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Mesoscopic Stern-Gerlach spin filter by nonuniform spin-orbit interaction

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A spin filtering in a two-dimensional electron system with nonuniform spin-orbit interactions (SOI) is theoretically studied. The strength of SOI is modulated perpendicular to the charge current. A spatial gradient of effective magnetic field due to the nonuniform SOI causes the Stern-Gerlach-type spin separation. The direction of the polarization is perpendicular to the current and parallel to the spatial gradient. Almost 100% spin polarization can be realized even without applying any external magnetic fields and without attaching ferromagnetic contacts. The spin polarization persists even in the presence of randomness.

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Considerable attention has been devoted to the manipulation of the electron spin in semiconductor systems.[1,2] To control the spin of electrons in semiconductors, the spin-orbit interaction (SOI) due to the lack of the inversion symmetry in two-dimensional electron system (2DES) is quite useful since its strength can be controlled by an additional gate voltage.[3,4] The spin field-effect transistor proposed by Datta and Das is a hybrid structure of ferromagnetic electrodes (FM) and a semiconductor (SM) 2DES channel.[5] This hybrid device, however, requires a complex and careful fabrication process. Furthermore, spin injection efficiency from FM into SM is poor because of a conductance mismatch between FM and SM.[6] Therefore, it is desired that spin-polarized carriers in semiconductor channels can be generated and manipulated without attaching any ferromagnetic contacts and without applying any external magnetic fields. Several devices based on the SOI are proposed to have spin-polarized carriers in a semiconductor channel.[7–13] From an experimental point of view, the spin-polarization mechanism should be robust against disorder.

One of the historical experiments of the spin separation is the Stern-Gerlach experiment.[14] They have considered a particle with spin propagating through the nonuniform magnetic field. Because the derivative of the magnetic field plays a role of the spin-dependent potential, particles with up and down spins are accelerated in opposite directions. However, for electrons, this effect is hard to observe because of the effect of the Lorentz force acting on a electron beam in transverse directions.[15]

The importance of the modulated SOI on the transport properties has recently been stressed.[16–20] Since the effect of SOI on the propagating electrons is, in some respect, similar to the effective magnetic field, the modulated SOI is expected to cause the Stern-Gerlach-like spin separation. In this paper, we theoretically demonstrate that a mesoscopic Stern-Gerlach spin filter is feasible by using a nonuniform spin-orbit interaction as shown in Fig. 1. The strength of the SOI is modulated along the direction perpendicular to the charge current. We demonstrate that nearly 100% spin polarization (perpendicular both to the current and the direction normal to 2DES) can be achieved without a ferromagnetic contact and an external magnetic field. The above results are obtained both from the wave-packet dynamics and from the transfer matrix calculation of the transmission coefficients. We have found that the large polarization is obtained when the electron propagates via the lowest channel where the transverse mode of the wave function contains only single antinode. We also investigate the effect of randomness. The results show that the polarization of the current survives as long as the randomness is not so strong.

We consider a 2DES in the x-y plane and the current is assumed to flow in the x direction, while a fixed boundary condition is imposed in the y direction. A square lattice is considered for modeling 2DES and only the nearest-neighbor hopping is taken into account. The tight-binding lattice spacing a and the hopping energy $V_0=\hbar^2/2m^*a^2$, where m^* is the effective electron mass are taken as the unit length and the unit energy, respectively. The region in which the SOI exists is $L_x \times L_y = 60 \times 30$, where L_x and L_y are the length and the width of the system. We attach ideal leads to both sides of this region. To include the SOI, we use the Ando model[21] described by the Hamiltonian,

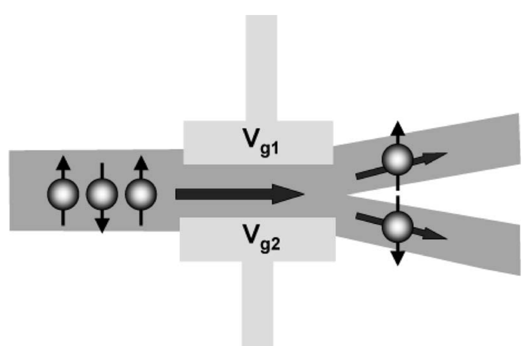

FIG. 1. Top view of Stern-Gerlach spin filter. V_{g1} and V_{g2} are gate voltages to produce a spatial gradient of spin-orbit interaction. Stern-Gerlach-type spin separation occurs when unpolarized electrons go through the nonuniform SOI region between the two gate electrodes.

