s-d–type exchange interactions in inhomogeneous ferromagnets

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Motivated by a need to understand spin-momentum transport in CPP (current perpendicular to the plane) structures, a quantum field theoretical treatment of spin-spin interactions in ferromagnets is presented. The *s-d* interaction of the conduction electrons and the magnetic medium is treated nonperturbatively from first principles in real space. The localized magnetic moments also interact with each other through a Heisenberg exchange potential. To take into account correlation effects, a second quantized formulation is used. The semiclassical limit is taken by using a coherent-state path-integral technique which also allows us to go beyond a linear-response approach. We derive a set of coupled equations of motion for the nonuniform magnetization, the spin current, and the two-point correlation functions of the magnetization. The rate of change of the magnetization is shown to obey a generalized Landau-Lifshitz equation that takes into account interaction with the conduction electrons. Within the relaxation time approximation it is shown that the polarization of the conduction electrons obeys a generalized diffusion equation. The diffusion tensor, which has off-diagonal terms due to the *s-d* exchange interaction, is now explicitly dependent on the magnetization of the medium. We also show that the magnetization fluctuations satisfy a diffusion-type equation. The derived equations are used in two illustrative examples.

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I. INTRODUCTION

Spin-spin interactions in ferromagnetic metals are of paramount importance in today's giant magnetoresistance (GMR) recording heads. There is also currently great interest in the magnetic recording industry in using spin currents, instead of magnetic fields, to switch the magnetization in a writer device. In this case a polarized electronic current is needed, such that the net spin of the polarization becomes effectively another magnetic source which induces an interaction with the magnetic moments of the medium. One widely used approximation is to separate the degrees of freedom of the current from those of the local magnetic moment. This latter separation is not justified in conducting metals but it nevertheless produces reasonable results in some cases.[1] This paper explores in some detail the consequences for the spin accumulation problem in ferromagnets of assuming that the interaction between the conduction electrons and the local moments is of the *s-d* exchange type. This interaction can give rise to what is now known as spin-momentum transfer (SMT) in magnetic multilayers. This latter mechanism has been predicted by Berger[1] and Slonczewski[2] and later verified experimentally by various groups.[3,4] Other interaction mechanisms between the conduction electrons and the magnetization vector have been proposed since the Berger-Slonczewski work.[5–9] In previous work, the interaction of the polarized current with the magnetization has not been treated self-consistently. In fact the equations of motion were based on those of a similar system, that of a current interacting with magnetic impurities.[10] We believe that this approach is not suitable for transition metals.

In this work, we start from a microscopic description of the conduction electrons and the ferromagnetic medium and then take the semiclassical limit to derive equations for macroscopic quantities of physical significance to experiment and to other phenomenological approaches. Although the derivations are somewhat complex, one can go to the main results [e.g., Eqs. (10) and (28) which are generalized Boltzmann-type equations] and see that the correct physics is contained in them.

Our results extend those of Ref. 8 and are in general relevant to problems of spin momentum transfer, domain walls, and spin-wave excitations.[11] We use many-body field theoretical methods to describe the system of magnetic moments plus conduction electrons. Even though only a single particle picture is needed, the methods we use permit us to treat the magnetic part of the problem and the conduction electrons on the same formal level. This allows us to derive transport equations for the conduction electrons and the local magnetic moments and include relaxation effects without recourse to more phenomenological modeling. Exchange effects, which are important in transition metals, are also included self-consistently. Finite-temperature properties are naturally included through the use of a path-integral formulation of the problem.[12] Including spin-dependent interactions in a transport problem means that we have to deal with many indexes. Path integrals are helpful with bookkeeping and hence simplify the discussion considerably as compared to the alternative approach of Ref. 10. Finally a path-integral representation helps in making consistent approximations to the derived transport equations.

Our treatment is nonperturbative and applies to systems far from equilibrium. One of the important results is that we are able to give an equation for the correlation functions of the magnetization from which a nonequilibrium fluctuation-dissipation result follows.

The paper is organized as follows. In Sec. II, we introduce the Hamiltonian of the interacting electron-magnetization system in second quantized form. Then we write the corresponding effective action of the Hamiltonian of the system. The details of this derivation are given in two appendixes. For the local magnetic moments we adopt a coherent state representation which is most suitable for a semiclassical treatment of the ferromagnet. From the effective action, we derive equations of motion for the magnetization (a modified Landau-Lifshitz equation) and the spin accumulation. These latter equations are then used to derive equations for the correlation functions of the magnetization. We also show that nonuniform magnetization of the medium gives rise to a spin accumulation effect similar to that due to interfaces. This is one of the main results of this work. Finally, we derive a diffusion-type equation for the magnetization fluctuations where the diffusion coefficient is dependent on the exchange integral.

In Sec. III, we apply the derived spin accumulation equations in two simple cases where the magnetization is nonuniform in the direction of flow of the current. In the last section, we summarize our results and discuss other related recent works on this subject that have appeared after this work was completed. In the Appendixes, we give details of the derivation of the effective action and the equations of motion.

II. THE *s-d* MODEL FOR INHOMOGENEOUS FERROMAGNETS

We start from a quantum field picture of the electronic current and the magnetic moments of the localized electrons in a thin slab of thickness comparable to the mean free path. In the following we do not include explicitly an electric field, but we assume that it is part of the spin-independent part of the Hamiltonian. Its inclusion has been done in Ref. 13. For the magnetic medium, a Heisenberg Hamiltonian is assumed. We explicitly take into account only exchange effects among the Heisenberg spins. Other important effects such as the demagnetization field term can be added phenomenologically. The conduction electrons will be assumed to be in thermal equilibrium with the magnetization.[14] Our work is a natural generalization of the model used by Langreth and Wilkins[10] to study spin resonance in dilute magnetic alloys. This latter model was used in Ref. 5 and 6 to study spin momentum transfer in magnetic multilayers. We will show that this magnetic impurity model is incomplete when it is applied to a conducting magnetic system.

The conduction electron field Ψ_s satisfies the usual anticommutation relations,

$$\{\Psi_s(\mathbf{r}),\Psi_{s'}(\mathbf{r}')\}=\{\Psi_s^\dagger(\mathbf{r}),\Psi_{s'}^\dagger(\mathbf{r}')\}=0,$$
$$\{\Psi_s(\mathbf{r}),\Psi_{s'}^\dagger(\mathbf{r}')\}=\delta_{ss'}\delta(\mathbf{r}-\mathbf{r}'), \tag{1}$$

where s is a spin index $\left(s=\pm\frac{1}{2}\right)$. The number density operator of the electrons is given by $\rho_{ss}(\mathbf{r})=\Psi_s^\dagger(\mathbf{r})\Psi_s(\mathbf{r})$. In this system, the electrons are treated as noninteracting, i.e., there is no Coulomb interaction, and hence the electron field can be expanded in terms of single particle wave functions:

$$\Psi_s(\mathbf{r})=\sum_i \phi_i(\mathbf{r})a_{s,i},\quad \Psi_s^\dagger(\mathbf{r})=\sum_i \phi_i^*(\mathbf{r})a_{s,i}^\dagger, \tag{2}$$

where $a_s(a_s^\dagger)$ is the annihilation (creation) operator for a particle of spin s. The total Hamiltonian of the system is composed of a conduction electron part, H_s, a magnetic part, H_d, and an interaction part, H_{sd},

$$H=H_s+H_d+H_{sd}. \tag{3}$$

The conduction electron part in a ferromagnet, such as Fe, is due to 4*s*-type electrons. The magnetic part is, however, due to 3*d*-type electrons. The magnetic medium is microscopically a lattice with a spin vector $\mathbf{S}_i$ at each lattice point i. Since we are interested in the continuum limit of this model, we can define a macroscopic spin vector for the medium

$$\mathbf{S}(\mathbf{r})=\sum_{i=1}^{N}\mathbf{S}_i\delta(\mathbf{r}-\mathbf{r}_i). \tag{4}$$

The magnetization vector is then simply given by the volume average of the global spin, $\mathbf{M}(\mathbf{x})=g\mu_B\mathbf{S}(\mathbf{x})/V$, where g is the spectroscopic splitting factor and μ_B is the Bohr magneton of the electron. The spin vector has an SU(2) representation and consequently so does the magnetization vector, e.g.,

$$[M_x(\mathbf{x}),M_y(\mathbf{x}')]=2ig\mu_B M_z(\mathbf{x})\delta(\mathbf{x}-\mathbf{x}'). \tag{5}$$

We employ units such that $g\mu_B=\hbar=1$. Hence the magnetization will have an *opposite* sign to that usually defined in the literature. The Hamiltonian of the spins is taken to be of the Heisenberg type,

$$H_d=-\frac{1}{2}\sum_{ij}J(\mathbf{r}_i-\mathbf{r}_j)\mathbf{S}_i\cdot\mathbf{S}_j-B\cdot\sum_i\mathbf{S}_i. \tag{6}$$

We take account of exchange only and assume the spins to be in an external field $\mathbf{B}$. The interaction between the electrons and the localized spins is taken to be of the *s-d* type, of the form

$$H_{sd}=-\frac{\lambda}{2}\int d\mathbf{x}[\mathbf{\Psi}^\dagger(\mathbf{x})\vec{\sigma}\mathbf{\Psi}(\mathbf{x})]\cdot\mathbf{S}(\mathbf{x}), \tag{7}$$

where λ is a coupling constant of the order of 0.1–1.0 eV and $\vec{\sigma}$ is a vector whose components are the Pauli matrices obeying,

$$[\sigma_i,\sigma_j]=2i\epsilon^{ijk}\sigma_k. \tag{8}$$

ϵ^{ijk} is the antisymmetric unit tensor.

