# Magnetic-field symmetry of pump currents of adiabatically driven mesoscopic structures

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(Received 3 February 2005; revised manuscript received 11 April 2005; published 13 July 2005)

We examine the scattering properties of a slowly and periodically driven mesoscopic sample using the Floquet function approach. One might expect that at sufficiently low driving frequencies it is only the frozen scattering matrix which is important. The frozen scattering matrix reflects the properties of the sample at a given instant of time. Indeed many aspects of adiabatic scattering can be described in terms of the frozen scattering matrix. However, we demonstrate that the Floquet scattering matrix, to first order in the driving frequency, is determined by an additional matrix which reflects the fact that the scatterer is time dependent. This low-frequency irreducible part of the Floquet matrix has symmetry properties with respect to time and/or a magnetic field direction reversal opposite to that of the frozen scattering matrix. Using the adiabatic decomposition of the Floquet scattering matrix we split the dc current flowing through the pump into several parts with well defined properties with respect to a magnetic field inversion.

DOI: 10.1103/PhysRevB.72.035324 PACS number(s): 73.23.−b, 72.10.−d, 73.40.Ei

## I. INTRODUCTION

The interplay of quantum-mechanical interference with quantized energy exchange results in a quantum pump effect which is investigated intensively both experimentally[1–5] and theoretically.[6–37] This phenomenon being promising for manipulating and controlling the passage of electrons through mesoscopic circuits is of fundamental interest. Adiabatic driving involves only low-energy exchange and avoids excitations into inelastic channels which degrade the quantum properties of the system. In this work we investigate the magnetic symmetry properties of the dc current of a quantum pump which might operate in the presence of applied voltages and temperature gradients.

The experimentally measured adiabatically pumped dc current[1] flowing through a chaotic cavity with periodically varying shape is symmetric in magnetic field $H$. That is in seeming contradiction with the theory[8,10,12,14,20,29] predicting that the pumped current has no definite symmetry under magnetic field reversal. As a result it was conjectured[38–40] that the current measured in Ref. 1 is caused by a classical rectification effect. Indeed subsequent measurements[3,5] confirmed that for slow one-parameter driving there is a symmetric in magnetic field induced current whose origin is classical rectification. Nevertheless one cannot exclude the possibility that the current measured in Ref. 1 also contains the contribution coming from the quantum pump effect. To check it, perhaps, it is necessary to investigate the system in a less symmetric setup, i.e., with reservoirs having different electrochemical potentials or temperatures. Further experimental and theoretical efforts to detect and distinguish the quantum pump effect are highly desirable in view of a possible application in quantum information processing devices.[41,42]

The aim of the present paper is to explore in detail the symmetry properties of the adiabatic current generated by the periodically driven mesoscopic conductor. To this end we represent the Floquet scattering matrix at low driving frequency $\omega$ as a sum of different terms with well defined symmetry properties (e.g., with respect to a magnetic field direction reversal). One term reflects the symmetry of a stationary scattering process while the other term vanishing at $\omega \to 0$ has symmetry properties opposite to a stationary scattering process. Based on such a representation we divide the dc current into parts with well defined symmetry properties. That opens up additional possibilities for the experimental detection of the quantum pump effect.

In particular, in the two terminal case, we find a voltage dependent contribution to the pumped current which is odd in magnetic field. At small voltage this current is linear in $V$. Thus for small magnetic fields the dc current has a component which is proportional to the product of frequency, magnetic field, and applied voltage. For comparison we recall that in the stationary case, for a two-terminal conductor, the current linear in voltage (or, alternatively, the conductance) is an even function of a magnetic field.[43,44] A current that is odd in magnetic field appears only in the nonlinear voltage regime[45,46] and is caused by electron-electron interactions. In contrast, in the nonstationary case considered here even noninteracting electrons can show a response that is odd in magnetic field and linear in applied voltage.

Recently the magnetic field symmetry of the dc current through an open quantum dot subject to a one-parameter potential oscillation has been investigated experimentally and theoretically as a function of frequency.[5] In contrast, in the present paper we consider a two-parameter oscillation and investigate the magnetic field symmetry of the dc current in the presence of adiabatic parametric quantum pumping.

The paper is organized as follows. In Sec. II we briefly consider the Floquet function approach to scattering of electrons at a periodically driven mesoscopic conductor and analyze the consequences of microscopic reversibility. We introduce an exact representation for the scattering matrix at low driving frequency $\omega$. According to this representation the Floquet scattering matrix elements (up to linear in $\omega$ terms) are proportional to the elements of both the stationary scattering matrix $\hat{S}_0$ and a residual Floquet matrix $\hat{A}$ which exhibits symmetry properties opposite to those of $\hat{S}_0$. The symmetry properties of $\hat{S}_0$ are dictated by microreversibility, and the residual Floquet matrix $\hat{A}$ reflects directly the breaking of these symmetries due to the driving of the sample. Using such a representation we analyze the magnetic field symmetry of the dc current flowing through the adiabatically driven scatterer in Sec. III. We show that in the two terminal case there is a dc current $I^{(od)}$ that is odd in magnetic field, linear in $\omega$ and dependent on the applied voltage. We conclude in Sec. IV.

## II. GENERAL APPROACH

We use the scattering matrix approach[7,44,47] which views the mesoscopic sample as a scatterer which causes transmission and reflection of incident carriers. The scatterer is assumed to be coupled to $N_r$ reservoirs via single channel ballistic leads which we will number by the Greek letters $\alpha, \beta$, etc.

We assume that in the stationary case electrons coming from the reservoirs and interacting with the scatterer are subject only to elastic scattering. Such (single particle) scattering can be described with the help of the scattering matrix $\hat{S}_0$. The index 0 denotes the stationary scattering matrix. In general $\hat{S}_0$ is a function of the electron energy $E$. This matrix collects all the quantum mechanical amplitudes for electrons coming from some lead $\beta$ to be scattered into the same or any other lead $\alpha$. These amplitudes are normalized in such a way that their squares define the corresponding particle fluxes (currents). If the electron velocities at a given energy are the same in all the leads we can use these amplitudes to relate the incident and out-going wave functions. For instance, let $\Psi_{0,\beta}^{(\mathrm{in})}(E,t)=e^{-i(E/\hbar)t}\psi_{0,\beta}^{(\mathrm{in})}(E)$, be the amplitude of a wave function describing electrons with energy $E$ incident in lead $\beta$. Then the amplitude of the wave function of particles outgoing in lead $\alpha, \Psi_{0,\alpha}^{(\mathrm{out})}(E,t)=e^{-i(E/\hbar)t}\psi_{0,\alpha}^{(\mathrm{out})}(E)$, is defined as follows:

$$\psi_{0,\alpha}^{(\mathrm{out})}(E) = \sum_{\beta=1}^{N_r} S_{0,\alpha\beta}(E)\psi_{0,\beta}^{(\mathrm{in})}(E). \tag{1}$$

Current conservation implies that the scattering matrix is a unitary matrix[48]

$$\hat{S}_0^{\dagger}\hat{S}_0 = \hat{S}_0\hat{S}_0^{\dagger} = \hat{I}, \tag{2}$$

where $\hat{I}$ is a unit matrix. In fact, the knowledge of the matrix $\hat{S}_0(E)$ is equivalent to the knowledge of the solution for the stationary Schrödinger equation.

