

# Coherent effect in normal-metal/ferromagnetic superconductor/normal-metal double tunnel junctions

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Taking simultaneously into account the electron-injected current from one normal-metal ( $N$ ) electrode and the hole-injected current from the other  $N$  electrode, we present a theory of the coherent quantum transport in  $N$ /ferromagnetic superconductor (FS)/ $N$  double tunnel junctions, and derive a general formula for the differential conductance. It is shown that the conductance spectrum exhibits an oscillatory behavior with the bias voltage, and the oscillation amplitude is reduced with both increasing temperature and increasing exchange energy in the FS. The exchange energy also leads to a Zeeman splitting of the conductance peaks and in the tunnel limit to the formation of a series of quasiparticle bound states in the FS.

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## I. INTRODUCTION

Coherent effects of the quasiparticle transport in double tunnel junctions consisting of a superconductor ( $S$ ) and normal metal ( $N$ ) have attracted much attention since the early experiments by Tomasch.<sup>1</sup> The coherent tunneling has been studied in  $S/N/S$  double tunnel junctions by considering current-carrying Andreev bound states,<sup>2,3</sup> and multiple Andreev reflection (AR).<sup>4-7</sup> The geometric resonance nature of differential conductance oscillations in the  $S/N/S$  (Refs. 8-10) and ferromagnet ( $F$ )/ $F/S$ <sup>11,12</sup> double tunnel junctions have been ascribed to the quasiparticle interference in the central film. Recently, the study of coherent quantum transport has been extended to  $F/S/F$  double tunnel junctions.<sup>13-15</sup> It was pointed out<sup>14</sup> that for an  $F/S/F$  double tunnel junction, if only the injection of electrons from the left  $F$  to  $S$  is taken into account, the current continuous condition cannot be satisfied, which arises from the creation and annihilation of Cooper pairs in  $S$ . To solve this difficulty, in the presence of a voltage drop between two  $F$  electrodes, not only the electron injection from one  $F$  electrode to  $S$ , but also hole injection from the other  $F$  electrode to  $S$  need to be taken into account.<sup>14</sup> Several important features have been revealed. The quantum interference effects of quasiparticle in the  $S$  interlayer give rise to oscillations of reflection and transmission probabilities as well as conductances with energy above the superconducting gap, and the AR and corresponding transmission coefficients show periodic vanishing phenomenon. In the tunnel limit, all the reflection and transmission coefficients exhibit some sharp peaks, corresponding to a series of bound states of quasiparticles in  $S$ . A similar but somewhat different approach<sup>15</sup> was applied to the same  $F/S/F$  structure, in which both electron and hole injections from the left  $F$  to  $S$  were taken into account. If the exchange splitting of  $F$  is taken to be zero, both the approaches<sup>14,15</sup> are equivalent to each other, reducing to the two different approaches to the  $N/S/N$  structures.<sup>16,17</sup>

The theories above dealt with the central film for  $S$ ,  $N$ , or  $F$ . Very recently, the coexistence of superconductivity and

ferromagnetism has been a most interesting subject in condensed matter physics.<sup>18-28</sup> It was predicted in the early 1960's by Fulde and Ferrell<sup>29</sup> and Larkin and Ovchinnikov<sup>30</sup> (FFLO) that pairing still can occur when electron momenta at the Fermi energy are different for two spin direction. Experimentally, CeCoIn<sub>5</sub> was proposed as candidates for observation of the FFLO state.<sup>31</sup> Unlike the conventional Cooper pairs in which two electrons have opposite spins and momenta ( $K\uparrow, -K\downarrow$ ), the Cooper pair in the FFLO state has a finite center-of-mass momentum  $Q$  of the order of  $2h_0/\hbar v_F$  and consequently leads to a spatially modulated superconducting order parameter, where  $h_0$  is the exchange energy corresponding to the half of the difference in the energy between the spin-up and spin-down bands, and  $v_F$  is the Fermi velocity. The FFLO state with  $[(K+Q/2)\uparrow, (-K+Q/2)\downarrow]$  was never observed in bulk  $F$  materials. It stems from the fact that in a bulk  $F$ ,  $h_0$  is at least two orders of magnitude larger than the energy gap  $\Delta_0$  of a bulk  $S$ , while the FFLO state can appear only in the region where  $h_0 < 0.754\Delta_0$  for the  $s$ -wave  $S$  (Refs. 29 and 30) or  $h_0 < 1.06\Delta_0$  for the  $d$ -wave  $S$ .<sup>32</sup> However, in a thin  $F/S$  bilayer, the effective exchange energy  $h_0$  and effective superconducting order parameter  $\Delta$  may be of the same order of magnitude, and thus the coexistence between superconductivity and ferromagnetism may be realized. On the assumption that the thickness of superconducting layer is smaller than the superconducting coherent length, and that of the ferromagnetic layer smaller than the length of the condensate penetration into the  $F$ , a thin  $F/S$  bilayer can be treated as a ferromagnetic superconductor (FS) film.<sup>23,28</sup>

