Nonlinear polaritons in antiferromagnetic/nonmagnetic superlattices

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We discuss nonlinear characteristics of surface and bulk magnetic polaritons in the AF/NM superlattices composed of nonlinear antiferromagnetic and linear nonmagnetic layers. The results obtained for nonlinear bulk modes show the unreciprocity $\omega(\mathbf{k}) \neq \omega(-\mathbf{k})$, and the existence of the attenuation or gain that indicates the modes unstable. The dispersion relations of surface polariton uncover clearly that there is a series of nonlinear surface eigenmodes, corresponding to the linear surface mode. The appearance of surface magnetic solitons may be possible in these superlattices.

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I. INTRODUCTION

In the last two decades, magnetostatic waves and magnetic polaritons in ferromagnetic (FM) and antiferromagnetic (AF) superlattices (SLs) have attracted interests of physicists and a lot of interesting results have been obtained.[1–6] However, these previous works were almost involved in the linear waves and polaritons of these systems, where a linear relation between the alternating magnetization $\mathbf{m}$ and magnetic field $\mathbf{h}$ in the magnetic mediums $\mathbf{m}=\overleftrightarrow{\chi}\cdot\mathbf{h}$ was used. Although this relation is available for a weak $\mathbf{h}$, it holds not any longer when $\mathbf{h}$ becomes strong. In this circumstance, a nonlinear response $\mathbf{m}=\overleftrightarrow{\chi}^{(1)}\cdot\mathbf{h}+\overleftrightarrow{\chi}^{(2)}:\mathbf{hh}+\overleftrightarrow{\chi}^{(3)}:\mathbf{hhh}+\cdots$ should be considered, instead of that linear relation.

The nonlinear magnetic phenomena and theories have been holding the attentions of physicists since the 1950s.[1,7] In recent years, the interest has been extended to nonlinear properties of magnetic SLs and multilayers.[8–12] These SLs also can be considered as a kind of magnetic photonic crystals.[13] Reference 9 discussed the nonlinear response and the transmission spectra of electromagnetic radiation of an AF/nonmagnetic superlattice (ANSL). In our previous works, we set up an effective-medium method to describe the effective nonlinear magnetic susceptibilities of magnetic SLs or multilayers, where the nonlinear surface magnetostatic waves of FM multilayers[11] and nonlinear bulk magnetic polaritons of the ANSL (Ref. 12) were discussed. In addition, the explicit expressions of the nonlinear magnetic susceptibility sectors of FMs and uniaxial AFs were presented. However, the effective-medium method is an approximated theory required to satisfy the condition of polariton wavelength much larger than the SL period. This method neglects the effects of the periodical interfaces in the SLs on the propagation of waves, which may be important if the wavelength is comparable to the period.

In this paper, we are concerned about such a case in which the effective-medium method is available no longer and examine the effect of nonlinearity on magnetic polaritons in the ANSL by means of a stricter method. Thus the results to be obtained with this method are not limited by the condition of wavelength.

The paper is organized as follows: Sec. II is devoted to deriving the nonlinear polariton solution in AF layers of the ANSL. In Sec. III, we solve the dispersion equations for the polaritons. The numerical results and discussion are put in Sec. IV, and the summary and conclusions in Sec. V.

II. NONLINEAR WAVE SOLUTION IN AF LAYERS

Let us consider an ANSL in a zero external field ($H_0=0$) made up of nonlinear AF layers with dielectric constant ε_1 and linear nonmagnetic (NM) layers with ε_2. The coordinate system and geometry are shown in Fig. 1, where the AF anisotropy axis is pointed along the z direction, and the two static sublattice-magnetizations are parallel and antiparallel to the z-axis, respectively. The y-axis is normal to the interface between a pair of adjacent layers. The surface waves propagate along the x-axis, and the bulk waves with the wave vector $\mathbf{k}=(k,Q,0)$ travel in the x-y plane. The thickness of

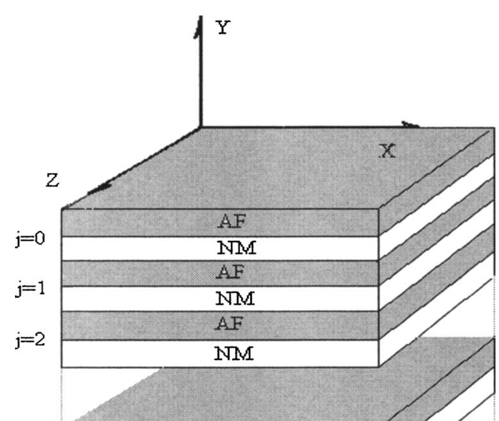

FIG. 1. Geometry and coordinate system. The superlattice is composed of antiferromagnetic layers (AFs) with thickness d_1 and nonmagnetic layers (NMs) with thickness d_2. The two sublattice magnetizations in AF layers are parallel and antiparallel to the z axis, respectively. The y-axis is pointed normal to the interfaces and the polariton waves propagate along the x-axis for the surface modes and in the x-y plane for the bulk modes. j is the index of supercell.

the AF layers is indicated by d_1 and that of NM layers by d_2, so that the period of the ANSL $D=d_1+d_2$. j represents the index of the ANSL period. For the surface modes, we take a semi-infinite ANSL occupying the space of $y<0$, but the space of $y>0$ is of vacuum. For the bulk modes, the ANSL is infinite.

First, we present the deriving process of the nonlinear wave equation for the propagation of magnetic polaritons in AF layers. The nonlinear wave fields should satisfy the Maxwell equations

$$\nabla \times \mathbf{e} = -\frac{\partial}{\partial t}\mathbf{b}, \quad \nabla \cdot \mathbf{b} = 0, \tag{1a}$$

$$\nabla \times \mathbf{h} = \frac{\partial}{\partial t}\mathbf{d}, \quad \nabla \cdot \mathbf{d} = 0, \tag{1b}$$

where we have taken the vacuum light-velocity $c=1$, meanwhile for the magnetic layers $\mathbf{d}=\varepsilon_1\mathbf{e}$ with ε_1 the relative dielectric constant and $\mathbf{b}=\mathbf{h}+\mathbf{m}$ with $\mathbf{m}=\mathbf{m}^{(1)}+\mathbf{m}^{(3)}$, the nonlinear magnetization within the third-order approximation. As we do not want to discuss the second- and third-harmonic generation and optical rectification, only the components with fundamental frequency ω are retained in the expression of $\mathbf{m}$. The first-order magnetization can be presented by $\mathbf{m}^{(1)}=\overleftrightarrow{\chi}^{(1)}\cdot\mathbf{h}$. Thus the nonlinear waves are governed by the wave equation resulting from Eq. (1),

