

# Deconfined quantum criticality of the $O(3)$ nonlinear $\sigma$ model in two spatial dimensions: A renormalization-group study

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We investigate the quantum phase transition of the  $O(3)$  nonlinear  $\sigma$  model *without Berry phase* in two spatial dimensions. Utilizing the  $CP^1$  representation of the nonlinear  $\sigma$  model, we obtain an effective action in terms of bosonic spinons interacting via compact  $U(1)$  gauge fields. Based on the effective field theory, we find that the bosonic spinons are deconfined to emerge at the quantum critical point of the nonlinear  $\sigma$  model. It is emphasized that the deconfinement of spinons is realized *in the absence of Berry phase*. This is in contrast to the previous study of Senthil *et al.* [Science **303**, 1490 (2004)], where the Berry phase plays a crucial role, resulting in the deconfinement of spinons. It is the reason why the deconfinement is obtained even in the absence of the Berry phase effect that the quantum critical point is described by the  $XY$  (“neutral”) fixed point, not the  $IXY$  (“charged”) fixed point. The  $IXY$  fixed point is shown to be unstable against instanton excitations and the instanton excitations are proliferated. *At the  $IXY$  fixed point it is the Berry phase effect that suppresses the instanton excitations, causing the deconfinement of spinons. On the other hand, the  $XY$  fixed point is found to be stable against instanton excitations because an effective internal charge is zero at the neutral  $XY$  fixed point.* As a result the deconfinement of spinons occurs at the quantum critical point of the  $O(3)$  nonlinear  $\sigma$  model in two dimensions.

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## I. MOTIVATION AND SUMMARY

Nature of quantum criticality is one of the central interests in modern condensed matter physics. Especially, *deconfined quantum criticality* has been proposed in various strongly correlated electron systems such as low dimensional quantum antiferromagnets<sup>1–8</sup> and Kondo systems.<sup>9–12</sup> In the present paper we investigate one deconfined quantum criticality based on the  $O(3)$  nonlinear  $\sigma$  model describing a quantum phase transition from antiferromagnetism to quantum disordered paramagnetism on two dimensional square lattices. This phase transition has been originally analyzed by Bernevig *et al.*<sup>1</sup> In the study the authors got to the conclusion that although the appropriate “off-critical” elementary degrees of freedom are given by either spin 1 excitons (gapped paramagnons) in the quantum disordered paramagnetism and spin 1 antiferromagnons in the antiferromagnetism, at the quantum critical point such excitations should *break up* into more elementary spin 1/2 excitations usually called *spinons*.<sup>1</sup> Thus, spinons emerge as true, deconfined, elementary excitations right at the quantum critical point. This is the precise meaning of the deconfined quantum criticality in the context of quantum antiferromagnetism. In Fig. 1 schematic phase diagram and proposed elementary excitations in the  $O(3)$  nonlinear  $\sigma$  model are shown.

This was challenged by Senthil *et al.*<sup>2</sup> They claimed that since the phase transition in Ref. 1 is supposed to fall into Landau-Ginzburg-Wilson (LGW) paradigm, the spectrum at the quantum critical point should be fully understandable only in terms of spin 1 bosonic degrees of freedom.<sup>2</sup> Senthil *et al.* proposed, as a possible candidate for a deconfined quantum critical point, a direct quantum phase transition between a Neel antiferromagnet and a valence bond solid (VBS) state. In particular, in the Neel state one gets spinon

condensation. In the paramagnetic phase instanton excitations [tunnelling events between energetically degenerate but topologically inequivalent vacua of the  $U(1)$  gauge field in the  $CP^1$  representation of the  $O(3)$  nonlinear  $\sigma$  model] should possibly arise, whose condensation does not allow spinon deconfinement. However, Senthil *et al.* argued that this is not the case at the quantum critical point, *where a Berry phase term makes instantons irrelevant* and accordingly, makes it possible to achieve spinon deconfinement.<sup>2</sup> Apparently, this would prove that it is not possible to get spinon deconfinement *without Berry phase*, which would invalidate the results of Bernevig *et al.*

In the present paper we show that such a contradiction does not exist. We focus our attention to the  $CP^1$  representation of the  $O(3)$  nonlinear  $\sigma$  model *without Berry phase* (that is, the system studied by Bernevig *et al.*), which leads to *the two flavor Abelian Higgs model*. In such a model the basic degrees of freedom are provided by a complex doublet of bosonic spinon fields, plus a compact  $U(1)$  gauge field giving long range interactions among spinons. Using a renormalization group (RG) analysis, we investigate the quantum critical point of the two flavor Abelian Higgs model. To perform an RG analysis, we move to the dual representation of

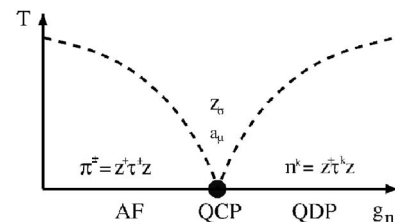


FIG. 1. Schematic phase diagram and proposed elementary excitations in the  $O(3)$  nonlinear  $\sigma$  model with the spin stiffness  $g_n^{-1}$ .

the  $CP^1$  action, in which the basic fields are the vortex fields representing spin 1/2 merons.<sup>2</sup> In the language of meron fields the phase where the meron fields have zero expectation value is associated with the Neel state, while the phase in which the meron fields take a nonzero expectation value (vortex condensation) corresponds to a featureless quantum disordered paramagnetic phase (here, not the VBS owing to the absence of Berry phase). In both phases processes in which an instanton is created with an attached vortex creation (annihilation) operator are relevant. This forbids spinon deconfinement in either off-critical phase.