For the dynamical problem with time-dependent scattering, scattering is characterized by the integral scattering operator which depends on two times.[20] One time argument relates to the incoming states and the second time argument to the outgoing states. In this paper we are dealing with a particular nonstationary case, namely, with a periodically driven scattering problem. We assume that the scattering potential (hence the scattering properties of a sample) is varied in time periodically with period $\mathcal{T}=2\pi/\omega$. Then, according to the Floquet theorem (see, e.g., Refs. 49–53), the solution for the time-dependent Schrödinger equation can be represented in a relatively simple form

$$\Psi(E,t) = e^{-i(E/\hbar)t}\sum_{n=-\infty}^{\infty} e^{-in\omega t}\psi(E_n). \tag{3}$$

Here $E$ is the Floquet energy; $\psi(E_n)$ is a general solution of the stationary Schrödinger equation corresponding to the energy $E_n=E+n\hbar\omega$.

Scattering on such an oscillatory scatterer can be described via the Floquet scattering matrix. In this work we are concerned with the low-frequency properties of this dynamic problem and the relevant Floquet matrix $\hat{S}_F$ describes the transitions between the propagating states only.[53] The elements $S_{F,\alpha\beta}(E_n,E)$ of this matrix are the quantum mechanical amplitudes (normalized for current) for an electron with energy $E$ to enter the scatterer through lead $\beta$ and to leave the scatterer with energy $E_n=E+n\hbar\omega$ through lead $\alpha$.

In particular, if the reservoirs are stationary then the incoming wave function is $\Psi_{0,\beta}^{(\mathrm{in})}(E,t)$ and the wave function for particles outgoing to lead $\alpha$ is of the form Eq. (3) with

$$\psi_{\alpha}^{(\mathrm{out})}(E_n) = \sum_{\beta=1}^{N_r}\sum_{m}\sqrt{\frac{k_m}{k_n}}S_{F,\alpha\beta}(E_n,E_m)\psi_{0,\beta}^{(\mathrm{in})}(E_m). \tag{4}$$

Here $k_n=\sqrt{2m_eE_n}/\hbar$ with $m_e$ being the electron mass. Physically Eq. (4) means that an electron interacting with an oscillating scatterer can gain or lose one or several energy quanta $n\hbar\omega$, $n=0,\pm 1,\pm 2,\ldots$, and thus an electron can change its energy by a discrete amount $n\hbar\omega$.

Current conservation implies again that also the matrix $\hat{S}_F$ is unitary. For the Floquet scattering matrix the analog of Eq. (2) reads

$$\sum_{\alpha}\sum_{n} S_{F,\alpha\beta}^{*}(E_n,E)S_{F,\alpha\gamma}(E_n,E_m) = \delta_{m0}\delta_{\beta\gamma}, \tag{5a}$$

$$\sum_{\beta}\sum_{n} S_{F,\alpha\beta}^{*}(E,E_n)S_{F,\gamma\beta}(E_m,E_n) = \delta_{m0}\delta_{\alpha\gamma}. \tag{5b}$$

Here the summation over $n$ goes only over those $n$ which correspond to a positive $E_n=E+n\hbar\omega$. In the low-frequency limit we have $\hbar\omega \ll E$, and thus $n$ extends from $-\infty$ to $+\infty$.

To find the Floquet scattering matrix one needs to solve a fully time-dependent Schrödinger equation. Compared to the stationary problem, this is a more difficult task and, generally, it can be done only numerically. On the other hand, the representation (3) seems effectively to reduce the periodically driven case to the stationary one. Therefore it is attractive to try to relate the Floquet scattering matrix $\hat{S}_F$ to the stationary scattering matrix $\hat{S}_0$.

### A. Adiabatic approximation

Let the stationary scattering matrix $\hat{S}_0(E,\{p\})$ depend on a set of parameters $p_i \in \{p\}, i=1,2,\ldots,N_p$ (e.g., the sample's shape, the strength of coupling to leads, the magnetic field, etc.). Varying these parameters one can change the scattering properties of a sample. We take these parameters to be periodic functions in time $p_i(t)=p_i(t+\mathcal{T})$, $\forall i$. Then the matrix $\hat{S}_0$ becomes time dependent $\hat{S}_0(E,t)=\hat{S}_0(E,\{p(t)\})$. In general the matrix $\hat{S}_0(t)$ does not describe the scattering of electrons by a time-dependent scatterer: only the Floquet scattering matrix $\hat{S}_F$ does. Nevertheless in the low-frequency limit $\omega \to 0$ there exists a connection between these two matrices. This connection becomes more evident if one represents the Floquet scattering matrix elements as a series in powers of $\omega$.

### *1. Zeroth order approximation*

To zeroth order in the driving frequency the elements of the Floquet scattering matrix $\hat{S}_F(E_n,E)$ can be approximated by the Fourier coefficients $\hat{S}_{0,n}$ of the stationary scattering matrix $\hat{S}_0$ as follows:[53]

$$\hat{S}_F(E_n,E)=\hat{S}_{0,n}(E)+O(\omega), \tag{6a}$$

$$\hat{S}_F(E,E_n)=\hat{S}_{0,-n}(E)+O(\omega). \tag{6b}$$

Here $O(\omega)$ denotes the rest which is at least first order in frequency $\omega$ and which is neglected in the zeroth order adiabatic approximation. The Fourier transformation used reads

$$\hat{S}_0(E,t)=\sum_{n=-\infty}^{\infty} e^{-in\omega t}\hat{S}_{0,n}(E), \tag{7a}$$

$$\hat{S}_{0,n}(E)=\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\omega t}\hat{S}_0(E,t). \tag{7b}$$

Before proceeding we check that this approximation is consistent with the current conservation condition. Substituting Eq. (6) into Eq. (5) and performing the inverse Fourier transformation we arrive at Eq. (2).

Equation (6) corresponds to the *frozen* scattering matrix approximation. Within this approximation the stationary scattering matrix (with parameters dependent on time) completely characterizes the time-dependent scattering. This approximation is exact if the scattering matrix $\hat{S}_0$ is independent of the electron energy $E$ within the relevant energy interval.[53]

### *2. First order approximation*

To first order in the pump frequency $\omega$ we can represent the Floquet matrix with the help of the frozen scattering matrix, its energy derivatives and a matrix $\hat{A}$. In general the matrix $\hat{A}$ cannot be expressed in terms of the stationary scattering matrix $\hat{S}_0$ and it has to be calculated (similar to $\hat{S}_0$ itself) in each particular case. The advantage of the representation which we introduce is that the matrix $\hat{A}$ has a much smaller number of elements than the Floquet scattering matrix. The matrix $\hat{A}$ depends on only one energy, $E$, and therefore it has $N_r^2$ elements like the stationary scattering matrix $\hat{S}_0$. In contrast, the Floquet scattering matrix $\hat{S}_F$ depends on two energies $E$ and $E_n=E+n\hbar\omega$, and therefore has $\sim(2n_{\max}+1)^2N_r^2$ relevant elements. Here $n_{\max}$ is the maximum number of energy quanta $\hbar\omega$ absorbed/emitted by an electron interacting with the scatterer which we should take into account to correctly describe the scattering process. For small amplitude driving we have $n_{\max}\approx 1$. In contrast, if the parameters vary with a large amplitude then $n_{\max}\gg 1$. We represent the Floquet matrix in the form[54]

$$\hat{S}_F(E_n,E)=\hat{S}_{0,n}(E)+\frac{n\hbar\omega}{2}\frac{\partial\hat{S}_{0,n}(E)}{\partial E}+\hbar\omega\hat{A}_n(E)+O(\omega^2), \tag{8a}$$

$$\hat{S}_F(E,E_n)=\hat{S}_{0,-n}(E)+\frac{n\hbar\omega}{2}\frac{\partial\hat{S}_{0,-n}(E)}{\partial E}+\hbar\omega\hat{A}_{-n}(E)+O(\omega^2). \tag{8b}$$

Note that the right-hand side (RHS) of Eq. (8a) is defined with respect to the incoming energy of carriers, while in Eq. (8b) the RHS is expressed in terms of the energy of outgoing particles. To first order in $\omega$, the case of interest here, these two representations are fully consistent. Going from one representation to the other, one needs to take into account that the contribution from the first term on the RHS depends on the choice of the reference energy. The second and the third terms being themselves proportional to $\omega$ do not depend on this choice.

