

## Pressure dependence of phonon interactions in liquid $^4\text{He}$

Damian H. S. Smith, Ruslan V. Vovk, Charles D. H. Williams, and Adrian F. G. Wyatt

*School of Physics, University of Exeter, Exeter, EX4 4QL, United Kingdom*

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We have measured the pressure dependence of a single phonon sheet and the interaction of two such sheets, in pure liquid  $^4\text{He}$ , from 0 to 21 bar and show it is dominated by three phonon processes (3pp). Pressure affects the 3pp scattering by changing the shape of the dispersion curve. The scattering varies from very strong at  $P=0$  to zero at  $P=19$  bar. The 3pp is small angle scattering at  $P=0$  and, as pressure is increased, the angles decrease further and eventually become zero. We find that the signal from a single phonon sheet increases considerably with pressure to a maximum at  $P=7.5$  bar and then decreases to a minimum at  $\sim 15$  bar, and becomes independent of pressure at  $>19$  bar. We discuss this behavior in terms of the creation of high-energy phonons and the expansion of the phonon sheet. The signal from the collision of two sheets, at  $8.8^\circ$  to each other, depends on the pressure dependences of the energy density in the individual sheets and of the 3pp. This gives direct evidence that the interaction between two phonon sheets is caused by 3pp scattering. Theory relevant to this experiment is given in the paper following this one, see Adamenko *et al.*, Phys. Rev. B **72**, 054507 (2005).

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### I. INTRODUCTION

Liquid  $^4\text{He}$  is a strongly interacting system of bosons that undergoes Bose-Einstein condensation at 2.17 K.<sup>1</sup> At absolute zero temperature it is thought that all the atoms are in a state that has a spectrum of momenta with about 0.1 probability for the zero momentum state. At higher temperatures there are excitations in the form of phonons and rotons. For  $T < 0.6$  K there is a negligible number of rotons, and for  $T < 0.08$  K the thermal phonon population is so small that injected phonons are not scattered by them. Liquid  $^4\text{He}$  is a superfluid at  $T < 2.17$  K and can be described by the two-fluid model in which the momentum density of the normal fraction is identified with the momentum density of the excitations, in the reference frame where the superfluid velocity is zero.<sup>2</sup> This defines the normal density  $\rho_n$ , as the momentum density is  $\rho_n(v_n - v_s)$ , and then the superfluid density is defined as  $\rho_s = \rho - \rho_n$ . Excitations are created if the superfluid velocity is above the critical velocity (in the absence of vorticity) and then superfluidity ceases.

The thermal population of phonons is in equilibrium due to interactions between the excitations. For phonons at low temperatures, this is predominantly through three phonon processes (3pp). This scattering does not conserve the phonon number and is allowed because the dispersion curve for phonons has an initial upward curvature,<sup>3</sup> the so-called anomalous dispersion. The properties of the excitations are fundamental to an understanding of liquid helium. They have been studied extensively when the phonons comprise an isotropic system, but it is only recently that anisotropic phonon systems have been created and analyzed. It turns out that anisotropy has a profound effect on the behavior of the phonons, and in this paper we report our investigation of the effects of pressure on anisotropic phonon systems.

Low-energy phonons with energy  $\epsilon/k \sim 1$  K can be created in liquid helium by a thin film heater and can form strongly interacting and anisotropic systems. These have many interesting properties such as creating another phonon

system of much higher-energy phonons with  $\epsilon/k > \epsilon_c/k = 10$  K,<sup>4</sup> forming phonon sheets<sup>5</sup> and creating hot lines when two sheets collide.<sup>6</sup> These phenomena can be understood in terms of the phonon scattering processes. There is good evidence that the low-energy phonons, *l*-phonons, scatter by three phonon processes (3pp).<sup>7</sup> Scattering by 3pp is fast, involves small angles, and allows spontaneous decay.<sup>8-11</sup> The high-energy phonons, *h*-phonons, at energies  $\epsilon > \epsilon_c$ , are created by four phonon processes (4pp).<sup>4</sup> Spontaneous decay processes for *h*-phonons are not allowed. The collision and interaction between two phonon sheets is also assumed to involve 3pp and it is important to get direct evidence for this. This paper is concerned with controlling the 3pp scattering with pressure,  $P$ , so we can test these assumptions.

Phonon scattering by 3pp and the existence of the critical energy  $\epsilon_c$  arises from the shape of the dispersion curve  $\epsilon(p, P)$ . At  $P=0$ , a plot of phonon energy versus momentum bends upward for small momentum and then bends down for larger momentum. If we write

$$\epsilon = c(P)p[1 + \psi(p, P)], \quad (1)$$

then  $\psi(p, P)$  is positive for  $0 < p < p_c(P)$ , and negative for  $p > p_c(P)$ , where  $p_c(P)$  is the momentum at  $\epsilon_c(P) = c(P)p_c(P)$ . When  $\psi(p, P) > 0$ , 3pp scattering is allowed, as energy and momentum can be conserved. We use values of  $\psi$  obtained from neutron scattering measurements of the dispersion curve.<sup>12</sup> We find, for example, when  $p < \sim 0.83p_c$  [the numerical factor depends on the exact shape of  $\psi(p, P)$  and is given here for  $P=0$ ], one phonon can decay into two phonons and *vice versa*. Momentum is conserved by the two product phonons having a nonzero angle between their momentum vectors, as the sum of the moduli of their momenta is more than the momentum of the initial phonon.