To analyze the quantum critical point of the system, we first resort to an effective low energy action in Eq. (9), where only phase fluctuations of the vortex fields are allowed and the instanton term is explicitly included. The parameters of such an action are the stiffness parameter of the vortex phase,  $\kappa$ , the instanton fugacity,  $y_m$ , and the phase stiffness of the dual Higgs field,  $\rho$ . An RG analysis permits us to write down the scaling equations in Eq. (12). When specified to the particular case  $D=3$  (that is, a planar model at zero temperature), such equations exhibit two quantum critical points. The former one is at  $\kappa^*=0$ ,  $y_m^*=0$  and  $\rho^*=0$ . *Such a critical point, dubbed inverted XY (IXY) fixed point or “charged” XY fixed point, is identified with the quantum phase transition studied by Senthil et al.*<sup>2</sup> The IXY fixed point is shown to be unstable against instanton excitations ( $y_m \neq 0$ ) and the instanton excitations are proliferated. Since condensation of vortices or instantons does not allow spinon unbinding, this is consistent with the conclusion of Senthil and coworkers, that is, with *the absence of spinon deconfinement without a Berry phase term*. The latter critical point is at  $\kappa^*=0$ ,  $y_m^*=0$  and  $\rho^* \neq 0$ . Remarkably, we find that this new fixed point remains stable against instanton excitations. From this analysis one sees that although off criticality the instantons are relevant everywhere, they become irrelevant at the quantum critical point. This allows spinon deconfinement, which is different from the conclusion by Senthil et al., since this critical point does not coincide with their one. We refer to this fixed point as the charge “neutral” XY one. *The XY fixed point is, instead, identified with the quantum critical point studied by Bernevig et al., thus showing that the deconfinement of spinons takes place even without the Berry phase*. As a result we find that the system is described by the critical field theory in Eq. (13) near the quantum critical point.

Recently, it was reported the result of Monte-Carlo simulation<sup>3</sup> supporting *the existence of deconfined spinons at the quantum critical point of the  $O(3)$  nonlinear  $\sigma$  model in the absence of the contribution of Berry phase*. They claimed that critical fluctuations of bosonic spinons at the quantum critical point result in the nonlocal action of the gauge field and this contribution causes the deconfinement of spinons.<sup>3</sup>

## II. EFFECTIVE ACTION FOR QUANTUM ANTIFERROMAGNETS WITH EASY PLANE ANISOTROPY: ABELIAN HIGGS MODEL WITH TWO FLAVORS

Low energy physics of two dimensional quantum antiferromagnets on square lattices is described by the  $O(3)$  non-

linear  $\sigma$  model *in the presence of Berry phase*<sup>2,3,13</sup>

$$S = S_n + S_B,$$

$$S_n = \int d^3x \frac{1}{2g_n} |\partial_\mu \mathbf{n}|^2,$$

$$S_B = iS \sum_r \epsilon_r \mathcal{A}_r. \quad (1)$$

Here  $\mathbf{n}$  is the unit three component vector representing the Néel order parameter.  $g_n^{-1}$  denotes the spin stiffness. The term  $S_B$  represents the contribution of Berry phase with  $\epsilon_r = (-1)^{r_x+r_y}$ .  $S$  in the Berry phase term is the value of spin 1/2 here.  $\mathcal{A}_r$  is the area enclosed by the curve mapped out by the time evolution of  $\mathbf{n}(\tau)$  on the unit sphere.<sup>2</sup> Representing the spin component in terms of bosonic spinons,  $\mathbf{n} = \frac{1}{2} z_\alpha^\dagger \sigma_{\alpha\beta} z_\beta$  called the  $CP^1$  representation, we obtain an effective bosonic quantum electrodynamics in two space and one time dimensions (QED<sub>3</sub>)<sup>2,3,13</sup>

$$S = S_B + \int d^3x \left[ \frac{1}{2g_n} |(\partial_\mu - ia_\mu) z_\sigma|^2 \right]. \quad (2)$$

Here  $z_\sigma$  is the bosonic spinon with  $\sigma=1, 2$  and  $a_\mu$ , the compact  $U(1)$  gauge field mediating long range interactions among spinons.  $S_B$  is the Berry phase action in association with the time component of the  $U(1)$  gauge field.<sup>2</sup> Following Senthil et al., we consider easy plane anisotropy. In the easy plane limit the bosonic spinor is represented to be  $z_\sigma = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = (1/\sqrt{2}) \begin{pmatrix} e^{i\phi_1} \\ e^{i\phi_2} \end{pmatrix}$ . Inserting this into the above action Eq. (2), we obtain an effective field theory for the  $SU(2)$  quantum antiferromagnet with the easy plane anisotropy,  $N_b=2$  *Abelian Higgs model* in the field theoretic language

$$S = \int d^3x \left[ \frac{\rho}{2} |\partial_\mu \phi_1 - a_\mu|^2 + \frac{\rho}{2} |\partial_\mu \phi_2 - a_\mu|^2 + \frac{1}{2e^2} |\partial \times a|^2 \right]. \quad (3)$$

Here  $N_b$  is the flavor number of bosonic spinons. As mentioned earlier, the flavors are two ( $N_b=2$ ).  $\rho \sim g_n^{-1}$  is the stiffness parameter of the phase fields of spinons. The kinetic energy of the gauge field is introduced with an internal gauge charge  $e$ . The kinetic energy can be generated by integration over high energy spinons. In (2+1)D this term does not affect the phase transitions of this model. This is because  $1/e^2$  has a negative scaling dimension and this kinetic energy term becomes irrelevant in the low energy limit. It is noted again that the Berry phase term  $S_B$  will not be considered any more. *The present paper investigates the deconfinement of spinons at the quantum critical point in the absence of the Berry phase effect*.