$$\nabla(\nabla\cdot\mathbf{h}) - \nabla^2\mathbf{h} - \varepsilon_1\omega^2\overleftrightarrow{\mu}\cdot\mathbf{h} = \varepsilon_1\omega^2\mathbf{m}^{(3)}, \tag{2}$$

with $\overleftrightarrow{\mu}=1+\overleftrightarrow{\chi}^{(1)}$ and the third-order magnetization indicated by

$$m_i^{(3)} = \sum_{jkl}\chi_{ijkl}^{(3)}h_jh_kh_l^*. \tag{3}$$

In expression (3) $i,j,k,l=x,y,z$ and $\overleftrightarrow{\chi}^{(1)}$, $\overleftrightarrow{\chi}^{(3)}$ are the first- and third-order magnetic susceptibilities whose nonzero elements have been presented elsewhere.[12] Here the elements to be used are

$$\chi_{xx}^{(1)} = \chi_{yy}^{(1)} = \chi_1 = \frac{2\omega_m\omega_a}{\omega_r-\omega^2}, \tag{4}$$

$$\chi_{xxxy}^{(3)} = -\chi_{yyyx}^{(3)} = -2\chi_{xxyx}^{(3)} = -2\chi_{xyxx}^{(3)} = 2\chi_{yyxy}^{(3)} = 2\chi_{yxyy}^{(3)} = \frac{-i8\omega_m^3\omega_a^2\omega^3}{M_0^2(\omega_r^2-\omega^2)^4}, \tag{5a}$$

$$\chi_{xyyx}^{(3)} = \chi_{yxxy}^{(3)} = -2\chi_{xxyy}^{(3)} = -2\chi_{xyxy}^{(3)} = -2\chi_{yyxx}^{(3)} = -2\chi_{yxyx}^{(3)} = \frac{4\omega_m^3\omega_a\omega^2(2\omega_r^2-\omega^2-3\omega_a^2)}{M_0^2(\omega_r^2-\omega^2)^4}, \tag{5b}$$

with $\omega_m=4\pi\gamma M_0$, $\omega_a=\gamma H_a$, and $\omega_e=\gamma H_e$. H_a, H_e and M_0 are the anisotropy, exchange fields, and sublattice magnetization, respectively. γ is the gyromagnetic ratio and $\omega_r=\sqrt{\omega_a(\omega_a+2\omega_e)}$ the AF resonance frequency. As an approximation, one may consider the field components h_i on the right-hand side of Eq. (3) as linear ones to find the nonlinear solution of $\mathbf{h}$ included in Eq. (2).

For the linear surface waves propagating along the x-axis and also for the linear bulk waves moving in the x-y plane, $\partial/\partial z=0$. Thus Eq. (2) is rewritten as

$$ik\frac{\partial}{\partial y}h_y - \frac{\partial^2}{\partial y^2}h_x - \varepsilon_1\omega^2\mu h_x = \varepsilon_1\omega^2\Gamma(y)(\chi_{xxxy}^{(3)}h_x - \chi_{xyyx}^{(3)}h_y), \tag{6a}$$

$$ik\frac{\partial}{\partial y}h_x + (k^2-\varepsilon_1\omega^2\mu)h_y = \varepsilon_1\omega^2\Gamma(y)(\chi_{xxxy}^{(3)}h_y - \chi_{xyyx}^{(3)}h_x), \tag{6b}$$

$$\left(k^2 - \frac{\partial^2}{\partial y^2} - \varepsilon_1\omega^2\right)h_z = \varepsilon_1\omega^2 m_z^{(3)}, \tag{6c}$$

with $\Gamma(y)=(h_xh_y^*-h_x^*h_y)$. Equation (6c) implies that h_z is a third-order small quantity and equal to zero in the circumstance of linearity (TM waves).

Secondly, we begin from the linear wave solution, or the first-order field, to look for the nonlinear wave solution in AF layers. In different regions, it is given below:[14,15]

$$\mathbf{h} = \mathbf{A}_0e^{-\alpha_0 y}e^{i(kx-\omega t)}, \quad \text{(in vacuum space)} \tag{7a}$$

$$\mathbf{h} = [\mathbf{A}e^{\alpha_1(y+jD)} + \mathbf{B}e^{-\alpha_1(y+jD)}]e^{-\beta jD}e^{i(kx-\omega t)}, \quad \text{(in the } j\text{th AF layer)} \tag{7b}$$

$$\mathbf{h} = [\mathbf{C}e^{\alpha_2(y+jD+d_1)} + \mathbf{D}e^{-\alpha_2(y+jD+d_1)}]e^{-\beta jD}e^{i(kx-\omega t)}, \quad \text{(in the } j\text{th NM layer)} \tag{7c}$$

where $\alpha_0 \geqslant 0$ and $\beta \geqslant 0$ are called the decay constants of the surface modes. However imaginary β ($\beta=iQ$) corresponds to the bulk modes, meanwhile Eq. (7a) is not necessary any longer. $j=0,1,2,3,\ldots$, for the semi-infinite ANSL. Letting the nonlinear terms on the right-hand side of Eq. (6) equal to zero, we obtain the expressions of α_0, α_1, and α_2,

$$\alpha_0^2 - k^2 + \omega^2 = 0, \tag{8a}$$

$$\alpha_1^2 - k^2 + \varepsilon_1\mu\omega^2 = 0, \tag{8b}$$

$$\alpha_2^2 - k^2 + \varepsilon_2\omega^2 = 0, \tag{8c}$$

respectively. At the same time, due to $\nabla\cdot\mathbf{b}=0$, one find the relations among the wave amplitudes, $A_y=-ikA_x/\alpha_1$ and $B_y=ikB_x/\alpha_1$. Substituting these relations and the solution into Eqs. (6a) and (6b), we find the equations satisfied by the nonlinear wave-field in the nth AF layer as follows:

$$ik\frac{\partial}{\partial y}h_y - \frac{\partial^2}{\partial y^2}h_x - \varepsilon_1\omega^2\mu h_x = A_xe^{i(kx-\omega t)}F(m)[A_1e^{\alpha_1(y+nD)} + B_1e^{-\alpha_1(y+nD)} + C_1e^{3\alpha_1(y+nD)} + D_1e^{-3\alpha_1(y+nD)}], \tag{9a}$$