In this paper, we present a theory of the coherent quantum transport in  $N/FS/N$  double tunnel junctions, by taking simultaneously into account four types of quasiparticle injection process: electron and hole injection from left  $N$  to FS, and corresponding to hole and electron injection from right  $N$  to FS, as shown in Figs. 1(a) and 1(c) and Figs. 1(b) and 1(d), we derive a general formula for the differential conductance in terms of the reflection and transmission coefficients at finite temperature. It differs from the zero temperature

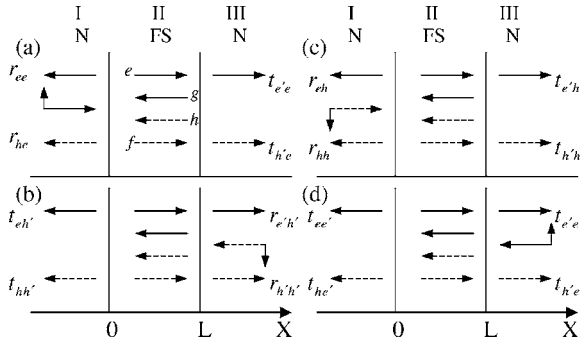


FIG. 1. Schematic illustration of reflections and transmissions of quasiparticles in  $N/FS/N$  structures, in which (a) and (c), respectively, stand for spin-up( $\uparrow$ ) electron and spin-down( $\downarrow$ ) hole incident on the interface  $x=0$  from the left-hand  $N$ ; (b) and (d), respectively, for spin-down( $\downarrow$ ) hole and spin-up( $\uparrow$ ) electron incident on the  $x=0$  from the right-hand  $N$ . Here horizontal arrows indicate quasiparticle transporting directions, solid lines represent electron in  $N$  or electronlike in FS, dashed lines represent hole in  $N$  or holelike in FS.

situation in which we consider only two types of process: the electron-injected current from one  $N$  electrode and the hole-injected current from the other  $N$  electrode.<sup>14</sup> In this case, the chemical potential in FS is determined by the current continuous condition, i.e., the current from the left  $N$  to FS via the left  $N/FS$  interface must be equal to that from the FS to the right  $N$  via the right interface. In the present coherent transport, the coexistence state between superconductivity and ferromagnetism means that there is an exchange splitting in the  $S$  caused by the ferromagnetic background, the quasiparticle interference in the FS and the resonant tunneling play an important role, exhibiting new quantum effects on the tunneling conductance in the  $N/FS/N$  structures.

## II. FERROMAGNETIC SUPERCONDUCTOR AND QUASIPARTICLE TRANSPORT COEFFICIENTS

Consider an  $N/FS/N$  double tunnel junctions, in which the left and right electrodes are made of the same  $N$ , and they are separated from the central FS by two thin insulating interfaces, respectively. The FS film may consist of a  $F/S$  bilayer on the assumption that the thickness of superconducting layer is smaller than the superconducting coherent length and that of the ferromagnetic layer smaller than the length of the condensate penetration into the  $F$ .<sup>23</sup> In this case, solution of superconducting order parameter may be regarded as being independent of the coordinates and the influence of the ferromagnetic layer on superconductivity is not local. As a result, a  $F/S$  bilayer is equivalent to a FS film with a homogeneous superconducting order parameter  $\Delta$  and an effective exchange field  $h_0$ . As has been given in Ref. 23,  $h_0$  is much smaller than that in an isolated ferromagnetic film and of the same order of magnitude as the effective value of  $\Delta$ .

We adopt the Bogoliubov–de Gennes (BdG) approach<sup>33</sup> to study the superconducting order parameter  $\Delta$  and transport of quasiparticles in the  $N/FS/N$  structure. In the absence of spin-flip scattering, the four-component BdG equations may be decoupled into two sets of two-component equation: one

for spin-up electronlike and spin-down holelike quasiparticle wave functions ( $u_\uparrow, v_\downarrow$ ), the other for ( $u_\downarrow, v_\uparrow$ ). The decoupled BdG equation has the form<sup>11,33</sup>

$$\begin{pmatrix} H_0 - \eta_\sigma h_0 & \Delta(T, h_0) \\ \Delta^*(T, h_0) & -H_0 + \eta_{\bar{\sigma}} h_0 \end{pmatrix} \begin{pmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{pmatrix} = E \begin{pmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{pmatrix}. \quad (1)$$