## III. ABELIAN HIGGS MODEL WITH ONE FLAVOR: RELEVANCE OF INSTANTON EXCITATIONS AT THE IXY FIXED POINT

We first review the results of Senthil et al.<sup>2</sup> Although the  $N_b=1$  Abelian Higgs model is considered in this section, this



FIG. 2. Feynman diagrams of the self-energy of Higgs fields and that of  $U(1)$  gauge fields.

consideration shows well how Berry phase plays a special role, causing the deconfinement of spinons. We note that this one flavor Abelian Higgs model was also utilized to show the relevance of Berry phase as a toy model in Ref. 2. We consider the following  $N_b=1$  Abelian Higgs model

$$S = \int d^3x \left[ \frac{\rho}{2} |\partial_\mu \phi - a_\mu|^2 + \frac{1}{2e^2} |\partial \times a|^2 \right]. \quad (4)$$

Here  $\phi$  is the phase of a Higgs field and  $a_\mu$ , the compact  $U(1)$  gauge field.  $\rho$  is the phase stiffness parameter and  $e$ , the internal electric charge of the Higgs field. This effective action is usually proposed to describe a superconductor to insulator transition of charged bosons.<sup>14</sup> In this paper we focus our attention on phase fluctuations instead of amplitude fluctuations of Higgs fields. In the case of noncompact  $U(1)$  gauge fields a charged fixed point to govern the superconducting transition is expected to exist.<sup>14</sup> RG equations are obtained to be in one loop level

$$\begin{aligned} \frac{d\rho}{dl} &= (D-2)\rho - \gamma e^2 \rho, \\ \frac{de^2}{dl} &= (4-D)e^2 - \lambda e^4 \end{aligned} \quad (5)$$

with  $\gamma=2/3\pi$  and  $\lambda=1/24\pi$ .<sup>15</sup>  $l$  is a usual scaling parameter and  $D$  denotes a dimension of space and time. We consider the case of  $D=3$ . The last term  $-\gamma e^2 \rho$  in the first equation originates from the self-energy correction of the Higgs field owing to gauge fluctuations while the term  $-\lambda e^4$  in the second equation results from that of the gauge field due to screening of the internal gauge charge by massless excitations of the Higgs fields.<sup>15</sup> In Fig. 2 these processes are explicitly shown by Feynman diagrams. In these RG equations there exist two fixed points; one is the  $XY$  (neutral) fixed point of  $e^{*2}=0$  and  $\rho^*=0$  and the other, the  $IXY$  (charged) fixed point of  $e^{*2}=1/\lambda$  and  $\rho^*=0$ . The  $XY$  fixed point is unstable against nonzero charge  $e^2 \neq 0$  and the RG flows in the parameter space of  $(\rho, e^2)$  converge into the  $IXY$  fixed point owing to  $1 - \gamma e^{*2} = 1 - \gamma/\lambda < 0$ . In other words, the quantum critical point of the superconductor to insulator transition is described by the  $IXY$  fixed point.<sup>14</sup>

In the case of compact  $U(1)$  gauge fields we must admit instanton excitations representing tunnelling events between topologically inequivalent gauge vacua. Performing the standard duality transformation,<sup>2,16,17</sup> we obtain an effective vortex action in the presence of instanton contributions

$$S_{dual} = \int d^3x \left[ \frac{\kappa}{2} |\partial_\mu \theta - c_\mu|^2 + \frac{1}{2\rho} |\partial \times c|^2 + \frac{e^2}{2} c_\mu^2 - y_m \cos \theta \right]. \quad (6)$$

Here  $\theta$  is the phase of a vortex field and  $c_\mu$ , the vortex gauge field mediating interactions between the vortices.  $\kappa$  is the stiffness parameter of the vortex phase field  $\theta$  and  $y_m \sim e^{-S_{inst}}$ , the instanton fugacity with an instanton action  $S_{inst} \sim 1/e^2$ .<sup>17</sup> The vortex gauge field  $c_\mu$  is massive owing to the massless  $U(1)$  gauge field  $a_\mu$  and it can be ignored in the low energy limit.<sup>18</sup> The last term  $-y_m \cos \theta$  appears as a result of instanton excitations.<sup>2,17</sup> When an instanton is created with a probability  $y_m$ , a magnetic flux should be emitted from the instanton owing to the Gauss law. In the presence of Higgs fields the magnetic flux is in the form of a vortex. Thus a vortex creation operator  $e^{-i\theta}$  is attached to an instanton in the form of  $y_m e^{-i\theta}$ . Performing the summation of instanton and anti-instanton excitations in the dilute approximation, the cos potential for vortex fluctuations is obtained.<sup>2,17</sup> In the above sine-Gordon action phase fluctuations of the vortex fields act as instanton (magnetic) potentials to the instantons (Dirac magnetic monopoles). Integrating over vortex phase fluctuations instead of performing the summation of instantons, we obtain Coulomb interactions  $\sim 1/x$  in  $(2+1)D$  between the instantons.<sup>17,19</sup> Interaction strength of the magnetic potential is proportional to  $\kappa^{-1}$ . Thus, the inverse stiffness parameter  $\kappa^{-1}$  plays the same role as the magnetic charge. Owing to the Coulomb interaction the instantons are expected to be deconfined. This implies that tunnelling events are very activated. Gauge fluctuations  $a_\mu$  are very strong and confinement of Higgs fields is obtained. In the following we shall see this using an RG analysis.

