

## Considerable enhancement of the critical current in a superconducting film by a magnetized magnetic strip

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We show that a magnetic strip on top of a superconducting strip magnetized in a specified direction may considerably enhance the critical current in the sample. At fixed magnetization of the magnet we observed the diode effect—the value of the critical current depends on the direction of the transport current. We explain these effects by a influence of the nonuniform magnetic field induced by the magnet on the current distribution in the superconducting strip. The experiment on a hybrid Nb/Co structure confirmed the predicted variation of the critical current with a changing value of magnetization and direction of the transport current.

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### I. INTRODUCTION

The best-known and useful property of superconductors is the ability to carry current without dissipation. Unfortunately, in type II superconductors the magnetic flux may enter a superconducting sample in the form of Abrikosov vortices, and their motion under the influence of the Lorentz force (produced by transport current) leads to dissipation of the energy.<sup>1</sup> There are two ways to prevent this motion: the inhomogeneities of the sample which pin the vortices<sup>2</sup> (so-called bulk pinning) and/or surface/geometrical barrier<sup>3–5</sup> which does not allow vortices to enter/exit the sample. For both cases there is a critical current  $I_c$  above which the pinning centers or the surface barrier do not hold vortices any more, so dissipation starts in the superconductor. For the bulk pinning case at  $I=I_c$  in any point of the sample there is equilibrium between the Lorentz force  $F_L=\Phi_0 j/c$  ( $\Phi_0$  is the magnetic flux quantum) and the pinning force  $F_p$ , and the local current density is equal to the pinning current density  $j(r)=j_p(r)$ . In the case of the surface barrier dissipation starts when the current density on the surface/edge exceeds the critical value  $j_s$  (for a defectless superconductor  $j_s$  is equal to the Ginzburg-Landau current density  $j_{GL}$ ) and in any point of the sample the current density has the same sign<sup>6–8</sup> (the last condition provides the vortex passage through the sample).

There is a theoretical limit for the critical current—it cannot be larger than the product of the Ginzburg-Landau current density and the cross-section  $S$  of the sample  $I_c^{\text{theor}}=j_{GL}S$ . In the bulk pinning case vortices become depinned normally at  $j_p \ll j_{GL}$  and  $j_p$  usually decreases with an increasing local magnetic field.<sup>2</sup> In a strip with the transport current the field is maximal on the edge of the sample and, hence, the pinning current density is minimal there. The situation is opposite to the above for the surface barrier mechanism—the current density is maximal on the edge and minimal inside the sample at  $I=I_c$ .<sup>6,8</sup> As a result, in both situations the real critical current is much smaller than  $I_c^{\text{theor}}$ .

One interesting way to increase the critical current (besides the attempts to increase the pinning current density by artificial pinning centers) is the use of magnetic or superconducting screens<sup>9–11</sup> around a superconducting sample. The main idea of this method is to make the current distribution

in the sample homogeneous due to screening of the current induced magnetic field. It was shown theoretically that for the surface barrier mechanism the critical current may reach the maximal possible value  $I_c^{\text{theor}}$  by this method.<sup>9</sup>

In our paper we propose another method for enhancement of the critical current by magnetized magnetic materials. We apply a nonuniform magnetic field induced by a magnetic strip to a superconducting film with a transport current. The easiest way to do that is to place the magnet on the top of the superconducting strip (Fig. 1). It is clear from the figure that, depending on the relative direction of the magnetization and the transport current, it leads to a decrease or increase of the total magnetic field inside the superconducting strip. It may result in smoothing of the current distribution and enhancement of the critical current.