The shape of the dispersion curve varies with pressure. The low-frequency sound velocity  $c(P)$  increases with pressure, due to the bulk modulus increasing faster than the density. Also  $\psi(p, P)$ , and hence  $\epsilon_c(P)$ , changes dramatically

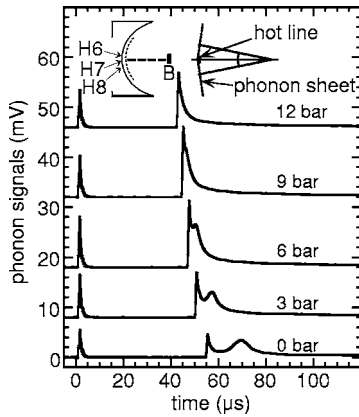


FIG. 1. Phonon signals as functions of time, for a single phonon sheet, are shown at different pressures. The traces have been offset vertically, but otherwise have the same scale. The inset top left shows the schematic arrangement of the heaters  $H_i$  on the glass lens, and the bolometer  $B$ . The inset top right shows, schematically, the phonon sheets and the hot line.

with pressure.<sup>13</sup> Indeed, by  $P=19$  bar,  $\psi(p, 19 \text{ bar}) < 0$  for all  $p$  and  $\epsilon_c(19 \text{ bar})=0$ .<sup>14,15</sup> So, for  $P > 19$  bar 3pp are not allowed.

Increasing the pressure gives a way of slowly turning off 3pp. We can investigate the behavior of single phonon sheets and collisions between sheets as the 3pp interactions, between low-energy phonons, are changed from very strong at  $P=0$ , to zero at  $P=19$  bar. We shall find, as we have previously assumed,<sup>4,16</sup> that 3pp scattering is central to the behavior of low-energy phonons. This is seen most clearly in the collisions between two phonon sheets.<sup>6</sup> When 3pp scattering is impossible, there is no interaction between two phonon sheets and they just pass through each other. The results show that single sheets have a large pressure dependence and new ideas are needed to explain them.

In this paper we describe the experiment in Sec. II, and give the results and discussion for single sheets in Sec. III and for colliding sheets in Sec. IV. We draw conclusions in Sec. V.

## II. THE EXPERIMENTAL ARRANGEMENT

We fabricated accurately positioned heaters by evaporating gold film heaters onto a cylindrical glass lens and then defining the individual heaters by thin lines scratched through the gold film; see the inset in Fig. 1. The lens had a radius of curvature of 12.96 mm and the line of the heaters was perpendicular to the cylindrical axis. The heaters were  $1 \text{ mm} \times 1 \text{ mm}$  and the angle between the normals of adjacent heaters was  $4.4^\circ$ . The symmetry axis was equidistant between heaters 6 and 7. Current pulses of 100 ns duration were applied to a heater, or a pair of heaters, from a pulse generator (LeCroy 9210) creating pulse powers in the range 3 to 25 mW.

The bolometer detector was at the center of curvature of the lens, and in the plane of the arc of heaters. This means that the center of the phonon beam from all heaters, hits the center of the bolometer at all pressures. This is important

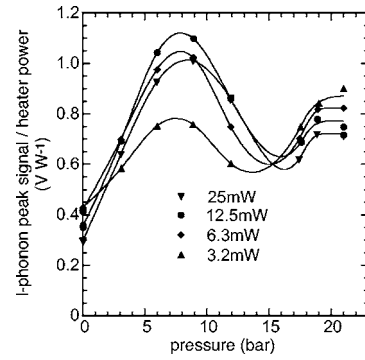


FIG. 2. The peak height of the low-energy phonon signal divided by the heater power, for a single phonon sheet, is shown as a function of pressure, for different heater powers. The heater pulse length is 100 ns.

because the lateral width of these beams are expected to change with pressure, and if the bolometer were not at the center of curvature, then it would detect phonons that are not in the center of the beam and so would give a misleading variation with pressure.

The bolometer was a zinc film,  $1 \text{ mm} \times 1 \text{ mm}$ , cut into a serpentine track with resistance at room temperature  $\sim 300 \Omega$ . At low temperatures this was held at  $\sim 50 \Omega$ , on the superconducting transition edge, by a feedback circuit.<sup>17</sup> The superconducting transition temperature was lowered to  $\sim 350 \text{ mK}$  with a constant external magnetic field. The change in feedback current to the bolometer was proportional to the power absorbed. The signal was amplified by a broadband, dc to 1 MHz, amplifier (EG&G 5113) and then recorded with a Tektronix DSA 601A. Many repetitions were averaged to improve the signal to noise ratio. The responsivity of the detection system is  $1.3 \times 10^7 \text{ V W}^{-1}$ .