Here the excitation energy  $E$  is measured relative to Fermi energy  $E_F$ ,  $\Delta(T, h_0)$  is the effective superconducting order parameter, and depends on the temperature  $T$  and effective exchange field  $h_0$ ,  $\eta_\sigma = 1$  for  $\sigma = \uparrow$ ,  $\eta_\sigma = -1$  for  $\sigma = \downarrow$ , where  $\bar{\sigma}$  stands for the spin opposite to  $\sigma$ .  $H_0(\mathbf{r}) = -\hbar^2 \nabla_{\mathbf{r}}^2 / 2m + V(\mathbf{r}) - E_F$  with  $V(\mathbf{r})$  the usual static potential. In our calculations, the two very thin insulating layers at  $x=0$  and  $x=L$  can be modeled to be two  $\delta$ -type barrier potentials  $U(x) = U_0[\delta(x) + \delta(x-L)]$ , where  $L$  is the thickness of the FS interlayer, and  $U_0$  depends on the product of barrier height and width. From the BdG equation, we get

$$u_\sigma^2 = \frac{1}{2} [1 + \sqrt{1 - \Delta^2(T, h_0) / (E + \eta_\sigma h_0)^2}], \quad (2)$$

$$v_{\bar{\sigma}}^2 = \frac{1}{2} [1 - \sqrt{1 - \Delta^2(T, h_0) / (E - \eta_{\bar{\sigma}} h_0)^2}]. \quad (3)$$

The wave vectors of the electronlike and holelike quasiparticles are given by

$$k_\sigma^e = \frac{\sqrt{2m}}{\hbar} [E_F + \sqrt{(E + \eta_\sigma h_0)^2 - \Delta^2(T, h_0)}]^{1/2}, \quad (4)$$

$$k_{\bar{\sigma}}^h = \frac{\sqrt{2m}}{\hbar} [E_F - \sqrt{(E - \eta_{\bar{\sigma}} h_0)^2 - \Delta^2(T, h_0)}]^{1/2}. \quad (5)$$

In the FS film the effective order parameter  $\Delta(T, h_0)$  can be determined by the self-consistent equation<sup>33</sup>

$$\Delta = g(\psi_\uparrow \psi_\downarrow), \quad (6)$$

where  $g$  is the strength of the effective attractive potential between the conducting electrons and

$$\psi_\sigma = \sum_k (\gamma_{k\sigma} u_{k\sigma} - \gamma_{k\bar{\sigma}}^* v_{k\sigma}^*), \quad (7)$$

with  $\gamma_{k\sigma}$  the Bogoliubov transformative operators. With the help of Eqs. (2), (3), (6), and (7) as well as that  $\gamma_{k\sigma}$  obeys, we obtain

$$1 = \frac{g}{2} \sum_k \left( \frac{1 - f_{k_\uparrow}}{\sqrt{\epsilon_{k_\uparrow}^2 + \Delta^2(T, h_0)}} - \frac{f_{k_\downarrow}}{\sqrt{\epsilon_{k_\downarrow}^2 + \Delta^2(T, h_0)}} \right), \quad (8)$$

where

$$f_{k\sigma} = \frac{1}{\exp[\beta(\sqrt{\epsilon_{k\sigma}^2 + \Delta^2(T, h_0)} - \eta_\sigma h_0)] + 1}, \quad (9)$$

with  $\epsilon_{k\sigma} = \hbar^2 (k_\sigma^e)^2 / (2m) - E_F$  and  $\beta = 1/k_B T$  the inverse temperature. From Eqs. (8) and (9), we get

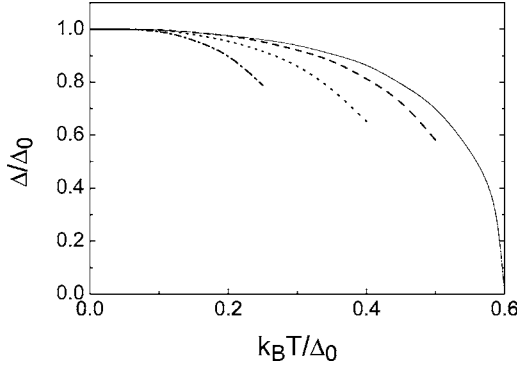


FIG. 2. Dependence of the effective order parameter in FS on the temperature for different exchange energies  $h_0/\Delta_0=0$  (solid line),  $h_0/\Delta_0=0.2$  (dashed line),  $h_0/\Delta_0=0.4$  (dotted line), and  $h_0/\Delta_0=0.6$  (dot-dashed line).

$$\ln \frac{\Delta_0}{\Delta(T, h_0)} = \int_0^{\hbar\omega_D} \frac{d\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2(T, h_0)}} \times \left( \frac{1}{\exp\{\beta[\sqrt{\varepsilon^2 + \Delta^2(T, h_0)} - h_0]\} + 1} + \frac{1}{\exp\{\beta[\sqrt{\varepsilon^2 + \Delta^2(T, h_0)} + h_0]\} + 1} \right). \quad (10)$$

Here  $\Delta_0$  is the superconducting gap at zero temperature and in the absence of exchange field, and  $\omega_D$  is the Debye frequency.