$$ik\frac{\partial}{\partial y}h_x + (k^2 - \varepsilon_1\omega^2\mu)h_y = A_x e^{i(kx-\omega t)}F(m)[A_2 e^{\alpha_1(y+nD)} + B_2 e^{-\alpha_1(y+nD)} + C_2 e^{3\alpha_1(y+nD)} + D_2 e^{-3\alpha_1(y+nD)}], \quad (9b)$$

where $F(m)=\exp(-mn\beta D)$ with $m=3$ for the surface modes, $m=1$ for the bulk modes. $A_1 \sim D_1$ and $A_2 \sim D_2$ are nonlinear coefficients. The solution of the two equations can be written as

$$h_x = A_x e^{i(kx-\omega t)}e^{-n\beta D}\{e^{\alpha_1(y+nD)} + \alpha' e^{-\alpha_1(y+nD)} + f_n[(y+nD)\alpha_1 L_1 e^{\alpha_1(y+nD)} + \alpha(y+nD)\alpha_1 L_2 e^{-\alpha_1(y+nD)} + L_3 e^{3\alpha_1(y+nD)} + L_4 e^{-3\alpha_1(y+nD)}]\}, \quad (10a)$$

and

$$h_y = -\frac{ik}{\alpha_1}A_x e^{-\beta nD}e^{i(kx-\omega t)}\{e^{\alpha_1(y+nD)} - \alpha' e^{-\alpha_1(y+nD)} + f_n[((y+nD)\alpha_1 L_1 + S)e^{\alpha_1(y+nD)} + \alpha(-(y+nD)\alpha_1 L_2 + T)e^{-\alpha_1(y+nD)} + L_3' e^{3\alpha_1(y+nD)} + L_4' e^{-3\alpha_1(y+nD)}]\}, \quad (10b)$$

in which $f_n=1$ for the bulk modes and $f_n=\exp(-2n\beta D)$ for the surface modes. The expressions of coefficients in Eqs. (9) and (10) are presented as follows:

(1) When α_1 is a real number, the coefficients in Eq. (9) are

$$A_1 = 2\alpha k A_m \chi_+, \quad B_1 = -2|\alpha|^2 k A_m \chi_-,$$
$$C_1 = 2kA_m\chi_-, \quad D_1 = -2\alpha|\alpha|^2 k A_m \chi_+, \quad (11a)$$

$$A_2 = 2i\alpha k A_m \delta_+, \quad B_2 = 2ik|\alpha|^2 A_m \delta_-,$$
$$C_2 = -2ikA_m\delta_-, \quad D_2 = -2ik\alpha|\alpha|^2 A_m \delta_+, \quad (11b)$$

with $\chi_\pm = i\alpha_1\chi^{(3)}_{xxxy} \pm k\chi^{(3)}_{xyyx}$, $\delta_\pm = ik\chi^{(3)}_{xxxy} \pm \alpha_1\chi^{(3)}_{xyyx}$, and $A_m = \varepsilon_1\omega^2|A|^2/[k^2+|\alpha_1|^2]$. The field strength $|A|^2=|A_x|^2+|A_y|^2 = [|\alpha_1|^2+k^2]|A_x|^2/|\alpha_1|^2$. From the boundary conditions of the linear field, one also can easily prove that α included in the formulas is

$$\alpha = B_x/A_x = (\alpha_0\mu + \alpha_1)/(\alpha_0\mu - \alpha_1) \quad (12a)$$

for the surface modes and

$$\alpha = \frac{e^{-\alpha_1 d_1}[\alpha_1\cosh(\alpha_2 d_2) - \mu\alpha_2\sinh(\alpha_2 d_2)] - \alpha_1 e^{-iQD}}{\alpha_1 e^{-iQD} - e^{\alpha_1 d_1}[\alpha_1\cosh(\alpha_2 d_2) + \mu\alpha_2\sinh(\alpha_2 d_2)]} \quad (12b)$$

for the bulk modes. The coefficients in Eq. (10) can written as

$$L_1 = \frac{1}{2\varepsilon_1\omega^2\mu}\left(A_1 - \frac{ik}{\alpha_1}A_2\right) = \frac{A_m k\alpha}{\mu\varepsilon_1\omega^2}\left(\chi_+ + \frac{k}{\alpha_1}\delta_+\right), \quad (13a)$$

$$L_2 = -\frac{1}{2\varepsilon_1\omega^2\mu\alpha}\left(B_1 + \frac{ik}{\alpha_1}B_2\right) = \frac{A_m\alpha^* k}{\mu\varepsilon_1\omega^2}\left(\chi_- + \frac{k}{\alpha_1}\delta_-\right), \quad (13b)$$

$$L_3 = \frac{1}{8\varepsilon_1\omega^2\mu}\left(C_1 - \frac{3ik}{\alpha_1}C_2\right) = \frac{A_m k}{4\mu\varepsilon_1\omega^2}\left(\chi_- - \frac{3k}{\alpha_1}\delta_-\right), \quad (13c)$$

$$L_4 = \frac{1}{8\varepsilon_1\omega^2\mu}\left(D_1 + \frac{3ik}{\alpha_1}D_2\right) = -\frac{A_m\alpha|\alpha|^2 k}{4\mu\varepsilon_1\omega^2}\left(\chi_+ - \frac{3k}{\alpha_1}\delta_+\right), \quad (13d)$$

$$L_3' = 3L_3 + \frac{i}{\alpha_1 k}C_2 = \frac{A_m}{4\mu\varepsilon_1\omega^2}\left[3k\chi_- - \frac{\delta_-}{\alpha_1}(k^2 + 8\alpha_1^2)\right], \quad (13e)$$

$$L_4' = -3L_4 + \frac{i}{\alpha_1 k}D_2 = \frac{A_m\alpha|\alpha|^2}{4\mu\varepsilon_1\omega^2}\left[3k\chi_+ - \frac{\delta_+}{\alpha_1}(k^2 + 8\alpha_1^2)\right], \quad (13f)$$

$$S = L_1 + \frac{i}{\alpha_1 k}A_2 = \frac{A_m\alpha}{\mu\varepsilon_1\omega^2}\left[k\chi_+ + \frac{\delta_+}{\alpha_1}(2\alpha_1^2 - k^2)\right], \quad (13g)$$

$$T = kL_2 + \frac{i}{\alpha_1\alpha k}B_2 = \frac{kA_m\alpha^*}{\mu\varepsilon_1\omega^2}\left[k\chi_- + \frac{\delta_-}{\alpha_1}(2\alpha_1^2 - k^2)\right], \quad (13h)$$