Note that the above mentioned methods are different from one used in the works (for example, see Ref. 12) where the influence of arrays of ferromagnetic nanoparticles on the critical current of a superconducting film was studied. This technique yielded a noticeable result only at a temperature close to the critical one, when the intrinsic pinning is weak, and theoretical estimations showed that it cannot provide a large pinning current density.<sup>13</sup> In contrast to that method, we apply the magnetic field of a ferromagnet to the *whole sample* rather than control the motion of individual vor-

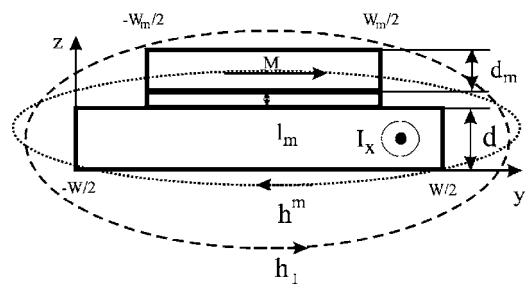


FIG. 1. Magnetic strip with thickness  $d_m$ , width  $w_m$  and magnetization  $M$  placed on the top of superconducting strip with thickness  $d$  and width  $w$ . Thickness of the isolating layer is  $l_m$ , and dashed and dotted lines show qualitatively the magnetic field lines from the magnet and the superconducting strip with transport current  $I$  in the X direction.

tices.<sup>14</sup> It allowed us to find the pronounced effect even at temperatures considerably lower than  $T_c$ .

In the paper we quantitatively study the value of the enhancement of the critical current for both the bulk pinning and the surface barrier mechanisms. We show theoretically that for real materials and realistic parameters it is possible to increase the critical current several times using this method. Besides we assume in our model that the magnetic field induced by the current is unable to change the magnetization of the magnetic material. It is valid in two cases: (i) if this field is smaller than the coercive field of the ferromagnetic material; (ii) if we apply the magnetic field in parallel to the strip to compensate for the current induced field and magnetize the magnet. The experiment on a Nb/Co structure with the parameters being far from optimal gave us an increase in the critical current by 20%.

The paper is organized as follows: In Secs. II and III we study theoretically the value and the conditions for the enhancement of critical current in the structure shown in Fig. 1 for two irreversibility mechanisms—the surface barrier and the bulk pinning, respectively. In Sec. IV we present the results of our experiment on a Nb/Co structure, and in Sec. V we discuss the restrictions and conditions for observing this effect in other superconducting materials.

## II. SURFACE BARRIER MECHANISM

First, let us consider the case when the critical current is determined by the surface barrier effect. As already mentioned above, the sample is in the critical state when on the edge the current density reaches the critical value  $j_s$  and nowhere inside the sample does  $j$  change sign.

To find the critical current for system shown in Fig. 1 we use the model equation<sup>6–8,15</sup>

$$\frac{dj(y)}{dy} + \frac{d}{2\pi} \int_{-w/2}^{w/2} \frac{j(y')}{y' - y} dy' = h_z^0 - n(y)\Phi_0, \quad (1)$$

which describes distribution of current  $j(y)$  and vortex density  $n(y)$ , averaged over the strip thickness and intervortex distance, in presence of transport current and external uniform magnetic field  $h_z^0$ . In the presence of an uniformly magnetized magnetic strip<sup>16</sup> we should add in the right-hand side of Eq. (1) the magnetic field  $h_z^m$ ,

$$h_z^m(y) = 2M/d(F(y, l_m) - F(y, l_m + d) - F(y, l_m + d_m) + F(y, d + l_m + d_m)), \quad (2)$$

$$F(y, a) = -\frac{a}{2} \log\left(\frac{a^2 + (y - w_m/2)^2}{a^2 + (y + w_m/2)^2}\right) + (w_m/2 - y) \arctan\left(\frac{a}{y - w_m/2}\right) + (w_m/2 + y) \arctan\left(\frac{a}{y + w_m/2}\right) \quad (3)$$

induced by magnetic strip the magnetized in the  $Y$  direction [ $\mathbf{M} = (0, \pm M, 0)$ ] and averaged over  $d$ . In Eq. (1) the distance