By solving Eq. (10) self-consistently, the dependence of the effective order parameter  $\Delta(T, h_0)$  in the FS layer on the temperature and effective exchange energy is obtained. At zero temperature,  $\Delta(0, h_0) = \Delta_0$  remains unchanged for  $h_0 < \Delta_0$ , independent of increasing  $h_0$ , as  $h_0$  is increased to  $\Delta_0$ ,  $\Delta$  suddenly drops to zero, exhibiting a first-order phase transition from the superconducting state to normal state. This zero-temperature solution that  $\Delta(0, h_0) = \Delta_0$  for  $h_0 < \Delta_0$  and  $\Delta(0, h_0) = 0$  for  $h_0 > \Delta_0$  has been obtained previously.<sup>29,30</sup> At finite temperatures, from Eq. (10), one would obtain multi-valued solutions for  $\Delta(T, h_0)$ . Among them we take only one branch of solutions, corresponding to the lowest thermodynamic potential.<sup>28</sup> It is found that for a finite exchange energy, the order parameter decrease as increasing temperature and has a sudden drop from a finite value to zero at the critical temperature  $T_c(h_0)$ , as shown in Fig. 2. Such a first-order phase transition arises from the presence of an exchange energy, which is similar to the superconducting transition in the presence of an applied magnetic field. While the exchange energy  $h_0=0$ , the superconducting transition remains second order as the solid line shown in Fig. 2. With increasing the exchange energy, the critical temperature  $T_c(h_0)$ , are also decreased and is always lower than  $T_c(h_0=0)$ .

To divide out the fast oscillation for the wave functions, following McMillan's method,<sup>10</sup> we introduce two envelope functions that are smooth on the atomic scale length  $\bar{u}_\sigma(x)$

$= u_\sigma(x) \exp(-ik_F x)$  and  $\bar{v}_\sigma(x) = v_\sigma(x) \exp(-ik_F x)$ . By neglecting the terms as  $d^2/dx^2$  which are of order  $\Delta_0/E_F$  with respect to the  $d/dx$  term, we obtain the reduced BdG equations for the quasiparticle wave functions

$$-\frac{i\hbar^2 k_F}{m} \frac{d}{dx} \bar{u}_\sigma(x) + \Delta^*(x) \bar{v}_\sigma(x) = E \bar{u}_\sigma(x), \quad (11)$$

$$\frac{i\hbar^2 k_F}{m} \frac{d}{dx} \bar{v}_\sigma(x) + \Delta(x) \bar{u}_\sigma(x) = E \bar{v}_\sigma(x), \quad (12)$$

where  $\Delta(x) = \Delta(T, h_0) \Theta(x) \Theta(L-x)$  and  $\Theta(x)$  is the Heaviside step function. Consider an electron for spin  $\sigma$  incident on the interface at  $x=0$  from the left-hand  $N$ . As shown in Fig. 1(a), there are four possible trajectories: normal reflection ( $r_{ee}$ ), AR ( $r_{he}$ ), transmission to the right-hand electrode as an electronlike quasiparticle ( $t_{e'e}$ ) and as a holelike quasiparticle ( $t_{h'e}$ ), where subscripts  $e$  ( $h$ ) and  $e'$  ( $h'$ ) indicate the electron (hole) in the left and right-hand  $N$  electrodes, respectively. From the Eqs. (11) and (12), the wave functions in three regions have the following form:

$$\Psi_I = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq_+ x} + r_{he} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{iq_- x} + r_{ee} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-iq_+ x} \quad (13)$$

for  $x < 0$ ,

$$\Psi_{II} = e \begin{pmatrix} u_\sigma \\ v_\sigma \end{pmatrix} e^{ik_\sigma^e x} + f \begin{pmatrix} v_\sigma \\ u_\sigma \end{pmatrix} e^{-ik_\sigma^h x} + g \begin{pmatrix} u_\sigma \\ v_\sigma \end{pmatrix} e^{-ik_\sigma^e x} + h \begin{pmatrix} v_\sigma \\ u_\sigma \end{pmatrix} e^{ik_\sigma^h x}, \quad (14)$$

for  $0 < x < L$  and

$$\Psi_{III} = t_{e'e} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq_+ x} + t_{h'e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-iq_- x}, \quad (15)$$