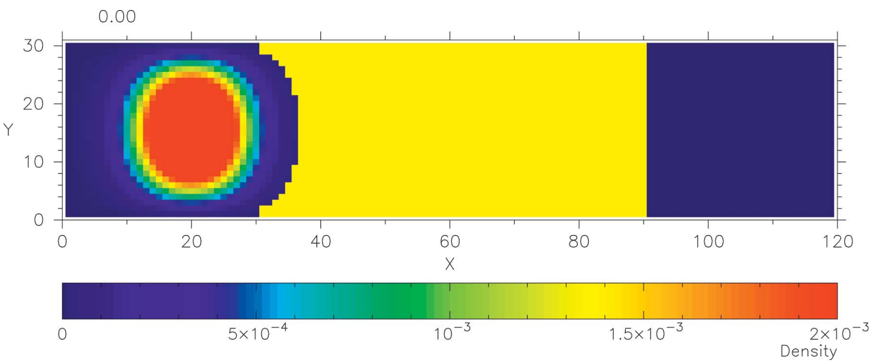

FIG. 2. (Color)Initial wave packet ($t=0$) propagating to the right. The yellow region indicates the area where SOI is present.

$$H=\sum_{i,\sigma} W_i c_{i\sigma}^\dagger c_{i\sigma} - \sum_{\langle i,j\rangle,\sigma,\sigma'} V_{i\sigma,j\sigma'} c_{i\sigma}^\dagger c_{j\sigma'}, \tag{1}$$

with

$$V_{i,i+\hat{\mathbf{x}}}=\begin{pmatrix} \cos\theta(y) & \sin\theta(y) \\ -\sin\theta(y) & \cos\theta(y) \end{pmatrix}, \tag{2}$$

and

$$V_{i,i+\hat{\mathbf{y}}}=\begin{pmatrix} \cos\theta(y) & -\mathrm{i}\sin\theta(y) \\ -\mathrm{i}\sin\theta(y) & \cos\theta(y) \end{pmatrix}, \tag{3}$$

where $c_{i\sigma}^\dagger(c_{i\sigma})$ is a creation (annihilation) operator of an electron on the site i with spin σ, and $\hat{\mathbf{x}}$ $(\hat{\mathbf{y}})$ the unit vector along x (y) direction. W_i is the random potential distributed uniformly in the range $[-W/2, W/2]$. Unless explicitly stated, we consider the impurity free case. The strength of SOI is characterized by $\theta(y)$

$$\theta(y)=\frac{2\theta_{\max}}{L_y}y-\theta_{\max}. \tag{4}$$

The Hamiltonian of the Rashba SOI (Ref. 22) is generally described as

$$H_{\mathrm{R}}=\frac{1}{\hbar}\left\{\alpha(y)p_x\sigma_y-\frac{[\alpha(y)p_y\sigma_x+p_y\alpha(y)\sigma_x]}{2}\right\}, \tag{5}$$

where $\alpha(y)$ is the strength of the SOI. The relation between $\alpha(y)$ and $\theta(y)$ is given by

$$\alpha(y)\simeq 2\theta(y)V_0 a \quad [\text{for } \theta(y)\ll 1]. \tag{6}$$

The spin separation mechanism we propose in this paper is as follows. Equation (5) means that the effective Zeeman field in the y direction appears when an electron propagates in the x direction. If the effective Zeeman field has a gradient in the y direction, up and down spin electrons are accelerated in opposite directions. Unlike the Stern-Gerlach experiment, this effect is expected to be easily observed even if the particles have charge. Since the spins are expected to align in the y direction, hereafter we concentrate on the polarization in the y direction.