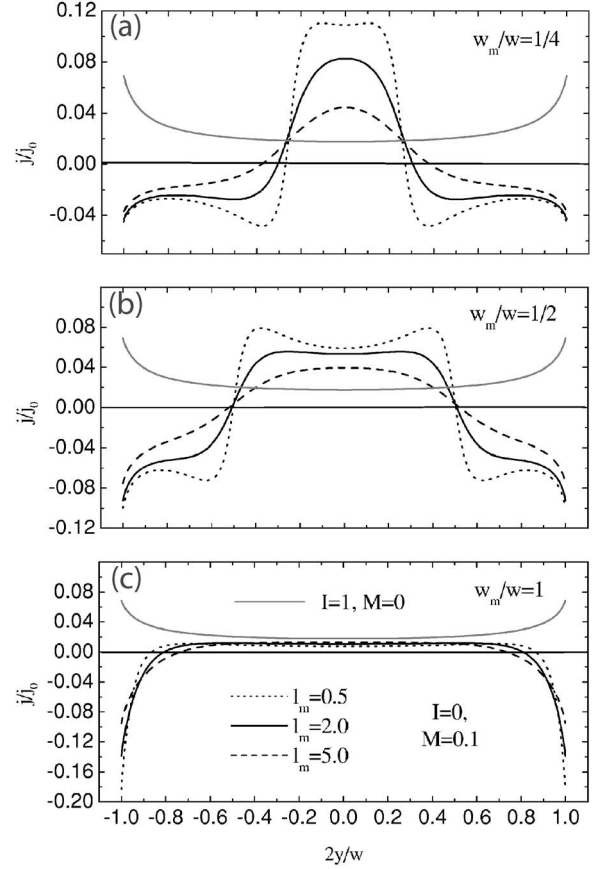


FIG. 2. Distribution of the current density in a superconducting strip ( $w=40, d=1$ ), induced by a magnetic strip ( $d_m=1, M=0.1, I=0$ ) for different widths  $w_m$  and separation distances  $l_m$ . Gray curve shows the current distribution in the superconducting strip with current  $I=1$  and zero magnetization  $M=0$ .

is measured in units of the London penetration depth  $\lambda$ , the current density is in units  $j_0 = c\Phi_0/8\pi^2\lambda^2\xi$  (where  $\xi$  is the coherence length). Magnetic field and magnetization are scaled in units of  $h_c = \Phi_0/2\pi\xi\lambda$ . In general Eq. (1) is valid for arbitrary thickness  $d$ ,<sup>6,8</sup> but one should be careful when applying to find the critical current of a thick strip with  $d > \lambda$  and a magnet on the top of it. Indeed, for such a sample the current distribution is strongly nonuniform over the strip thickness, which leads to a nonuniform force (it is stronger on the top and weaker at the bottom of a superconducting strip) acting on the vortex. We may say that in this limit the results obtained from solution of Eqs. (1) and (2) should be considered as a semiquantitative estimation.

We consider the case of an applied zero magnetic field in the  $Z$  direction,  $h_z^0=0$ , assuming that the edge of the sample is defect-free ( $j_s = j_{GL} = \sqrt{4/27}$ ). In Fig. 2 we plotted the distributions of the current density induced by a magnetized magnetic strip (at different  $w_m$  and  $l_m$ ) or by a transport current. It is obvious that there is an optimal (for every specific ratio  $w/\lambda_{\text{eff}}$ ) distance  $l_m$  and width  $w_m$  at which the enhancement of  $I_c$  would be strongest. The reason is that, if the magnetic strip is very close to and/or narrower than the superconducting strip the total current distribution (from the magnet and the transport current) may be more nonuniform