Ignoring the vortex gauge field  $c_\mu$ , we obtain the RG equations of the usual sine-Gordon model<sup>4,5</sup>

$$\begin{aligned} \frac{d\kappa}{dl} &= (D-2)\kappa + \beta y_m^2 \frac{1}{\kappa}, \\ \frac{dy_m}{dl} &= \left( D - \alpha \frac{1}{\kappa} \right) y_m, \end{aligned} \quad (7)$$

with positive numerical constants,  $\beta$  and  $\alpha$ .<sup>5</sup> In our consideration their precise values are not important. In these two equations there exist no stable fixed points in  $(2+1)D$  while in  $(1+1)D$  there is a line of fixed points describing the Kosterlitz-Thouless transition as well known.<sup>4</sup> The fixed point of  $\kappa^*=0$  and  $y_m^*=0$  corresponds to the  $IXY$  fixed point of the original Abelian Higgs model. The  $IXY$  fixed point is not stable against instanton excitations  $y_m \neq 0$ . Both the phase stiffness  $\kappa$  and the instanton fugacity  $y_m$  become larger and larger at low energy. If we rewrite the RG equation of the stiffness parameter  $\kappa$  in terms of the magnetic charge  $g$  corresponding to the inverse stiffness parameter  $\kappa^{-1}$  in the presence of Higgs fields, i.e.,  $g = \kappa^{-1}$ , we obtain the same RG equation with Ref. 5 for the magnetic charge,  $dg/dl = -(D-2)g - \beta y_m^3 g^3$ . The effective magnetic charge  $g$  becomes smaller and smaller to be zero owing to the negative bare scaling dimension  $-(D-2)$  in the presence of screening of magnetic charges by instanton excitations. The negative bare

scaling dimension  $-1$  of the magnetic charge in  $D=3$  results from the bare Coulomb interaction  $\sim 1/x$  between instantons. The screening effect is represented by the last term  $-\beta y_m^3 g^3$  and this leads the Coulomb potential to be the Yukawa-type potential  $\sim e^{-x/\lambda}/x$  where  $\lambda$  is the screening length in association with the instanton fugacity. The zero magnetic charge leads the instanton fugacity to go to infinity at low energy. In other words, the instantons are more activated. Depth of the cos potential in Eq. (6) becomes deeper and deeper. Thus, the phase of vortex fields is pinned at one ground position of the cos potential. We conclude that instanton excitations induce vortex condensation  $\langle e^{i\theta} \rangle \neq 0$  and instantons remain deconfined as the case of the pure  $U(1)$  gauge theory in  $(2+1)D$ .<sup>19</sup> Confinement of charged bosons is realized. This is the result in the absence of the Berry phase effect.

If there exists a Berry phase term, the  $IXY$  fixed point can be stable against instanton excitations.<sup>2</sup> The contribution of Berry phase to instantons is given by  $\mathcal{L}_B = i(\pi/2) \sum_n \zeta_n \Delta Q_n$ .<sup>2,3</sup> Here  $n$  labels dual lattices of original lattices.  $\Delta Q_n$  represents an instanton excitation at the dual site  $n$ .  $\zeta_n$  is a fixed integer field and it is given by 0,1,2,3 depending on whether the dual lattice coordinate is (even, even), (even, odd), (odd, even) or (odd, odd).<sup>2,3</sup> *Performing the duality transformation in the presence of this Berry phase term,*<sup>2</sup> we find that the Berry phase gives rise to spatial oscillation in the instanton induced term  $-y_m \cos \theta$ . Another way to say this is that the Berry phase gives destructive interference to instanton excitations. Instantons acquire Berry phases depending on instanton positions and summation of the instantons results in spatial dependence in the cos potential.<sup>2</sup> Thus, the contribution of instanton excitations makes a partition function vanish unless the instanton excitations are quadrupled, i.e.,  $\Delta Q_n \equiv 0 \pmod{4}$ . *Only quadrupled instanton excitations contribute to the partition function and this effect is proven to be irrelevant at the quantum critical point described by the  $IXY$  fixed point.*<sup>2,20</sup> It is the result of Senthil *et al.* in the case of the  $N_b=1$  Abelian Higgs model. In the case of the  $N_b=2$  Abelian Higgs model the same argument can be applied to the  $IXY$  fixed point. As a result the deconfinement of spinons is realized owing to the Berry phase effect. However, we find that one different thing appears in the case of the  $N_b=2$  Abelian Higgs model. We show that there exists another fixed point called the charge neutral  $XY$  fixed point. *The neutral  $XY$  fixed point is shown to be stable against instanton excitations in the absence of Berry phase.*

#### IV. RENORMALIZATION-GROUP ANALYSIS OF ABELIAN HIGGS MODEL WITH TWO FLAVORS: IRRELEVANCE OF INSTANTON EXCITATIONS AT THE $XY$ FIXED POINT

We return to the main problem,  $N_b=2$  Abelian Higgs model Eq. (3). In two spatial dimensions it is well known that the  $O(3)$  nonlinear  $\sigma$  model in the absence of Berry phase,  $S_n$  in Eq. (1) shows a continuous phase transition be-

tween an antiferromagnetically ordered state with  $O(3)$  symmetry breaking and a quantum disordered phase with no symmetry breaking at zero temperature, depending on the spin stiffness parameter  $g_n^{-1}$ .<sup>13</sup> Thus, Eq. (3) derived from Eq. (1) is naturally expected to exhibit the second order quantum phase transition between the two phases, depending on the phase stiffness parameter  $\rho$ . At a sufficiently large stiffness above the critical stiffness  $\rho_c$  the Neel state would emerge. This ordered phase is represented by condensation of spinons,  $\langle z_\sigma \rangle \sim \langle e^{i\phi_\sigma} \rangle \neq 0$ . The spinon condensation leads the  $U(1)$  gauge field to be massive via the Anderson-Higgs mechanism. Integrating over the massive  $U(1)$  gauge field, we obtain an effective field theory in terms of spinon and antispinon confined objects,  $z_1^\dagger z_2 \sim e^{-i(\phi_1 - \phi_2)}$  corresponding to antiferromagnons of spin 1. In the context of the gauge theory this phase corresponds to the Higgs-confinement phase.<sup>21</sup> At a sufficiently small stiffness below the critical stiffness quantum fluctuations of the phase fields  $\phi_\sigma$  destroy the antiferromagnetic long range order and a quantum disordered phase appears with gapped spinons,  $\langle z_\sigma \rangle \sim \langle e^{i\phi_\sigma} \rangle = 0$ . The massive spinon excitations would be confined to form spinon and anti-spinon composites corresponding to massive spin excitons or gapped paramagnons of spin 1.