In Appendix A, we use functional methods to derive the effective action for the system studied here. We find that it is a functional of the average magnetization $\mathbf{M}$, the electron propagators $\mathcal{G}$, and the magnon propagators $\mathcal{M}$,

FIG. 1. (Color online) Interaction terms between the conduction electrons and the localized magnetic moments. The smooth curve represents the conduction electron propagator. The curved line represents the spin-spin correlation functions.

$$\Gamma[\mathbf{M},\mathcal{G},\mathcal{M}] = \mathcal{A}_0[\mathbf{M}] + \frac{i}{2}\ln\det\mathcal{M}^{-1} - i\ln\det\mathcal{G}^{-1} + \frac{1}{2}\int d\mathbf{x}d\mathbf{y}\left[\frac{\delta^2\mathcal{A}_0}{\delta M_{\alpha\, i}(\mathbf{x})M_{\beta\, j}(\mathbf{y})}\mathcal{M}^{ji}_{\beta\alpha}(\mathbf{y},\mathbf{x})\right] + \int d\mathbf{x}d\mathbf{y}\left[\frac{\delta^2\mathcal{A}_0}{\delta\Psi^{\dagger}_{\alpha\, s}(\mathbf{x})\delta\Psi_{\beta\, s'}(\mathbf{y})}\mathcal{G}^{ss'}_{\beta\alpha}(\mathbf{y},\mathbf{x})\right] + \frac{\lambda^2}{2}g^{\alpha\alpha'\alpha''}g^{\beta\beta'\beta''}\frac{\sigma^i_{s_4s_1}}{2}\frac{\sigma^j_{s_2s_3}}{2} \times \int d\mathbf{x}d\mathbf{y}[\mathcal{G}^{s_1s_2}_{\alpha\beta}(\mathbf{x},\mathbf{y})\mathcal{M}^{ij}_{\alpha''\beta''}(\mathbf{y},\mathbf{x})\mathcal{G}^{s_3s_4}_{\alpha'\beta'}(\mathbf{y},\mathbf{x})] + O(\lambda^4). \tag{9}$$

This is the central result of the paper from which we derive approximate Boltzmann equations for the magnetization and the spin accumulation valid in the ballistic and the diffusive regimes.

The functional $\mathcal{A}_0$ is the functional $\mathcal{A}$, Eq. (A11), with all the source terms set to zero. The tensor g^{ijk} is equal to 1 if $i=j=k=1$ and equal to -1 if $i=j=k=2$ and zero, otherwise. The last term in the above equation has a simple interpretation in terms of Feynman diagrams, Fig. 1. It describes electron scattering off the magnons with and without spin flipping. This term is important at high temperatures and for magnetizations out of equilibrium. A similar term to this one has been recently introduced by Levy and Zhang to represent scattering at the normal-ferromagnetic interface. They discussed the need for it on physical grounds.[15–17] We will show below that our formalism provides us with a self-consistent way to calculate the physical effects of this scattering term. In Appendix B, we derive the following equation for the spin distribution function $\mathfrak{M}$:

$$\left[\partial_T + \frac{p}{m}\cdot\partial_X\right]\mathfrak{M}^k(T,X,p) + \epsilon^{klp}[B^l + \lambda M^l(T,X)]\mathfrak{M}^p(T,X,p) = -\frac{\mathfrak{M}^k(T,X,p) - \mathfrak{M}^k_{eq}}{\tau_k} - \frac{\mathfrak{M}^k(T,X,p) - \mathfrak{M}^k_0}{\tau_p}. \tag{10}$$

It is worthwhile to pause here and consider the content of this equation. The first term on the left-hand side is the total time derivative, with independent variables $(T,\mathbf{X},\mathbf{p})$ and $d\mathbf{p}/dt=0$ (we consider a nonzero electric field elsewhere[13]). The other term on the left-hand side is the contribution of torque due to the local moments and the right-hand side is an approximate expression for the collision operator. In the absence of gradients, the polarization of the conduction electrons is along the local effective field and there will be no spin currents. This will not be the case if there is an electric field present. The spin accumulation effects studied here are solely due to inhomogeneous magnetization of the medium. It is important to observe that this effect is present even in the absence of a current as is the case for the spin accumulation effects due to interfaces.[1] The fluid equations we derive from Eq. (10) are valid provided the averaging we perform is valid, which is an average over translational momentum. This is expected to be the case in a collisional limit, but is by no means restricted to this limit. There is a good analogy to the [magnetohydrodynamic (MHD)] fluid equations which are derived to describe a plasma confined by a magnetic field. The MHD equations are most commonly used to describe plasmas in which the collision frequency is much less than the gyrofrequency. As we shall see, the form of the fluid equations we derive has much in common with the MHD equations. Next we multiply Eq. (10) by the velocity and then average over it. To obtain the usual Fick's law for spin diffusion, we assume that the momentum relaxation time is small and hence the right-hand side of Eq. (10) is larger than the effects due to the local magnetization **M**. In this case we have

$$\mathfrak{J}^{kl}(T,X) = -D^k\partial_{X_l}\mathfrak{M}^k_c(T,X), \tag{11}$$

where the "diffusion" constants are given in terms of an average Fermi velocity v_F by

$$D^k = \frac{1}{3}v_F^2\tau_k^{eff}, \tag{12}$$

where $1/\tau_k^{eff} = 1/\tau_k + 1/\tau_p$. To get this result we made use of the following approximation for the velocities of the conduction electrons:

$$\partial_{X_l}\int\frac{d\hat{p}}{4\pi}v^lv^j\mathfrak{M}^k(T,X,p) \simeq \frac{1}{3}v_F^2\partial_{X_j}\mathfrak{M}^k_c(T,X). \tag{13}$$

Finally using Eqs. (10) and (11), we find that the classical magnetization of the conduction electrons obeys a diffusion equation for each one of its components,

$$[\partial_T - D^k\nabla^2]\mathfrak{M}^k_c(T,X) = -\frac{1}{\tau_k}[\mathfrak{M}^k_c(T,X) - \mathfrak{M}^k_{eq}(X)] - [(\mathbf{B}+\lambda\mathbf{M})\times\mathfrak{M}_c]^k. \tag{14}$$

This equation is, however, rotationally invariant and does not show the reduced symmetry of the ferromagnetic state. To get a more realistic equation we improve on Fick's law by keeping all terms in Eq. (10) and treat exchange effects between the conduction electrons and the magnetization more carefully. This amounts to taking into account the *s-d*-exchange term in the electron propagators.

For slow variations in time, we obtain a modified Fick's law for the spin accumulation $\mathfrak{M}_c$, Eq. (B4), that takes into account the variation of the local magnetization in space and in orientation, the flux in the jth direction of the k-component of the spin being

$$\mathfrak{J}^{kj}(T,\mathbf{X}) = -\mathfrak{D}^{kp}\partial_{X_j}\mathfrak{M}_c^p(T,\mathbf{X}), \tag{15}$$

where now the diffusion constant becomes a tensor. It is defined in terms of a matrix A

$$\mathfrak{D}^{kp}(\mathbf{X}) = D^p(A^{-1})^{kp}(\mathbf{X}). \tag{16}$$

Although there is a summation over p in Eq. (15), there is no summation over p in Eq. (16). The matrix A depends locally on the effective magnetization field $\mathbf{H}$,

$$A(\mathbf{X}) = \begin{bmatrix} 1 & -\tau_x H_z & \tau_x H_y \\ \tau_y H_z & 1 & -\tau_y H_x \\ -\tau_z H_y & \tau_z H_x & 1 \end{bmatrix}. \tag{17}$$

In our approximation, the effective local field is simply

$$\mathbf{H} = \mathbf{B} + \lambda\mathbf{M}(\mathbf{x}). \tag{18}$$

Now in the steady state, the equation satisfied by the average magnetic moment $\mathfrak{M}_c$ becomes a generalized diffusion equation

$$\sum_{p,l} \partial_{X_l}[\mathfrak{D}^{kp}(X)\partial_{X_l}\mathfrak{M}_c^p(\mathbf{X})] = \frac{1}{\tau_k}(\mathfrak{M}_c^k - \mathfrak{M}_{eq}^k) + \lambda[\mathbf{M}\times\mathfrak{M}_c]^k. \tag{19}$$

The tensor character of the diffusion term in Eq. (19) is not due to anisotropic transport—the flux in the j direction in Eq. (15) is due to a gradient with respect to X_j. Rather, the p-component of $\mathfrak{M}_c$ is rotated into the kth direction by the effective field $\mathbf{H}$, while transport takes place in the direction of the gradient. The diffusion tensor, Eq. (16), has striking similarities to the diffusion tensor of charged species in a magnetized plasma.[18] If we restrict ourselves to the case where the local effective field is constant and along the z-axis only, then the transverse diffusion coefficients are similar to those found by Hirst[19] and Kaplan[20] using very different methods from the one presented here but their solution is, however, not completely self-consistent as discussed in Ref. 11. Their work showed that in the direction perpendicular to the effective field, diffusion of polarization of the electron gas is much slower than along the field. The off-diagonal terms have their origin in the indirect exchange interaction among the conduction electrons and must be taken into account in a transition metal.