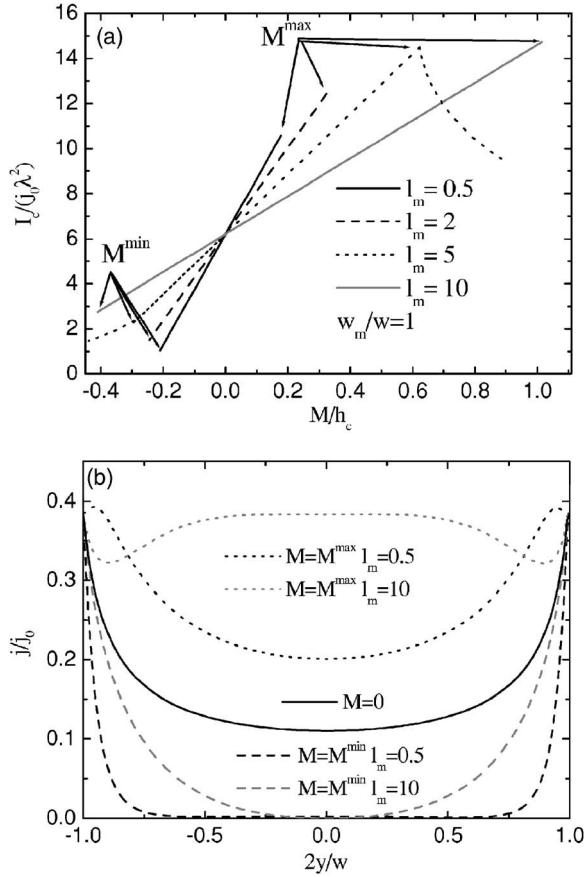


FIG. 3. Dependence of the critical current (a) in the superconducting strip with a strong surface barrier effect as a function of the magnetization of the magnetic strip magnetized in the  $Y$  direction. In the range  $(M_{\min}, M_{\max})$  there are no vortices in the strip at  $I < I_c$ . In part (b) we plotted the current distribution at  $I = I_c$  for  $M = M_{\max}$  and  $M = M_{\min}$ . Parameters of the hybrid system are the same as shown in Fig. 2(c). Dotted curves in (a) show the qualitative behavior of  $I_c(M)$  at  $M > M_{\max}$  and  $M < M_{\min}$ .

than that from the transport current alone (see Fig. 2). Usually the enhancement effect is maximal when  $w_m \approx w$  and for our parameters ( $w=40, d=1, d_m=1$ ) the thickness of isolating layer of about  $\approx 5$  is optimal for reaching  $I_c^{\text{theor}}$ .

In Fig. 3(a) we presented the dependence of the critical current (in the  $X$  direction) on the sign and value of the magnetization  $M$  at different  $l_m$  and  $w_m$ . It always has a linear dependence on the  $M$ , if there are no vortices in the sample, with a slope depending on  $d_m$ ,  $l_m$ , and  $w_m$ . At  $M = M_{\max}$  the critical current is maximal for the given geometrical parameters of superconducting and magnetic strips because at  $M > M_{\max}$  the vortices start to nucleate somewhere inside the sample rather than at the edge [where  $j$  is maximal, see Fig. 3(b)] and  $I_c$  decreases. At  $M < M_{\min}$  vortices appear in the superconducting sample even at  $I < I_c$  and the critical current decreases more slowly than by the linear law, with a further decreasing  $M$ . At the parameters choice of the superconducting strip it is possible to increase its critical current by the proposed method more than 2.5 times (it almost reaches the theoretical limit  $I_c^{\text{theor}} = j_{GL} w d \approx 15.4$ ).

It is clear that if we change the direction of the current (at fixed magnetization), we change the direction of variation of

$I_c$  (see Fig. 1). We may write that  $I_c^+(M) = I_c^-(-M)$  where  $I_c^+(M)$  is the critical current in the  $X$  direction and  $I_c^-(M)$  is the critical current in the opposite direction.

### III. BULK PINNING MECHANISM

In contrast to the previous case, a superconducting strip is filled up with vortices in the critical state (at  $I = I_c$ ). Their distribution is determined by the condition that in every point of the sample the current density is equal to the pinning current density. The equation for distribution of the magnetic field and the current density is a kind of the Bio-Savar's Law

$$h_z(y) = h_z^0 + h_z^m(y) + \frac{d}{2\pi} \int_{-w/2}^{w/2} \frac{j_p(y')}{y - y'} dy', \quad (4)$$

in which the pinning current density depends on the local magnetic field  $j_p(y) = j_p(h_z(y))$ . In our numerical calculations we use the well-known Kim-Anderson model<sup>2,17</sup>

$$j_p(h_z) = \frac{j_{p0}}{1 + |h_z|/h_p} \quad (5)$$

which we inserted in Eq. (4).