It should be noted that the quantum disordered phase considered above is fully symmetric and thus featureless *owing to the absence of Berry phase*. If the Berry phase term is introduced in the featureless quantum disordered phase, the Berry phase leads the quantum disordered phase to be a valence bond solid (VBS).<sup>2</sup> This VBS exhibits translational symmetry breaking.<sup>2</sup> In the VBS massive spinons are also confined to form massive meson excitations. These are spin excitons, spin singlet to triplet excitations.<sup>2</sup> In the context of the gauge theory the VBS corresponds to a confinement phase owing to the condensation of instantons as the above quantum disordered phase. In both the antiferromagnetism and the quantum disordered paramagnetism *in the absence of Berry phase* or the VBS *in the presence of Berry phase*, the spinons are always confined and thus, fractional spin  $1/2$  excitations, spinons are not found.

Now we examine the quantum critical point. In the presence of the Berry phase effect it was already discussed in the previous section that the fractional spin  $1/2$  spinon excitations can be deconfined to emerge at the quantum critical point described by the  $IXY$  fixed point. On the other hand, in the absence of Berry phase the  $IXY$  fixed point was shown to be unstable against instanton excitations. Only the confinement of spinons is expected to arise. In this respect a new scenario for deconfinement of spinons is necessary in the present case. It seems to be natural to consider a new fixed point instead of the  $IXY$  fixed point. *Indeed, a new stable fixed point, the charge neutral  $XY$  fixed point is found in the  $N_b=2$  Abelian Higgs model.* The deconfinement scenario in the present paper is completely different from the previous one.<sup>2</sup>

Performing the standard duality transformation of the  $N_b=2$  Abelian Higgs model Eq. (3), we obtain an effective vortex action in the presence of instantons<sup>2,17</sup>

$$\begin{aligned}
 S_{dual} = \int d^3x & \left[ |(\partial_\mu - ic_{1\mu})\Phi_1|^2 + |(\partial_\mu - ic_{2\mu})\Phi_2|^2 \right. \\
 & + m^2(|\Phi_1|^2 + |\Phi_2|^2) + \frac{u}{2}(|\Phi_1|^4 + |\Phi_2|^4) + \frac{1}{2\rho}|\partial \times c_1|^2 \\
 & \left. + \frac{1}{2\rho}|\partial \times c_2|^2 + \frac{e^2}{2}|c_{1\mu} + c_{2\mu}|^2 - z_m(\Phi_1^\dagger\Phi_2^\dagger + \Phi_1\Phi_2) \right]. \quad (8)
 \end{aligned}$$

Here  $\Phi_{1(2)}$  represents the vortex field and  $c_{1(2)\mu}$  the vortex gauge field mediating interactions between vortices.  $m$  is the mass of vortex fields and  $u$ , the coupling strength of local interactions between vortices. The vortex mass is given by  $m^2 \sim \rho - \rho_c$ , where  $\rho_c$  is the critical stiffness parameter.  $z_m \sim e^{-S_{inst}}$  is the instanton fugacity with an instanton action  $S_{inst} \sim 1/e^2$ . Vortex excitations can be considered to be merons (half Skyrmions).<sup>2</sup>  $\Phi_1$  is the vortex in the  $z_1 \sim e^{i\phi_1}$  spinon field and it carries down spin  $\mathbf{n}^z = -1/2$  in the core.<sup>22</sup>  $\Phi_2$  is that in the  $z_2 \sim e^{i\phi_2}$  spinon field and it carries up spin  $\mathbf{n}^z = 1/2$  in the core.<sup>22</sup> Physical picture of the meron excitations is well described in Ref. 2. When an instanton is created with a probability  $z_m$ , a magnetic flux should be emitted from the instanton owing to the Gauss law. Since there exist two kinds of vortices, the vortex creation operator  $\Phi_1^\dagger\Phi_2^\dagger$  is attached to the instanton.<sup>2,17</sup> Here we should not forget the spin degrees of freedom in the meron fields. Then, we can see that the operator  $\Phi_1^\dagger\Phi_2^\dagger$  represents a Skyrmion excitation.<sup>23</sup> The spin up meron  $\Phi_2$  turns into the spin down meron  $\Phi_1$  and vice versa.<sup>23</sup> In this respect an instanton excitation represents a tunnelling event between the spin down and spin up merons, corresponding to a Skyrmion (hedgehog) configuration of the Néel vector fields,  $\mathbf{n}$ . In this dual vortex formulation it is the main problem whether the instanton induced term representing Skyrmion excitations is relevant or not. If this term is relevant in the RG sense, only Skyrmion excitations can appear. The Skyrmion excitations change spin 1 ( $\mathbf{n}_2^z - \mathbf{n}_1^z = 1$ ) in the vortex core. As a result only spin 1 excitations are possible and fractionalized spinon excitations do not occur. In other words, a spin up meron is confined with a spin down meron to appear only in the form of a Skyrmion. Only if the Skyrmion excitation term becomes irrelevant, the meron excitations of spin 1/2 can emerge. The confinement (deconfinement) of merons in the dual vortex description corresponds to the confinement (deconfinement) of spinons in the original Higgs field representation. It is known that the instanton induced term is relevant in both antiferromagnetism and quantum disordered paramagnetism.<sup>2</sup> We study the relevance of the instanton induced term using an RG analysis. In the case of the  $N_b=1$  Abelian Higgs model we have already seen the relevance of instanton excitations.