for  $x > L$ . Here  $q_\pm = \sqrt{2m(E_F \pm E)/\hbar}$  are the different wave vectors for the electron and hole in  $N$ . All the coefficients in Eqs. (13)–(15) can be determined by boundary conditions at  $x=0$  and  $x=L$ . The matching conditions for the wave function are given by  $\Psi_{II}(0) = \Psi_I(0)$ ,  $(d\Psi_{II}/dx)_{x=0} - (d\Psi_I/dx)_{x=0} = 2mU_0 \Psi_I(0)/\hbar^2$ ,  $\Psi_{II}(L) = \Psi_{III}(L)$ , and  $(d\Psi_{II}/dx)_{x=L} - (d\Psi_{III}/dx)_{x=L} = 2mU_0 \Psi_{III}(L)/\hbar^2$ . Since analytical results for these coefficients are tedious, we only give expressions for  $r_{he}$ ,  $r_{ee}$ ,  $t_{e'e}$ , and  $t_{h'e}$  in the Appendix. From them we get  $R_{he} = q_- |r_{he}|^2 / q_+$ ,  $R_{ee} = |r_{ee}|^2$ ,  $T_{e'e} = |t_{e'e}|^2$ , and  $T_{h'e} = q_- |t_{h'e}|^2 / q_+$ , respectively, corresponding to the AR, the normal reflection coefficients, the transmission coefficients of electronlike and holelike quasiparticle. For a spin  $\bar{\sigma}$  hole incident on the interface at  $x=L$  from the right-hand  $N$ . As shown in Fig. 1(b),  $r_{e'h'}$ ,  $r_{h'h'}$ ,  $t_{eh'}$  and  $t_{hh'}$  can be obtained by a similar calculation and their expression are also given in the Appendix. As a result,  $R_{e'h'} = q_+ |r_{e'h'}|^2 / q_-$ ,  $R_{h'h'} = |r_{h'h'}|^2$ ,  $T_{eh'} = q_+ |t_{eh'}|^2 / q_-$ , and  $T_{hh'} = |t_{hh'}|^2$ . From electron-hole scattering symmetries as well as our calculation results, we have  $R_{ee} = R_{e'e'}$ ,  $R_{hh} = R_{h'h'}$ ,  $R_{eh} = R_{he} = R_{e'h'}$ ,  $R_{h'e} = R_{h'e'}$ , and  $T_{e'e} = T_{e'e'}$ ,  $T_{h'h} = T_{h'h'}$ ,  $T_{h'e} = T_{eh'}$ ,  $T_{e'h} = T_{he'}$ . It is easily shown analytically that all the coefficients of electron-hole transformation such as  $R_{he}$  and  $T_{h'e}$  are proportional to  $\sin^2[(k_\sigma^e - k_\sigma^h)L/2]$ , which vanish if  $(k_\sigma^e - k_\sigma^h)L = 2n\pi$  with  $n$

arbitrary positive integer. From the expressions for  $k_\sigma^e$  and  $k_\sigma^h$  given above, this condition is equivalent to

$$\left[ \frac{E + \eta_\sigma h_0}{\Delta(T, h_0)} \right]^2 = \left[ \frac{2\pi n E_F / \Delta(T, h_0)}{k_F L} \right]^2 + 1, \quad (16)$$

under which there is neither AR nor hole (electron) transmission so that the quasiparticles pass directly from one  $N$  electrode to the other, not converting to the Cooper pair in FS.

### III. TUNNELING CONDUCTANCE

Once all the transmission and reflection probabilities are obtained, we can calculate currents in response to a difference in chemical potentials between the two  $N$ 's. Assume  $\mu_L$  and  $\mu_R$  to be the chemical potential of the left- and right- $N$  electrodes, respectively, and  $\mu$  the chemical potential of FS. Under the bias voltage  $V$  ( $eV = \mu_L - \mu_R$ ) applied to the  $N/FS/N$  structure, and taking into account the four processes shown in Fig. 1, we get the current from the left-hand  $N$  into FS as

$$\begin{aligned} I_L = \frac{e}{h} \sum_\sigma \int_0^\infty dE [ & f_0(E - e\phi_1)(1 - R_{ee} + R_{he}) + f_0(E - e\phi_2) \\ & \times (T_{hh'} - T_{eh'}) + f_0(E + e\phi_1)(-1 - R_{eh} + R_{hh}) \\ & + f_0(E + e\phi_2)(T_{he'} - T_{ee'}) ], \end{aligned} \quad (17)$$

where  $h$  is the Planck constant,  $f_0(E)$  is the Fermi distribution function,  $e\phi_1 = \mu_L - \mu$ , and  $e\phi_2 = \mu - \mu_R$ . Similarly, the current from FS to the right  $N$  is given by

$$\begin{aligned} I_R = \frac{e}{h} \sum_\sigma \int_0^\infty dE [ & f_0(E - e\phi_1)(T_{e'e} - T_{h'e}) + f_0(E - e\phi_2) \\ & \times (1 - R_{h'h'} + R_{e'h'}) + f_0(E + e\phi_1)(T_{e'h} - T_{h'h}) \\ & + f_0(E + e\phi_2)(-1 - R_{h'e'} + R_{e'e'}) ]. \end{aligned} \quad (18)$$