(2) If α_1 is imaginary, i.e., $\alpha_1 = i\lambda$, these coefficients should be changed into

$$A_1 = -2|\alpha|^2 k A_m\chi_+, \quad B_1 = 2\alpha k A_m \chi_-,$$
$$C_1 = -2k\alpha^* A_m\chi_-, \quad D_1 = 2k\alpha^2 A_m\chi_+, \quad (14a)$$

$$A_2 = -2ik|\alpha|^2 A_m\delta_+, \quad B_2 = -2ik\alpha A_m\delta_-,$$
$$C_2 = 2ik\alpha^* A_m\delta_-, \quad D_2 = 2ik\alpha^2 A_m\delta_+. \quad (14b)$$

$$L_1 = -\frac{A_m|\alpha|^2 k}{\varepsilon_1\omega^2\mu}\left(\chi_+ + \frac{k}{\alpha_1}\delta_+\right), \quad L_2 = -\frac{A_m k}{\varepsilon_1\omega^2\mu}\left(\chi_- + \frac{k}{\alpha_1}\delta_-\right), \quad (15a)$$

$$L_3 = -\frac{A_m\alpha^* k}{4\varepsilon_1\omega^2\mu}\left(\chi_- - \frac{3k}{\alpha_1}\delta_-\right), \quad L_4 = \frac{A_m\alpha^2 k}{4\varepsilon_1\omega^2\mu}\left(\chi_+ - \frac{3k}{\alpha_1}\delta_+\right), \quad (15b)$$

$$L_3' = -\frac{A_m\alpha^*}{4\varepsilon_1\omega^2\mu}\left[3k\chi_- - \frac{\delta_-}{\alpha_1}(k^2 + 8\alpha_1^2)\right],$$
$$L_4' = -\frac{A_m\alpha^2}{4\varepsilon_1\omega^2\mu}\left[3k\chi_+ - \frac{\delta_+}{\alpha_1}(k^2 + 8\alpha_1^2)\right], \quad (15c)$$

$$S = -\frac{A_m|\alpha|^2}{\varepsilon_1\omega^2\mu}\left[k\chi_+ + \frac{\delta_+}{\alpha_1}(2\alpha_1^2 - k^2)\right],$$
$$T = -\frac{A_m}{\varepsilon_1\omega^2\mu}\left[k\chi_- + \frac{\delta_-}{\alpha_1}(2\alpha_1^2 - k^2)\right]. \quad (15d)$$

Note that all these coefficients contain implicitly the factor

$\Delta = |A|^2/4\pi M_0^2$, so we say that they are of the second-order. For simplicity in the process of deriving dispersion equations, we introduce three second-order quantities,

$$\eta_1(y+nD) = (y+nD)\alpha_1 L_1 e^{\alpha_1(y+nD)} + \alpha(y+nD)\alpha_1 L_2 e^{-\alpha_1(y+nD)} + L_3 e^{3\alpha_1(y+nD)} + L_4 e^{-3\alpha_1(y+nD)}, \quad (16a)$$

$$\eta_2(y+nD) = [(y+nD)\alpha_1 L_1' + S]e^{\alpha_1(y+nD)} + \alpha[-(y+nD)\alpha_1 L_2 + T]e^{-\alpha_1(y+nD)} + L_3' e^{3\alpha_1(y+nD)} + L_4' e^{-3\alpha_1(y+nD)}, \quad (16b)$$

and

$$\theta(y+nD) = \frac{i\alpha_1}{\varepsilon_1\omega^2 k}[A_2 e^{\alpha_1(y+nD)} + B_2 e^{-\alpha_1(y+nD)} + C_2 e^{3\alpha_1(y+nD)} + D_2 e^{-3\alpha_1(y+nD)}]. \quad (16c)$$

Thus the nonlinear magnetic field can be rewritten as

$$\mathbf{h} = A_x\left\{[e^{\alpha_1(y+nD)} + \alpha' e^{-\alpha_1(y+nD)} + \eta_1(y+nD)f_n]\mathbf{e}_x - \frac{ik}{\alpha_1}[e^{\alpha_1(y+nD)} - \alpha' e^{-\alpha_1(y+nD)} + \eta_2(y+nD)f_n]\mathbf{e}_y\right\}e^{-\beta nD}e^{i(kx-\omega t)}, \quad (17a)$$

and the third-order magnetization is equal to

$$m_y^{(3)} = -\frac{ik}{\alpha_1}A_x\theta(y+nD)f_n e^{i(kx-\omega t)}e^{-n\beta D}. \quad (17b)$$

The two formulas will be applied for solving the dispersion equations of the nonlinear surface and bulk polaritons from the boundary conditions satisfied by the wave fields.

III. NONLINEAR DISPERSION RELATIONS

Seeking the dispersion relations of AF polaritons should begin from the boundary conditions of the magnetic field h_x and magnetic induction field b_y continuous at the interfaces and surface ($y=-nD, -nD-d_1$, and 0). The results (17a) and (17b) related to the nth AF layer, as well as the solutions (7a) in the vacuum and (7c) in the nth NM layer will be used to determine the dispersion relations. In the following several paragraphs, we shall calculate the dispersion relations of the surface and bulk modes, respectively.

A. Bulk dispersion equation

For the bulk polaritons, there are 6 amplitude coefficients in the wave solutions (7c) and ((17a) and (17b)), A_x, α', C_x, C_y, D_x, and D_y. The magnetic induction $b_y = \mu h_y + m_y^{(3)}$ in AF layers and $b_y = h_y$ in NM layers. The boundary conditions of b_y and h_x continuous at the interfaces ($y=-nD$ and $-nD-d_1$) imply four equations, and $\nabla\cdot\mathbf{h}=0$ in a NM layer leads to two additional relations $C_y = -ikC_x/\alpha_2$ and $D_y = ikD_x/\alpha_2$. Thus we have

$$A_x[1+\alpha'+\eta_1(0)f_n] = (C_x e^{-\alpha_2 d_2} + D_x e^{\alpha_2 d_2})e^{iQD}, \quad (18a)$$

$$\frac{A_x}{\alpha_1}[\mu(1-\alpha'+\eta_2(0)f_n)+\theta(0)f_n] = \frac{1}{\alpha_2}(C_x e^{-\alpha_2 d_2} - D_x e^{\alpha_2 d_2})e^{iQD}, \quad (18b)$$

$$A_x[e^{-\alpha_1 d_1} + \alpha' e^{\alpha_1 d_1} + \eta_1(-d_1)f_n] = C_x + D_x, \quad (18c)$$

$$\frac{A_x}{\alpha_1}[\mu(e^{-\alpha_1 d_1} - \alpha' e^{\alpha_1 d_1} + \eta_2(-d_1)f_n) + \theta(-d_1)f_n] = \frac{1}{\alpha_2}(C_x - D_x). \quad (18d)$$