In the framework of model Eq. (5) we may claim that the use of a magnetic strip could largely increase  $I_c$  if the current induced field  $h_l$  is comparable with or larger than  $h_p$  on the edge of the superconducting strip. When the magnetic field induced by the magnetic strip compensates  $h_l$ , it causes an increase in the pinning current density and in the critical current. Figure 4 illustrates this phenomenon (here we considered only the case  $w_m = w$  which provides maximal enhancement of  $I_c$ ). When we decrease the field  $h_p$  (keeping other parameters constant), the effect of field compensation becomes more pronounced (see the inset in Fig. 5) and the critical current approaches a maximum value for this model:  $I_c^{\text{max}} = j_{p0} w d$ . Also, as for the surface barrier mechanism, there is an optimal distance  $l_m$ , where the effect is strongest (see Fig. 5) for all other parameters being constant.

### IV. EXPERIMENT

Actually, the idea of using a magnetized strip to enhance the critical current in a superconducting strip was originated from the experimental work,<sup>18</sup> where the effect of a chain of ferromagnetic particles on  $I_c$  of a superconducting bridge was studied. The critical current in the  $X$  direction increased when the magnetic particles were magnetized in the  $Y$  direction.<sup>18</sup> We explain that result by the influence of the magnetic field induced by the magnetic particles (in the way considered above) and to check this hypothesis we made an experiment with a simpler geometry.

In this work, for experimental investigation of the effect of an inhomogeneous magnetic field of the magnet on the critical current of a superconducting film, we fabricated a narrow Nb bridge with a Co line positioned under the center of the bridge. Figures 6(a) and 6(b) show an AFM image of the structure under study. The bridge was characterized by the following parameters: the thickness  $d$  was about 100 nm, the lateral dimension of the constriction width  $w = 2 \mu\text{m}$  and

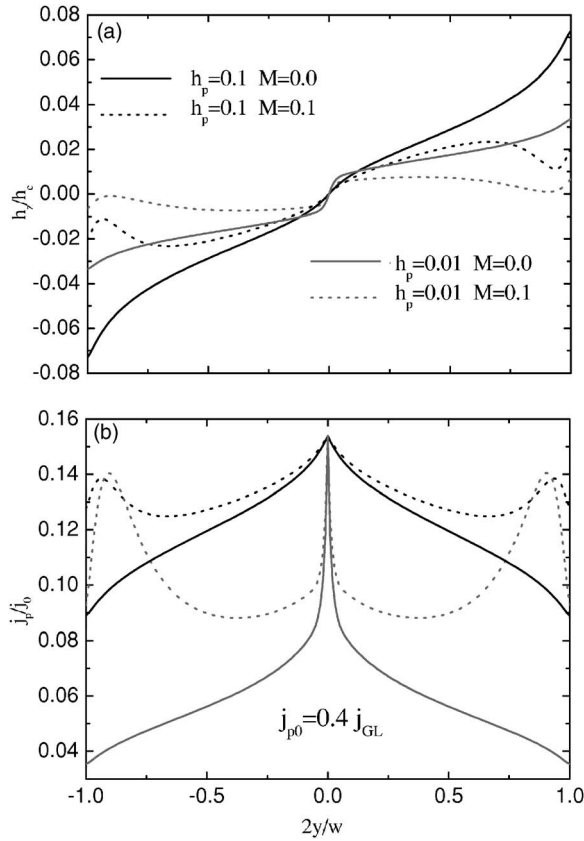


FIG. 4. Magnetic field (a) and current density distribution (b) in the superconducting strip for different values of strength of pinning [see Eq. (5) for other values of  $h_p$ ] and magnetization  $M$ . With a decreasing  $h_p$  the effect of the magnetic strip becomes stronger.

the length  $L=12 \mu\text{m}$ , the critical temperature was about 9.2 K. The ferromagnetic Co strip was obtained by electron lithography<sup>19</sup> and had the following dimensions: width  $w_m=0.4 \mu\text{m}$ , length  $L=14 \mu\text{m}$ , and thickness  $d_m=100 \text{ nm}$ . The ferromagnetic and superconducting strips were separated by