The current continuous condition requires  $I_L = I_R$ , from which  $\mu$  is determined to be  $\mu = (\mu_L + \mu_R)/2$ . Using the probability conservation conditions  $R_{ee} + R_{he} + T_{e'e} + T_{h'e} = 1$ ,  $R_{eh} + R_{hh} + T_{e'h} + T_{h'h} = 1$ ,  $R_{e'e'} + R_{h'e'} + T_{ee'} + T_{he'} = 1$ , and  $R_{e'h'} + R_{h'h'} + T_{eh'} + T_{hh'} = 1$ , we obtain<sup>15,16</sup>

$$\begin{aligned} I = \frac{e}{h} \sum_\sigma \int_0^\infty dE (R_{he} + R_{eh} + T_{e'e} + T_{h'h}) \\ \times \left[ f_0\left(E - \frac{eV}{2}\right) - f_0\left(E + \frac{eV}{2}\right) \right]. \end{aligned} \quad (19)$$

The differential conductance is given by

$$G = \frac{G_0}{16k_B T} \sum_\sigma \int_0^\infty dE (R_{he} + R_{eh} + T_{e'e} + T_{h'h}) W, \quad (20)$$

with  $G_0 = 2e^2/h$  and

$$W = \cosh^{-2}\left(\frac{2E - eV}{4k_B T}\right) + \cosh^{-2}\left(\frac{2E + eV}{4k_B T}\right). \quad (21)$$

At zero temperature, the differential conductance is reduced to

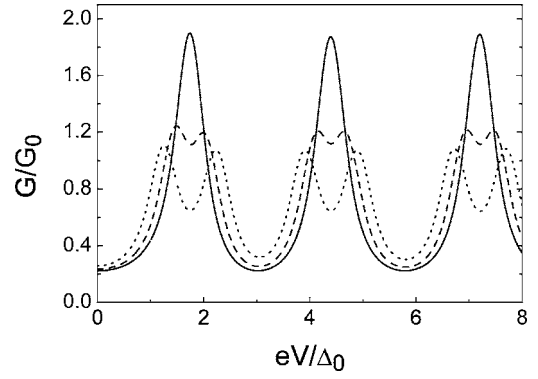


FIG. 3. Differential conductance as a function of the bias voltage at zero temperature for different exchange energies  $h_0/\Delta_0=0$  (solid line),  $h_0/\Delta_0=0.3$  (dashed line), and  $h_0/\Delta_0=0.5$  (dotted line). Here  $E_F/\Delta_0=2000$ ,  $k_F L=1000$ , and  $z=1.0$ .

$$G = \frac{G_0}{2} \sum_\sigma (R_{he} + R_{eh} + T_{e'e} + T_{h'h})_{E=eV}. \quad (22)$$

Figures 3 and 4 shows the differential conductance  $G$  as a function of bias voltage  $eV/\Delta_0$  for different exchange energy  $h_0/\Delta_0$  and different barrier strength  $z$  at zero temperature, respectively. Several interesting features can be found. First, the conductance exhibits an oscillatory behavior, the oscillation period being given by Eq. (16). These oscillation phenomena arises from the quantum interference effects of quasiparticles in FS at the Fermi level. Second, with increasing the exchange energy, the conductance peaks are gradually split into two peaks, as shown in Fig. 3, the energy difference between the two splitting peaks is equal to  $2h_0$ . For the case of a finite exchange energy there is exchange splitting in the energy spectrum between the spin-up and spin-down energy subbands, and consequently it leads to the Zeeman splitting of conductance peaks. The observation of such Zeeman splitting in the conductance spectrum can be taken as a evidence for the coexistence between superconductivity and ferromagnetism. Third, with increasing barrier strength, each split peak becomes two sharp peaks, corresponding to a series of bound states of quasiparticles in FS. In the tunnel limit for large  $z$ , The positions of these peaks are determined by  $k_\sigma^e L$

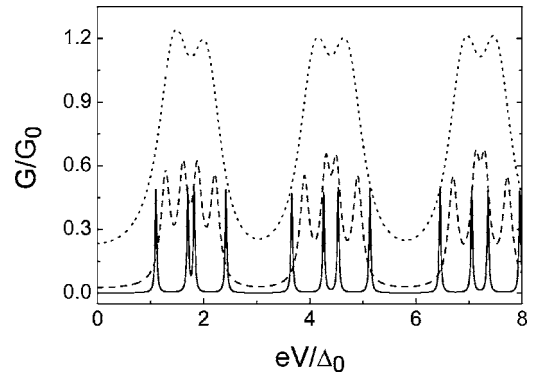


FIG. 4. Differential conductance as a function of the bias voltage at zero temperature for different barrier strength:  $z=1.0$  (dotted line),  $z=2.0$  (dashed line), and  $z=3.0$  (solid line). Here  $E_F/\Delta_0=2000$ ,  $k_F L=1000$ , and  $h_0/\Delta_0=0.3$ .