To solve the above equations of motion, we retain a subset of the terms arising in the full spin propagator, using an approximation similar to the random phase approximation in the calculation of the ground state energy of an interacting electron gas.[21] The zero order propagator is taken to be that of the electrons in the external $\mathbf{B}$ field and the localized spins interacting through the exchange interaction in the presence of the $\mathbf{B}$ field. Since the magnetic moments constitute a many-body problem, a full solution is not possible in general. Hence an explicit solution to the problem requires first a

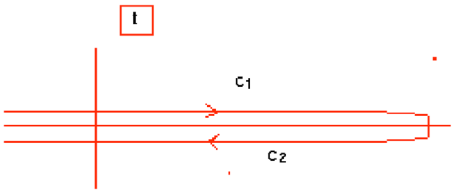

FIG. 2. (Color online) Closed time path: branch 1 corresponds to forward propagation in time while branch 2 is that for backward propagation.

calculation of the background magnetization. The magnetization satisfies a generalized Landau-Lifshitz equation which follows from Eq. (B1). First we observe that when the external sources are turned off, we have

$$\mathbf{M}_1(\mathbf{x}) = \mathbf{M}_2(\mathbf{x}) = \mathbf{M}(\mathbf{x}), \tag{20}$$

where $\mathbf{M}$ is the average, i.e., classical, magnetization. Equation (B1) is a system of two equations for $\mathbf{M}_1$ and $\mathbf{M}_2$, the magnetization vector along the paths C_1 and C_2, respectively, Fig. 2. It is the averaging of these two equations that gives the equation of motion for the average magnetization,

$$\partial_t\mathbf{M}(\mathbf{x}) = \mathbf{M}(\mathbf{x}) \otimes \left(\frac{1}{2}J\nabla^2\mathbf{M}(\mathbf{x}) + \mathbf{B} + \frac{\lambda}{2}\vec{\sigma}_{s's}\frac{1}{2}[\mathcal{G}_{11}^{ss'}(\mathbf{x},\mathbf{x}^+) + \mathcal{G}_{22}^{ss'}(\mathbf{x},\mathbf{x}^-)]\right), \tag{21}$$

where

$$\mathcal{G}_{11}^{ss'}(\mathbf{x},\mathbf{x}^+) = \mathcal{G}_{11}^{ss'}(\mathbf{x},\mathbf{y})|_{y\to x^+}. \tag{22}$$

The last term is simply the spin of the current. Recalling that at equal times and equal positions, all different Green's functions are related, the equation of motion for $\mathbf{M}$ simply becomes

$$\partial_t\mathbf{M}(\mathbf{x}) = \mathbf{M}(\mathbf{x}) \otimes \left(\frac{1}{2}J\nabla^2\mathbf{M}(\mathbf{x}) + \mathbf{B} + \lambda\mathfrak{M}(\mathbf{x})\right). \tag{23}$$

The last term gives rise to dissipation and a contribution to the precession for magnetic multilayers.[8] As we will see below, this term becomes J-dependent in the nonuniform case. This latter equation, Eq. (23), is the equivalent of the Landau-Lifshitz equation (LL) for the magnetization in the presence of a current. This form is still valid even in the presence of an electric field.

The solution of Eq. (B2), the equation for the electron propagators, can be represented in terms of Feynman diagrams, Fig. 3. First, we define the propagator of a noninteracting electron in an external magnetic field $\mathbf{B}$ and zero electric field,

FIG. 3. (Color online) Series expansion of the conduction electron propagator in powers of λ.

$$\mathcal{G}^{(0)\alpha\beta}_{ss'}(\mathbf{x},\mathbf{y}) = \left\{ (i\partial_{t_y} - \in_\alpha) + \frac{1}{2}\sigma^i_{ss'}B^i \right\}^{-1} \delta^{\alpha\beta}_{ss'}(\mathbf{x},\mathbf{y}), \quad (24)$$

and expand $\mathcal{G}$ in powers of λ using Eq. (B2). Keeping only terms up to order λ^2, we have

$$\sigma\mathcal{G} = \sigma\mathcal{G}^{(0)} - \lambda\sigma\mathcal{G}^{(0)}\mathbf{M}\cdot\sigma\mathcal{G}^{(0)} + \frac{1}{2}\lambda^2\sigma\mathcal{G}^{(0)}\mathbf{M}\cdot\sigma\mathcal{G}^{(0)}\mathbf{M}\cdot\sigma\mathcal{G}^{(0)} + \lambda^2\mathcal{G}^{(0)}\sigma\cdot\mathcal{M}\cdot\sigma\mathcal{G}^{(0)}\sigma\mathcal{G}^{(0)}\cdots. \quad (25)$$

This is a matrix equation and hence integrations over time, space, and spin degrees of freedom are implicit in the above notation. Recently Mills calculated the damping contribution to order λ.[22,14] One of his conclusions is that this contribution is dependent on the symmetry of the system in this case. This follows from the fact that the spin propagator of the conduction electrons in a ferromagnet is not $O(3)$-invariant, to first order in λ and higher since it depends explicitly on **M**. Equation (B3) gives the dependence of correlations on the exchange interaction and on the *s-d* interaction between the current and the medium. Our assumption is that exchange interactions are much stronger than the *s-d* interaction. Hence to lowest order, we neglect the latter in the equation for the fluctuations. To understand the meaning of such an equation, we study the case with strong exchange interactions, i.e., we take the average magnetization to be a constant and assume the external **B** field to be small. In this case Eq. (B3) becomes

$$\partial_{t_y}\mathcal{M}^{\times k}_{\alpha\beta}(\mathbf{y},\mathbf{z}) - [\mathbf{M}\cdot\partial_{t_y}\mathcal{M}^{\times k}_{\alpha\beta}(\mathbf{y},\mathbf{z})]\mathbf{M} = i\epsilon^{\alpha\beta}(\mathbf{n}\times\mathbf{M}) + \int d\mathbf{x}[J(\mathbf{x}-\mathbf{y})\mathcal{M}^{\times k}_{\alpha\beta}(\mathbf{x},\mathbf{z})\times\mathbf{M}], \quad (26)$$

where for each k, the unit vector **n** has components $n^i = \delta^{ik}$. The notation $\mathcal{M}^{\times k}$ is for a vector with components $\mathcal{M}^{ik}$, $i = 1,2,3$. Now if we average over the variable **z**, we get an equation that gives the time variation of the fluctuations of the magnetization around **M**. These fluctuations will in turn cause fluctuations in the current through the last term in Eq. (26).

Next we show how this latter equation gives rise to a Boltzmann-type equation for the magnetization fluctuations $\mathcal{M}^{il}$. First we expand $\mathcal{M}^{\times k}(\mathbf{x},\mathbf{z})$ around the position $\vec{\mathbf{y}}$,

$$\mathcal{M}^{\times k}(\mathbf{x},\mathbf{z}) = \mathcal{M}^{\times k}(\mathbf{y},\mathbf{z}) + \partial_x\mathcal{M}^{\times k}(\mathbf{x},\mathbf{z})\big|_{\mathbf{x}=\mathbf{y}}\Delta\mathbf{x} + \frac{1}{2}\partial_{\mathbf{x}}\partial_{\mathbf{x}}\mathcal{M}^{\times k}(\mathbf{x},\mathbf{z})\big|_{\mathbf{x}=\mathbf{y}}\Delta\mathbf{x}\Delta\mathbf{x} + \cdots, \quad (27)$$

where $\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{y}}$. If we put this back in Eq. (27), we get a diffusion-type equation for all components of the magnetization fluctuations

$$\partial_{t_y}\mathcal{M}^{\times k}_{\alpha\beta}(\mathbf{y},\mathbf{z}) - [\mathbf{M}\cdot\partial_{t_y}\mathcal{M}^{\times k}_{\alpha\beta}(\mathbf{y},\mathbf{z})]\mathbf{M} = -\frac{1}{2}J_2(\mathbf{y})\mathbf{M}\times\nabla^2_y\mathcal{M}^{\times k}_{\alpha\beta}(\mathbf{y},\mathbf{z}) + i\epsilon^{\alpha\beta}\mathbf{n}\mathbf{M} - J_0(y)\mathbf{M}\times\mathcal{M}^{\times k}_{\alpha\beta}(\mathbf{y},\mathbf{z}), \quad (28)$$

where $J_0(\mathbf{y})$ and $J_2(\mathbf{y})$ are the zeroth and second moments of the exchange coupling,

$$J_0(\mathbf{y}) = \int d\mathbf{x}J(\mathbf{x}-\mathbf{y}), \quad (29)$$

$$J_2(\mathbf{y}) = \frac{1}{3}\int d\mathbf{x}J(\mathbf{x}-\mathbf{y})\Delta x^2. \quad (30)$$

These integrals converge since the exchange coupling is short ranged. The first moment vanishes since we are assuming isotropic exchange coupling. Treatment of the coupling of the conduction electrons to the ambient magnetization at low temperatures or temperatures close to T_c must include Eq. (28) to account for the fluctuations of the magnetization.

III. APPLICATIONS

In this section, we mainly show how this formalism can be applied to multilayers. We will not solve for the correlations of the magnetization in this paper, i.e., we will not work with Eq. (27) which is one of the main results in this paper and the main reason for introducing the machinery used in this work. Nevertheless, we still have new equations for the spin accumulation in nonuniform magnetization. We will study their solutions in the usual linear normal-ferromagnet-normal structure and in rings. The question of correlations is treated elsewhere.[14]

In Ref. 13, we showed how our results extend those of Zhang, Levy, and Fert[8] by taking into account the indirect exchange effect of the magnetization on the conduction electrons. This is an important effect in transition magnetic metals and cannot be ignored.[6] In the following we study two types of structures with inhomogeneous magnetization. First we examine CPP-type structures with very thin paramagnetic spacers and no interfacial scattering. Second we consider structures which are topologically equivalent to a torus. These examples clearly show the origin of spin accumulation to be directly related to inhomogeneities in the magnetization. It is also obvious from these examples that domain walls are another physical example where the results presented here can be applied.[11] The interface will not be represented by a step function in the examples below and will instead take the shape shown in Fig. 4 which plots the mean field $a(x) = \tau\lambda M(x)$ due to the magnetization.