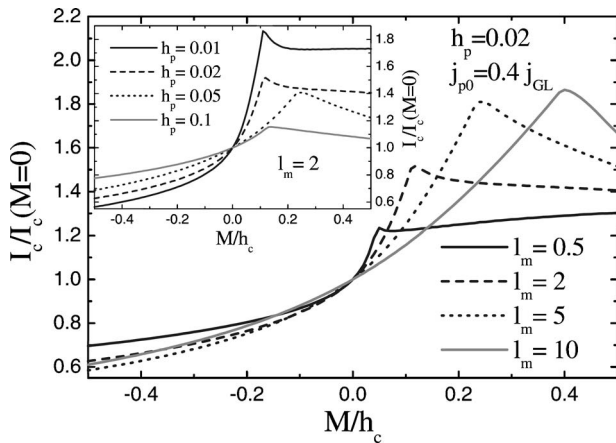


FIG. 5. Dependence of the critical current (in bulk pinning model) on the magnetization of the magnetic strip for different distances  $l_m$ . The parameters of the strips are the same as in Fig. 2(c). In the inset we show the variation of  $I_c(M)$  with a change in pinning (for different  $h_p$ ).

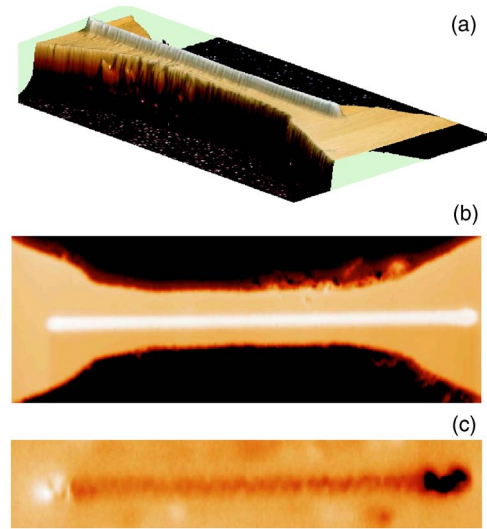


FIG. 6. (Color online) 3D (a) and 2D (b) AFM images of our niobium bridge with a cobalt strip on the top. In (c) we show the MFM image of our cobalt magnet in the demagnetized state.

a thin ( $l_m=50 \text{ nm}$ ) layer of insulator material (to prevent the proximity effect). The magnetic state of the Co line was monitored by a Solver scanning probe microscope at room temperature in the “flying” mode. Figure 6(c) shows an MFM image of the sample. The stripe domain structure of the Co strip (with the residual magnetization close to zero) is clearly visible.

The measurements were performed at a temperature  $T=4.2 \text{ K}$  by the standard four probe method. The dependence of the critical current (along the X axis) on external uniform magnetic field applied in the Y direction was measured. Note that the critical current of the blank Nb bridge (without magnet) is independent of the external magnetic field in the X and Y direction up to 3 kOe.

Figure 7 shows the results of measuring  $I_c(H)$  for the positive and the negative transport current, respectively. We observed variation of the critical current when the magnetic strip is magnetized in the Y direction. There are two effects. First, the critical current  $I_c^+(H)$  enhances with an increasing external magnetic field. Second, there is a strong asymmetry for different directions of the current, so-called diode effect (see current-voltage characteristic in the inset in Fig. 7). The value of the diode effect is about 180% in the dc regime. Actually, the weak effect was also found when we magnetized the strip in the X direction (variation of  $I_c$  was about 5%). We explain it by appearance of an uncontrolled components of the magnetic field induced by nonuniform magnetization distribution of the magnetic strip [Fig. 6(c)].

In our experiment we also observed a hysteretic dependence of the critical current on the magnetic field in both X and Y directions. It occurs already after the first sweeping up and down the applied magnetic field. We attribute it with the hysteresis in the process of magnetization of the cobalt strip. From these measurements we found a coercive field of our magnetic strip,  $h_{\text{coer}} \approx 180 \text{ Oe}$ , in the Y direction at  $T=4.2 \text{ K}$ . Probably, the changes in the domain structure of a ferromagnetic strip could be fixed by measuring the critical