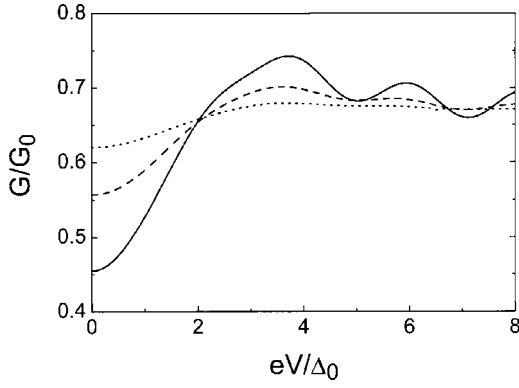


FIG. 5. Differential conductance as a function of the bias voltage for different temperatures:  $k_B T/\Delta_0=0.3$  (solid line),  $k_B T/\Delta_0=0.4$  (dashed line), and  $k_B T/\Delta_0=0.5$  (dotted line). Here  $E_F/\Delta_0=2000$ ,  $k_F L=1000$ ,  $z=1.0$ , and  $h_0/\Delta_0=0.4$ .

$=n\pi$  and  $k_{\bar{\sigma}}^h=n\pi$ , as has been discussed in Ref. 14. These bound states are the results of quantum interference between electronlike quasiparticles in the FS well and those between holelike ones, respectively. For  $z=0$ , however, the conductance does not exhibit an oscillatory behavior, this value of  $G/G_0$  is a constant and equal to two. The result stems from the fact that the coefficients in Eq. (22) satisfy  $R_{he}+T_{e'e}=1$  and  $R_{eh}+T_{h'h}=1$ .

For a finite temperature, the differential conductance is calculated in terms of Eqs. (20) and (10). The dependence of the differential conductance on bias voltage for different temperature and exchange energy are plotted in Figs. 5 and 6. It is found that the  $G$  exhibits a damped oscillatory behavior with bias voltage, and the oscillation amplitude is reduced with both increasing temperature and increasing exchange energy in the FS. One also finds that there is no a Zeeman splitting of the conductance peaks at finite temperature. It is shown that temperature can destroy the coherence and therefore destroy the peaks.

#### IV. CONCLUSION

In summary we have studied the coherent tunneling conductance in the  $N/FS/N$  double tunnel junctions. The ex-

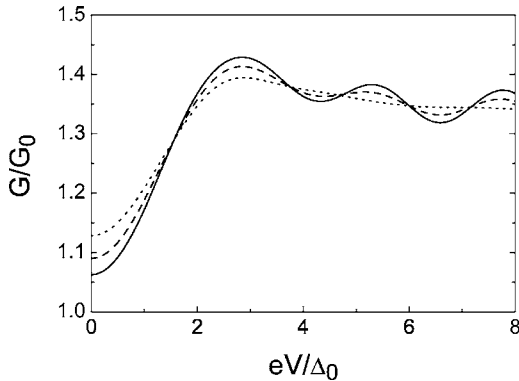


FIG. 6. Differential conductance as a function of the bias voltage at finite temperature for different exchange energies  $h_0/\Delta_0=0$  (solid line),  $h_0/\Delta_0=0.2$  (dashed line), and  $h_0/\Delta_0=0.3$  (dotted line). Here  $E_F/\Delta_0=2000$ ,  $k_F L=1000$ ,  $z=0.5$ , and  $k_B T/\Delta_0=0.3$ .

pression for the tunneling current through the junction is derived by simultaneously taking into account the electron-injected current from one  $N$  electrode and the hole-injected current from the other  $N$  electrode. One way to form a FS film is to constitute a thin  $F/S$  bilayer, where the thickness of superconducting layer is smaller than the superconducting coherent length and that of the ferromagnetic layer smaller than the length of the condensate penetration into the  $F$ . The quantum interference effects of quasiparticle in the  $FS$  interlayer give rise to oscillations of the tunneling conductance with bias voltage, and the oscillation amplitude is reduced with both increasing temperature and increasing exchange energy in the FS, the exchange energy in FS also leads to a Zeeman splitting of conductance peaks. In the tunnel limits of strong barrier strength, a series of bound states of quasiparticles will be formed in the FS. The  $N/FS/M$  structures can be made with the development of nanofabrication technique and the improvement of experimental methods. It is expected that the theoretical results obtained will be confirmed in the future experiment.