For a local magnetization which is a function only of distance **x** in the direction of the current, $\mathbf{M} = M(x)\mathbf{z}$, the spin accumulation obeys the simplified equations

$$D_{xx}\frac{d^2m_x(x)}{dx^2} + D_{xy}\frac{d^2m_y(x)}{dx^2} - 2\frac{D^2_{xy}}{Da(x)}\frac{da(x)}{dx}\frac{dm_x(x)}{dx} + \left(D_{xx} - 2\frac{D^2_{xy}}{D}\right)\frac{da(x)}{dx}\frac{dm_y(x)}{dx} = \frac{m_x(x)}{\tau_{sf}} - \frac{a(x)m_y(x)}{\tau_{sf}}, \quad (31)$$

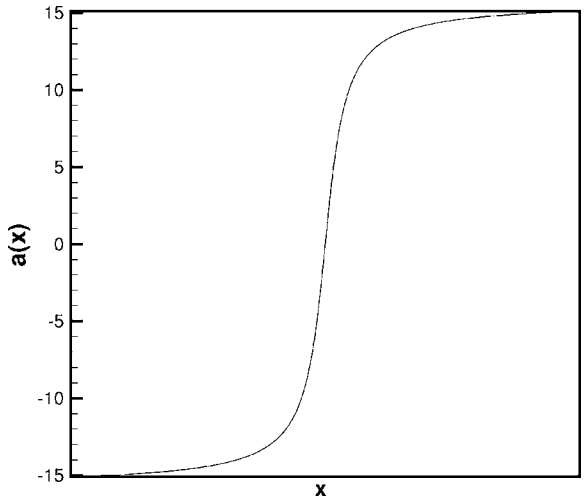

FIG. 4. Profile of the interface [or molecular field $a(x) = \tau\lambda M(x)$] used in the text. The current flows perpendicular to the interface. The nonmagnetic spacer is taken to have zero thickness.

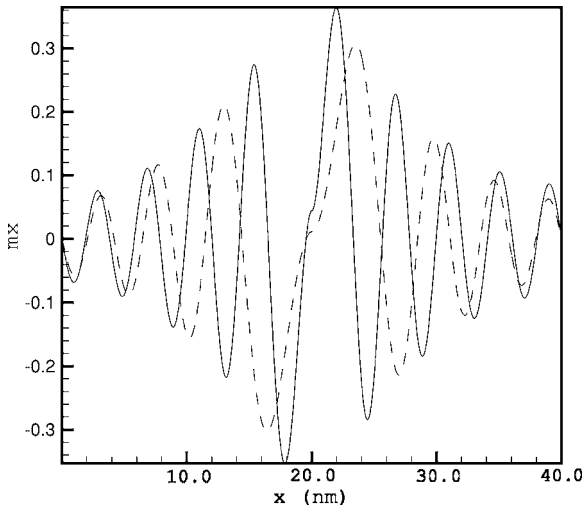

FIG. 5. The x-component of the spin accumulation as a function of the interface inhomogeneities. On the left, $M=-M_0\mathbf{z}$ and on the right $M=M_0\mathbf{z}$. The continous curve corresponds to the sharper interface of size 1.0 nm. The other dashed curve corresponds to an interface of 4.0 nm. The relative angle between the two magnetizations is $\pi/3$ and the size of each side is 20 nm.

$$-D_{xy}\frac{d^2m_x(x)}{dx^2}+D_{yy}\frac{d^2m_y(x)}{dx^2}-\left(D_{yy}-2\frac{D_{xy}^2}{D}\right)\frac{da(x)}{dx}\frac{dm_x(x)}{dx}-2\frac{D_{xy}^2}{Da(x)}\frac{da(x)}{dx}\frac{dm_y(x)}{dx}=\frac{m_y(x)}{\tau_{sf}}+\frac{a(x)m_x(x)}{\tau_{sf}},\tag{32}$$

$$D\frac{d^2m_z(x)}{dx^2}=\frac{m_z}{\tau_{sf}},\tag{33}$$

where

$$a(x)=\tau\lambda M(x).\tag{34}$$

The coefficients D_{xx}, D_{yy}, D_{xy}, and D_{yx} are functions of the local magnetization, the scattering rates, and the exchange constant

$$D_p=D_{xx}=D_{yy}=\frac{D}{1+[\tau\lambda M(x)]^2},\tag{35}$$

$$D_{yx}=-D_{xy}=D\frac{\tau\lambda M(x)}{1+[\tau\lambda M(x)]^2}.\tag{36}$$

Even in this simple approximation of the equations of motions, these equations are different than those in Refs. 19 and 20 which did not treat the torque on the conduction electrons self-consistently.[11] Moreover, the renormalization of the diffusion coefficients and the off-diagonal parts do not appear in the magnetic impurity problem of Ref. 10. The above system of equations will be solved for different configurations of the magnetization **M**. We adopt the following parameters for our calculations: The spin diffusion length $l_{sdl}=\sqrt{D\tau_{sf}}=100$ nm, $D=10^{-3}$ m^2/s, $D=100D_{xx}$, and $\lambda=0.1$ eV. It should be noted that in these equations, the torque term has the opposite sign to that which appears in, e.g., Zhang *et al.*[8] since we have taken the electron charge to be positive in our definitions of the magnetic moments. An important feature of the solutions of these equations not present in Refs. 5 and 8 is the oscillation of the spin components around the local magnetization.

First we consider a configuration with in-plane magnetization. The magnetization is assumed to vary with position along the direction of the current. The spacer has practically zero thickness, which is a reasonable approximation for most GMR devices. Figure 4 shows $a(x)$ for a typical interface. We do not explicitly include a nonmagnetic spacer but we set the magnetization to zero at the center. At the ends it is parallel to the local z axis, the local direction of the equilibrium magnetization. The transverse components of the spin accumulation are set to zero at the outer ends. The respective z axes make a nonzero angle in order for the spin accumulation to be nonzero. First we demonstrate the effect of inhomogeneities on the spin accumulation; we keep the relative angle between the magnetizations the same but we vary the size of the "domain wall," i.e., the transition region of the local mean field. It is seen from Figs. 5 and 6 that the larger the inhomogeneities the larger is the spin accumulation. This effect is independent of the relative orientations and was not predicted before since the z component of the spin accumulation is no longer independent of the transverse components in general, i.e., all three equations for the spin accumulations are coupled. Therefore spin accumulation can be enhanced by having a layer with constant direction magnetization but with spatial inhomogeneities. Such a structure can be achieved by, e.g., controlled doping that changes the magnetic saturation along the direction of the current. The solution of Eq. (32) shows that the spin accumulation is largest for the case where the two magnetizations are orthogonal to each other and the size of the sample is smallest. In all these results, the equilibrium spin accumulation is normalized to -1 on the left-hand side and normalized to $+1$ on the right-

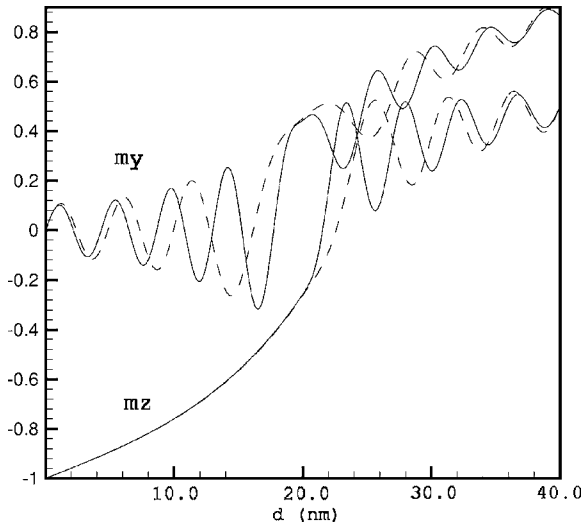

FIG. 6. The y and z components of the spin accumulation as a function of the interface inhomogeneities for the same configuration of **M** as in Fig. 5.

hand side. The components of the spin accumulation are taken with respect to a global frame, that of the layer on the left.

As our second example, we choose a "ring" structure. This can be part of a long solenoid with a square cross section. Therefore we now solve our equations with periodic boundary conditions. In each side of the square cross section, the profile of the magnetization within a period is shown in Fig. 7. The relative angle of the magnetization between neighboring sides is 90°. We study the spin accumulation as a function of the length of the sides. In Fig. 8 we plot the three components of the total spin accumulation. The equilibrium spin accumulation is taken to be normalized to one. The solutions show the expected behavior. The spin accumulation tries to reach its equilibrium value near the middle of

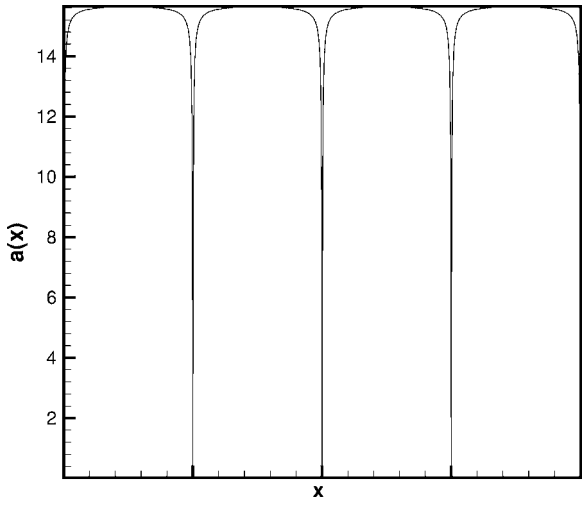

FIG. 7. Profile of the local magnetization along the different sides of the square ring (or torus) in one period. The magnetization is in the y-z plane whereas the current is in the x direction. The relative angle of the magnetization between neighboring sides is 90°.