It is possible to make a spatial gradient of the SOI in the y direction, e.g., by using two gate electrodes that partially cover the channel, and the change in the SOI strength between the two electrodes $\Delta\alpha$ has been experimentally obtained to be $0.4–0.8\times10^{-11}$ eV m.[3,4] Let us assume $a=30$ nm and $m^*=0.05m_0$, where m_0 is the free electron mass. Then the system area is 1.8×0.9 μm^2, $V_0\simeq0.85$ m eV, and $\Delta\alpha\simeq0.64\times10^{-11}$ eV m by setting $\Delta\theta(y)=0.04\pi$. We therefore choose $\theta_{\max}$ to be 0.02π.

To investigate the electron transport in the nonuniform SOI system, we calculate the time evolution of the wave packet by the equation-of-motion method based on the exponential product formula.[23] The charge density $\Sigma_\sigma(|\langle\uparrow|\psi_\sigma\rangle|^2+|\langle\downarrow|\psi_\sigma\rangle|^2)$ of the initial wave packet is shown in Fig. 2. The initial wave packet with spin σ is assumed to be

$$\psi_\sigma(t=0)=A\sin\left(\frac{\pi y}{L_y+1}\right)\exp\left(\mathrm{i}k_x x-\frac{\delta k_x^2 x^2}{4}\right)\chi_\sigma, \tag{7}$$

with

$$\chi_\uparrow=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix}, \quad \chi_\downarrow=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}. \tag{8}$$

We set $k_x=0.5$ and $\delta k_x=0.2$.

The wave packet after time evolution is shown in Fig. 3. The charge density splits into the upper and the lower parts with its spin polarizations opposite. The maximum values of $\Sigma_\sigma(|\langle\uparrow|\psi_\sigma\rangle|^2+|\langle\downarrow|\psi_\sigma\rangle|^2)$ and that of $|\Sigma_\sigma(|\langle\uparrow|\psi_\sigma\rangle|^2-|\langle\downarrow|\psi_\sigma\rangle|^2)|$ are almost the same, which suggests that nearly 100% spin filtering has been achieved.

To reinforce the above finding, we have also calculated the polarization of the current by the Landauer formula.[24] Figure 4 shows the polarization of the current as a function of the electron Fermi energy E in units of the hopping energy. The inset of this figure is the schematic view of the system. Three-terminal geometry has been employed to distinguish the current through the upper and lower part of the system. The system area including the nonuniform SOI is again 60×30 and the width of the upper and lower leads in the right-hand side is 5. We assume that the chemical potential of the reservoir attached to the left-hand side lead is $E+eV$ with V the voltage, while both of the upper and lower leads in the right-hand side are attached to the reservoirs with chemical potential E. The transmission coefficients are calculated via the transfer matrix method[25] extended to include spins.[26] In order to realize the multiterminal system, the static potential is assumed in the middle of the right leads. We confirmed that the result of the transfer matrix method agrees with the result of the Green function method[27] up to four digits.