In Eq. (8) we have introduced a new matrix $\hat{A}(E,t)$ with Fourier coefficients $\hat{A}_n(E)$. The current conservation condition (5) leads to the following equation for the matrix $\hat{A}(E,t)$:[54]

$$\hbar\omega[\hat{S}_0^{\dagger}(E,t)\hat{A}(E,t)+\hat{A}^{\dagger}(E,t)\hat{S}_0(E,t)]=\frac{1}{2}\mathcal{P}\{\hat{S}_0^{\dagger};\hat{S}_0\}, \tag{9a}$$

$$\mathcal{P}\{\hat{S}_0^{\dagger};\hat{S}_0\}=i\hbar\left(\frac{\partial\hat{S}_0^{\dagger}}{\partial t}\frac{\partial\hat{S}_0}{\partial E}-\frac{\partial\hat{S}_0^{\dagger}}{\partial E}\frac{\partial\hat{S}_0}{\partial t}\right). \tag{9b}$$

Note the matrix $\mathcal{P}\{\hat{S}_0^{\dagger};\hat{S}_0\}$ is traceless. Another but equivalent representation can be obtained from Eq. (9a) multiplying both sides from the left by $\hat{S}_0$ and from the right by $\hat{S}_0^{\dagger}$, and by taking into account that because of the unitarity condition, Eq. (2), we have $S_0 d[S_0^{\dagger}]=-d[S_0]S_0^{\dagger}$.

We remark that Eq. (9) tells us that the expansion in powers of $\omega$ is, in fact, an expansion in powers of $\hbar\omega/\delta E$, where $\delta E$ is the energy scale over which the scattering matrix $\hat{S}_0(E)$ changes significantly. Therefore, the frequency $\omega$ can be considered as slow and the expansion (8) can be relevant if

$$\hbar\omega \ll \delta E. \tag{10}$$

Consequently, to characterize scattering with an accuracy of order $\omega$ one needs to determine the matrix $\hat{A}$. Equation (9) defines only the anticommutator of two matrices, $\hat{S}_0$ and $\hat{A}$, and it is insufficient to determine the matrix $\hat{A}$.

By analogy with Eq. (6) we can express the Floquet scattering matrix elements up to first order in driving frequency in terms of the Fourier coefficients of some effective matrix. We introduce two matrices $\hat{S}_{\text{in}}$ and $\hat{S}_{\text{out}}$ defined with respect to incoming and outgoing energies, respectively,

$$\hat{S}_{\text{in}}(E,t) = \hat{S}_0(E,t) + \frac{i\hbar}{2}\frac{\partial^2 \hat{S}_0}{\partial t\,\partial E} + \hbar\omega\hat{A}(E,t), \tag{11a}$$

$$\hat{S}_{\text{out}}(E,t) = \hat{S}_0(E,t) - \frac{i\hbar}{2}\frac{\partial^2 \hat{S}_0}{\partial t\,\partial E} + \hbar\omega\hat{A}(E,t). \tag{11b}$$

Performing the Fourier transformation of Eqs. (11) and comparing the result with Eqs. (8) we find

$$\hat{S}_F(E_n,E) = \hat{S}_{\text{in},n}(E) + O(\omega^2), \tag{12a}$$

$$\hat{S}_F(E,E_n) = \hat{S}_{\text{out},-n}(E) + O(\omega^2). \tag{12b}$$

We emphasize that the matrices $\hat{S}_{\text{in}}(t)$ and $\hat{S}_{\text{out}}(t)$ are not *scattering* matrices because they are not unitary: Their Fourier coefficients just define the corresponding matrix elements of the Floquet scattering matrix according to Eq. (12). Nevertheless these matrices conserve the current "on average," i.e., after integrating over the time period $\mathcal{T}$:

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \hat{S}_{\text{in}}^{\dagger}(E,t)\hat{S}_{\text{in}}(E,t) = \hat{I} + O(\omega^2), \tag{13a}$$

$$\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \hat{S}_{\text{out}}^{\dagger}(E,t)\hat{S}_{\text{out}}(E,t) = \hat{I} + O(\omega^2). \tag{13b}$$

Now we use Eq. (9) to analyze the general properties of the matrix $\hat{A}$ which are due to the microreversibility of the Schrödinger equation with a periodically oscillating potential.

### B. Microreversibility and magnetic field symmetry of the Floquet scattering matrix

We start with the stationary case when the single particle Hamiltonian (and correspondingly the scattering matrix) is independent of time and recall some properties of the stationary scattering matrix.[44,48]

The microreversibility of the equation of motion (i.e., the Schrödinger equation) puts some constraints onto the scattering matrix. To make the notation more convenient let us arrange the incoming/outgoing wave functions at all the leads into the vector column

$$\hat{\psi} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_{N_r} \end{pmatrix}. \tag{14}$$

Then Eq. (1) can be written in the compact form

$$\hat{\psi}^{(\text{out})} = \hat{S}_0\hat{\psi}^{(\text{in})}. \tag{15}$$

The microreversibility condition (i.e., the invariance with respect to the time inversion) for the spinless case under consideration leaves the solution of the scattering problem invariant under the simultaneous inversion of the direction of movement, the inversion of a possibly present magnetic field $H$, and the replacement $\Psi \to \Psi^*$. Therefore, the evolution of the two wave functions, namely, $\Psi(E,H,t)$ and $\Psi^*(E,-H,-t)$, is exactly the same and is described by the same scattering matrix $\hat{S}_0$. Taking into account that the inversion of the direction of movement turns the outgoing waves to incoming ones and vice versa we can write the following equations for the starting solution and its transform:

$$\hat{\psi}^{(\text{out})}(E,H) = \hat{S}_0(E,H)\hat{\psi}^{(\text{in})}(E,H), \tag{16a}$$

$$[\hat{\psi}^{(\text{in})}(E,-H)]^* = \hat{S}_0(E,H)[\hat{\psi}^{(\text{out})}(E,-H)]^*. \tag{16b}$$

From the unitarity condition (2) it follows that $\hat{S}_0^{-1} = \hat{S}_0^{\dagger}$. Therefore we can rewrite Eq. (16a) as follows: $\hat{\psi}^{(\text{in})}(E,H) = \hat{S}_0^{\dagger}(E,H)\hat{\psi}^{(\text{out})}(E,H)$. Comparing the last with Eq. (16b) we arrive at the required condition[44]

$$\hat{S}_0(-H) = \hat{S}_0^{T}(H), \tag{17}$$

where the upper index "$T$" denotes transposition.

Next we consider a periodically driven scattering problem. As we saw microreversibility requires the scattering matrix to be symmetric with respect to the interchange of incoming and outgoing channels. For the Floquet scattering matrix these channels are characterized by both the lead index and the number $n$ showing how many energy quanta $\hbar\omega$ an electron absorbs/emits during the scattering process. In addition, to get the required symmetry condition, we have to take into account that the parameters $p_i$ of the Hamiltonian depend on time. We suppose they change periodically in time with the same frequency $\omega$, and with possible relative phase shifts $\varphi_i$:

$$p_i(t) = p_{i,0} + p_{i,1}\cos(\omega t + \varphi_i). \tag{18}$$

In such a case time reversal implies the inversion of the sign of all the phase shifts $\varphi_i$. Therefore, the Floquet scattering matrix elements are subject to the following fundamental symmetry:

$$S_{F,\alpha\beta}(E,E_n;H,\varphi) = S_{F,\beta\alpha}(E_n,E;-H,-\varphi) \tag{19a}$$

or, in a matrix form,

$$\hat{S}_F(E;-H,-\varphi) = \hat{S}_F^{T}(E;H,\varphi). \tag{19b}$$

Here $E$ is the Floquet energy [see Eq. (2)]; $\varphi$ denotes the set of all the $\varphi_i$.