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#### APPENDIX: EXPRESSIONS FOR REFLECTION AND TRANSMISSION COEFFICIENTS

Using the boundary conditions on the wave functions given by Eqs. (13)–(15) and carrying out a little tedious algebra, we find

$$r_{he} = -2u_{\sigma}v_{\bar{\sigma}} \sin[(k_{\sigma}^e - k_{\bar{\sigma}}^h)L/2] \{ (u_{\sigma}^2 + v_{\bar{\sigma}}^2) \sin[(k_{\sigma}^e - k_{\bar{\sigma}}^h)L/2] + i(u_{\sigma}^2 - v_{\bar{\sigma}}^2)(1 + 2z^2) \cos[(k_{\sigma}^e - k_{\bar{\sigma}}^h)L/2] \} / M, \quad (A1)$$

$$r_{ee} = z \{ i(u_{\sigma}^2 - v_{\bar{\sigma}}^2)^2 [(iz - 1)^3 e^{i(k_{\sigma}^e + k_{\bar{\sigma}}^h)L} - z^2(1 + iz)e^{-i(k_{\sigma}^e + k_{\bar{\sigma}}^h)L}] + 2u_{\sigma}^2v_{\bar{\sigma}}^2 [\cos(k_{\sigma}^e - k_{\bar{\sigma}}^h)L - 1](i + z)(1 + 2z^2) + z^2(i + z)Q + (i - z)(1 - iz)^2 W \} / M, \quad (A2)$$

$$t_{e'e} = -(u_{\sigma}^2 - v_{\bar{\sigma}}^2) e^{-iq_+L} \{ [(iz - 1)^2 e^{ik_{\bar{\sigma}}^h L} + z^2 e^{-ik_{\bar{\sigma}}^h L}] u_{\sigma}^2 - [(iz - 1)^2 e^{ik_{\sigma}^e L} + z^2 e^{-ik_{\sigma}^e L}] v_{\bar{\sigma}}^2 \} / M, \quad (A3)$$

$$t_{h'e} = 4iz u_{\sigma} v_{\bar{\sigma}} (u_{\sigma}^2 - v_{\bar{\sigma}}^2) e^{iq_-L} \{ \sin[(k_{\sigma}^e + k_{\bar{\sigma}}^h)L/2] - z \cos[(k_{\sigma}^e + k_{\bar{\sigma}}^h)L/2] \} \sin[(k_{\sigma}^e - k_{\bar{\sigma}}^h)L/2] / M, \quad (A4)$$

with

$$Q = 2u_{\sigma}^2 v_{\sigma}^2 - u_{\sigma}^4 e^{i(k_{\sigma}^e - k_{\sigma}^h)L} - v_{\sigma}^4 e^{-i(k_{\sigma}^e - k_{\sigma}^h)L}, \quad (\text{A5})$$

$$W = 2u_{\sigma}^2 v_{\sigma}^2 - u_{\sigma}^4 e^{-i(k_{\sigma}^e - k_{\sigma}^h)L} - v_{\sigma}^4 e^{i(k_{\sigma}^e - k_{\sigma}^h)L}, \quad (\text{A6})$$

$$M = z^2(u_{\sigma}^2 - v_{\sigma}^2)^2[(i+z)^2 e^{i(k_{\sigma}^e + k_{\sigma}^h)L} + (i-z)^2 e^{-i(k_{\sigma}^e + k_{\sigma}^h)L}] \\ + 4u_{\sigma}^2 v_{\sigma}^2 z^2(1+z^2)[\cos(k_{\sigma}^e - k_{\sigma}^h)L - 1] + Qz^4 + (1+z^2)^2 W. \quad (\text{A7})$$

These coefficients in Fig. 1(b) can be similarly obtained as

$$r_{e'h'} = -r_{he}, \quad (\text{A8})$$

$$r_{h'h'} = ze^{2iqL}\{(u_{\sigma}^2 - v_{\sigma}^2)^2 i[(1-iz)z^2 e^{i(k_{\sigma}^e + k_{\sigma}^h)L} \\ + (1+iz)^3 e^{-i(k_{\sigma}^e + k_{\sigma}^h)L}] + 2u_{\sigma}^2 v_{\sigma}^2 [\cos(k_{\sigma}^e - k_{\sigma}^h)L - 1](i-z) \\ \times (1+2z^2) + z^2(z-i)Q + (1+iz)^2(i+z)W\}/M, \quad (\text{A9})$$

$$t_{hh'} = -(u_{\sigma}^2 - v_{\sigma}^2)e^{iqL}\{[(1+iz)^2 e^{-ik_{\sigma}^e L} + z^2 e^{ik_{\sigma}^e L}]u_{\sigma}^2 \\ - [(1+iz)^2 e^{-ik_{\sigma}^h L} + z^2 e^{ik_{\sigma}^h L}]v_{\sigma}^2\}/M, \quad (\text{A10})$$

and

$$t_{eh'} = t_{h'e}, \quad (\text{A11})$$

where  $z = mU_0/(\hbar^2 k_F)$ .

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