1}{x-i\omega+R_+}\right\} = -2\frac{(M_j-b_jR_-)\sqrt{R_- - i\omega}\arctan\sqrt{\frac{z}{R_- - i\omega}}}{\{B^2(R_- - i\omega)^2+\omega^2\}}$$
$$+2\frac{(M_j-b_jR_+)\sqrt{R_+ - i\omega}\arctan\sqrt{\frac{z}{R_+ - i\omega}}}{\{B^2(R_+ - i\omega)^2+\omega^2\}}$$
$$+\frac{\{iM_jB+b_j\omega(B-1)\}\arctan\sqrt{\frac{iBz}{\omega}}}{B^{3/2}\sqrt{-i\omega}}\left\{\frac{1}{iBR_- + \omega(B-1)} - \frac{1}{iBR_+ + \omega(B-1)}\right\}$$
$$+\frac{\{iM_jB+b_j\omega(B+1)\}\arctan\sqrt{\frac{-iBz}{\omega}}}{B^{3/2}\sqrt{i\omega}}\left\{\frac{1}{iBR_- + \omega(B+1)} - \frac{1}{iBR_+ + \omega(B+1)}\right\}. \quad (24)$$

Here $x=Dq^2$, $z=\max x=\tau^{-1}$ as before, and we approximated the term $\ln(q_m/q)$ by its average, taking

$$B = 1 + e^2N_F < \ln(q_m^2/q^2) > \; > 1. \quad (25)$$

Then, we consider the low-T limit $z \gg T > |\omega|$, obtaining

$$L_j(\omega) = \frac{M_j\pi}{\sqrt{2\omega B^3}}\left\{\frac{1}{R_-} - \frac{1}{R_+}\right\} + \text{const}, \quad (26)$$

and we evaluate

$$\int_{-T}^{T} L_j(\omega)d\omega. \quad (27)$$

Thus we obtain the self-energy equations

$$\Sigma_j = \frac{T^{3/2}a_{1\mathrm{D}}M_j}{X}\left\{\frac{1}{R_-} - \frac{1}{R_+}\right\}, \quad (28)$$

with $j=1$, 2, and $a_{1\mathrm{D}}=\sqrt{2}c_o(B-1)^2/(N_FB^{3/2})$. The solution is

$$\Sigma_j = s_jT^{3/2}, \quad s_j = \frac{\sqrt{2}b_j\tau_Sc_o(B-1)^2}{N_FB^{3/2}}. \quad (29)$$

In 3D we take $V(q,\omega)=(4\pi e^2/q^2)[(Dq^2-i\omega)/(P-i\omega)]$, with $P=4\pi e^2N_FD$. To calculate the self-energies of Eq. (8), we first evaluate the integrals

$$K_j(q) = \int_{-T}^{T} d\omega \frac{b_j(Dq^2-i\omega)+M_j}{P^2+\omega^2}\left\{\frac{1}{Dq^2-i\omega+R_-} - \frac{1}{Dq^2-i\omega+R_+}\right\} \simeq 2T\frac{b_jDq^2+M_j}{P^2}\left\{\frac{1}{Dq^2+R_-} - \frac{1}{Dq^2+R_+}\right\}. \quad (30)$$

Then, evaluating

$$\int_{\sqrt{T/D}}^{\sqrt{z/D}} K_j(q)\{Dq^2-P\}dq, \quad (31)$$

yields the self-energy equations

$$\Sigma_j = \frac{T^2a_{3\mathrm{D}}}{X}\left\{\frac{(M_j-b_jR_+)(P+R_+)}{\sqrt{DR_+}}\arctan\sqrt{\frac{z}{R_+}} - \frac{(M_j-b_jR_-)(P+R_-)}{\sqrt{DR_-}}\arctan\sqrt{\frac{z}{R_-}} + b_j(R_+-R_-)\sqrt{\frac{z}{D}}\right\}. \quad (32)$$

Here $a_{3\mathrm{D}}=4c_oe^2/(\pi P^2)$. We obtain the solution

$$\Sigma_j = s_jT^2, \quad s_j = \frac{b_ja_{3\mathrm{D}}}{2\sqrt{D}}\left\{\sqrt{z} - \sqrt{2\tau_S}\left(\frac{1}{2\tau_S}+P\right)\arctan\sqrt{2\tau_Sz}\right\}. \quad (33)$$

From the above we see that $\tau_{\phi o}^{-1}(T\to 0)$ is much smaller than both τ^{-1} and $\tau_{sp}^{-1}=\tau_{SO}^{-1}+\tau_S^{-1}$. Actually, τ_{sp}^{-1} dominates over $\tau_{\phi o}^{-1}(T\to 0)$ in the denominator of C_0, causing a "saturation" of this contribution.