Next we derive the symmetry conditions for the matrix $\hat{A}$ entering Eq. (8). Our definition of the phases $\varphi_i$ [see Eq. (18)] implies that the frozen scattering matrix $\hat{S}_0(E,t)$ (i.e., the stationary scattering matrix with parameters dependent on time $\hat{S}_0(E,t) = \hat{S}_0[E,p_i(t)]$) possesses the following symmetry:

$$\hat{S}_0(E,-t;H,-\varphi) = \hat{S}_0(E,t;H,\varphi). \tag{20}$$

Equation (9) gives us

$$\hat{A}(E,-t;H,-\varphi) = -\hat{A}(E,t;H,\varphi). \tag{21}$$

Correspondingly, for the Fourier coefficients, we have the following:

$$\hat{S}_{0,n}(E;H,-\varphi) = \hat{S}_{0,-n}(E;H,\varphi), \tag{22a}$$

$$\hat{A}_n(E;H,-\varphi) = -\hat{A}_{-n}(E;H,\varphi). \tag{22b}$$

Substituting the equations given above into the adiabatic expansion (8) and taking into account the microreversibility condition (19), we find the required symmetry condition for the matrix $\hat{A}(t)$:

$$\hat{A}(-H) = -\hat{A}^T(H). \tag{23}$$

In particular, in the absence of magnetic fields, $H=0$, the diagonal elements of $\hat{A}$ vanish. That was previously shown in Ref. 54. Alternatively Eq. (23) can be obtained directly from Eq. (9) exploiting the symmetry condition Eq. (17) and the unitarity of the frozen scattering matrix $\hat{S}_0(E,t)$.

The symmetry properties of the residual Floquet matrix $\hat{A}$ are completely different from that of the stationary scattering matrix $\hat{S}_0$. The residual Floquet matrix $\hat{A}$ reflects directly the most important differences between an adiabatic scattering process at a periodically evolving scatterer and a strictly stationary scattering process.

Notice, in contrast to some previous considerations (see, e.g., Refs. 14 and 29) we do not impose any restrictions on the spatial symmetry of the quantum pump. Therefore our central result, (23) is quite general and holds for any slowly oscillating scatterer. In the Appendix we illustrate how one can calculate the residual scattering matrix $\hat{A}$ for several simple scatterers.

## III. MAGNETIC FIELD SYMMETRY OF THE DC CURRENT FLOWING THROUGH THE SLOWLY DRIVEN SCATTERER

Now we use the results of the previous section to analyze the dc current through the mesoscopic sample with periodically varying parameters. We will consider two mechanisms which can give rise to such a current. The first mechanism is a quantum pump effect consisting in rectifying of time-dependent currents generated by the non stationary scatterer.[54] Second we permit a constant in time difference of electrochemical potentials/temperatures between the different reservoirs. The last is important, because the widely investigated situation with reservoirs being at the same electrochemical potential actually hides some physics underlying the quantum pump effect.

The dc current $I_\alpha$ flowing from the scatterer to the reservoir in the lead $\alpha$ can be calculated as follows:[53]

$$I_\alpha = \frac{e}{h}\int_0^\infty dE\left\{\sum_{\beta=1}^{N_r}\sum_n |S_{F,\alpha\beta}(E_n,E)|^2 f_{0,\beta}(E) - f_{0,\alpha}(E)\right\}. \tag{24}$$

Here $f_{0,\alpha}$ is the electron distribution function for the reservoir $\alpha$. We assume that the reservoirs are in a stationary equilibrium state with possibly different electrochemical potentials $\mu_\alpha$ and temperatures $T_\alpha$. Then $f_{0,\alpha}$ is the Fermi distribution function

$$f_{0,\alpha}(E) = \frac{1}{1+e^{(E-\mu_\alpha)/k_B T_\alpha}}, \tag{25}$$

with $k_B$ being the Boltzmann constant. Substituting the adiabatic expansion (8) into Eq. (24) and performing the inverse Fourier transformation we find the current up to linear in $\omega$ terms as follows:[54]

$$I_\alpha = \int_0^\infty dE \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_\beta \left\{ f_{0,\beta}(E)\frac{dI_{\alpha\beta}(E,t)}{dE} + \frac{e}{h}|S_{0,\alpha\beta}(E,t)|^2[f_{0,\beta}(E) - f_{0,\alpha}(E)]\right\}, \tag{26}$$

where $dI_{\alpha\beta}/dE$ is a spectral current driven by the non stationary scatterer from lead $\beta$ into lead $\alpha$:

$$\frac{dI_{\alpha\beta}}{dE} = \frac{e}{h}\left(2\hbar\omega \mathrm{Re}[S^*_{0,\alpha\beta}A_{\alpha\beta}] + \frac{1}{2}\mathcal{P}\{S_{0,\alpha\beta};S^*_{0,\alpha\beta}\}\right). \tag{27}$$

Here $\mathrm{Re}[X]$ is the real part of $X$; the function $\mathcal{P}\{X;Y\}$ is defined in Eq. (9b). The spectral currents $dI_{\alpha\beta}/dE$ are subject to the following conservation law:[54]

$$\sum_{\alpha=1}^{N_r} \frac{dI_{\alpha\beta}(E,t)}{dE} = 0. \tag{28}$$

Using Eq. (28) and the unitarity of the frozen scattering matrix $\Sigma_\alpha |S_{0,\alpha\beta}|^2 = \Sigma_\beta |S_{0,\alpha\beta}|^2 = 1$, one can easily check that the current $I_\alpha$ is conserved: $\Sigma_\alpha I_\alpha = 0$. Further, using the symmetry conditions (17) and (23), and rearranging the terms in Eq. (26) we divide the current into the even $I_\alpha^{(\mathrm{ev})}(H) = I_\alpha^{(\mathrm{ev})}(-H)$ and odd $I_\alpha^{(\mathrm{od})}(H) = -I_\alpha^{(\mathrm{od})}(-H)$, in magnetic field parts

$$I_\alpha^{(\mathrm{ev})}(H) = \frac{e}{h}\int_0^\infty dE \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_\beta \left\{ [f_{0,\beta} - f_{0,\alpha}] \times \left(\frac{|S_{0,\alpha\beta}|^2 + |S_{0,\beta\alpha}|^2}{2} + \hbar\omega \mathrm{Re}[S^*_{0,\alpha\beta}A_{\alpha\beta} - S^*_{0,\beta\alpha}A_{\beta\alpha}]\right) + [f_{0,\beta} + f_{0,\alpha}]\frac{\mathcal{P}\{S_{0,\alpha\beta};S^*_{0,\alpha\beta}\} + \mathcal{P}\{S_{0,\beta\alpha};S^*_{0,\beta\alpha}\}}{4}\right\}, \tag{29a}$$

$$I_{\alpha}^{(\text{od})}(H) = \frac{e}{h}\int_0^{\infty} dE \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\beta} \Bigg\{ [f_{0,\beta} - f_{0,\alpha}] \times \left( \frac{|S_{0,\alpha\beta}|^2 - |S_{0,\beta\alpha}|^2}{2} + \frac{\mathcal{P}\{S_{0,\alpha\beta}; S_{0,\alpha\beta}^*\} - \mathcal{P}\{S_{0,\beta\alpha}; S_{0,\beta\alpha}^*\}}{4} \right) + [f_{0,\beta} + f_{0,\alpha}]\hbar\omega \mathrm{Re}[S_{0,\alpha\beta}^* A_{\alpha\beta} + S_{0,\beta\alpha}^* A_{\beta\alpha}] \Bigg\}. \tag{29b}$$

To show that both these currents are separately conserved, i.e., that $\Sigma_{\alpha} I_{\alpha}^{(\text{ev})}=0$ and $\Sigma_{\alpha} I_{\alpha}^{(\text{od})}=0$, one can use the relations[54]

$$4\hbar\omega \sum_{\alpha=1}^{N_r} \mathrm{Re}[S_{0,\alpha\beta}^* A_{\alpha\beta}] = \mathcal{P}\{\hat{S}_0^{\dagger}; \hat{S}_0\}_{\beta\beta}, \tag{30a}$$

$$4\hbar\omega \sum_{\beta=1}^{N_r} \mathrm{Re}[S_{0,\alpha\beta}^* A_{\alpha\beta}] = \mathcal{P}\{\hat{S}_0; \hat{S}_0^{\dagger}\}_{\alpha\alpha}, \tag{30b}$$

which follow from Eq. (9a).