In passing, we briefly discuss vortex descriptions for possible quantum phases. In the case of  $\rho > \rho_c$  a vortex vacuum  $\langle \Phi_\sigma \rangle = 0$  is energetically favorable. This corresponds to antiferromagnetism where spinons are condensed,  $\langle z_\sigma \rangle \neq 0$ . As mentioned above, the Skyrmion excitation term is relevant and thus  $\langle \Phi_1\Phi_2 \rangle \neq 0$  is obtained. In the opposite case vortex

condensation  $\langle \Phi_\sigma \rangle \neq 0$  is expected to occur. This naturally leads to  $\langle \Phi_1\Phi_2 \rangle \neq 0$ . The vortex condensation results in quantum disordered paramagnetism where spinons are gapped,  $\langle z_\sigma \rangle = 0$  and confined. The quantum critical point emerges at  $\rho = \rho_c$ . In the following we show that instanton excitations become irrelevant at the quantum critical point, thus causing  $\langle \Phi_1\Phi_2 \rangle = 0$ . This implies deconfinement of meron excitations,  $\Phi_1$  and  $\Phi_2$ , corresponding to that of spinon excitations,  $z_1$  and  $z_2$ .

In Eq. (8) the mass term  $(e^2/2)|c_{1\mu} + c_{2\mu}|^2$  resulting from the massless  $U(1)$  gauge field  $a_\mu$  permits us to set  $c_{2\mu} = -c_{1\mu} \equiv -c_\mu$  in the low energy limit. As a result we obtain the following dual vortex action in the low energy limit

$$\begin{aligned}
 S_{dual} = \int d^3x & \left[ \frac{\kappa}{2}|\partial_\mu\theta_1 - c_\mu|^2 + \frac{\kappa}{2}|\partial_\mu\theta_2 + c_\mu|^2 + \frac{1}{2\rho}|\partial \times c|^2 \right. \\
 & \left. - y_m \cos(\theta_1 + \theta_2) \right]. \quad (9)
 \end{aligned}$$

Here  $\theta_{1(2)}$  is the phase field of the vortex field  $\Phi_{1(2)}$ .  $\kappa$  is the stiffness parameter of the vortex phase fields and  $y_m = 2\bar{\Phi}_1\bar{\Phi}_2 z_m$ , the renormalized instanton fugacity with the amplitude of vortex condensation  $\bar{\Phi}_{1(2)} = |\langle \Phi_{1(2)} \rangle|$ . We replaced  $\rho/2$  with  $\rho$ . In the above dual action one massless vortex gauge field  $c_\mu$  appears in contrast to the case of  $N_b=1$ , Eq. (6) where there is no massless vortex gauge field. We note that in the vortex vacuum the massless vortex gauge fields correspond to magnon excitations in the antiferromagnetic long range order. In the following we show that existence of the massless vortex gauge field causes the instanton fugacity  $y_m$  to be zero at the quantum critical point even in the absence of Berry phase.

We first discuss two limiting cases in Eq. (9); one is  $\rho \rightarrow 0$  which allows us to ignore the vortex gauge field and the other,  $y_m \rightarrow 0$  which permits us to ignore the instanton excitations. First, ignoring the vortex gauge field in Eq. (9), we obtain the following RG equations

$$\frac{d\kappa}{dl} = (D-2)\kappa + \beta y_m^2 \frac{2}{\kappa}, \quad (10)$$

$$\frac{dy_m}{dl} = \left( D - \alpha \frac{2}{\kappa} \right) y_m.$$

In the case when the vortex gauge field is ignored, the vortex Lagrangian Eq. (9) is the same as the Lagrangian Eq. (6) except the fact that the flavor number is two in Eq. (9). The effective magnetic charge,  $g = \kappa^{-1}$  is screened by two kinds of vortices. If we rewrite the first RG equation in Eq. (10) in terms of the effective magnetic charge  $g$ , we obtain  $dg/dl = -(D-2)g - 2\beta y_m^2 g^3$ . As shown by the second term, two kinds of vortices screen out the magnetic charge. Eq. (10) is the same as Eq. (7) except the factor 2. In an appendix we briefly sketch how Eq. (10) is derived from Eq. (9) in the absence of the vortex gauge field  $c_\mu$ . In these RG equations both  $\kappa$  and  $y_m$  become larger and larger in the low energy limit as the case of the  $N_b=1$  Abelian Higgs model [Eq. (7)]. There exist no stable fixed points. Instanton excitations are

relevant and only the confinement of meron fields  $\theta_{1(2)}$  (the confinement of spinon fields  $\phi_{1(2)}$ ) is expected to occur. Next, ignoring the instanton excitations, i.e., the compactness of the  $U(1)$  gauge field  $a_\mu$  in Eq. (9), we obtain the same form of Lagrangian as Eq. (4) and get similar RG equations with Eq. (5)<sup>15</sup>

$$\begin{aligned}\frac{d\kappa}{dl} &= (D-2)\kappa - \gamma\rho\kappa, \\ \frac{d\rho}{dl} &= (4-D)\rho - 2\lambda\rho^2.\end{aligned}\quad (11)$$

The factor 2 in the second equation results from the screening effect by two flavors of the vortex fields. In the above we have two fixed points; one is the  $IXY$  fixed point of  $\rho^*=0$  and  $\kappa^*=0$  which is unstable against nonzero value of  $\rho$  and the other, the stable  $XY$  fixed point of  $\rho^*=1/2\lambda$  and  $\kappa^*=0$ .<sup>24</sup> The stability is guaranteed by  $1-\gamma\rho^*=1-\gamma/2\lambda<0$ . Consider one to one correspondence of the RG equations between Eq. (5) and Eq. (11). Note that if we ignore the instanton excitations in Eq. (7), we obtain the fixed point of  $\kappa^*=0$ . This fixed point of the dual vortex action Eq. (6) in the absence of instantons corresponds to the  $IXY$  fixed point of a superconductor to insulator transition in the original Higgs field representation Eq. (4). What Eq. (7) and Eq. (10) tell us is that the  $IXY$  fixed point becomes unstable when we admit the instanton excitations. *The presence of the additional vortex gauge field also makes the  $IXY$  fixed point unstable even in the absence of instanton excitations, resulting in the stable  $XY$  fixed point in the case of the same stiffness parameter for the two phase fields. It is the key question in this paper whether the  $XY$  fixed point in Eq. (11) remains stable or not after including the instanton excitations.*