From these four equations, we find the dispersion relation of the nonlinear bulk polaritons,

$$\cos(QD) - \cosh(\alpha_1 d_1)\cosh(\alpha_2 d_2) - \frac{\alpha_1^2 + \alpha_2^2\mu^2}{2\alpha_1\alpha_2\mu}\sinh(\alpha_1 d_1)\sinh(\alpha_2 d_2) = \frac{1}{4}N, \quad (19)$$

with the nonlinear factor N described by

$$N = \eta_1(0)\left[-e^{-iQD} + \cosh(\alpha_2 d_2)e^{\alpha_1 d_1} + \frac{\alpha_1}{\mu\alpha_2}\sinh(\alpha_2 d_2)e^{\alpha_1 d_1}\right] + \left[\eta_2(0) + \frac{\theta(0)}{\mu}\right]\left[-e^{-iQD} + \cosh(\alpha_2 d_2)e^{\alpha_1 d_1} + \frac{\alpha_2\mu}{\alpha_1}\sinh(\alpha_2 d_2)e^{\alpha_1 d_1}\right] + \eta_1(-d_1)\left[-e^{\alpha_1 d_1}e^{iQD} + \cosh(\alpha_2 d_2) - \frac{\alpha_1}{\mu\alpha_2}\sinh(\alpha_2 d_2)\right] + \left[\eta_2(-d_1) + \frac{\theta(-d_1)}{\mu}\right]\left[-e^{\alpha_1 d_1}e^{iQD} + \cosh(\alpha_2 d_2) - \frac{\alpha_2\mu}{\alpha_1}\sinh(\alpha_2 d_2)\right]. \quad (20)$$

Due to the nonlinear interaction, the nonlinear term $N/4$ appears in the dispersion equation of the polaritons and is directly proportional to Δ. This term is a second-order quantity and makes a small correct to the dispersion properties of the linear bulk polaritons.

Generally speaking, this nonlinear dispersion equation is a complex relation. However in some special circumstances it may be a real one. Here we illustrate it with an example. If $Q=0$, the bulk wave moves along the x-axis and the dispersion equation is a real equation for real α_1. For such a dispersion equation, ω has a real solution, otherwise the solution of ω is a complex number with the real part ω^{NL}, so-called the nonlinear mode frequency, and the imaginary part $\Delta\tau$, the attenuation or gain coefficient. In addition, it is very interesting that the unreciprocity of the bulk modes, $\omega(\mathbf{k}) \neq \omega(-\mathbf{k})$ with $\mathbf{k}=(k,Q,0)$, is seen, due to the existence of $\exp(-iQD)$ in the nonlinear term $N/4$ as a function of QD with the period 2π.

B. Surface dispersion relations

For the surface modes, note $f_n=\exp(-2n\beta D)$ and take the transformation $iQ\to\beta$ in Eqs. (17a) and (17b), we can find

$$\cosh(\beta D)-\cosh(\alpha_1 d_1)\cosh(\alpha_2 d_2) - \frac{\alpha_1^2+\alpha_2^2\mu^2}{2\alpha_1\alpha_2\mu}\sinh(\alpha_1 d_1)\sinh(\alpha_2 d_2)=\frac{1}{4}N' e^{-2\beta n D} \tag{21}$$

in which N' can be obtained directly from Eq. (20) with the same transformation. This nonlinear term is directly proportional to the multiple of Δ and $\exp(-2n\beta D)$, so in the same condition the nonlinearity makes a larger ω contribution to the bulk modes than the surface modes. We can use the linear expression of $\exp(-\beta D)$ to reduce the nonlinear term on the right-hand side of Eq. (21), but have to derive its nonlinear expression to describe $\cosh(\beta D)$ on the left-hand side, since its nonlinear part may has the same numerical order as that of $N'\exp(-2n\beta D)/4$. So we need another equation to determine it. Applying the boundary conditions at the surface, $y=0$ and $n=0$, we can find

$$\alpha_1[1+\alpha'+\eta_1(0)]=-\{\mu[1-\alpha'+\eta_2(0)]+\theta(0)\}\alpha_0. \tag{22}$$

Combining this with Eqs. (18a)–(18c), the equation determining β is found,

$$e^{\beta D}=\frac{[1+\alpha'+\eta_1(0)f_n]\cosh(\alpha_2 \mathrm{d}_2)+\alpha_2\mu\{1-\alpha'+[\eta_2(0)+\theta(0)/\mu]f_{\mathrm{n}}\}\sinh(\alpha_2 \mathrm{d}_2)/\alpha_1}{\mathrm{e}^{-\alpha_1 d_1}+\alpha' e^{\alpha_1 d_1}+\eta(-d_1)f_n}, \tag{23}$$

with

$$\alpha'=\frac{1}{\alpha_0\mu-\alpha_1}\{\alpha_0\mu+\alpha_1+\alpha_1\eta_1(0)+\alpha_0[\mu\eta_2(0)+\theta(0)]\}. \tag{24}$$

$f_n=\exp(-2n\beta D)$ in Eq. (23) also can be considered as a linear quantity since it always appears in the multiply of it and Δ. We also should note that there is a series of nonlinear surface eigenmodes as n can be any integer value equal to or larger than 1. Actually the nonlinear contribution decreases rapidly as n is increased, so only for small n, the nonlinear effect is important. In addition, increasing Δ and decreasing n have a similar effect in numerical calculation.