In a general multiterminal situation, i.e., if not all the reservoirs are at the same potential (temperature), the main contributions to both the even $I_{\alpha}^{(\text{ev})}$ and the odd $I_{\alpha}^{(\text{od})}$ currents are proportional to the conductances $(e^2/h)|S_{0,\alpha\beta}|^2$ averaged over time. The nonstationarity results only in small corrections. However in the two terminal case the odd in magnetic field dc current $I^{(\text{od})}$ has no contribution coming from the conductances, see the Appendix. The current $I^{(\text{od})}$ is linear in $\omega$ and it is entirely due to the non-adiabaticity of the pump scattering processes. Therefore a two-terminal setup is the most appropriate for the detection of an adiabatic quantum pump effect.

Notice, the odd in magnetic field current $I_{\alpha}^{(\text{od})}(H)$ consists of two parts. One of them is present even if all the reservoirs have the same chemical potentials and temperatures. While the second contribution arises if the electron reservoirs are at different conditions. For instance, if there is a small dc voltage $V$ then the second contribution to $I_{\alpha}^{(\text{od})}(H)$ is linear in both the voltage $V$ and the pump frequency $\omega$.

## IV. CONCLUSION

In this work we analyze the scattering properties of a periodically driven mesoscopic scatterer. Traversing such a scatterer an electron can gain or lose one or several energy quanta $\hbar\omega$ and thus can change its energy. Therefore, generally the scattering matrix of a periodically driven mesoscopic scatterer depends on two energies, incoming and outgoing. We show that at low driving frequency $\omega \to 0$ one can introduce effective matrices depending on only one energy, either incoming or outgoing [see Eq. (11)], which approximates accurately the Floquet scattering matrix up to terms of order $\omega$ [see Eq. (12)]. We introduce two effective matrices $\hat{S}_{\text{in}}$ and $\hat{S}_{\text{out}}$, which are not unitary. Nevertheless each of them conserves the current after averaging over a driving cycle.

The matrices $\hat{S}_{\text{in}}$ and $\hat{S}_{\text{out}}$ are the sum of a frozen scattering matrix and a matrix which determines the linear in $\omega$ part. The last is responsible for the quantum pump effect[8] and it consists of two contributions. The first one is the second derivative of the frozen scattering matrix $\hat{S}_0(t)$. The second contribution is defined by an in principle new matrix $\hat{A}$. In particular, the matrix $\hat{A}$ has a symmetry with respect to magnetic field reversal, Eq. (23), that is opposite to that of the stationary (frozen) scattering matrix, Eq. (17). In contrast to the stationary scattering matrix $\hat{S}_0$ the residual Floquet matrix $\hat{A}$ reflects directly the chirality of the pumping process.

Using the adiabatic representation Eq. (12) for the Floquet scattering matrix we examine the dc current flowing through the two terminal (many channels) mesoscopic sample. We divide the current into parts with definite symmetry properties with respect to a magnetic field and/or a voltage inversion.

As it is known in the stationary case the dc current through the coherent two terminal sample is an even function of a magnetic field. On the other hand, the periodically driven scatterer shows an odd in magnetic field, linear in $\omega$ current, (A24b), which is due to the quantum pump effect. The odd in applied voltage part of this current is proportional to the residual Floquet matrix $\hat{A}$. The matrix $\hat{A}$ reflects the interplay of absorbing/emitting of energy quanta $\hbar\omega$ with quantum mechanical interference inside the scatterer. For instance, for a pointlike scatterer (without the space for interference inside) the matrix $\hat{A}$ is identically zero. Our work suggests that additional experiments which investigate a driven mesoscopic conductor in a less symmetric setup, i.e., with reservoirs having different electrochemical potentials or temperatures, might be useful to reveal the presence of a quantum pump effect.

## ACKNOWLEDGMENTS

We thank M. L. Polianski for a critical reading of the manuscript. This work was supported by the Swiss National Science Foundation.

## APPENDIX

### The residual scattering matrix $\hat{A}$

In this Appendix we illustrate how one can calculate the linear in $\omega$ corrections to the frozen scattering matrix in the same fashion as the stationary scattering matrix $\hat{S}_0$. According to Eq. (12), at $\omega \to 0$ the Floquet scattering matrix elements are the Fourier coefficients of some matrices $\hat{S}_{\text{in}}/\hat{S}_{\text{out}}$, Eq. (11) which depend on the stationary scattering matrix $\hat{S}_0$ and on the matrix $\hat{A}$. The matrix $\hat{S}_{\text{in}}/\hat{S}_{\text{out}}$ does not possess a definite symmetry, i.e., with respect to a time and/or a magnetic field direction reversal. While the stationary scattering matrix $\hat{S}_0$ and the matrix $\hat{A}$ do. The last circumstance is the motivation why we expressed the current $I_{\alpha}$ in terms of $\hat{S}_0$

and $\hat{A}$ instead of $\hat{S}_{\mathrm{in}}/\hat{S}_{\mathrm{out}}$. On the other hand, for the calculation of the Floquet scattering matrix elements it is more convenient to work in terms of the matrix $\hat{S}_{\mathrm{in}}$ or $\hat{S}_{\mathrm{out}}$.

### *Single δ-function barrier*

Here we consider the Floquet scattering matrix in the limit $\omega \to 0$ of an oscillating pointlike scatterer coupled to two reservoirs via one-channel leads. As we will show for such a scatterer

$$\hat{A} = 0, \tag{A1}$$

and low-frequency scattering up to linear in $\omega$ terms is entirely described by the frozen scattering matrix $\hat{S}_0(t)$, see Eqs. (11) and (12) at $\hat{A}=0$.

To find $\hat{S}_F$ we have to solve the Schrödinger equation with the potential $V(x,t)$ being the delta function $\delta(x)$ multiplied by the amplitude oscillating in time

$$i\hbar \frac{\partial \Psi}{\partial t} = \left( -\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial x^2} + V(x,t) \right)\Psi,$$

$$V(x,t) = \delta(x)[V_0 + 2V_1 \cos(\omega t + \varphi)]. \tag{A2}$$

According to the Floquet theorem the solution of the above equation has the form of Eq. (3). Away from the point $x=0$ the functions $\psi(E_n)$ are the plain waves

$$\psi(E_n) = a_n e^{ik_n x} + b_n e^{-ik_n x}. \tag{A3}$$

The coefficients $a_n$, $b_n$ are determined from the boundary condition at $x=0$:

$$\Psi(x=+0) = \Psi(x=-0),$$

$$\left.\frac{\partial \Psi}{\partial x}\right|_{x=+0} - \left.\frac{\partial \Psi}{\partial x}\right|_{x=-0} = \frac{2m_e}{\hbar^2} V(t)\Psi(x=0). \tag{A4}$$