The experimental cell was cooled by a dilution refrigerator to  $\sim 50 \text{ mK}$ . The cell was filled with isotopically pure  $^4\text{He}$ .<sup>18</sup> The pressure is measured with a Druck pressure gauge and absolute pressures are quoted.

## III. A SINGLE PHONON SHEET

### A. Results for a single sheet

For the pressure measurements we used the three most central heaters, H6, H7, and H8. They all gave similar results. Typical bolometer signals, at various pressures, are shown in Fig. 1, for a heater pulse of 100 ns duration and 12.5 mW power. The signals at low pressures show two distinct contributions, the narrow  $l$ -phonon peak that arrives first and the broader  $h$ -phonon peak that arrives later. It is clear that as the pressure is increased, the signal arrives earlier due to the velocity increasing with pressure. The separation between  $l$ - and  $h$ -phonon peaks reduces with pressure due to the deviation,  $\psi(p, P)$ , from a linear dispersion curve, decreasing. Also, we see that the  $l$ -phonon peak height varies with pressure. This is shown in Fig. 2 for four heater powers, all with 100 ns pulse length.

In Fig. 2 the peak  $l$ -phonon signal,  $S$ , divided by the heater power,  $W_H$ , is plotted against pressure. The integral of

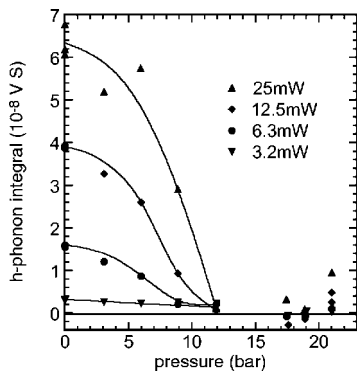


FIG. 3. The integrated high-energy phonon signal, for a single phonon sheet, is shown as a function of pressure for different heater powers. The heater pulse length is 100 ns.

the  $l$ -phonon signal shows the same general behavior as the peak height. A number of features are apparent. We see that the  $l$ -phonon signal rises with increasing pressure to  $P \sim 7.5$  bar, then it decreases, reaching a minimum at  $\sim 15$  bar. Thereafter, it rises a little and saturates within the random error on the data. This pattern is shown for all heater powers, but for the lowest heater power, 3.2 mW, the magnitude of the changes with pressure, are much smaller. At all pressures the  $l$ -phonon signal is approximately proportional to the heater power for all heater powers, with the exception of the lowest power in the middle of the pressure range. Finally, we see in Fig. 2 that  $S/W_H$  at 21 bar is a little more than twice its value at 0 bar.

As the  $h$ -phonon signal is extended in time, we consider the integral of the  $h$ -phonon signal. However, there is a difficulty as the  $h$ - and  $l$ -phonons are not well separated in time at the higher pressures. So we adopt the following scheme. The signal at 18.9 bar should be only due to  $l$ -phonons, and this is integrated over the period of  $50 \mu\text{s}$  from the start of the signal. This long period is chosen so that it will include most of the  $h$ -phonons when they are present at lower pressures. To obtain a value for the integrated  $l$ -phonons, at lower pressures, the integral at 18.9 bar is scaled according to the  $l$ -phonon peak height at each pressure. Also at each pressure, the total signal is integrated over  $50 \mu\text{s}$  from the start of the  $l$ -phonon signal, and the integrated contribution from the  $l$ -phonons, as calculated above, is subtracted from the total integral. This difference we take as the integrated  $h$ -phonon signal. Clearly, there is the possibility of a systematic error, however, we believe the result that there is a general decrease in the  $h$ -phonons with pressure, is very reliable. In Fig. 3, we show the integrated  $h$ -phonon signal as a function of pressure for different heater powers. The signal decreases with pressure for all powers, which indicates that there is a strong overall decrease in the production of  $h$ -phonons as pressure is increased.

### B. Discussion of the effects of pressure

To understand the pressure dependence of the detected signal, it is necessary to survey the various effects of pressure on the heater, phonon scattering in the helium, and the

bolometer in order to see which has the dominant effect. The transmission of phonons from the heater to the helium is efficient and transfers most of the heater pulse energy into the helium rather than the substrate. Phonon transmission between solids can be modeled well by classical acoustic transmission. The reason why heat does not readily go into the substrate is because the velocity of sound in the gold is very low compared to the substrate, this gives a critical cone for transmission and so most of the phonons in the gold are totally internally reflected at the gold-substrate interface. The emissivity for gold-sapphire is 0.035 and 0.079 for transverse and longitudinal phonons, respectively.<sup>19</sup> These values are to be compared with that for gold-helium, 0.2 for  $T_H \sim 3$  K.<sup>20</sup> The effective emissivity for gold-glass is likely to be smaller than for gold-sapphire because of the short mean-free path in glass, which means that phonons are scattered back into the gold. Most of the energy goes into the liquid helium via the background channel<sup>21-23</sup> and, although there is no evidence on the pressure dependence of this channel, it might be expected that a slightly higher fraction of the energy goes into the liquid helium when the pressure is increased from zero, as the helium becomes better acoustically matched to the heater. So the effect of pressure on energy emission from the heater is, at most, small.