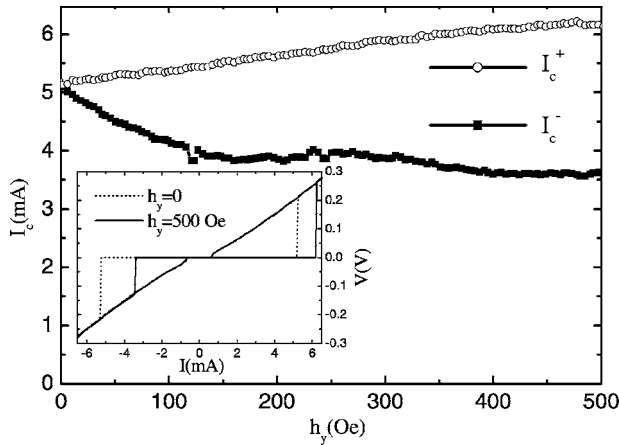


FIG. 7. Experimental dependence of  $I_c^+$  (critical current in the  $X$  direction) and  $I_c^-$  (critical current in the opposite direction) on the applied magnetic field in the  $Y$  direction. At  $h_y=0$  the magnetic strip was demagnetized (remanent magnetization is equal to zero). In the inset we presented the dc  $I$ - $V$  characteristics of our hybrid system at  $h_y=0$  and  $h_y=500$  Oe, which show a pronounced diode effect at large  $h_y$ .

current of the superconducting bridge. This problem deserves separate investigation (both theoretical<sup>16</sup> and experimental) and it will be done elsewhere.

## V. DISCUSSION AND CONCLUSION

The increase in  $I_c$  was about 20% for our specific geometrical parameters (it is close to the value observed in Ref. 18 for a chain of magnetic particles). If we use the model of surface barrier and parameters typical for dirty Nb (Ref. 20) ( $\lambda \sim 100$ – $200$  nm,  $\xi \sim 20$ – $10$  nm,  $h_c \sim 1600$  Oe) we find the same maximal increase in the critical current for our hybrid system. For the bulk pinning model [with  $j_{p0} \sim 4 \times 10^6$  A/sm<sup>2</sup> and  $h_p \sim 250$  Oe found from the best fit to experimental results for  $I_c(h_z^0)$ ] we found a much smaller theoretical enhancement of the critical current. These results lead us to believe that the surface barrier plays an essential role in our experiment, at least at  $h_z^0 = 0$ . It is in agreement with the results of Ref. 20, where the importance of the surface barrier effect was experimentally proved for similar Nb bridges at low magnetic fields. We are planning to continue our research on the wide superconducting and magnetic strips to optimize enhancement of the critical current.

Our experiment shows that it is possible to observe the diode effect in such a structure and control its value by variation of the applied magnetic field (Fig. 7). We may theoretically estimate, using the standard model of viscous motion of vortices and the heat balance equation, that the diode effect exists for our structure at frequency  $\nu \lesssim 10^5$  Hz of the applied ac current.

In the theoretical model we neglected the effect of the current induced magnetic field on magnetization of the magnetic strip. In our experiment it does not play any role because we applied a parallel magnetic field to the sample. This field did not affect the critical current of the sample directly

because the parallel critical field is about  $h_c d/2\lambda \sim 3200$ – $6400$  Oe for our thin Nb bridge.

In our calculations within the surface barrier model we assumed that the vortices enter the sample when  $j_{\text{edge}} = j_{\text{GL}}$ . Actually, in a real situation there always are some surface defects which favor the vortex entrance and diminish the surface critical current density.<sup>21–23</sup> This does not affect the main result because the current density is still maximal at the edge and minimal in the center of the strip with surface defects at  $I = I_c$ . Enhancement of  $I_c$  may be even larger because  $j$  inside the sample may be larger than the current density at the edge (but cannot exceed  $j_{\text{GL}}$ ).

The larger the ratio  $w/\lambda_{\text{eff}}$ , the higher nonuniformity of the distribution of the current density over the strip width (see analytical expression in Ref. 24 for an arbitrary value of  $w/\lambda_{\text{eff}}$ ). It means that covering by magnetic material is more effective for wide films with  $w/\lambda_{\text{eff}} \gg 1$ .