First, to find $S_{F,LL}$ and $S_{F,RL}$ we consider the plain wave of a unit amplitude with energy $E$ coming from the left (we directed the $x$ axis from the left to the right):

$$\Psi^{(\mathrm{in})}(E,t) = e^{-i(E/\hbar)t} e^{ikx}. \tag{A5}$$

Here $E=\hbar^2 k^2/(2m_e)$. Then the coefficients $a_n^{(\mathrm{out})}$ and $b_n^{(\mathrm{out})}$ for an outgoing wave

$$\Psi^{(\mathrm{out})} = e^{-i(E/\hbar)t} \sum_n e^{-in\omega t}[\theta(x) a_n^{(\mathrm{out})} e^{ik_n x} + \theta(-x) b_n^{(\mathrm{out})} e^{-ik_n x}] \tag{A6}$$

[here $\theta(x)$ is the Heaviside step function: $\theta(x)=1$ at $x>0$ and $\theta(x)=0$ at $x<0$] define the Floquet scattering matrix elements as follows:

$$S_{F,RL}(E_n,E) = \sqrt{\frac{k_n}{k}} a_n^{(\mathrm{out})},$$

$$S_{F,LL}(E_n,E) = \sqrt{\frac{k_n}{k}} b_n^{(\mathrm{out})}. \tag{A7}$$

Substituting the whole wave function $\Psi = \Psi^{(\mathrm{in})} + \Psi^{(\mathrm{out})}$ into the boundary condition (A4) we get the following relations between the different $a_n^{(\mathrm{out})}$ and $b_n^{(\mathrm{out})}$:

$$(k_n + i\kappa_0) a_n^{(\mathrm{out})} = k_n \delta_{n0} - i(\kappa_1 a_{n-1}^{(\mathrm{out})} + \kappa_{-1} a_{n+1}^{(\mathrm{out})}),$$

$$b_n^{(\mathrm{out})} = a_n^{(\mathrm{out})} - \delta_{n0}. \tag{A8}$$

Here we have introduced the following parameters:

$$\kappa_0 = \frac{m_e}{\hbar^2} V_0, \quad \kappa_{\pm 1} = \frac{m_e}{\hbar^2} V_1 e^{\mp i\varphi}. \tag{A9}$$

We solve Eq. (A8) in the adiabatic limit $\omega \to 0$ of interest here. In this limit we can safely expand the wave vector $k_n$ as follows:

$$k_n = k + \frac{n\omega}{v} + O(\omega^2), \tag{A10}$$

where $v=\hbar k/m_e$ is an electron velocity. In addition we use the adiabatic expansion Eq. (12a) and express $\hat{S}_F(E_n,E)$ in terms of the Fourier coefficients of the matrix $\hat{S}_{\mathrm{in}}(E)$. Substituting Eqs. (A7), (A10), and (12a) into Eq. (A8), and ignoring all the terms of order $\omega^2$ and higher we can write

$$(k + i\kappa_0) S_{\mathrm{in},RL,n} + \left( \frac{1}{2} - i\frac{\kappa_0}{2k} \right) \frac{n\omega}{v} S_{0,RL,n}$$
$$= k\delta_{n0} - i(\kappa_1 S_{\mathrm{in},RL,n-1} + \kappa_{-1} S_{\mathrm{in},RL,n+1})$$
$$+ i\frac{\omega}{2vk}[\kappa_1 (n-1) S_{0,RL,n-1} + \kappa_{-1}(n+1) S_{0,RL,n+1}],$$

$$S_{\mathrm{in},LL,n}(E) = S_{\mathrm{in},RL,n}(E) - \delta_{n0}\sqrt{\frac{k_n}{k}}.$$

Performing the inverse Fourier transformation we find the equation for the time-dependent matrix elements of the matrix $\hat{S}_{\mathrm{in}}(E,t)$:

$$S_{\mathrm{in},RL}(E,t) = \frac{k}{k + i\kappa(t)} - \frac{i}{2vk}\frac{k - i\kappa(t)}{k + i\kappa(t)}\frac{\partial S_{0,RL}(E,t)}{\partial t},$$

$$S_{\mathrm{in},LL}(E,t) = S_{\mathrm{in},RL}(E,t) - 1. \tag{A11}$$

Here $\kappa(t) = m_e V(t)/\hbar^2$. We solve these equations perturbatively in the small parameter proportional to $\partial/\partial t \sim \omega \to 0$. To find the matrix elements $S_{\mathrm{in},RR}$ and $S_{\mathrm{in},LR}$ one can either exploit the symmetry condition or solve the same problem but with the unit wave incoming from the right: $\Psi^{(\mathrm{in})}(E,t) = e^{-i(E/\hbar)t} e^{-ikx}$. Up to terms linear in $\omega$, the solution of both problems reads

$$\hat{S}_{\mathrm{in}}(E,t) = \hat{S}_0(E,t) + \frac{i\hbar}{2}\frac{\partial^2 \hat{S}_0(E,t)}{\partial t\, \partial E}. \tag{A12}$$

Here we used $\partial k/\partial E = 1/(\hbar v)$. The stationary matrix is well known:

$$\hat{S}_0 = \frac{k}{k+i\kappa}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \hat{I}. \quad \text{(A13)}$$

Comparing Eqs. (11a) and (A12) we arrive at the announced result, Eq. (A1). Thus, to describe the low frequency scattering on point-like scatterer it is enough to know only the frozen scattering matrix.

Alternatively, one can use Eq. (9) to show that the matrix $\hat{A}$ vanishes for the oscillating $\delta$-function potential. It is because the commutator $\mathcal{P}\{\hat{S}_0^\dagger;\hat{S}_0\}$ is identically zero for the scattering matrix (A13). We can conclude that a point scatterer cannot generate a quantum pump effect since it cannot rectify ac currents [the spectral density $dI_{\alpha\beta}/dE$, Eq. (27), vanishes]. An oscillating scatterer does of course generate ac currents, but these currents are total time derivatives of the charge near the barrier[54] and thus cannot contribute to a dc current.

Note, that the deviation of the effective scattering matrix $\hat{S}_{\text{in}}(E,t)$, Eq. (A12) from the frozen scattering matrix $\hat{S}_0(E,t)$, Eq. (A13), is as small as, at least, $\hbar\omega/E$. For the opaque barrier the deviation is even smaller due to the factor $k/\kappa \ll 1$. For the small oscillating amplitude case the deviation is additionally damped by the factor $\kappa_1/\kappa_0 \ll 1$.

### *Scatterer composed of two point-like barriers*

In this subsection we consider an example of a spatially "extended" scatterer which consists of two pointlike scatterers placed at $x=0$ and $x=L$, respectively. This system is coupled to two reservoirs via single channel leads.

The scattering properties of point scatterers are assumed to be oscillating in time with the same frequency $\omega$. Scattering at the left and on the right barriers is described via the Floquet scattering matrices $\hat{S}_F^L$ and $\hat{S}_F^R$, respectively. Scattering on the whole system is described via the Floquet scattering matrix $\hat{S}_F$.