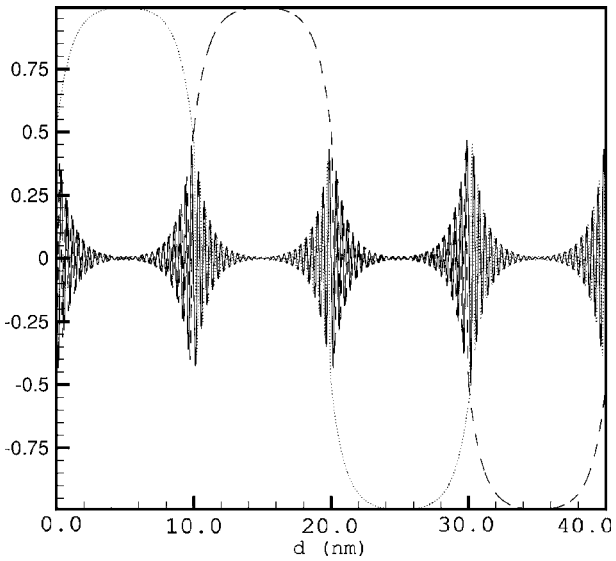

FIG. 8. Spin accumulation in a square strucure. The length of each side is $L=10$ nm. The magnetization is in-plane in each side. The x-component of the spin accumulation is perpendicular to the interface and it oscillates as it decays away from each interface. The z-component of the spin accumulation in side 1, $0<d<10$ nm, is along the magnetization in side 1. In side 2, the magnetization is along the y axis. Dotted curve: z-component, dashed curve: y-component.

each side. We find that the spin accumulation is largest when the length of each side is smallest, as expected. This geometry shows how spin accumulation can be transported over large distances and also modulated by controlling the size of the cross section, similar to what happens in a regular transformer except here we are working with spin charge. Therefore a magnetic ring is able to support a spin current locally in addition to the charge current.[23,13]

IV. CONCLUSION

We have introduced a many-body formalism based on path-integral techniques capable of handling a system of both local magnetic moments and conduction electrons in a self-consistent manner. Transport properties can be obtained through the calculation of the two-point functions of the current and the magnetization, respectively. One of the important outcomes of this treatment is that we were able to derive a set of new equations that are needed when the magnetization of the medium is no longer homogeneous. First we showed that the polarization of the current is no longer homogeneous and satisfies a generalized diffusion equation where the diffusion tensor is dependent on the direction of the magnetization. We have hence shown that exchange effects are important in a ferromagnet and need to be taken into account properly. The fluctuations of the magnetization were also shown to obey a diffusion-type equation which depends on the direction of the local magnetization. This latter important result is made possible through the use of a path integral technique. In addition we recover a Boltzmann equation for the current which follows from Eq. (B8) and a

Landau-Lifshitz equation for the average of the magnetization, Eq. (23). We have also shown how the nonuniform magnetization affects both the conduction electrons and the spin-momentum transfer term. We gave a simple physical picture for our main results. We finally showed how our results can be applied in various configurations. Our results show that spin accumulation can be enhanced by inhomogeneities at the interface.

In the examples we focused on spin accumulation and we did not deal with its effect on the dynamics of the local magnetization. We showed how to recover the Landau-Lifshitz equation but we discussed the effect of the collisions on the classical magnetization only in qualitative terms. The use of the relaxation time approximation is another shortcoming of this work. Its improvement will complicate the treatment further but we believe these issues should be addressed in the future.

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APPENDIX A: THE *s-d* GENERATING FUNCTIONAL, VARIATIONAL PRINCIPLE, AND EFFECTIVE ACTION

In this appendix, we introduce the *s-d* generating functional from which we derive equations of motion for the magnetization and the spin current. This *s-d* functional is defined in terms of the density matrix ρ of the system.[10,12] One of the advantages of this method is that it allows equal treatment of thermal effects and nonthermal effects. We will give a general outline of the method as we apply it to the particular *s-d* exchange system since the steps used in the derivation of the equations of motion are different from the Green's function method in Ref. 24.

To motivate the structure of the functional we are about to introduce, we recall the structure of the density matrix elements. One way to calculate density matrix elements is through a path-integral representation.[25–28] We have recently used such a formalism to treat the problem of fluctuations and dissipation in coherent magnetization.[29,30] In a transport problem, we instead introduce a functional of the density matrix. This functional is then made to depend on new virtual sources η_1, η_1^*, η_2, η_2^*, $\mathbf{J}_1$, and $\mathbf{J}_2$. These sources are coupled to the conduction electrons' field and the magnetic moments of the medium which will enable us to generate all kinds of correlation functions and their time evolution. The functional is given in terms of a trace formula,

$$\mathbb{Z}[\eta_1,\eta_1^*,\eta_2,\eta_2^*,\mathbf{J}_1,\mathbf{J}_2,\rho]=\mathrm{Tr}\left\{\rho(t_0)\left(\mathcal{T}^{-1}\exp\left[-i\int d\mathbf{x}[\eta_2^*(\mathbf{x})\cdot\boldsymbol{\Psi}(\mathbf{x})+\boldsymbol{\Psi}^+(\mathbf{x})\cdot\eta_2(\mathbf{x})+\mathbf{J}_2(\mathbf{x})\cdot\mathbf{S}(\mathbf{x})]\right]\right)\right.$$
$$\left.\times\left(\mathcal{T}\exp\left[i\int d\mathbf{x}[\eta_1^*(\mathbf{x})\cdot\boldsymbol{\Psi}(\mathbf{x})+\boldsymbol{\Psi}^+(\mathbf{x})\cdot\eta_1(\mathbf{x})+\mathbf{J}_1(\mathbf{x})\cdot\mathbf{S}(\mathbf{x})]\right]\right)\right\}, \tag{A1}$$

where $\mathcal{T}$ is the time-ordering operator. It orders operators with the earliest time argument to the right. $\mathcal{T}^{-1}$ is the inverse of $\mathcal{T}$. The external sources η_1 and η_2 are two-component classical (i.e., Grassmann) sources where η_1 (η_2) and η_1^* (η_2^*) are treated as independent. The operators are all written in the Heisenberg representation. The coefficients of the expansion of the functional $\mathbb{Z}$ in terms of its arguments give all possible correlation functions of the system. The Hilbert space for the conduction electrons and the local magnetic moments of the medium is the product of the corresponding Hilbert spaces, $|\boldsymbol{\Phi},\boldsymbol{\Omega}\rangle\equiv|\Phi\rangle\otimes|\Omega\rangle$, where $|\boldsymbol{\Phi}\rangle$ is a many-body fermionic state representing the conduction electrons and $|\boldsymbol{\Omega}\rangle$ is a magnetic moment state. The magnetic moment states will be represented in terms of spin-coherent states (SCS)[31] (and references therein). Since the operators are initially taken to be in the Heisenberg picture, then in the presence of the additional external sources, η_1, η_2, and $\mathbf{J}$, the states are no longer time independent. Now we write the functional formula in terms of a path integral. Since we are not interested in the transient behavior of the interaction between the current and the magnetic moments, we assume that the external electric field was turned on a long time ago and we will eventually set $t_0=-\infty$. Reference 14 treats the case where t_0 is kept finite in a finite size thin film. Moreover, we assume that the density matrix is separable initially, $\rho(t=-\infty)=\rho_s(t=-\infty)\otimes\rho_d(t=-\infty)$, where ρ_s is the density matrix of the conduction *s*-electrons and ρ_d is that of the local magnetic moments.

Now let $|\boldsymbol{\Phi}_i,\boldsymbol{\Omega}_i\rangle$ be an initial overcomplete set of states for the operators $\boldsymbol{\Psi}(\mathbf{r},t_0)\otimes 1$ and $1\otimes\mathbf{S}(\mathbf{r},t_0)$. Similarly, we let $|\boldsymbol{\Phi}_c,\boldsymbol{\Omega}_c\rangle$ be an overcomplete set of states for the operators $\boldsymbol{\Psi}(\mathbf{r},t_c)\otimes 1$ and $1\otimes\mathbf{S}(\mathbf{r},t_c)$ at the time t_c. At each intermediate time, we define similar states. Then the functional $\mathbb{Z}$ can be written as follows:

$$\mathbb{Z}[\eta_1,\eta_1^*,\eta_2,\eta_2^*,\mathbf{J}_1,\mathbf{J}_2,\rho]=\int \mathfrak{D}\mathbf{\Phi}_i^*\mathfrak{D}\mathbf{\Phi}_i\mathfrak{D}\mathbf{\Omega}_i\int \mathfrak{D}\mathbf{\Phi}_c^*\mathfrak{D}\mathbf{\Phi}_c\mathfrak{D}\mathbf{\Omega}_c\int \mathfrak{D}\mathbf{\Phi}_i'^*\mathfrak{D}\mathbf{\Phi}_i'\mathfrak{D}\mathbf{\Omega}_i'\langle\mathbf{\Phi}_i',\mathbf{\Omega}_i'|\rho|\mathbf{\Phi}_i,\mathbf{\Omega}_i\rangle$$
$$\times\langle\mathbf{\Phi}_i,\mathbf{\Omega}_i|\mathcal{T}^{-1}\exp\left[-i\int d\mathbf{x}[\eta_2^*(\mathbf{x})\cdot\mathbf{\Psi}(\mathbf{x})+\mathbf{\Psi}^+(\mathbf{x})\cdot\eta_2(\mathbf{x})+\mathbf{J}_2(\mathbf{x})\cdot\mathbf{S}(\mathbf{x})]\right]|\mathbf{\Phi}_c,\mathbf{\Omega}_c\rangle$$
$$\times\langle\mathbf{\Phi}_c,\mathbf{\Omega}_c|\mathcal{T}\exp\left[i\int d\mathbf{x}[\eta_1^*(\mathbf{x})\cdot\mathbf{\Psi}(\mathbf{x})+\mathbf{\Psi}^+(\mathbf{x})\cdot\eta_1(\mathbf{x})+\mathbf{J}_1(\mathbf{x})\cdot\mathbf{S}(\mathbf{x})]\right]|\mathbf{\Phi}_i',\mathbf{\Omega}_i'\rangle. \quad \text{(A2)}$$