Admitting both the massless vortex gauge fields and the instanton excitations, we obtain the following RG equations as a combined form of Eq. (10) and Eq. (11)

$$\begin{aligned}\frac{d\kappa}{dl} &= (D-2)\kappa + \beta y_m^2 \frac{2}{\kappa} - \gamma\rho\kappa, \\ \frac{d\rho}{dl} &= (4-D)\rho - 2\lambda\rho^2, \\ \frac{dy_m}{dl} &= \left(D - \alpha \frac{2}{\kappa}\right) y_m.\end{aligned}\quad (12)$$

In these RG equations the  $XY$  fixed point of  $\rho^*=1/2\lambda$ ,  $\kappa^*=0$  and  $y_m^*=0$  is only the stable one against instanton excitations while the  $IXY$  fixed point of  $\rho^*=0$ ,  $\kappa^*=0$  and  $y_m^*=0$  is unstable against both the vortex gauge field excitations  $\rho \neq 0$  and the instanton excitations  $y_m \neq 0$ . It is instructive to rewrite the first RG equation in terms of the effective magnetic charge  $g=\kappa^{-1}$ . It is obtained to be  $dg/dl=-(D-2)g-2\beta y_m^2 g^3+\gamma\rho g$ . *At the  $XY$  fixed point the effective magnetic charge  $g^*=\kappa^{*-1}$  becomes infinite because of  $-(1-\gamma\rho^*)>0$  as the case of noncompact gauge fields.* Since the  $XY$  fixed point is the charge neutral fixed point, it seems to be natural that the effective magnetic charge in the presence of Higgs

fields is infinite at the  $XY$  fixed point.<sup>25</sup> This infinitely large effective magnetic charge makes the instanton excitations irrelevant, i.e.,  $y_m \rightarrow 0$ .

In both the Néel and quantum disordered phases the instanton induced term plays a special role, resulting in only the Skyrmion excitations. Away from the quantum critical point (the  $XY$  fixed point) we find that the instanton fugacity goes to infinity in the low energy limit [Eq. (12)]. Thus, depth of the cos potential in Eq. (9) becomes infinitely deep in the low energy limit and one ground position is chosen for the  $\theta_1+\theta_2$  field. This implies that fluctuations of the  $\theta_1$  field are strongly correlated with those of the  $\theta_2$  field, permanently causing an only ground state of the cos potential for the  $\theta_1+\theta_2$  field. This leads to  $\langle\Phi_1\Phi_2\rangle \neq 0$  in both phases of Eq. (8). Thus, the meron excitations are not possible and only spin 1 excitations are expected to occur.<sup>2,17</sup> However, at the quantum critical point the instanton excitations become irrelevant as shown in Eq. (12). The cos potential in Eq. (9) can be safely ignored at the quantum critical point and the  $\theta_1$  field can fluctuate “independently” with the  $\theta_2$  field. Here “ ” is used in the sense that the  $\theta_1$  field is coupled to the  $\theta_2$  field via the noncompact  $U(1)$  gauge field  $c_\mu$ . As a result the meron excitations carrying fractionalized spin 1/2 are expected to appear.

Now we can reach the critical field theory at the  $XY$  fixed point based on the results of Eq. (12). Inserting the fixed point values of  $\rho^*=1/2\lambda$ ,  $\kappa^*=0$  and  $y_m^*=0$  into Eq. (9), we obtain the critical field theory at the  $XY$  fixed point,  $\mathcal{L}_{dual} = (\kappa^*/2)|\partial_\mu\theta_1 - c_\mu|^2 + (\kappa^*/2)|\partial_\mu\theta_2 + c_\mu|^2 + (1/2\rho^*)|\partial \times c|^2 - y_m^* \cos(\theta_1 + \theta_2) = (1/2\rho^*)|\partial \times c|^2$ . However, this critical field theory is not satisfactory in the sense that there are no terms representing critical fluctuations of vortex fields (merons). *There should be deconfined critical meron fluctuations.* It seems to be natural to introduce the contribution of critical vortex fluctuations coupled to the noncompact vortex gauge fields  $c_\mu$ ,  $\mathcal{L}_v = |(\partial_\mu - ic_\mu)\Phi_1|^2 + |(\partial_\mu + ic_\mu)\Phi_2|^2 + m^{*2}(|\Phi_1|^2 + |\Phi_2|^2) + (u^*/2)(|\Phi_1|^4 + |\Phi_2|^4)$ . Here  $\Phi_{1(2)}$  represents the meron field.  $m^*$  and  $u^*$  are the fixed point values of the mass and self-interaction strength of vortex fields, respectively, at the quantum critical point. Notice that the fixed point value of the vortex mass should be zero,  $m^{*2}=0$ . This zero vortex mass trivially leads to the zero fixed point value of the vortex stiffness parameter, i.e.,  $\kappa^*=0$  at the quantum critical point. This can be easily checked by the relation  $\kappa^* \sim -m^{*2}/u^*=0$  in the mean field (tree) level. The fixed point value  $u^*$  is not explicitly shown in the present paper since we utilize the effective phase action Eq. (9). It is certain that its fixed point value is finite.<sup>14</sup> As a result we reach the following critical field theory

$$\begin{aligned}\mathcal{L}_c &= |(\partial_\mu - ic_\mu)\Phi_1|^2 + |(\partial_\mu + ic_\mu)\Phi_2|^2 + \frac{u^*}{2}(|\Phi_1|^4 + |\Phi_2|^4) \\ &+ \frac{1}{2\rho^*}|\partial \times c|^2.\end{aligned}\quad (13)$$

*The deconfined quantum critical point in the  $O(3)$  nonlinear  $\sigma$  model with the easy plane anisotropy is described by the critical field theory Eq. (13) in terms of the merons interact-*

ing via the noncompact  $U(1)$  gauge fields.