By analogy with the previous example we consider scattering of a unit wave coming from the left, Eq. (A5). The whole wave function is of a Floquet function type (3) with

$$\psi(E_n) = \begin{cases} \delta_{n0}e^{ikx} + \sqrt{\dfrac{k}{k_n}} S_{F,LL}(E_n,E)e^{-ik_n x}, & x<0, \\ a_n e^{ik_n x} + b_n e^{-ik_n x}, & 0<x<L, \\ \sqrt{\dfrac{k}{k_n}} S_{F,RL}(E_n,E)e^{ik_n(x-L)}, & x>L. \end{cases} \quad \text{(A14)}$$

To find the unknown coefficients we use the boundary conditions which we formulate in terms of scattering matrices $\hat{S}_F^L$ and $\hat{S}_F^R$ assumed to be known:

$$S_{F,LL}(E_n,E) = S_{F,LL}^L(E_n,E) + \sum_m S_{F,LR}^L(E_n,E_m)\sqrt{\frac{k_m}{k_n}} b_m,$$

$$\frac{k_n}{k} a_n = S_{F,RL}^L(E_n,E) + \sum_m S_{F,RR}^L(E_n,E_m)\sqrt{\frac{k_m}{k_n}} b_m,$$

$$\frac{k_n}{k} b_n e^{-ik_n L} = \sum_m S_{F,LL}^R(E_n,E_m)\sqrt{\frac{k_m}{k_n}} a_m e^{ik_m L},$$

$$S_{F,RL}(E_n,E) = \sum_m S_{F,RL}^R(E_n,E_m)\sqrt{\frac{k_m}{k_n}} a_m e^{ik_m L}. \quad \text{(A15)}$$

To simplify this system of equations we use the adiabatic approximation (12) for the Floquet scattering matrices. For this approximation to be valid, the energy quantum $\hbar\omega$ should be small compared with the relevant energy scale for the problem [see Eq. (9)].

In the case under consideration, there are several energy scales. The first one is determined by the energy $E$ of an incoming electron. This scale relates to the deviation of the effective scattering matrices $\hat{S}_{\text{in}}^L$ and $\hat{S}_{\text{in}}^R$ for pointlike scatterers from the corresponding frozen ones. This deviation is of the order of $\hbar\omega/E$. Another energy scale $\delta E$ relates to the spatial size of the system $L$ and arises from the quantum mechanical interference in the region between the scatterers at $0<x<L$. In our case, Eq. (A15), the interference effect is described via the factors $e^{ik_m L}$ which we will expand as follows:

$$e^{\pm ik_m L} = e^{\pm ikL}\left(1 \pm im\frac{\omega}{\omega_L} + O(\omega^2)\right). \quad \text{(A16)}$$

Here $\omega_L = v/L$ defines the distance $\Delta E \sim \hbar\omega_L$ between the quantum levels if the system is decoupled from the reservoirs. The second term in the brackets on the right-hand side of Eq. (A16) is due to an interplay of a quantum-mechanical interference with a quantized energy exchange between the scatterer and an electron traversing it.

The system can be treated as spatially "extended" if $L \gg \lambda_E$, where $\lambda_E = h/\sqrt{2m_e E}$ is the de Broglie wave length for an electron with energy $E$. In such a case the nonadiabatic corrections to the frozen scattering matrix are at least of order $\hbar\omega/\Delta E$. Note if the energy $E$ is close to the energy of a transmission resonance then the corrections will be of order $\hbar\omega/\Gamma$, where $\Gamma$ is the width of the transmission resonance. In contrast, if $L \ll \lambda_E$ then the scatterer can be viewed as pointlike and the nonadiabatic corrections will be as small as $\hbar\omega/E$. Therefore, assuming $L \gg \lambda_E$ we can safely ignore the corrections of order $\hbar\omega/E$ and concentrate on the larger corrections of order $\hbar\omega/\delta E$ with $\delta E = \min\{\Delta E, \Gamma\}$. Since we ignore the terms of order $\hbar\omega/E$ we can replace the Floquet scattering matrices for pointlike scatterers by the corresponding frozen scattering matrices $\hat{S}_F^{R/L}(E_n,E_m) = \hat{S}_{0,n-m}^{R/L} + O(\hbar\omega/E)$ [see Eq. (8)]. To avoid a possible misunderstanding we do not write the energy $E$ as an argument of $\hat{S}_0^{R/L}$ emphasizing that these matrices can be treated as energy independent on the scale of order $\delta E$. Nevertheless they can depend on energy over a much larger scale, say, of order $E$. On the other hand, since we keep the terms of order $\hbar\omega/\delta E$ we use the adiabatic approximation $\hat{S}_F(E_n,E) = \hat{S}_{\text{in},n}(E) + O(\omega^2)$ [see Eq. (12)] for the Floquet scattering matrix of the whole structure.

Using these approximations and substituting Eq. (A16) into the system of equations (A15) and performing the inverse Fourier transformation we arrive at the following time-dependent equations valid up to first order in $\partial/\partial t$:

$$S_{\mathrm{in},LL}(E,t) = \hat{S}^L_{0,LL}(t) + S^L_{0,LR}(t)b(t),$$

$$a(t) = S^L_{0,RL}(t) + S^L_{0,RR}(t)b(t),$$

$$e^{-ikL}\left(b(t) + \frac{1}{\omega_L}\frac{db(t)}{dt}\right) = S^R_{0,LL}(t)e^{ikL}\left(a(t) - \frac{1}{\omega_L}\frac{da(t)}{dt}\right),$$

$$S_{\mathrm{in},RL}(E,t) = S^R_{0,RL}(t)e^{ikL}\left(a(t) - \frac{1}{\omega_L}\frac{da(t)}{dt}\right). \qquad \text{(A17)}$$

Here we introduced the functions $a(t)$ and $b(t)$ defined as follows ($x=a,b$):

$$x(t) = \sum_n e^{-in\omega t}\sqrt{\frac{k_n}{k}}x_n. \qquad \text{(A18)}$$

We consider the terms $da/dt$ and $db/dt$ as small perturbations and solve the system of equations (A17) up to linear order in these corrections terms.

Note, that without the terms $da/dt$ and $db/dt$ the system of equations (A17) is exactly the system of equations which defines the matrix elements of the frozen (stationary) scattering matrix (with the evident replacement $\hat{S}_{\mathrm{in}} \to \hat{S}_0$).

Analogously, to calculate $S_{\mathrm{in},RR}$ and $S_{\mathrm{in},LR}$ we consider the same problem but with the unit wave coming from the right: $\Psi^{(\mathrm{in})}(E,t)=e^{-i(E/\hbar)t}e^{-ik(x-L)}$. It is convenient to represent the results in the matrix form

$$\hat{S}_{\mathrm{in}}(E,t) = \hat{S}_0 - \frac{1}{\omega_L}\hat{M}_L\hat{M}^{-1}\frac{\partial}{\partial t}[\hat{M}^{-1}\hat{M}_R]. \qquad \text{(A19)}$$

Here $\bar{S}_0$ is the frozen scattering matrix

$$\hat{S}_0(E,t) = \hat{M}_0 + \hat{M}_L\hat{M}^{-1}\hat{M}_R. \qquad \text{(A20)}$$

The matrices $\hat{M}$ are all expressed in terms of the scattering matrix elements for the left and right scatterers. They depend on energy through the factor $e^{ikL}$ and on time through the matrices $\hat{S}^L_0$ and $\hat{S}^R_0$:

$$\hat{M}_0 = \begin{pmatrix} S^L_{0,LL} & 0 \\ 0 & S^R_{0,RR} \end{pmatrix}, \quad \hat{M} = \begin{pmatrix} 1 & -S^L_{0,RR} \\ -S^R_{0,LL}e^{i2kL} & 1 \end{pmatrix},$$

$$\hat{M}_L = \begin{pmatrix} 0 & S^L_{0,LR} \\ S^R_{0,RL}e^{ikL} & 0 \end{pmatrix}, \quad \hat{M}_R = \begin{pmatrix} S^L_{0,RL} & 0 \\ 0 & S^R_{0,LR}e^{ikL} \end{pmatrix}. \qquad \text{(A21)}$$

Our aim is to calculate the matrix

$$\hbar\omega\hat{A} = \hat{S}_{\mathrm{in}} - \hat{S}_0 - \frac{i\hbar}{2}\frac{\partial^2\hat{S}_0}{\partial t\,\partial E}$$