Because the nonlinear terms in Eqs. (19) and (21) all contain $\chi_{ijkl}^{(3)}$ directly proportional to $1/(\omega_r^2-\omega^2)^4$, the nonlinear effects may be too strong for us to use the third-order approximation for the nonlinear magnetization when ω is near to ω_r. In this situation we will take a smaller value of Δ to assure of the availability of this approximation.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We take the FeF_2/ZnF_2 superlattice as an example for numerical calculations, where the physical parameters $H_a=200$ kG, $H_e=540$ kG, $4\pi M_0=7.04$ kG, and $\gamma=1.97\times10^{10}$ $\mathrm{rads^{-1}(kG)^{-1}}$ lead to the AF resonance frequency $\omega_r/2\pi c=53$ $\mathrm{cm^{-1}}$. The unit of the wave vector is represented by $\omega_r/c=3.32\times10^2$ $\mathrm{rad(cm)^{-1}}$, but $\omega_r/2\pi c$ is selected as that of frequency. $D=d_1+d_2$ is measured in $2\pi c/\omega_r=1.9\times10^{-2}$ cm. The relative dielectric constants in the AF layers and NM layers are presented by $\varepsilon_1=5.5$ and $\varepsilon_2=8.0$. In the numberical caculation, we apply the SL period $D=1.9\times10^{-2}$ cm, and take $n=1$ for the surface modes. When $f_1=d_1/D$ is considered as an adjustive parameter, d_1 and d_2 are two fixed values for given f_1 and D. The nonlinear factor $\Delta=|A/(4\pi M_0)|^2$ is the relative strength of the wave field. The nonlinear shift in frequency is defined as $\Delta\omega=(\omega^{\mathrm{NL}}-\omega)/\omega_r$, where the nonlinear frequency ω^{NL} and attenuation or gain coefficient $\Delta\tau$ as the real and imagine parts of the frequency solution from the nonlinear dispersion equations both are solved numerically. ω is determined by the linear dispersion relations.

First of all we present the linear polariton spectrum,[14,15] as shown in Fig. 2, where there are three bulk-mode bands and a surface mode between the two lower bulk-mode bands. We put the middle band in a separated figure, Fig. 2(b), since it is very narrow. These three frequency bands all take the dispersion curves with $QD=0$ and π as their boundaries. In terms of the shape of a band, the second-order derivative of linear frequency with respect to k, $\partial^2\omega/\partial k^2$ for a mode in it can be

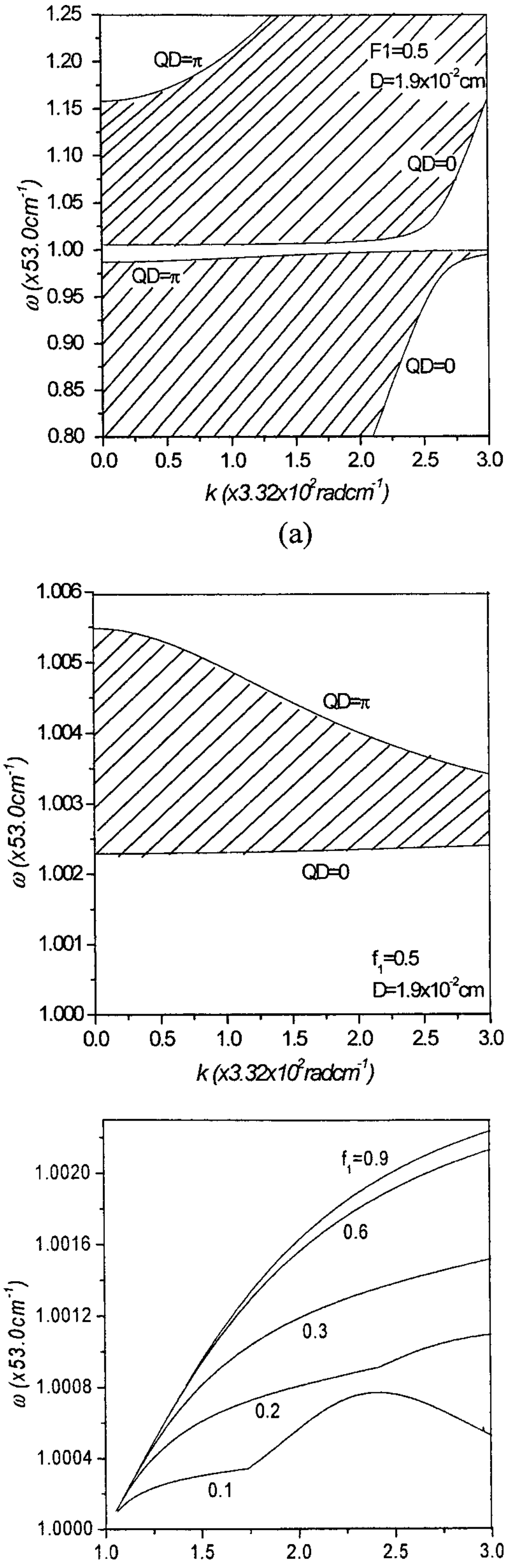

FIG. 2. Linear frequency spectrum of the polaritons. Because of the symmetry of dispersion curves with respective to $k=0$, we present only the dispersion patern in the range of $k>0$. (a) shows the top and bottom bands, and (b) presents the middle band. The surface mode is illustrated in (c).

roughly estimated to be positive or negative. If a dispersion curve bends downwards, it corresponds to $\partial^2\omega/\partial k^2<0$, otherwise to $\partial^2\omega/\partial k^2>0$. Thus when the nonlinear shift $\Delta\omega$ is presented, one can judge that the bulk-polariton solitons can exist or not, according to the Lighthill criterion $\Delta\omega\cdot\partial^2\omega/\partial k^2\leqslant 0$ for the existence of solitons.[16] One confirms from the figures that $\partial^2\omega/\partial k^2>0$ for modes in the top band, $\partial^2\omega/\partial k^2<0$ in the bottom band, but $\partial^2\omega/\partial k^2<0$ or $\partial^2\omega/\partial k^2>0$ in the middle band, depending on k and QD. Due to the reciprocity $\omega(\mathbf{k})=\omega(-\mathbf{k})$ as one feature of the bulk spectrum of a periodical structure in the linear circumstance, one will see the same results in the other zones as in the zone $0<QD<\pi/D$.

Afterwards we shall discuss the nonlinear frequency shift and attenuation. Note the nonlinear shift in frequency $\Delta\omega$ and attenuation $\Delta\tau$ are merely small corrections to the linear dispersion, so the linear frequency spectrum is thought of as a reference to discuss the nonlinear properties.