The transmission of phonons from a beam into the bolometer has a low probability. Typically, the probability is around  $10^{-3}$  for phonons with energy  $\epsilon/k \sim 1$  K.<sup>24,26</sup> The transmission is both by the acoustic channel and the background channel. For a rough bolometer surface the background channel is more important as the incident phonons are approximately in one direction and have a range of local incident angles on the rough surface. To estimate the pressure dependence of the transmission we first consider the acoustic channel. The transmission probability  $t$  is  $t \sim 4z_h z_z / (z_h + z_z)^2$  as only phonons nearly normal to the interface can propagate from the liquid helium to the zinc, where  $z_i = \rho_i c_i$  and  $\rho_i$  and  $c_i$  are the density and sound velocity, and subscripts  $h$  and  $z$  refer to the helium and the zinc film of the bolometer, respectively. For modest pressure changes,  $z_z$  is unchanged, but  $z_h$  increases by  $\sim 27\%$  for  $0 < P < 7$  bar and  $\sim 60\%$  for  $0 < P < 20$  bar. There is no information on the pressure dependence of the background channel, but if we assume that it is, at most, the same as the acoustic channel, then as  $z_z \gg z_h$ ,  $t \sim 4z_h/z_z$ , and the responsivity of the bolometer only changes by  $< 27\%$  up to 7 bar. As the measured increase in the  $l$ -phonon signal between 0 and 7 bar, is a factor between 2 and 3, depending on pulse power, see Fig. 2, we rule out the bolometer being responsible for this large change, unless the  $l$ -phonon energies change with pressure.

For the higher-energy  $h$ -phonons going into the bolometer, the situation is more subtle because the  $h$ -phonon energy,  $\sim \epsilon_c$ , changes with pressure. The main transmission channel for these phonons is via the background channel that is strongly dependent on the energy of the phonons. At  $P = 0$  bar, the transmission probability increases linearly with the phonon energy  $\epsilon$ , to  $5 \times 10^{-3}$  for  $\epsilon/k = 5$  K and then remains constant.<sup>23</sup> When the energy of the  $h$ -phonons decreases below  $\epsilon/k = 5$  K, due to  $\epsilon_c$  decreasing with increasing pressure, the response of the bolometer decreases for the same energy flux. At 10 bar  $\epsilon_c/k = 5$  K, so the decrease in the

$h$ -phonon signal with pressure for  $P > \sim 10$  bar, due to the decrease in bolometer responsivity with decreasing phonon energy, is more than the decrease in the energy flux of the  $h$ -phonons. As the signal is decreasing rapidly in this range, see Fig. 3, the change in responsivity makes only a small effect, so again the bolometer is not responsible for the large observed change with pressure.

Phonon scattering in liquid helium depends strongly on pressure. The thermalization of the  $l$ -phonons in the phonon sheet is very rapid, as any one phonon is interacting with a population of other phonons. The interaction rate is then much higher than the spontaneous decay rate. Calculations have shown that the relaxation time for a dilute distribution of phonons, with a spectrum corresponding to 2 K, in liquid helium at  $T=0$  and  $P=0$ , relaxes to a near-equilibrium one in  $10^{-10}$  s.<sup>10,11</sup> The relaxation rate drops to zero by 19 bar. Also, the maximum 3pp half-energy angle decreases with pressure, from  $<11^\circ$  at  $P=0$  (Ref. 16) to 0 at 19 bar, as the upward dispersion reduces and the scattering becomes more collinear. The 4pp scattering rate for  $\epsilon > \epsilon_c$  changes because  $\epsilon_c(P)$  decreases and because the dispersion curve becomes more linear around  $\epsilon_c$ . There are no theoretical calculations of 4pp rates as a function of pressure, at present. It is, however, clear that the 4pp scattering rates do not vanish at any pressure. This means that at some pressure,  $P < 19$  bar, the 3pp rate is equal to the 4pp creation rate and then, at higher pressures, the creation rate of  $h$ -phonons is limited by the 3pp rate.

At  $P=0$ , the phonons emitted by the heater into the liquid helium form a phonon sheet of strongly interacting  $l$ -phonons that occupy a small solid angle in momentum space.<sup>5</sup> The sheet temperature  $T$  and the occupied solid angle  $\Omega^{(l)}$  we believe are initially determined by the heater pulse power and, subsequently,  $\Omega^{(l)}$  remains constant but  $T$  decreases due to  $h$ -phonon creation and any lateral expansion of the sheet as it propagates. Lateral expansion has been predicted theoretically.<sup>25</sup> We suppose that the behavior at  $P=0$  persists at  $P > 0$ , with only quantitative changes to  $\Omega^{(l)}$  and  $T$ , and continues up to a pressure where the 3pp scattering angles are too small to create a strongly interacting system of phonons. We argue below that this pressure is  $\sim 7$  bar.