Besides the spin-scattering rate terms appearing in the denominator of C_0, the dephasing rate probed in experiments is also set by the terms $R_\pm$—see Eq. (22), pure magnetic scattering case. As mentioned in Appendix C, it is expected that in the presence of the Coulomb interaction the simple diffusion pole in C_2—see Eq. (C1)—survives intact, in the pure spin-orbit scattering case, thus yielding absence of dephasing saturation.

We note that the idea that magnetic scattering may cause saturation has been recently suggested in Ref. 13. The situation here is to be contrasted with the absence of spin disorder, where it has been shown that $\tau_\phi^{-1}(T\to 0)\to 0$, e.g., in 2D $\tau_\phi^{-1}(T\to 0)\propto T\to 0$.[14–21] Moreover, we should point out that other self-energy processes, which are first order in the interaction $V(q,\omega)$, e.g., with V crossing diagonally the Cooperon—see e.g., Ref. 17, are not expected to modify *qualitatively* these results. However, Ref. 15 found that in 2D $\tau_\phi^{-1}(T\to 0)\propto T\ln(1/T)$. It turned out that the $\ln(T)$ factor was superfluous, and due to the ommission of other self-energy processes, which are shown in Ref. 17.

Now, the total correction to the conductivity can be written as

$$\delta\sigma_{\rm tot}=\delta\sigma_o+\delta\sigma_I, \tag{34}$$

where the first term is the Cooperon contribution of Eq. (21) and the second term due to interactions, involving additional and more complicated terms—e.g., see Refs. 28 and 29. If $\delta\sigma_I(T\to 0)$ contains nonsaturating terms, then the picture so far presented should change.

The majority of experiments show saturation of τ_ϕ^{-1} in the low-temperature limit. The samples in which saturation is observed probably contain magnetic impurities, even in minute quantities. The dimensionality of the samples is not a decisive factor for the appearance of saturation. We believe that the observed saturation can be understood in the frame of our results above, and should be due to magnetic scattering. Including the apparent lack of saturation in certain samples—e.g., Refs. 6 and 13. In such cases it is difficult to say whether sufficiently low temperatures have been reached for saturation to be observable. The relevant T, below which saturation can be observed, is proportional to the strength of the magnetic scattering rate, and lack of saturation was observed in the cleaner samples—e.g., Ref. 13.

In summary, we have demonstrated that, within the frame of our approach, dephasing saturation arises from magnetic disorder and interactions, with the role of the former being decisive. Already, without considering interactions,[26] the Cooperon C_0 has a finite correction of the simple diffusion pole—see Eqs. (2)—which is equivalent to a "saturating" dephasing rate, but not for the pure spin-orbit scattering case—see Eqs. (C1). We treat the effects of interactions both on the spin-independent C_0 and on the spin-dependent $C_{1,2}$. Based on symmetry, a lack of saturation is expected for pure spin-orbit scattering. The magnitude of the "dephasing rate" at $T\to 0$ is set by the magnetic scattering rate. It appears that our results are in agreement with relevant experiments.

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APPENDIX A

We give a derivation of the spin-dependent Cooperons C_1^o, C_2^o—without Coulomb interactions—as presented in Ref. 26. C_1, C_2 obey the equations (see Fig. 1)

$$C_1^o=\Gamma_1+\Gamma_1 PC_1^o+\Gamma_2 PC_2^o, \tag{A1}$$

$$C_2^o=\Gamma_2+\Gamma_2 PC_1^o+\Gamma_1 PC_2^o, \tag{A2}$$

where $\Gamma_1=(\tau_o^{-1}+\tau_{SO}^{-1}/3-\tau_S^{-1}/3)/(2\pi N_F)$ and $\Gamma_2=(\tau_S^{-1}-\tau_{SO}^{-1})/(3\pi N_F)$. τ_o^{-1} is the spin-independent impurity scattering rate, so that $\tau^{-1}=\tau_o^{-1}+\tau_{SO}^{-1}+\tau_S^{-1}$. Also $P(q,\omega)=\Sigma_k G_R(k+q,\epsilon+\omega)G_A(k,\epsilon)=2\pi N_F\tau(1+i\omega\tau-Dq^2)$. Then

$$C_j^o=\frac{\Gamma_j}{\Delta}, \tag{A3}$$

with $\Delta=(1-\Gamma_1 P)^2-(\Gamma_2 P)^2$. Now, taking the case $\tau_S^{-1}=0$—pure spin-orbit case—we obtain $\Delta=B(B+a)$, with $a=4\tau/(3\tau_{SO})$ and $B=Dq^2-i\omega$. Then

$$C_j^o=\frac{\Gamma_j}{a}\left\{\frac{1}{B}-\frac{1}{B+a}\right\}. \tag{A4}$$

These are Eqs. (C1) in Appendix C. Equations (2) for pure magnetic scattering follow likewise.