Let us consider the properties of the nonlinear bulk modes. Due to the existence of three bulk-mode bands, we shall discuss them separately. Figure 3 is offered to illustrate the bottom band. (a) and (b) show the nonlinear shift in frequency as a function of the component of the wave vector k for $Q=0$ and π/D, respectively. (a) with $Q=0$ tells us that for $f_1=0.3$ and 0.5, the nonlinear shift is downward or negative in the region of smaller k, but becomes positive from negative with the increase of k. For a SL with thicker AF layers, for example, $f_1=0.7$, the shift is always positive. Figure 3(b) for $QD=\pi$ shows that the shift in frequency always is positive for various values of f_1. This figure also indicates that the shift increases more quickly with k for the ANSL with small f_1. Roughly speaking, the frequency shift given in (b) is tens of times as big as that in (a). (c) and (d) indicate the nonlinear frequency shift and attenuation versus Q for a fixed k and several values of f_1. We see from (c) the clear unreciprocity since the dispersion properties in the zone $2\pi/D>Q>\pi/D$ differ from those in $0>Q>-\pi/D$. $\Delta\tau$ as a function of QD is shown in (d). A negative $\Delta\tau$ is called the attenuation, otherwise the gain. The nonlinear polariton waves can be attenuated and also gained, depending on QD, which means that the waves are unstable. Combining with the linear results, we conclude that the existence of the bulk soliton solution is possible in this band.

For the top bulk band, we first discuss $\Delta\omega$ versus k for $Q=0$, but we do not present figures for $Q=\pi/D$ since the shift is very small for the upper boundary of this band. Now we are going to examine the meanings of Fig. 4(a) shows that $\Delta\omega$ always is positive and possesses its maximum. (b) and (c) show $\Delta\omega$ and $\Delta\tau$ as a function of QD, respectively. In the vicinities near to $QD=0$ and 2π, the shift is positive, but negative in the two narrow regions near to $QD=2.0$ and 4.5. Although $\Delta\tau$ is positive, more obvious in only the two small regions corresponding to negative frequency-shift. We also confirm the unreciprocity from (b).

The properties of bulk nonlinear polaritons in the middle band are illustrated in Fig. 5 for $QD=0$, where the attenuation is vanishing since we find that the dispersion equation always is real in numerical calculations. Here we take one different value of nonlinear parameter $\Delta=1.0\times10^{-4}$ since the mode frequency in this band is very near to the resonant

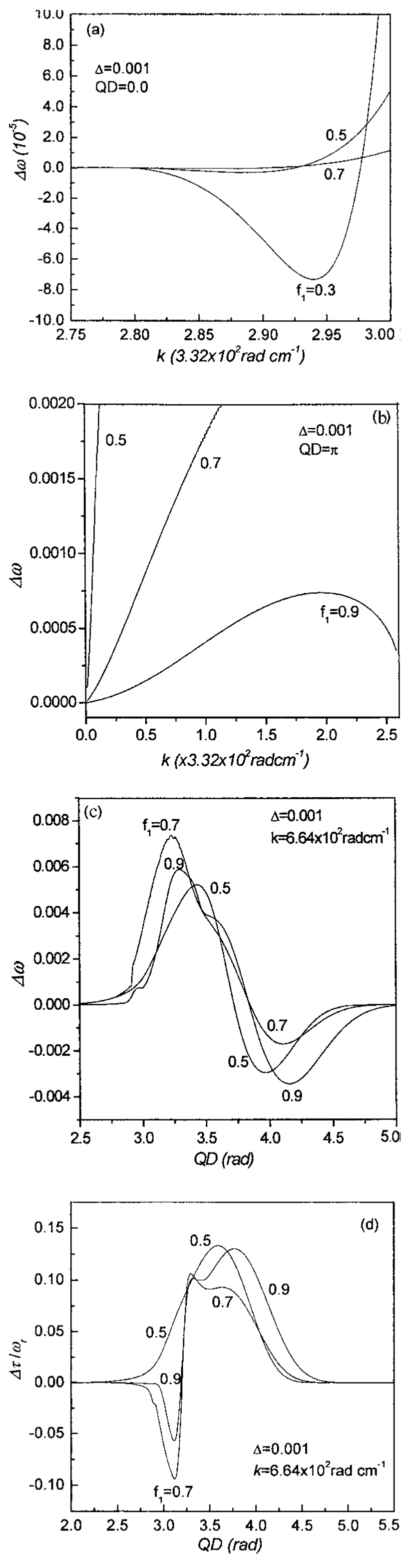

FIG. 3. Nonlinear shift infrequency with respect to the bottom linear band and attenuation. (a) and (b) show the shift as a function of k for various relative thicknesses of magnetic layers f_1, corresponding to $QD=0$ and π, respectively. (c) indicates the shift versus QD for a fixed k and different f_1 and (d) illustrates the nonlinear attenuation as a function of QD for k and f_1 as the same as those in (c).