The situation is more complicated when both the surface barrier and the bulk pinning play an essential role. Nevertheless, the current distribution would be still nonuniform<sup>25</sup> and we expect the predicted effect to exist. We believe that the only qualitative agreement between our theory and experiment is consequence of our simplification when we considered separate influence of the surface barrier and bulk pinning. To obtain the quantitative agreement one needs take into account the combined effect of those irreversibility mechanisms on the critical current of ferromagnetic-superconducting bilayer.

The materials with a large magnetization which can be used in experiments are cobalt or iron with  $M_{\text{sat}} \approx 1800$  Oe. A good candidate for observing the predicted effect in the case of surface barrier mechanism is amorphous MoGe. The pinning current density may be as small as  $10^2$  A/cm<sup>2</sup>, and the experiment shows a pronounced surface barrier effect (with  $j_s \sim j_{\text{GL}} \sim 10^6$  A/sm<sup>2</sup>) on the critical current (see Ref. 24). For this material  $h_c \sim 800$  Oe (with  $\xi \sim 7.5$  nm,  $\lambda \sim 560$  nm) and, in dimensionless units  $M_{\text{sat}}/h_c \sim 2.2$ . It means that the predicted effect may be easily observed in this material [see Fig. 3(a)]. It is known that the surface barrier plays an important role for YBaCuO high temperature superconductors.<sup>26,27</sup> In this material  $j_{\text{GL}} \sim 10^8$  A/sm<sup>2</sup>,  $h_c \sim 10^4$  Oe (with  $\xi \sim 2$  nm,  $\lambda \sim 200$  nm),  $M_{\text{sat}}/h_c \sim 0.2$  and one can also observe enhancement of the critical current. As in our experiment, a parallel magnetic field can be applied to the hybrid system to magnetize the magnetic material and diminish the effect of the current induced field (the  $Y$  component of the magnetic field induced by the transport current can be overestimated as  $j_{\text{GL}}d/2 \sim h_c/8$  at  $d \sim \lambda$  and, hence, can change the magnetization of the cobalt magnet with the coercive field of about 180 Oe for our magnetic strip).

Soon after submission of our paper we learned about the work of Toubou *et al.* on the recent experiment on the LaSrMnO/YBaCuO bilayer.<sup>28</sup> In this work the asymmetry in the current-voltage characteristics (and variation of the critical current) was found by application of a proper magnetic field, but the value of the effect was small. The  $M_{\text{sat}}$  of the LaSrMnO layer was three times weaker than in our Co layer and  $h_c$  of YBaCuO is by an order of magnitude larger than  $h_c$  for Nb (see estimation above). Besides, the width of the stud-

ied YBaCuO strip was comparable with  $\lambda_{\text{eff}}$  and, hence, even in the absence of the ferromagnetic layer the current density distribution was almost uniform over the superconducting layer. In our opinion, these factors caused small influence of the magnet on the critical current in Ref. 28.

The magnetic field induced by the magnetic strip decays as  $\sim 1/r^2$  (at  $r \gg w_m$ ) and the one induced by the superconducting strip with transport current as  $\sim 1/r$  (at  $r \gg w$ ) only. Actually, it is the most important property of our hybrid system useful for applications (for a superconducting magnet, for example), a magnetized strip can strongly enhance the critical current of a superconducting sample and slightly modify the field structure far from the superconducting strip.

We made all numerical calculations for  $d_m = 1$ . By increasing the thickness of a magnetic strip it is possible to obtain the same enhancement of the critical current at a lower magnetization. As long as  $d_m \ll w_m$  we can obtain the same effect if  $d_m \cdot M = \text{const}$ .

Just as with the magnetic and superconducting screens of different shapes considered in Refs. 10 and 11, we believe

that there is an optimal form of a magnetic strip that provides for a stronger compensation of a current induced magnetic field than rectangular shape. It is clear that by increasing the thickness of a magnetic strip at the edges one can enhance the magnetic field at the edges of a superconducting strip and retain it in the middle. That allows one to control the current distribution in the superconducting strip in a more flexible way.

## ACKNOWLEDGMENTS

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