In conclusion of this section, we believe that the behavior of the  $l$ -phonon signal with pressure, for  $0 < P < 19$  bar, is determined by the change in the shape of the dispersion curve with pressure and the effect that this has on the 3pp scattering angles and rates, and the consequential effect that this has on the formation and behavior of the sheet. The creation rate of  $h$ -phonons is important, and the responsivity of the bolometer has only a small effect on the pressure dependence of the  $l$ -phonon signal.

### C. Discussion of a single sheet

We now describe the processes that we believe are the explanation of the pressure dependence of the  $l$ -phonon signal. The most surprising features of the results in Fig. 2 are the large increase up to 7 bar and the subsequent decrease to the minimum at  $P \sim 15$  bar.

We start with the injection of phonons into the helium by the heater. The heater temperature can be estimated from the

measured boundary resistances<sup>21</sup> and varies between  $\sim 1.7$  and  $\sim 2.6$  K for heater powers between 3.2 and 25 mW. The spectrum of phonons transmitted into the helium will be at lower energies than those in the heater, because the main transmission channel is the background channel where phonon energy is not conserved. Phonon energies are approximately halved on transmission, as one phonon in the heater creates approximately two phonons in the helium.<sup>27</sup>

The phonons are emitted into a wide angular range because the dominant background channel emits a wide angular distribution. Also, the surface of the heater film is likely to be rough on the nm scale. This angular distribution could be measured at 20 bar, where the phonons travel ballistically to the detector, but this has not yet been done, however, we know that it must be done with short pulses, as long pulses behave differently.<sup>28</sup>

At zero pressure the emitted phonons form a sheet of strongly interacting phonons. This occurs on a much shorter time scale than the propagation time. The angular distribution of these phonons has been measured after a propagation path of  $16.7 \text{ mm}^5$  and has a mesa shape. The flat top of the angular distribution indicates a constant energy density over an area greater than the area of the heater,  $1 \text{ mm}^2$ . The sides of the mesa indicate that the energy density decreases with angle. In real space, the sheet is planar in the center, over an area equal to that of the heater, and then is gently curved, with the radius of curvature equal to the distance from the heater.

The shape of the mesa is mainly due to the creation of  $h$ -phonons within the sheet.<sup>4</sup> These phonons leave the sheet because their group velocity is lower than that of the sheet. This causes the energy, and hence the energy density of the sheet, to decrease. The creation rate of  $h$ -phonons depends strongly on the temperature of the phonon sheet and drops to a low value at  $\sim 0.7$  K at  $P=0$ .<sup>16</sup> So, after a distance of  $\sim 17$  mm, all parts of the sheet that had a temperature higher than  $\sim 0.7$  K have cooled to approximately this temperature. This region of constant temperature forms the top of the mesa.<sup>5</sup> The sides of the mesa are at a lower temperature.

The energy density in the sheet,  $E^{(l)}$ , is proportional to  $\Omega^{(l)}T^4$ , so it not only depends on temperature but is also proportional to the solid angle  $\Omega^{(l)}$  of the occupied cone of states in momentum space. This effect of  $\Omega^{(l)}$  can clearly be seen in the energy density of sheets created with different heater powers, after they have all cooled to the same temperature of  $\sim 0.7$  K.<sup>5</sup> The fact that the measured energy density, in the center of the sheet at distances far from the heater, was found to increase with the heater power shows that this was mainly due to different values of  $\Omega^{(l)}$ , as the sheets were all at nearly the same temperature. This indicates that  $\Omega^{(l)}$  is formed in the initial creation of the sheet and remains at the initial value thereafter, and the value of  $\Omega^{(l)}$  increases with the heater power. The mechanism for the formation of the initial phonon sheet and  $\Omega^{(l)}$  is, as yet, unknown.

The behavior described above is for  $P=0$ . We believe that the behavior in the range 0–7 bar is similar to that at 0 bar. The fact that the sheet energy density increases with pressure in this pressure range, indicates that the creation of  $h$ -phonons and any lateral expansion diminish with pressure. This is supported by Fig. 3, which shows that the  $h$ -phonon

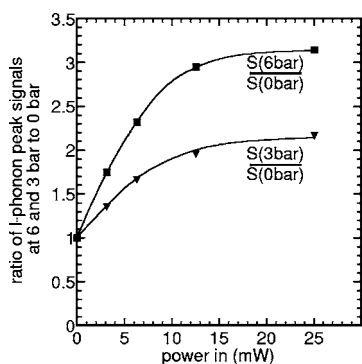


FIG. 4. The ratio of the peak low-energy phonon signals at 6 and 0 bar, and the ratio for 3 and 0 bar, for a single phonon sheet, are shown as functions of the heater power. The heater pulse length is 100 ns. Note that the curves extrapolate to 1 at zero power and saturate at higher powers.

signal decreases with pressure. This also implies that the  $h$ -phonon creation rate decreases with pressure.