The polarization of the current is defined as

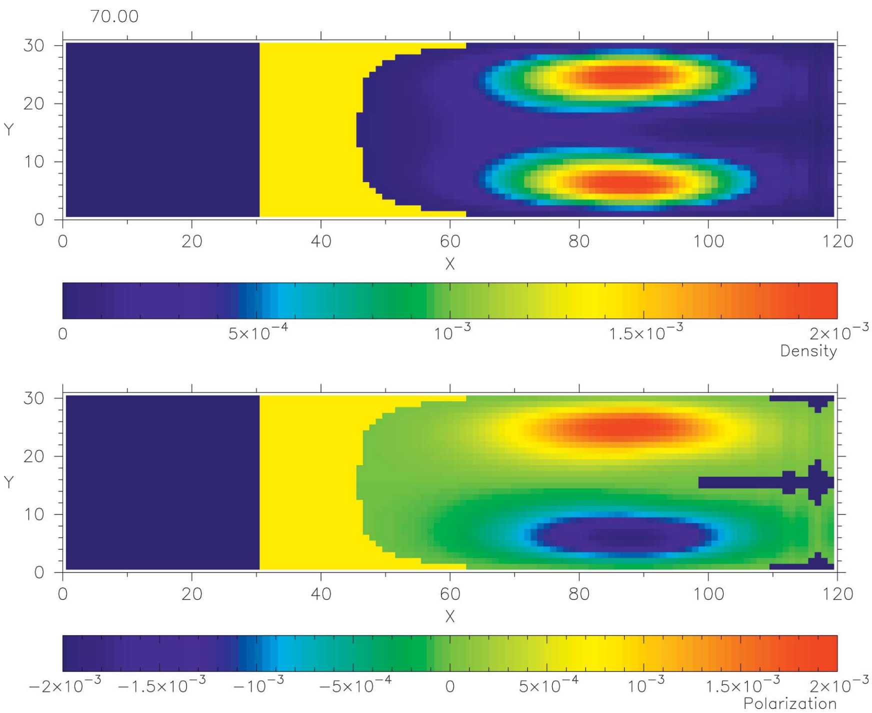

FIG. 3. (Color) Wave packet after time evolution ($t=70\hbar V_0^{-1}$). The strength of the SOI is modulated in the y direction. Upper: the charge density $\Sigma_\sigma(|\langle\uparrow|\psi_\sigma\rangle|^2+|\langle\downarrow|\psi_\sigma\rangle|^2)$. Lower: the corresponding polarization, $\Sigma_\sigma(|\langle\uparrow|\psi_\sigma\rangle|^2-|\langle\downarrow|\psi_\sigma\rangle|^2)$.

$$P_y = \frac{T_\uparrow - T_\downarrow}{T_\uparrow + T_\downarrow}, \tag{9}$$

where $T_\sigma=(e^2/h)\Sigma_{\sigma'}|t_{\sigma,\sigma'}|^2$. Here $t_{\sigma,\sigma'}$ is the transmission coefficient from the left lead, with spin σ' to the right leads with spin σ. We focus on the component from the lowest channel of the left lead, which corresponds to the result of the time evolution of the wave packet. As shown in Fig. 4, the spin-filtering effect is clearly obtained in the nonuniform SOI system. It should be noted that the polarization along the z direction can be obtained in the uniform SOI system by considering multiterminal geometry.[8] However, the polarization has the strong energy dependence and the energy region where the large polarization occurs is very narrow.

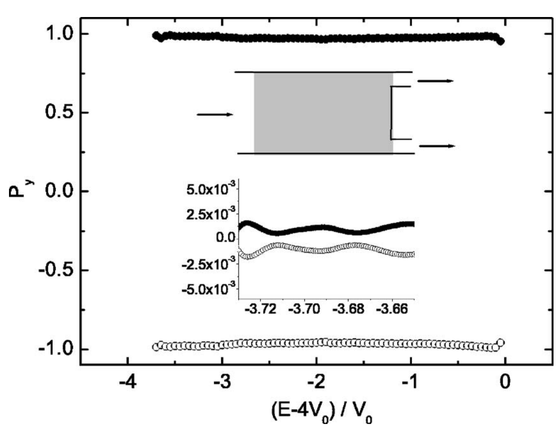

FIG. 4. Polarization P_y [Eq. (9)] of the current as a function of the Fermi energy. The filled (open) circle shows the polarization of the upper (lower) lead in the right hand side. The main figure shows the component from the lowest channel of the left lead. The inset shows the result in which the whole channel is taken into account. The schematic view of the three-terminal geometry is also shown in the inset.