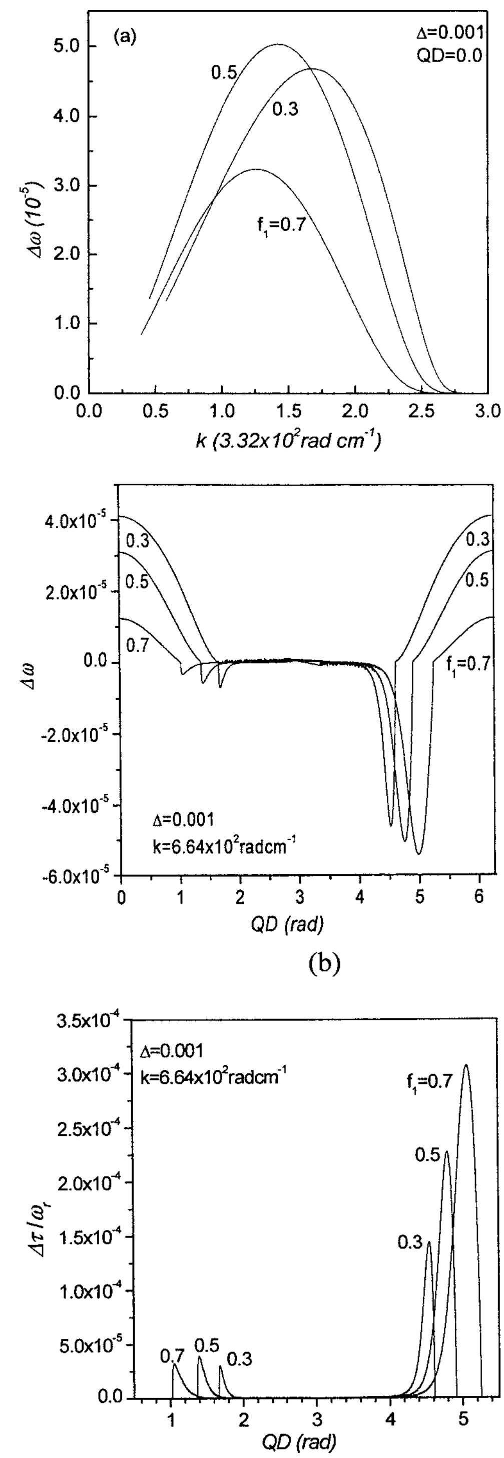

FIG. 4. Nonlinear shift in frequency and attenuation of modes in the top band. (a) shows the shift as a function of k for various thicknesses of magnetic layers f_1. (b) indicates the shift versus QD for a fixed k and different f_1 and (c) illustrates the nonlinear attenuation as a function of QD with k and f_1 as the same as those in (b).

frequency ω_r, hence the nonlinearity is very strong even for a smaller field-strength. The figure shows the positive frequency shift that increases basically with k. In this band, the bulk polaritons for various values of QD are similar in features.

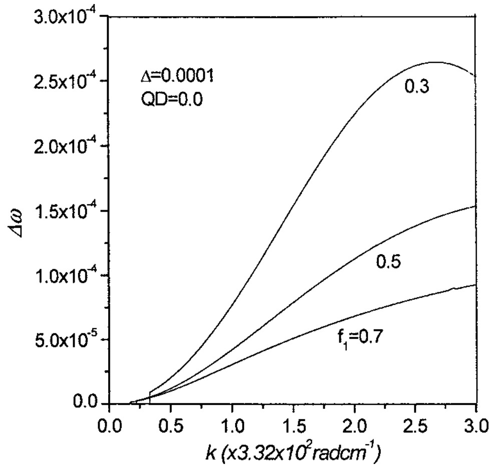

FIG. 5. Nonlinear frequency shift $\Delta\omega$ of the modes in the middle band versus k for $QD=0$, $\Delta=1.0\times10^{-4}$ and various values of f_1.

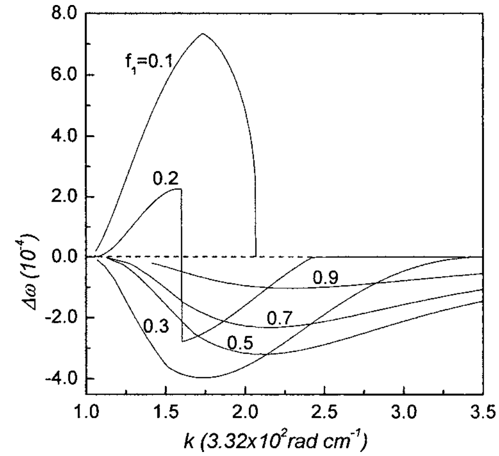

FIG. 6. Nonlinear frequency shift $\Delta\omega$ of the surface mode versus k for $\Delta=1.0\times10^{-4}$ and various values of f_1.

Finally, we examine the surface magnetic polariton in the case of nonlinearity, which is shown in Fig. 6. Similar to those in the middle bulk band, the surface-mode frequency also is very close to ω_r, as a result, the nonlinear effect also is stronger. Considering the reasonableness of the theory, we also use $\Delta=1.0\times10^{-4}$. The attenuation $\Delta\tau=0$ as the dispersion equations are real. The shift $\Delta\omega$ is negative for $f_1 \geqslant 0.3$, but positive for $f_1=0.1$. For $f_1=0.2$, it is positive and increases with k in the range of small k, but negative in the range of large k and its absolute value decreases as k is increased. Although there can be a series of surface eigenmodes in the nonlinear situation, the obvious nonlinear effect can be seen only for $n=1$, so that we present only the corresponding mode. One should note that the Lighthill criterion is satisfied for $f_1=0.1$ and 0.2; as a result, the surface soliton may form from the surface magnetic polariton.

V. SUMMARY AND CONCLUSIONS

We have discussed the nonlinear magnetic polaritons of the ANSL in zero-field $H_0=0$ with the nonlinear antiferromagnetic susceptibilities obtained in the third-order approximation.[12] We have cited briefly the linear frequency spectrum in order to understand the properties of the nonlinear polaritons.

For the discussion of nonlinear dispersion, we assume that AF layers in the ANSL are nonlinear, and meanwhile NM layers are linear. We first derive the nonlinear wave solution of the polaritons in AF layers and introduce the linear wave solution in NM layers, and then we look for the dispersion equations of the nonlinear polaritons.

The dispersion equations for the surface mode are real ones and we see a series of the nonlinear surface eigenmodes. It is very interesting that the surface soliton may exist in the system. The nonlinear shift in frequency can be upward and also downward, depending on the relative thickness of AF layers.

In general speaking, the dispersion relation for the nonlinear bulk modes, except modes in the middle band, is a complex equation. It means that the solution of frequency directly resulting from this relation should be a complex number for a given $\mathbf{k}$, whose real part is just the mode frequency, and imaginary part is called the attenuation. Because of the appearance of $\exp(iQD)$ in the nonlinear dispersion relation of the bulk modes, the frequency band between $QD=0$ and π is equal no longer to the band between $QD=\pi$ and 2π, which implies the unreciprocity of the nonlinear bulk modes, $\omega(\mathbf{k})\neq\omega(-\mathbf{k})$ that also appears in the complicated one-dimensional photonic crystals.[17] We find that the nonlinear frequency shift and attenuation are very obvious and they can be positive and also be negative, depending on values of $\mathbf{k}$ and f_1. The attenuation $\Delta\tau\neq0$ means that the nonlinear waves are unstable. The soliton solution may be found since the Lighthill criterion can be fulfilled in the two bands. In the middle bulk band, the mode attenuation is vanishing, the nonlinearity is very evident, and the nonlinear shift is positive.

In this paper, the nonlinear parameter is taken as $\Delta=1.0\times10^{-3}$ or 1.0×10^{-4}. These two values correspond to the field amplitudes of about 0.022 (A/m) and 0.00704 (A/m), respectively. One can easily realize these amplitudes in relevant experiments. The ANSL used here also can be considered as a one-dimensional photonic crystal; but this photonic crystal is composed of antiferromagnetic and nonmagnetic layers. These kinds of photonic crystals has been given a lot of attention.[13]

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