In Fig. 4 we show the ratio of the  $l$ -phonon signals at 6 and 0 bar, and also at 3 and 0 bar, as functions of heater power,  $W_H$ . We see that the ratio extrapolates to unity at zero power then rises with power and saturates at the highest powers. The extrapolation to unity as  $W_h \rightarrow 0$  indicates that there is no change in the  $h$ -phonon creation rate, or expansion, with pressure as  $W_H \rightarrow 0$ , at pressures  $< 7$  bar. This implies that there is no  $h$ -phonon creation or expansion as  $W_H \rightarrow 0$ . We expect there to be no  $h$ -phonon creation at the lowest powers because then the  $l$ -phonon temperature is low and the creation rate of energy in the  $h$ -phonons falls off as  $\sim \exp(-13/T)K$ .<sup>29</sup> The saturation of the ratio at the highest powers indicates that  $h$ -phonon creation and expansion become independent of power, at high powers, but at values that depend on pressure.

At  $P > 7.5$  bar the signal rapidly decreases with pressure and we enter a new regime. We suggest that this is because the sheet area starts to expand from its value at 7.5 bar. We speculate that this is because the formation mechanism for  $\Omega^{(l)}$  starts to break down at this pressure. We believe this happens because the 3pp interactions become nearly collinear, which creates a pseudoballistic regime, with phonons propagating in approximately straight lines, but nevertheless, interacting strongly. As a consequence, the phonon sheet broadens as pressure increases and it tends to the angular distribution of the phonons emitted by the heater. This angular distribution would be attained completely at 19 bar, where there are no interactions.

#### IV. COLLIDING PHONON SHEETS

The importance of 3pp scattering can be established by colliding phonon sheets. Two phonon sheets can be created by simultaneously pulsing two heaters. We used H6 and H8, which have an angular separation of  $8.8^\circ$ . When two phonon sheets collide, a hot line is created and the  $l$ - and  $h$ -phonon signals increase.<sup>6</sup> The  $l$ -phonon signal is maximized at each pressure by slightly delaying one of the heater pulses with

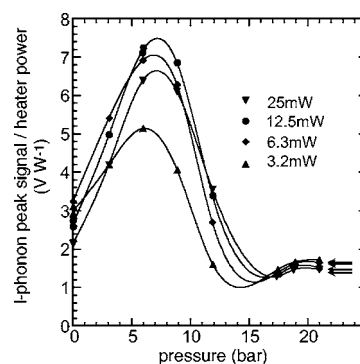


FIG. 5. The peak height of the low-energy phonon signal divided by the heater power, for two simultaneous phonon sheets, is shown as a function of pressure, for different heater powers. The heater pulse length is 100 ns. The arrows indicate the sum of the signals from two separate sheets at 21 bar. As these are the same as the signal from the corresponding two simultaneous phonon sheets, at 21 bar, we see that the two sheets do not interact at this pressure.

respect to the other. This ensures that the hot line is fully incident on the bolometer. The maximum delay was  $< 1 \mu s$ . The fact that the response of the hot line to pressure is similar to that of a single sheet gives confidence that the single sheets were also symmetrically disposed about the bolometer.

The pressure dependence of these signals is shown in Fig. 5. The  $l$ -phonon signal shows a similar behavior to that of a single sheet. With increasing pressure, the signal rises to a peak at  $\sim 7.5$  bar and then falls, to below its value at  $P=0$ , at  $\sim 15$  bar. This is a greater fall than for a single sheet. At higher pressures, it rises a little to a constant value that is just equal to the sum of the two signals from the separate sheets at 19 bar.

This behavior of the  $l$ -phonons from the colliding sheets is determined by the energy density of the sheets,  $E^{(l)}$ , when the sheets are near the bolometer, and the strength of the 3pp scattering rate, both of which change with pressure. The 3pp rate can become so low that the sheets do not interact, even though the energy density in the sheets is not zero. The size of the  $l$ -phonon signal from the hot line is determined by the rate of energy being fed into the hot line by the sheets. This rate is proportional to  $E^{(l)} \tan(\alpha/2)$ , where  $\alpha$  is the angle between the sheets. The angle between the phonons in the sheets must be around the 3pp scattering angle for there to be a strong interaction between the phonons in the two sheets. This means that the angle between the sheets must not be much more than the 3pp scattering angle. We kept  $\alpha$  constant at  $8.8^\circ$ , where the sheets are strongly interacting, and varied the pressure.