In this calculation, we consider the current in the lowest channel of the left lead where there are no nodes in the wave function along the transverse direction [see Eq. (7)]. In order to obtain the high polarization of the current, we found that the lowest channel is expected only in the left lead while the channel mixing is allowed in the sample. The inset of Fig. 4 shows the polarization, which takes into account the whole channel. The spin separation effect of the higher channel becomes weaker due to the fact that the transverse wave function has several antinodes along the y direction. Each antinode splits but the trajectory is quite different, which causes the cancellation of the polarization. One can avoid this difficulty by attaching a narrow lead or fabricate a point contact in the left-hand side so that only the first channel opens. The complex trajectory is mainly due to the scattering by the hard wall located to separate the channel to the upper and lower leads. Using the soft wall potential may improve the situation.

From the experimental point of view, we have to consider the effect of randomness. Figure 5 shows that the average of the polarization in the presence of impurities. The energy of the electron is fixed to $E/V_0=3.0$ and the average over 10 000 samples has been performed. From this figure, we see that the finite polarization of the current persists even in the presence of impurities. Further increase of the randomness significantly destroys the polarization. The strength of the disorder is related to the mean free path in 2DES without spin-flip scattering as

$$W = \left(\frac{6\lambda_F^3}{\pi^3 a^2 L_m}\right)^{1/2} E, \tag{10}$$

where λ_F denotes the Fermi wavelength and L_m the mean free path.[28] The inset of Fig. 5 shows the replot of the polar-

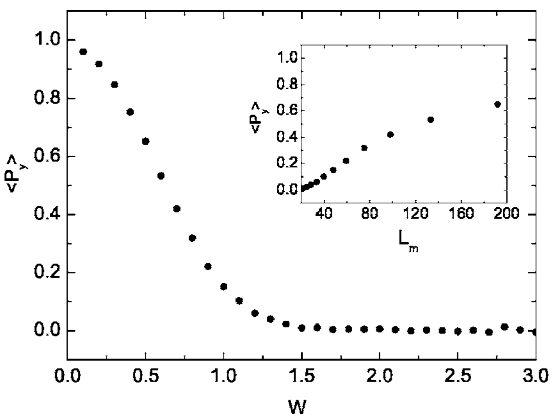

FIG. 5. The polarization of the current of the lower lead as a function of the strength of disorder. The inset shows the polarization as a function of the mean free path. The average over 10 000 samples has been performed.

ization as a function of L_{m}, which indicates that finite polarization remains as long as L_{m} exceeds the system length. The electron mean free path in the InGaAs 2DES channel exceeds 1 μm in the wide range of the gate voltage,[4] and the spin-filtering effect by nonuniform SOI strength is expected to survive in actually fabricated devices. We have also changed the distribution of the impurities to the binary distribution, where the site energy W_i takes only two values, 0 and $W>0$. In this case, the variance of the potential fluctuation becomes $p(1-p)W^2$ with p the probability that W_i takes W. We set $p\approx 0.092$ so that $p(1-p)=1/12$ corresponding to the uniform distribution considered above. No significant difference is observed by this change of distribution. We have also confirmed that distributing attractive impurities, i.e., $W<0$ does not influence the results either.

In conclusion, we have investigated the spin filtering due to the nonuniform spin-orbit interaction (SOI) system. The strength of SOI is modulated perpendicular to the charge current, which yields the gradient of the effective Zeeman field, and the spin separation occurs as in the Stern-Gerlach experiment. Both the time evolution of the wave packet and the transmission coefficient indicate the large polarization that survives even in the presence of impurities. In this mechanism, the direction of the spin polarization can be easily switched by the gate voltages between the two gate electrodes. When the fully polarized current (which can be produced by the device proposed here) is injected to this system, one expects that the current flows along one side of the system and the currents in the upper and lower leads become totally different, i.e., the information of the spin polarization is transformed into conductance. Therefore this system can also be used as a detector of the polarized current. It should be emphasized that this effect is expected to survive even at relatively high temperature. This is because the mechanism proposed here does not rely on the quantum interference, which is easily destroyed by dephasing. Also the weak dependence of P_y on energy in the lowest channel suggests that smearing of the Fermi distribution function is not important.

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