Hence we can now formally write the functional $\mathbb{Z}$ as a time-ordered path integral around a closed path in time starting at $t=t_0$, passing through $t=t_c$ and then going back to $t=t_0$ (see Fig. 2). This functional then has a path-integral representation similar to the equilibrium case,[12,32]

$$\mathbb{Z}[\eta^*,\eta,\mathbf{J},\rho]=\int \mathfrak{D}\mathbf{\Psi}^*\mathfrak{D}\mathbf{\Psi}\mathfrak{D}\mathbf{m}\exp\{i\mathcal{A}[\mathbf{\Psi}^*,\mathbf{\Psi},\mathbf{m},\eta^*,\eta,\mathbf{J}]\}$$
$$\times\langle\mathbf{\Psi}_2,\mathbf{m}_2|\rho|\mathbf{\Psi}_1,\mathbf{m}_1\rangle, \quad \text{(A3)}$$

where we have used the following notation for the now classical tensor fields $\mathbf{m}$ and $\mathbf{\Psi}:\mathbf{m}\equiv(\mathbf{m}_+,\mathbf{m}_-),\mathbf{\Psi}\equiv(\mathbf{\Phi}_+,\mathbf{\Phi}_-)$, where $+$ and $-$ stand for the component that is propagating forward and backward in time, respectively. The field $\mathbf{m}$ is therefore a 2×3 tensor, while $\mathbf{\Psi}$ is a 2×2 tensor. Similarly, we write the source terms in terms of tensors, $\eta=(\eta_1,\eta_2),\mathbf{J}=(\mathbf{J}_1,\mathbf{J}_2)$. Hence η becomes a 2×2 tensor and $\mathbf{J}$ a 2×3 tensor. This notation greatly simplifies the manipulation of the path integral. The action $\mathcal{A}$ is given as the difference of two actions; one due to the fields propagating forward in time and the other due to fields propagating backward in time,

$$\mathcal{A}[\mathbf{\Psi}^*,\mathbf{\Psi},\mathbf{m},\eta^*,\eta,\mathbf{J}]=\mathcal{A}[\mathbf{\Psi}_1^*,\mathbf{\Psi}_1,\mathbf{m}_1,\eta_1^*,\eta_1,\mathbf{J}_1]-\mathcal{A}[\mathbf{\Psi}_2^*,\mathbf{\Psi}_2,\mathbf{m}_2,\eta_2^*,\eta_2,\mathbf{J}_2]. \quad \text{(A4)}$$

Both terms on the right are obtained in the usual way. The electron contribution is standard. The magnetic moment contribution can be obtained in the same way, but it involves a geometrical part coming from the SU (2) symmetry. Hence the forward part of the action is given by

$$\mathcal{A}[\mathbf{\Psi}_1^*,\mathbf{\Psi}_1,\mathbf{m}_1,\eta_1^*,\eta_1,\mathbf{J}_1]=\mathcal{A}_{WZ}[\mathbf{m}_1]-\int d\mathbf{x}H_d[\mathbf{m}_1(\mathbf{x})]+\int d\mathbf{x}\left\{i\mathbf{\Psi}_{1\alpha}^\dagger(\mathbf{x})\frac{\partial}{\partial t}\mathbf{\Psi}_{1\alpha}(\mathbf{x})-H_{s+sd}(\mathbf{\Psi}_{1\alpha}^\dagger,\mathbf{\Psi}_{1\alpha},\mathbf{m}_1)\right\}+\int d\mathbf{x}\{\eta_{1\alpha}^*(\mathbf{x})\mathbf{\Psi}_{1\alpha}(\mathbf{x})+\mathbf{\Psi}_{1\alpha}^\dagger(\mathbf{x})\eta_{1\alpha}(\mathbf{x})+\mathbf{J}_1(\mathbf{x})\cdot\mathbf{m}_1(\mathbf{x})\}, \quad \text{(A5)}$$

where a summation over α, the spin index, is implied. The $\mathcal{A}_{WZ}$ is the Wess-Zumino part of the action $\mathcal{A}$. Because of the boundary conditions on the spin fields at the left ends of the time path at $t=-\infty$ (KMS-type conditions), this WZ term has the same form as in the equilibrium case where the path of integration is along the imaginary-time branch from $t=0$ to $t=-i\beta$,[29,33]

$$\mathcal{A}_{WZ}=\int_0^1 d\tau\int_C dt\mathbf{m}(t,\tau)[\partial_t\mathbf{m}(t,\tau)\times\partial_\tau\mathbf{m}(t,\tau)]. \quad \text{(A6)}$$

The vector map $\mathbf{m}(t,\tau)$ is a parametrization of the surface enclosed by the trajectory of the magnetization,

$$\mathbf{m}(t,0)=\mathbf{m}_1(t),t\in C_1=\mathbf{m}_2(t),t\in C_2,$$
$$\mathbf{m}(t,1)=\mathbf{m}_0, \quad \text{(A7)}$$
$$\mathbf{m}(-\infty+i0^+,\tau)=\mathbf{m}(-\infty+i0^-,\tau).$$

$\mathbf{m}_0$ is a distinguished vector and is usually taken along the quantization axis. Next we expand the initial density matrix elements in terms of the initial configurations of the conduction electron field and the magnetization field. Therefore we are led to define a new functional $\mathbb{F}$ which may describe any initial correlations between the conduction electrons and the local magnetic moments,

$$\langle\mathbf{\Psi}_2,\mathbf{m}_2|\rho|\mathbf{\Psi}_1,\mathbf{m}_1\rangle=\exp\{i\mathbb{F}[\mathbf{\Psi}^\dagger,\mathbf{\Psi},\mathbf{m}]\}. \quad \text{(A8)}$$

Since we are assuming that the density matrix of the whole system is decoupled at $t=t_0$, then all cross terms in the expansion are zero. Keeping only terms up to second order, the expansion is

$$\mathbb{F}[\mathbf{m},\mathbf{\Psi},\mathbf{\Psi}^\dagger]=\mathbf{C}^{(0)}+\int d\mathbf{x}\epsilon^{\alpha\beta}\mathbf{C}_\alpha^{(1)}(\mathbf{x})\cdot\mathbf{m}_\beta(\mathbf{x})+\frac{1}{2}\int d\mathbf{x}d\mathbf{y}\,\epsilon^{\alpha\gamma}\epsilon^{\beta\lambda}\mathbf{m}_\alpha(\mathbf{x})\cdot\mathbf{C}_{\gamma\lambda}^{(2)}(\mathbf{x},\mathbf{y})\cdot\mathbf{m}_\beta(\mathbf{y})+\int d\mathbf{x}d\mathbf{y}\,\epsilon^{\alpha\gamma}\epsilon^{\beta\lambda}\mathbf{\Psi}_\alpha^\dagger(\mathbf{x})\cdot\mathbf{Q}_{\gamma\lambda}(\mathbf{x},\mathbf{y})\cdot\mathbf{\Psi}_\beta(\mathbf{y}). \quad \text{(A9)}$$

The tensor ϵ is defined such that $\epsilon_{11}=-\epsilon_{22}=1$, and $\epsilon_{12}=\epsilon_{21}=0$. The functional coefficients. $\mathbf{C}^{(0)}$, $\mathbf{C}^{(1)}$, $\mathbf{C}^{(2)}$, and $\mathbf{Q}$ are as yet unknown. The notation used should be clear; for example the last term involves summations over the path index and

the spin index, $\epsilon^{\alpha\gamma}\epsilon^{\beta\lambda}\Psi^{\dagger}_{\alpha}\cdot\mathbf{Q}_{\gamma\lambda}\cdot\Psi_{\beta}=\epsilon^{\alpha\gamma}\epsilon^{\beta\lambda}\Psi^{\dagger}_{s\alpha}Q^{ss'}_{\gamma\lambda}\Psi_{s',\beta}$, where the upper indexes on $\mathbf{Q}$ are for spin up and spin down. Inserting this expansion back in Eq. (A3), we end up with the following expression for the action $\mathcal{A}$:

$$\mathcal{A}[\Psi^{*},\Psi,\mathbf{m},\eta^{\dagger},\eta,\mathbf{J},\mathbf{Q},\mathbf{C}]=\epsilon^{\alpha\beta}\left\{\mathcal{A}_{WZ}[\mathbf{m}_{\beta}]-\int d\mathbf{x}H_{d}[\mathbf{m}_{\beta}(\mathbf{x})]\right\}+\epsilon^{\alpha\beta}\int d\mathbf{x}\left\{i\Psi^{\dagger}_{\beta}(\mathbf{x})\frac{\partial}{\partial t}\Psi_{\beta}(\mathbf{x})-H_{s+sd}(\Psi^{\dagger}_{\beta},\Psi_{\beta},\mathbf{m}_{\beta})\right\}$$
$$+\epsilon^{\alpha\beta}\int d\mathbf{x}\{\eta^{\dagger}_{\beta}(\mathbf{x})\Psi_{\beta}(\mathbf{x})+\Psi^{\dagger}_{\beta}(\mathbf{x})\eta_{\beta}(\mathbf{x})+\mathbf{J}_{\beta}(\mathbf{x})\cdot\mathbf{m}_{\beta}(\mathbf{x})\}$$
$$+\frac{1}{2}\epsilon^{\alpha\gamma}\epsilon^{\beta\lambda}\int d\mathbf{x}d\mathbf{y}\mathbf{m}_{\alpha}(\mathbf{x})\cdot\mathbf{C}_{\gamma\lambda}(\mathbf{x},\mathbf{y})\cdot\mathbf{m}_{\beta}(\mathbf{y})+\epsilon^{\alpha\gamma}\epsilon^{\beta\lambda}\int d\mathbf{x}d\mathbf{y}\Psi^{\dagger}_{\alpha}(\mathbf{x})\cdot\mathbf{Q}_{\gamma\lambda}(\mathbf{x},\mathbf{y})\cdot\Psi_{\beta}(\mathbf{y}),\tag{A10}$$