[see Eq. (11a)] for the double scatterer structure under consideration. From Eq. (A20) it follows that

$$\frac{i\hbar}{2}\frac{\partial^2\hat{S}_0}{\partial t\,\partial E} = -\frac{1}{2\omega_L}\frac{\partial}{\partial t}[\hat{M}_L(\hat{M}^{-1})^2\hat{M}_R].$$

Then using Eq. (A19) we obtain

$$\hbar\omega\hat{A}(E,t) = \frac{1}{2\omega_L}\left\{\frac{\partial}{\partial t}[\hat{M}_L\hat{M}^{-1}]\hat{M}^{-1}\hat{M}_R - \hat{M}_L\hat{M}^{-1}\frac{\partial}{\partial t}[\hat{M}^{-1}\hat{M}_R]\right\}. \qquad \text{(A22)}$$

Expressing the matrix elements of $\hat{A}$ in terms of the matrix elements of $S^L_0$ and $S^R_0$ we get

$$\hbar\omega A_{LL} = \frac{S_{0,LL} - S^L_{0,LL}}{\omega_L\Delta}\frac{\partial}{\partial t}\left[\ln\left(\frac{S^L_{0,LR}}{S^L_{0,RL}}\right)\right],$$

$$\hbar\omega A_{LR} = \frac{S_{0,LR}}{2\omega_L\Delta}\left\{(2-\Delta)\frac{\partial}{\partial t}\left[\ln\left(\frac{S^L_{0,LR}}{S^R_{0,LR}}\right)\right] + (1-\Delta)\frac{\partial}{\partial t}\left[\ln\left(\frac{S^R_{0,LL}}{S^L_{0,RR}}\right)\right]\right\},$$

$$\hbar\omega A_{RL} = -\frac{S_{0,RL}}{2\omega_L\Delta}\left\{(2-\Delta)\frac{\partial}{\partial t}\left[\ln\left(\frac{S^L_{0,RL}}{S^R_{0,RL}}\right)\right] + (1-\Delta)\frac{\partial}{\partial t}\left[\ln\left(\frac{S^R_{0,LL}}{S^L_{0,RR}}\right)\right]\right\},$$

$$\hbar\omega A_{RR} = \frac{S_{0,RR} - S^R_{0,RR}}{\omega_L\Delta}\frac{\partial}{\partial t}\left[\ln\left(\frac{S^R_{0,RL}}{S^R_{0,LR}}\right)\right]. \qquad \text{(A23)}$$

Here $\Delta = 1 - S^L_{0,RR}S^R_{0,LL}e^{i2kL}$. We see that for the case considered the matrix elements of $\hat{A}$ are proportional to time derivatives as it should be. If there is no magnetic field $H=0$, then the scattering matrices $\hat{S}^L_0$ and $\hat{S}^R_0$ are symmetric in the lead indices $S^{L/R}_{0,\alpha\beta}=S^{L/R}_{0,\beta\alpha}$. In such a case $A_{\alpha\alpha}=0$ and $A_{LR}=-A_{RL}$ that is in agreement with Eq. (23).

### Two terminal many channel scatterer

Next we show that for a two-terminal pump the odd in magnetic field current is independent of the conductance of the pump. Let us consider the scatterer connected to only two reservoirs via, possibly many channel, ballistic leads. We will mark the quantities related to the left and to the right reservoirs via the lower indices "$L$" and "$R$," respectively. Let the left lead have $N_L$ channels, and the right lead have $N_R$ channels: $N_L+N_R=N_r$. We define the currents flowing to the left $I_L$ and to the right $I_R=-I_L$, and the distribution functions for the left $f_{0,L}$ and for the right $f_{0,R}$ reservoirs as follows:

$$I_L = \sum_{\alpha=1}^{N_L} I_\alpha,\; f_{0,\alpha} = f_{0,L},\; 1 \leqslant \alpha \leqslant N_L,$$

$$I_R = \sum_{\alpha=N_L+1}^{N_r} I_\alpha,\ f_{0,\alpha} = f_{0,R},\ N_L + 1 \leqslant \alpha \leqslant N_r.$$

By analogy we redefine the quantities dependent on two indices. For example, the reflection to the left $R_{LL}$ and the spectral current $dI_{RL}/dE$ driven from the left to the right are defined as follows:

$$R_{LL} = \sum_{\alpha=1}^{N_L} \sum_{\beta=1}^{N_L} |S_{0,\alpha\beta}|^2,$$

$$\frac{dI_{RL}}{dE} = \sum_{\alpha=N_L+1}^{N_r} \sum_{\beta=1}^{N_L} \frac{dI_{\alpha\beta}}{dE}.$$

Note that the two terminal transmission is symmetric in reservoirs indices $T_{LR}=T_{RL}$ and it is even in magnetic field. That can be easily seen from their definition, similar to the one given above for $R_{LL}$, and from the unitarity of the scattering matrix $\hat{S}_0$, Eq. (2). In addition from Eq. (28) we get $dI_{L\xi}/dE + dI_{R\xi}/dE = 0$, for $\xi = L, R$. Using the identity[54]

$$\frac{dI_{LL}}{dE} + \frac{dI_{LR}}{dE} \equiv \frac{dI_L}{dE} = \sum_{\alpha=1}^{N_L} i\frac{e}{2\pi}\left(\frac{\partial \hat{S}_0}{\partial t}\frac{\partial \hat{S}_0^\dagger}{\partial E} - \frac{\partial \hat{S}_0}{\partial E}\frac{\partial \hat{S}_0^\dagger}{\partial t}\right)_{\alpha\alpha},$$

performing necessary summations in Eqs. (29), and integrating by parts over time and over energy, we get

$$I_L^{(\mathrm{ev})} = \frac{e}{h}\int_0^\infty dE \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}}\Bigg\{[f_{0,R} - f_{0,L}]\Bigg(T_{LR} + \sum_{\alpha=1}^{N_L}\sum_{\beta=N_L+1}^{N_r} \frac{\mathcal{P}\{S_{0,\alpha\beta}; S_{0,\alpha\beta}^*\} + \mathcal{P}\{S_{0,\beta\alpha}; S_{0,\beta\alpha}^*\}}{4}\Bigg) + \left(-\frac{\partial}{\partial E}[f_{0,R} + f_{0,L}]\right)\frac{i\hbar}{4}\sum_{\alpha=1}^{N_L}\left(\frac{\partial \hat{S}_0}{\partial t}\hat{S}_0^\dagger + \hat{S}_0^\dagger \frac{\partial \hat{S}_0}{\partial t}\right)_{\alpha\alpha}\Bigg\}, \tag{A24a}$$

$$I_L^{(\mathrm{od})} = e\int_0^\infty dE \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}}\Bigg\{\frac{-i}{8\pi}\frac{\partial[f_{0,R} + f_{0,L}]}{\partial E}\sum_{\alpha=1}^{N_L}\left(\frac{\partial \hat{S}_0}{\partial t}\hat{S}_0^\dagger - \hat{S}_0^\dagger\frac{\partial \hat{S}_0}{\partial t}\right)_{\alpha\alpha} + \frac{\omega}{2\pi}[f_{0,R} - f_{0,L}]\sum_{\alpha=1}^{N_L}\sum_{\beta=N_L+1}^{N_r}\mathrm{Re}[S_{0,\alpha\beta}^* A_{\alpha\beta} + S_{0,\beta\alpha}^* A_{\beta\alpha}]\Bigg\}. \tag{A24b}$$

For low driving frequencies $\omega \to 0$ we see that in the two terminal case the part of the dc current that is odd in magnetic field $I^{(\mathrm{od})}(H) = -I^{(\mathrm{od})}(-H)$, is linear in $\omega$ irrespective of whether the reservoirs are at the same conditions ($f_{0,L} = f_{0,R}$) or not ($f_{0,L} \neq f_{0,R}$).


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