APPENDIX B

In the limit $\epsilon_F\tau\gg 1$ Eqs. (4) and (5) reduce to

$$C_1=C_1^o+C_1(\Sigma_1 C_1^o+\Sigma_2 C_2^o)+C_2(\Sigma_1 C_2^o+\Sigma_2 C_1^o), \tag{B1}$$

$$C_2=C_2^o+C_1(\Sigma_2 C_1^o+\Sigma_1 C_2^o)+C_2(\Sigma_1 C_1^o+\Sigma_2 C_2^o). \tag{B2}$$

Then, setting $u_1=1-(\Sigma_1 C_1^o+\Sigma_2 C_2^o)$ and $u_2=\Sigma_2 C_1^o+\Sigma_1 C_2^o$,

$$\det=u_1^2-u_2^2,\quad D_1=C_1^o u_1+C_2^o u_2,\quad D_2=C_2^o u_1+C_1^o u_2, \tag{B3}$$

we have

$$C_j=D_j/\det,\quad j=1,2. \tag{B4}$$

Taking

$$c=\frac{1}{B+m_1}-\frac{1}{B+m_2},\quad B=Dq^2-i\omega,\quad m_1=2/(3\tau_S),$$
$$m_2=2/\tau_S, \tag{B5}$$

we obtain

$$D_j=b_j c+(-1)^j c^2(b_1^2-b_2^2)\Sigma_j. \tag{B6}$$

Then

$$C_j=\frac{d_j}{N_+N_-}, \tag{B7}$$

with $d_j=b_j m_0(B+m_1)(B+m_2)+(-1)^j m_0{}^2(b_1^2-b_2^2)\Sigma_j$, $N_\pm=(B+m_1)(B+m_2)-m_0(b_1\Sigma_1+b_2\Sigma_2)\pm m_0(b_2\Sigma_1+b_1\Sigma_2)$ and $m_0=m_2-m_1$. We should emphasize that so far this algebra is exact.

In the following, we only keep terms of order B, obtaining

$$d_j=b_j m_0[m_1m_2+B(m_1+m_2)]+(-1)^j m_0{}^2(b_1^2-b_2^2)\Sigma_j, \tag{B8}$$

$$N_+=m_1m_2+B(m_1+m_2)+m_0(b_1-b_2)(\Sigma_2-\Sigma_1), \tag{B9}$$

$$N_-=m_1m_2+B(m_1+m_2)-m_0(b_1+b_2)(\Sigma_2+\Sigma_1). \tag{B10}$$

Making use of

$$\frac{1}{N_+N_-}=\frac{1}{m_0X}\left(\frac{1}{N_-}-\frac{1}{N_+}\right), \tag{B11}$$

we obtain directly Eqs. (12). The derivation above is equally straightforward for the case of spin-obit disorder.

APPENDIX C

In this Appendix we discuss the case of finite spin-orbit impurity scattering, i.e., $\tau_{SO}^{-1}>0$, $\tau_S^{-1}=0$. The Cooperons are given by[26,27]

$$C_0^o(q,\omega)=\frac{1}{2\pi N_F\tau^2}\frac{1}{Dq^2-i\omega+4/(3\tau_{\rm SO})},$$

$$C_{1,2}^o(q,\omega)=b'_{1,2}\left\{\frac{1}{Dq^2-i\omega}-\frac{1}{Dq^2-i\omega+4/(3\tau_{\rm SO})}\right\}, \tag{C1}$$

with $b'_1=(3\tau_{SO}/\tau-2)/(2u)$, $b'_2=-1/u$, $u=4\pi N_F\tau^2$.

Diagonalizing the system formed by Eqs. (4) and (5) yields for the Cooperons $C_{1,2}$

$$C_{1,2}(q,\omega)=S'_{1,2}\left\{\frac{1}{Dq^2-i\omega+r_-}-\frac{1}{Dq^2-i\omega+r_+}\right\}. \tag{C2}$$

Here $r_+=(b'_1-b'_2)(\Sigma_2-\Sigma_1)$, $r_-=-(b'_1+b'_2)(\Sigma_2+\Sigma_1)$, $S'_j=\{b'_j(Dq^2-i\omega)+M'_j\}/X'$, $X'=2(b'_1\Sigma_2+b'_2\Sigma_1)$, $M'_1=(b_2'^2-b_1'^2)\Sigma_1$, $M'_2=-(b_2'^2-b_1'^2)\Sigma_2$. The derivation of Eqs. (C2) is the same as for the magnetic impurity case given in Appendix B.