where we have made an obvious redefinition of the coefficients. The functional $\mathbb{Z}$ now becomes of the standard form[32]

$$\mathbb{Z}[\eta^{\dagger},\eta,\mathbf{J},\mathbf{Q},\mathbf{C}]=\oint\mathfrak{D}\Psi^{*}\mathfrak{D}\Psi\mathfrak{D}\mathbf{m}\times\exp\{i\mathcal{A}[\Psi^{\dagger},\Psi,\mathbf{m},\eta^{\dagger},\eta,\mathbf{J},\mathbf{Q},\mathbf{C}]\}.\tag{A11}$$

The integral notation emphasizes that the path in time is closed, Fig. 2. Therefore we now can apply the usual field theoretical methods to extract the equations of motion from this functional. From the correlation functions, it is clear that the functional

$$\mathbb{W}[\eta^{\dagger},\eta,\mathbf{J},\mathbf{Q},\mathbf{C}]=-i\ln\mathbb{Z}[\eta^{\dagger},\eta,\mathbf{J},\mathbf{Q},\mathbf{C}]\tag{A12}$$

is the generator that we need to derive the irreducible Green's functions of the system. To get the average value of the conduction electron field or the magnetization, we differentiate with respect to the coefficients in the linear terms. For the conduction electrons, we have

$$\frac{\delta\mathbb{W}}{\delta\eta^{\dagger}_{\alpha s}(\mathbf{x})}=\epsilon^{\alpha\beta}\langle\Psi_{\beta s}(\mathbf{x})\rangle,\tag{A13}$$

and for the magnetization, we get

$$\frac{\delta\mathbb{W}}{\delta J_{\alpha i}(\mathbf{x})}=\epsilon^{\alpha\beta}\langle m_{\beta i}(\mathbf{x})\rangle.\tag{A14}$$

The average of the conduction electrons field, a Fermi-type field, is set to zero while we set the average of the magnetization to be

$$M_{\alpha i}(\mathbf{x})=\langle m_{\alpha i}(\mathbf{x})\rangle.\tag{A15}$$

Given the above definitions, Eqs. (A13) and (A14), the two-point correlation terms are easily obtained,

$$\frac{1}{\mathbb{Z}}\frac{\delta\mathbb{Z}}{\delta Q^{ss'}_{\lambda\gamma}(\mathbf{x},\mathbf{y})}=i\frac{\delta\ln\mathbb{W}}{\delta Q^{ss'}_{\lambda\gamma}(\mathbf{x},\mathbf{y})}=i\epsilon^{\alpha\lambda}\epsilon^{\beta\gamma}\langle\Psi^{\dagger}_{\alpha s}(\mathbf{x})\Psi_{\beta s'}(\mathbf{y})\rangle,\tag{A16}$$

$$\frac{\delta\mathbb{W}}{\delta C^{ij}_{\gamma\lambda}(\mathbf{x},\mathbf{y})}=\epsilon^{\alpha\gamma}\epsilon^{\beta\lambda}\frac{1}{2}\langle m_{\alpha i}(\mathbf{x})m_{\beta j}(\mathbf{y})\rangle,\tag{A17}$$

where s, s' are for spin up and spin down and i, j are for the spin field components. The indices $\alpha,\beta,\ldots$ denote the branch of time in Fig. 2. Mixed correlation functions can be obtained in the same way. Clearly, solving for the two-point propagators is the least we can do to have a meaningful solution that includes relaxation effects. Knowing these propagators amounts to knowing the particle density, the spin density, the current density, and the scattering amplitudes, among others. Since we assume that the conduction electrons' field has no mean value, its two-point propagator is then explicitly given by time-ordered products,

$$\mathcal{G}^{ss'}(\mathbf{x},\mathbf{y})=\langle\mathcal{T}_{c}[\Psi_{s}(\mathbf{x})\Psi^{+}_{s'}(\mathbf{y})]\rangle,\tag{A18}$$

where $\mathcal{T}_{c}$ is the time-ordered operator on the closed time path. From the above expressions, it is clear that the function $\mathcal{G}_{21}$ is the "less than" Green's function and $\mathcal{G}_{12}$ is the "greater than" Green's function. $\mathcal{G}_{11}$ is the Feynman propagator, while $\mathcal{G}_{22}$ is the Dyson propagator.[24] These Green's functions are not all independent. From their definitions, we can see that

$$\mathcal{G}^{ss'}_{11}(\mathbf{x},\mathbf{y})+\mathcal{G}^{ss'}_{22}(\mathbf{x},\mathbf{y})=\mathcal{G}^{ss'}_{12}(\mathbf{x},\mathbf{y})+\mathcal{G}^{ss'}_{21}(\mathbf{x},\mathbf{y}).\tag{A19}$$

The Green's function $\mathcal{G}_{12}$ is of special interest since it is related to the average of the density operator of the conduction electrons. The two-point functions for the magnetization are similarly given by

$$\mathcal{M}^{ij}(\mathbf{x},\mathbf{y})=\langle\mathcal{T}_{c}[S_{i}(\mathbf{x})S_{j}(\mathbf{y})]\rangle-\langle S_{i}(\mathbf{x})\rangle\langle S_{j}(\mathbf{y})\rangle.\tag{A20}$$

These Green's functions are easily related to the retarded and advanced Green's functions. Since we are considering a situation which is not far from equilibrium, we will follow closely the treatment in Ref. 34. Therefore, as in the equilibrium case, we relate the "less than" functions to the distribution function of electrons and spin in the semiclassical limit. Since the functions $\mathbf{J}$, $\mathbf{Q}$, $\mathbf{C}$ are not bound to a simple physical interpretation, we make the following Legendre transformation:

$$\Gamma[\mathbf{M}_{\alpha i}(\mathbf{x}),\mathcal{G}^{ss'}_{\alpha\beta}(\mathbf{x},\mathbf{y}),\mathcal{M}^{ij}_{\alpha\beta}(\mathbf{x},\mathbf{y})] = \mathbb{W}[\mathbf{J}_{\alpha i},\mathbf{Q}^{ss'}_{\alpha\beta},\mathbf{C}^{ij}_{\alpha\beta}] - \int d\mathbf{x} J_{\alpha i}(\mathbf{x}) M_{\alpha i}(\mathbf{x}) - \int d\mathbf{x} d\mathbf{y} Q^{ss'}_{\alpha\beta}(\mathbf{x},\mathbf{y}) \mathcal{G}^{s's}_{\beta\alpha}(\mathbf{y},\mathbf{x}) - \frac{1}{2}\int d\mathbf{x} d\mathbf{y} \times C^{ij}_{\alpha\beta}(\mathbf{x},\mathbf{y})[\mathcal{M}^{ji}_{\beta\alpha}(\mathbf{y},\mathbf{x}) + M_{\beta j}(\mathbf{y}) M_{\alpha i}(\mathbf{x})]. \quad \text{(A21)}$$

We end up with a functional Γ that is expressed solely in terms of magnetization and correlation functions of the current and the localized spins. The equations of motion are then found by differentiating Γ with respect to its arguments,

$$\frac{\delta\Gamma}{\delta M_{\alpha i}(\mathbf{x})} = -J_{\alpha i}(\mathbf{x}) - \int d\mathbf{y} C^{ij}_{\alpha\beta}(\mathbf{x},\mathbf{y}) M_{\beta j}(\mathbf{y}), \quad \text{(A22)}$$

$$\frac{\delta\Gamma}{\delta \mathcal{G}^{ss'}_{\alpha\beta}(\mathbf{x},\mathbf{y})} = -Q^{s's}_{\beta\alpha}(\mathbf{y},\mathbf{x}), \quad \text{(A23)}$$

$$\frac{\delta\Gamma}{\delta \mathcal{M}^{ij}_{\alpha\beta}(\mathbf{x},\mathbf{y})} = -\frac{1}{2} C^{ji}_{\beta\alpha}(\mathbf{y},\mathbf{x}). \quad \text{(A24)}$$

Using the standard tools of field theory[27] we solve for $\mathbf{J}$, $\mathbf{Q}$, $\mathbf{C}$ in terms of $\mathbf{M}$, $\mathcal{G}$, and $\mathcal{M}$. A discussion of Wick's theorem is beyond the scope of this paper. Omitting terms of order λ^4 and higher, we have the approximate effective action for the conduction electrons and the localized magnetic moments, Eq. (9).