For  $0 < P < 7$  bar, the  $l$ -phonon signal from the hot line follows the energy density in the single sheet,  $E^{(l)}$ . This behavior is exemplified by comparing heater powers 3.2 mW and 6.3 mW; the weaker response to pressure at 3.2 mW can be seen in both the single sheet and the colliding sheets. For  $7 < P < 15$  bar, the hot line decreases with  $E^{(l)}$ , as can be seen in Figs. 2 and 5, and the hot line signal falls faster than the signal from a single sheet. This is because the falloff in the 3pp scattering rate also contributes to the rapid decrease

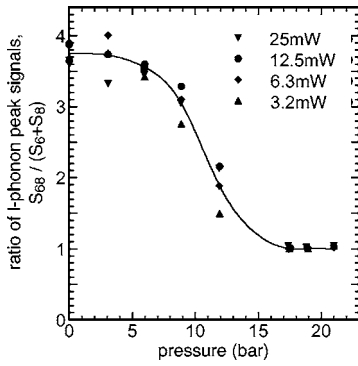


FIG. 6. The ratio of the peak height of the low-energy phonon signal, for two simultaneous phonon sheets,  $S_{68}$ , to the sum of peak heights for separate phonon sheets,  $S_6 + S_8$ , is shown as a function of pressure, for different heater powers to heaters  $H_6$  and  $H_8$ . The heater pulse length is 100 ns. Note that the ratio goes to unity at high pressures and the ratios at different powers, behave similarly.

in the hot line signal between 10 and 15 bar: The  $l$ -phonon signal from the hot line alone is essentially zero by 19 bar, even though  $E^{(l)}$  is still large. For  $P > \sim 19$  bar, the two sheets do not interact, but just pass through each other so that the signal from two simultaneous pulses is equal to the sum of the signals from separate pulses, as shown in Fig. 5 and more clearly in Fig. 6.

In Fig. 6, we show the ratio of the  $l$ -phonons in the hot line to the sum of the  $l$ -phonons in the separate sheets, as a function of pressure for the four heater powers. We see that the ratio is constant at  $\sim 3.8$  for  $0 < P < 7$  bar and then it decreases to unity at  $\sim 19$  bar, where there is no interaction between the sheets. The behavior is independent of heater power and does not show the strong variation with pressure that is shown by both the single sheet and the hot line. The behavior in Fig. 6 confirms that the 3pp scattering causes the interaction between the sheets.

## V. CONCLUSIONS

We have measured the signal from single phonon sheets and from two sheets colliding at  $8.8^\circ$  to each other, as a function of pressure, for four different powers, in order to see the effect of turning off the three phonon processes. We found that pressure has a large effect on single sheets, with the signal rising between 2 and 3 times to a peak at  $\sim 7.5$  bar and then falling to about twice the value at 0 bar, by 19 bar. For colliding sheets there is a similar increase to 7.5 bar and then the signal falls to the sum of the signals from separate sheets by 19 bar, showing that the sheets have stopped interacting.

These pressure measurements have confirmed our previous ideas concerning the creation and behavior of phonon sheets and collisions between sheets, and they have extended our understanding and raised new questions. The most basic idea that 3pp scattering between  $l$ -phonons is central to their behavior, has been demonstrated beyond doubt. This is most clearly seen in Fig. 6, where we see that two phonon sheets do not collide, but pass through each other at high pressures when the 3pp is not allowed.

Figure 3 shows the  $h$ -phonon creation drops from its maximum at 0 bar to zero at 12 bar. This decrease in  $h$ -phonon energy means that more energy is left in the  $l$ -phonon system and in the range 0 to 7.5 bar we see the  $l$ -phonon signal increase substantially. There could also be some lateral expansion of the phonon sheet, but this has not yet been experimentally established, although it has been theoretically predicted.<sup>25</sup> In Fig. 4 we show that the ratio of the  $l$ -phonons at 6 and 0 bar vary as a function of heater power. The ratio tends to unity as  $W_h \rightarrow 0$  and saturates at high powers. We suggested that at very low powers there is no  $h$ -phonon creation or expansion at any pressure  $< 7$  bar, and at high powers the  $h$ -phonon creation and expansion become independent of power, although they can have different values at different pressures.

When  $P > 7.5$  bar, we enter a new regime. The signal decreases rapidly with pressure, and we have suggested that this is due to the sheet expanding. We speculated that this happens because the process, that forms  $\Omega^{(l)}$  and  $T$ , breaks down when  $\theta_{3pp}$  becomes too small. Then the interactions are between nearly collinear phonons, which makes the propagation pseudoballistic. This causes the angular distribution to widen with pressure and the angular distribution reaches that emitted by the heater, at 19 bar, where 3pp cease.

The results in Fig. 3 show that the high-energy phonon creation decreases with pressure. There is some uncertainty in the details of this graph because of the difficulty of separating the  $l$  and  $h$ -phonons at higher pressures; see Fig. 1. This uncertainty may be resolved in the future, by using a detector that is only sensitive to the  $h$ -phonons.

A direct test of 3pp scattering comes from the interaction of two sheets at a small angle to each other. The  $l$ -phonon signal, from the hot line that is formed, depends on both the energy density of the sheets when they are near the bolometer, and the strength of the 3pp scattering. The overall similarity of the graphs for the single sheets and for the colliding sheets, see Figs. 2 and 5, shows the strong dependence of the  $l$ -phonon signal from the hot line, on the energy density in the single sheets. This can be best seen in Fig. 6, where the ratio of the  $l$ -phonon signals from the colliding sheets to the sum of the separate sheets, as a function of pressure, is shown. This ratio eliminates the peak at  $\sim 7.5$  bar seen in these quantities individually, see Figs. 2 and 5, showing that the behavior at this pressure is similar in both of them.