The diffusion pole in Eqs. (C1) survives, by symmetry, in the presence of the spin-conserving Coulomb interaction, which implies $r_+=0$ or $r_-=0$ in Eqs. (C2). The 2D weak localization correction to the conductivity is

$$\delta\sigma_o=-\frac{e^2Du}{\pi}\frac{eH}{\pi}\times\sum_{n=0}^{N_H}\left\{\frac{1}{2\pi N_F\tau^2}\frac{1}{4DeH(n+1/2)+4/(3\tau_{\rm SO})+\tau_{\phi o}^{-1}}+\frac{4b'_2eDH(n+1/2)+M'_2}{X}\times\left(\frac{1}{4DeH(n+1/2)+r_-}-\frac{1}{4DeH(n+1/2)+r_+}\right)\right\}, \tag{C3}$$

with $N_H=1/(4De\tau H)$.

Fitting this expression to Eq. (23) with either $r_+=0$ or $r_-=0$ yields absence of dephasing saturation. This is the case for all dimensionalities for pure spin-orbit scattering. As mentioned in the text, this is not the case for finite magnetic scattering.

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[1] J. J. Lin and J. Bird, J. Phys.: Condens. Matter **14**, R501 (2002).
[2] P. Mohanty, E. M. Q. Jariwala, and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997).
[3] P. Mohanty, Physica B **280**, 446 (2000).
[4] J. Bird, A. D. C. Grassie, M. Lakrimi, K. M. Hutchings, J. J. Harris, and C. T. Foxon, J. Phys.: Condens. Matter **2**, 7847 (1990).
[5] M. Noguchi, T. Ikoma, T. Odagiri, H. Sakakibara, and S. N. Wang, J. Appl. Phys. **80**, 5138 (1996).
[6] D. Natelson, R. L. Willett, K. W. West, and L. N. Pfeiffer, Phys. Rev. Lett. **86**, 1821 (2001).
[7] A. G. Huibers, J. A. Folk, S. R. Patel, C. M. Marcus, C. I. Duruõz, and J. S. Harris, Phys. Rev. Lett. **83**, 5090 (1999).
[8] D. P. Pivin, A. Andresen, J. P. Bird, and D. K. Ferry, Phys. Rev. Lett. **82**, 4687 (1999).
[9] A. B. Gougam, F. Pierre, H. Pothier, D. Esteve, and N. O. Birge, J. Low Temp. Phys. **118**, 447 (2000).
[10] J. J. Lin and K. Y. Kao, J. Phys.: Condens. Matter **13**, L119 (2001).
[11] J. J. Lin, T. J. Li, and Y. L. Zhong, J. Phys. Soc. Jpn. **72**, 7 (2003), and references therein.
[12] F. Pierre and N. O. Birge, Phys. Rev. Lett. **89**, 206804 (2002).
[13] F. Pierre, A. B. Gougam, A. Anthore, H. Pothier, D. Esteve, and N. O. Birge, Phys. Rev. B **68**, 085413 (2003).
[14] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitskii, J. Phys. C **15**, 7367 (1982).
[15] H. Fukuyama and E. Abrahams, Phys. Rev. B **27**, 5976 (1983).
[16] I. L. Aleiner, B. L. Altshuler, and M. E. Gershenzon, Waves Random Media **9**, 201 (1999).
[17] I. L. Aleiner, B. L. Altshuler, and M. G. Vavilov, J. Low Temp. Phys. **126**, 1377 (2002).
[18] R. Raimondi, P. Schwab, and C. Castellani, Phys. Rev. B **60**, 5818 (1999).
[19] A. Völker and P. Kopietz, Phys. Rev. B **61**, 13 508 (2000).
[20] A. Völker and P. Kopietz, Phys. Rev. B **65**, 045112 (2002).
[21] J. von Delft, J. Phys. Soc. Jpn. **72**, 24 (2003).
[22] D. S. Golubev and A. D. Zaikin, Phys. Rev. B **59**, 9195 (1999).
[23] D. S. Golubev, A. D. Zaikin, and G. Schön, J. Low Temp. Phys. **126**, 1355 (2002).
[24] Y. Imry, H. Fukuyama and P. Schwab, Europhys. Lett. **47**, 608 (1999).
[25] A. Zawadowski, J. von Delft, and D. C. Ralph, Phys. Rev. Lett. **83**, 2632 (1999).
[26] G. Bergmann, Phys. Rep. **107**, 1 (1984).
[27] The prefactors $b_{1,2}$ and $b'_{1,2}$ in Eqs. (2) and (C1) are correct herein. In Ref. 26 they contain a superfluous prefactor.
[28] C. Castellani, C. Di Castro, P. A. Lee, and M. Ma, Phys. Rev. B **30**, 527 (1984).
[29] G. Kastrinakis, Europhys. Lett. **42**, 345 (1998).