APPENDIX B: THE FLUID EQUATIONS FOR THE SPIN ACCUMULATION

The equations of motion for $\mathbf{M}$, $\mathcal{G}$, $\mathcal{M}$ are obtained by minimizing Γ and setting the external sources to zero with the appropriate initial conditions. Within the above stated approximations, the magnetization of the medium obeys the following equation of motion:

$$\epsilon^{\alpha\beta}\epsilon^{ilk} M_{\beta l}(\mathbf{x}) \partial_t M_{\beta k}(\mathbf{x}) + \epsilon^{\alpha\beta}\delta_{\alpha\beta} B_i(\mathbf{x}) + \frac{1}{2} J \epsilon^{\alpha\beta}\nabla^2 M_{\beta i}(\mathbf{x}) + \frac{\lambda}{2}\epsilon^{\alpha'\alpha}\sigma^i_{s's}\mathcal{G}^{ss'}_{\alpha'\alpha}(\mathbf{x},\mathbf{x}^\alpha) = 0. \quad \text{(B1)}$$

Here we have taken the long-wavelength limit to get the familiar exchange term through a coarse-graining procedure where each cell is taken to have a maximum spin of S. The last term on the left is clearly the interaction with the conduction electrons' magnetic moments to *all* orders in λ. The equation of motion for the conduction electrons is

$$\left[[i\partial_{t_y} - \in(y)]\delta_{s's} + \frac{\mu}{2}\sigma^i_{s's}B^i + \frac{\lambda}{2}\sigma_{ss'}M_{\alpha i}(\mathbf{y})\right]\mathcal{G}^{ss''}_{\gamma\alpha}(\mathbf{y},\mathbf{z}) + \lambda^2 g^{\alpha\alpha'\alpha''} g^{\beta\beta'\beta''}\frac{\sigma^i_{s_4 s}}{2}\frac{\sigma^j_{s's_3}}{2}\int d\mathbf{x}[\mathcal{G}^{s_3 s_4}_{\alpha'\beta'}(\mathbf{y},\mathbf{x}) \times \mathcal{G}^{s''s}_{\gamma\alpha}(\mathbf{z},\mathbf{x})\mathcal{M}^{ij}_{\alpha''\beta''}(\mathbf{x},\mathbf{y})] = -i\delta^{s's''}_{\gamma\beta}(\mathbf{z}-\mathbf{y}), \quad \text{(B2)}$$

where $\in(y)$ is the spin-independent energy of the conduction electron. The term of first order in λ describes the *full* exchange interaction between the magnetic moments of the localized electrons and those of the current. The structure of this equation is familiar from the theory of correlation functions due to Coulomb interactions.[21] There, the propagator $\mathcal{M}(\mathbf{x},\mathbf{y})$ is replaced by the Hartree propagator. Therefore the solution of this equation should follow by analogy with the treatment in Ref. 21. The final equation is the equation of motion for the magnetic correlation functions,

$$\epsilon^{\alpha\beta}\epsilon^{ijk}\{\partial_t M_{\beta k}(\mathbf{x}) - M_{\beta k}(\mathbf{x})\partial_t\}\mathcal{M}^{k'j}_{\alpha'\beta}(\mathbf{z},\mathbf{x}) + \epsilon^{\alpha\beta}\int d\mathbf{y}[J(\mathbf{x}-\mathbf{y})\mathcal{M}^{k'i}_{\alpha'\beta}(\mathbf{z},\mathbf{y})] + \lambda^2 g^{\alpha_1\alpha_2\beta} g^{\beta_1\beta_2\alpha}\frac{\sigma^j_{s_4 s_1}}{2}\frac{\sigma^i_{s_2 s_3}}{2}\int d\mathbf{y}[\mathcal{M}^{k'j}_{\alpha'\beta}(\mathbf{z},\mathbf{x}) \times \mathcal{G}^{ss'}_{\alpha_1\beta_1}(\mathbf{x},\mathbf{y})\mathcal{G}^{s's}_{\alpha_2\beta_2}(\mathbf{y},\mathbf{x})] = i\delta^{k'i}_{\alpha'\alpha}(\mathbf{x}-\mathbf{z}). \quad \text{(B3)}$$

The integrals are all four-dimensional and hence we have defined $J(\mathbf{x}-\mathbf{y}) = J(\vec{\mathbf{x}}-\vec{\mathbf{y}})\delta(t_x - t_y)$. It is important to observe that up to this point, the propagators $\mathcal{M}$ and $\mathcal{G}$ are the *true* propagators of the theory. Hence the above equations are nonperturbative in nature. Since we are interested in how the magnetic moment of the current influences that of the medium (or vice versa), we define

$$\mathfrak{M}^i_{\alpha\beta}(\mathbf{x},\mathbf{y}) = \frac{1}{2}\sigma^i_{ss'}\mathcal{G}^{s's}_{\alpha\beta}(\mathbf{x},\mathbf{y}) \quad \text{(B4)}$$

to be the conduction electron spin propagator. The spin "charge" of the current is easily seen to follow from $\mathfrak{M}_{\alpha\beta}$ by setting $\alpha=1$, $\beta=2$ and letting $\mathbf{y}\to\mathbf{x}^+$,

$$\mathfrak{M}^i(\mathbf{x}) = \frac{1}{2}\sigma^i_{ss'}\mathcal{G}^{s's}_{12}(\mathbf{x},\mathbf{x}). \quad \text{(B5)}$$

However, we will find it useful to go to the center of mass and relative coordinates (Wigner coordinate system) to make contact with the classical quantities,

$$\mathbf{X} = \frac{1}{2}(\vec{\mathbf{x}}+\vec{\mathbf{y}}), \quad T = \frac{1}{2}(t_x + t_y),$$

$$\mathbf{x}_\Delta = \vec{\mathbf{x}} - \vec{\mathbf{y}}, \quad t_\Delta = t_x - t_y. \quad \text{(B6)}$$

In this new coordinate system, the magnetic moment of the conduction electrons $\mathfrak{M}^i(\mathbf{x})$ becomes a function of the macroscopic variables $\mathbf{X}$ and T only,

$$\mathfrak{M}^i(\mathbf{x},\mathbf{y}) = \int\frac{d\omega}{2\pi}\int\frac{d\mathbf{p}}{(2\pi)^3}\exp[i\omega t_\Delta - i\mathbf{p}\cdot\mathbf{x}_\Delta]\mathfrak{M}^i(\mathbf{X},T;\omega,\mathbf{p}). \quad \text{(B7)}$$

To get the equation of motion for $\mathfrak{M}$, the spin charge of the conduction electrons, we first multiply Eq. (B2) from the

left-hand side by $\sigma^{l}_{ss''}$ and sum over the spin degrees of freedom. We end up with an equation for the polarized current propagator,

$$\epsilon^{\alpha\beta}\delta_{ss'}[i\partial_{t_y} - \in_{\alpha}]\mathfrak{M}_{\gamma\alpha}(\mathbf{y},\mathbf{z}) + \frac{i\epsilon^{\alpha\beta}}{2}[\mathbf{B} + \lambda\mathbf{M}(\mathbf{y})] \times \mathfrak{M}_{\gamma\alpha}(\mathbf{y},\mathbf{z}) + \lambda^2 g^{\alpha\alpha'\alpha''} g^{\beta\beta'\beta''} \frac{\sigma^{j}_{s_4 s}}{2}\frac{\sigma^{j}_{s' s_3}}{2}\int d\mathbf{x}[\mathcal{G}^{s_3 s_4}_{\alpha'\beta'}(\mathbf{y},\mathbf{x}) \times \mathcal{M}^{ij}_{\alpha''\beta''}(\mathbf{x},\mathbf{y})\sigma^{l}_{s''s'}\mathcal{G}^{s''s}_{\gamma\alpha}(\mathbf{z},\mathbf{x})] = 0. \tag{B8}$$

The last term on the left provides for the relaxation of the spin moment of the conduction electrons. To derive the equation for the polarization of the current, in the following we use the relaxation time approximation and replace the last term in Eq. (B8), the collision integral, by a local term. The classical polarization of the current $\mathfrak{M}_{\mathbf{c}}$ is found by first assuming that the lth component, $\mathfrak{M}^{l}(\mathbf{x})$, has the following form:

$$\mathfrak{M}^{l}(T,X,p,\omega) = \delta[\omega - p^2/2m - V(T,X)]\mathfrak{M}^{l}(T,X,p), \tag{B9}$$

where we have set $\in(p) = p^2/2m + V(T,X)$. Then by averaging over the fast degrees of freedom, we have by definition

$$\mathfrak{M}^{l}_{c}(T,X) = v\int \frac{d\omega}{2\pi}\frac{d\mathbf{p}}{(2\pi)^3}\mathfrak{M}^{l}(T,X,p,\omega), \tag{B10}$$

where v is the volume of the system. The spin current $\mathfrak{J}$ is defined in the usual way. It has a tensorial character because of the vector character of the spin charge,

$$\mathfrak{J}^{kl}(T,X) = \frac{v}{m}\int \frac{d\mathbf{p}}{(2\pi)^3}\mathfrak{M}^{k}(T,X,p)p^{l}. \tag{B11}$$

The equation of motion for $\mathfrak{M}_c$ is found by first going to the center of mass coordinates and using the quasiparticle approximation.[24] We find for each k-component

$$\left[\partial_T + \frac{p}{m}\cdot\partial_X\right]\mathfrak{M}^{k}(T,X,p) + \epsilon^{klp}[B^{l} + \lambda M^{l}(T,X)]\mathfrak{M}^{p}(T,X,p) = -\frac{\mathfrak{M}^{k}(T,X,p) - \mathfrak{M}^{k}_{eq}}{\tau_k} - \frac{\mathfrak{M}^{k}(T,X,p) - \mathfrak{M}^{k}_{0}}{\tau_p}, \tag{B12}$$

where $\tau_k(\tau_p)$ is the relaxation time for spin flip (momentum) scattering processes.[35,36] By definition the average of the last term over the momentum is zero. This way of writing the collision term is valid only in the absence of spin-momentum coupling terms such as $\mathbf{L}\cdot\mathbf{S}$ coupling.

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