The decrease in the pressure range  $7.5 < P < 15$  bar is much more for the hot line than for the single sheet, as can be seen in Figs. 2 and 5, and in the ratio shown in Fig. 6. This is direct evidence of the 3pp becoming weak at high pressure, which stops the two sheets interacting. At  $P > 19$  bar the the signal from the two sheets is just equal to the sum of the two sheets separately, which shows that no hot line is formed.

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- <sup>1</sup>F. London, *Nature (London)* **141**, 643 (1938).
- <sup>2</sup>L. Landau, *J. Phys.* **5**, 71 (1941); see also I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (Addison-Wesley, New York, 1988).
- <sup>3</sup>N. E. Phillips, C. G. Waterfield, and J. K. Hoffer, *Phys. Rev. Lett.* **25**, 1260 (1970).
- <sup>4</sup>I. N. Adamenko, K. E. Nemchenko, A. V. Zhukov, M. A. H. Tucker, and A. F. G. Wyatt, *Phys. Rev. Lett.* **82**, 1482 (1999).
- <sup>5</sup>R. V. Vovk, C. D. H. Williams, and A. F. G. Wyatt, *Phys. Rev. B* **68**, 134508 (2003).
- <sup>6</sup>R. V. Vovk, C. D. H. Williams, and A. F. G. Wyatt, *Phys. Rev. Lett.* **91**, 235302 (2003).
- <sup>7</sup>R. A. Sherlock, N. G. Mills, and A. F. G. Wyatt, *J. Phys. C* **8**, 2575 (1975).
- <sup>8</sup>H. J. Maris and W. E. Massey, *Phys. Rev. Lett.* **25**, 220 (1970).
- <sup>9</sup>S. Havlin and M. Luban, *Phys. Lett.* **42**, 133 (1972).
- <sup>10</sup>M. A. H. Tucker, A. F. G. Wyatt, I. N. Adamenko, A. V. Zhukov, and K. E. Nemchenko, *Low Temp. Phys.* **25**, 488 (1999) [*Fiz. Nizk. Temp.* **25**, 657 (1999) (in Russian)].
- <sup>11</sup>I. N. Adamenko, K. E. Nemchenko, A. V. Zhukov, M. A. H. Tucker, and A. F. G. Wyatt, *Physica B* **284–288**, 35 (2000).
- <sup>12</sup>W. G. Stirling, in *75th Jubilee Conference on Liquid Helium—4*, edited by J. G. M. Armitage (World Scientific, Singapore, 1983), p. 109.
- <sup>13</sup>J. Jäckle and K. W. Kerr, *Phys. Rev. Lett.* **27**, 654 (1971).
- <sup>14</sup>R. C. Dynes and V. Narayanamurti, *Phys. Rev. Lett.* **33**, 1195 (1974).
- <sup>15</sup>A. F. G. Wyatt, N. A. Lockerbie, and R. A. Sherlock, *Phys. Rev. Lett.* **33**, 1425 (1974).
- <sup>16</sup>A. F. G. Wyatt, M. A. H. Tucker, I. N. Adamenko, K. E. Nemchenko, and A. V. Zhukov, *Phys. Rev. B* **62**, 9402 (2000).
- <sup>17</sup>R. A. Sherlock and A. F. G. Wyatt, *J. Phys. E* **16**, 673 (1983).
- <sup>18</sup>P. C. Hendry and P. V. E. McClintock, *Cryogenics* **27**, 131 (1987).
- <sup>19</sup>O. Weis, *J. Phys. Colloq.* **C-4**, 49 (1974).
- <sup>20</sup>J. Wolter and R. E. Horstman, *Phys. Lett.* **61A**, 238 (1977).
- <sup>21</sup>L. J. Challis, *J. Phys. C* **7**, 481 (1974).
- <sup>22</sup>R. A. Sherlock, N. G. Mills, and A. F. G. Wyatt, *J. Phys. C* **8**, 300 (1975).
- <sup>23</sup>T. W. Bradshaw and A. F. G. Wyatt, *J. Phys. C* **16**, 651 (1983).
- <sup>24</sup>G. J. Page and A. F. G. Wyatt, *J. Phys. C* **11**, 4927 (1978).
- <sup>25</sup>I. N. Adamenko, K. E. Nemchenko, V. A. Slipko, and A. F. G. Wyatt, *Phys. Rev. B* **68**, 134507 (2003).
- <sup>26</sup>M. Brown and A. F. G. Wyatt, *J. Chem. Phys.* **15**, 4717 (2003).
- <sup>27</sup>A. F. G. Wyatt and G. N. Crisp, *J. Phys. Chem. Solids* **39**, C6–244 (1978).
- <sup>28</sup>M. A. H. Tucker and A. F. G. Wyatt, *J. Phys. (Paris), Colloq.* **C6**, Supp. 8, 2825 (1994).
- <sup>29</sup>I. N. Adamenko, K. E. Nemchenko, and A. F. G. Wyatt, *J. Low Temp. Phys.* **126**, 1471 (2002).