We reemphasize the main difference between our deconfinement scenario and the previous one.<sup>2</sup> In the earlier study<sup>2</sup> the fixed point to govern critical dynamics was not clearly pointed out. But, the study<sup>2</sup> infers that the fixed point is the charged fixed point ( $IXY$  fixed point). *At the charged fixed point the Berry phase plays a special role (causing destructive interference for instanton excitations and making the instantons irrelevant) to result in the deconfinement of critical bosonic spinons.* On the other hand, our present study claims that the true fixed point of the quantum phase transition in the  $O(3)$  nonlinear  $\sigma$  model with the easy plane anisotropy is not the charged fixed point but the charge neutral fixed point ( $XY$  fixed point). *At the neutral  $XY$  fixed point the fixed point value of the internal charge is zero and thus its corresponding magnetic charge is infinite, causing the irrelevance of instantons even in the absence of Berry phase.* The deconfinement of spinons is expected to occur at the quantum critical point.

One may suspect that it is physically meaningful to consider the  $O(3)$  nonlinear  $\sigma$  model without Berry phase. In a different angle this doubt is associated with the question *when the contribution of Berry phase can be ignored.* The following two cases may be the candidates; one is the case of *double layered antiferromagnets* and the other, *the presence of disorders.* It is well known that in two leg ladders the contribution of Berry phase cancels between the legs.<sup>26</sup> The same mechanism works in the double layered quantum antiferromagnets.<sup>27</sup> In this case the mechanism of spinon deconfinement proposed by Senthil *et al.* cannot be applied. Instead, our mechanism may be applicable. One problem is that in the double layered antiferromagnet there exist more flavors than those in the one layer system. However, it is certain that massless vortex gauge fields still remain. The presence of massless vortex gauge fields is expected to cause the deconfinement of spinons. More cautious studies are required near future.

The presence of nonmagnetic disorders leads to random depletion of spins. This results in two important effects. First, the random depletion introduces a random Berry phase term to the nonlinear  $\sigma$  model.<sup>28</sup> Second, it causes a random exchange coupling between spins.<sup>28</sup> We expect that the random Berry phase term is difficult to suppress instanton excitations. The contribution of Berry phase to instantons is given by  $\mathcal{L}_B = i(\pi/2)\sum_n \zeta_n \Delta Q_n^2$ , as mentioned earlier. Remember that in the absence of randomness  $\zeta_n$  is a fixed integer field and it is given by 0, 1, 2, 3 depending on whether the dual lattice coordinate is (even, even), (even, odd), (odd, even) or (odd, odd).<sup>2</sup> *The presence of disorders introduces randomness to  $\zeta_n$ .* In other words, the random depletion of spins results in  $\langle \zeta_n \rangle = 0$ , where  $\langle \dots \rangle$  denotes the average over disorders. The effect of random Berry phase would not be sufficient to suppress the instanton excitations. Thus, the mechanism of spinon deconfinement by Senthil *et al.* would not work in the presence of disorders. One problem is the effect of random exchange couplings. In the limit of weak randomness we may treat the effect of disorders as a random mass term of spinons in the  $CP^1$  representation.<sup>29</sup> Then, the problem becomes whether the randomness is relevant or not.

When the random mass is relevant, the spinons would be localized near the disorders. The quantum criticality is expected to disappear. As a result the spinons would be confined to form antiferromagnetic spin fluctuations of spin 1. This consideration is consistent with increase of antiferromagnetic correlations when nonmagnetic impurities are doped into nonmagnetic states.<sup>28</sup> Our preliminary calculation shows that the deconfined quantum criticality is sustained against sufficiently weak disorders, which is completely consistent with the case of fermionic  $QED_3$  describing the algebraic spin liquid.<sup>6</sup> In this case the mechanism of the deconfined quantum criticality is due to the  $XY$  fixed point resulting from massless vortex gauge fields. The role of nonmagnetic disorders in the deconfined quantum criticality is under our current investigation.

## V. CONCLUSION

In the present study we investigated the deconfinement of bosonic spinons at the quantum critical point of the  $O(3)$  nonlinear  $\sigma$  model without Berry phase in the easy plane limit. The low energy effective field theory in the  $CP^1$  representation is given by the  $N_b=2$  Abelian Higgs model with  $N_b$ , the flavor number of bosonic spinons. *The quantum critical point of the  $N_b=2$  noncompact Abelian Higgs model corresponds to the  $XY$  fixed point while that of the  $N_b=1$  noncompact Abelian Higgs model, the  $IXY$  fixed point. This difference originates from the existence of massless vortex gauge fields in the case of  $N_b=2$ .* We showed that the instanton fugacity becomes zero at the  $XY$  fixed point and thus, instanton excitations do not destabilize the  $XY$  fixed point. As a consequence we find the critical field theory [Eq. (13)] in terms of fractional particles (merons) coupled to noncompact  $U(1)$  gauge fields at the quantum critical point of the  $N_b=2$  Abelian Higgs model in  $(2+1)D$ . On the other hand, the  $IXY$  fixed point was shown to be unstable against instanton excitations. In order to obtain deconfined spinons at the  $IXY$  fixed point, the contribution of Berry phase seems to be crucial.

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## APPENDIX A

We briefly sketch how to derive Eq. (10) from Eq. (9) in the absence of the vortex gauge field. This derivation is based on Ref. 13. We consider the two flavor sine-Gordon action

$$S = \int d^D x \left[ \frac{\kappa}{2} (\partial_\mu \theta_1)^2 + \frac{\kappa}{2} (\partial_\mu \theta_2)^2 - y_m \cos(\theta_1 + \theta_2) \right]. \quad (A1)$$

Here we utilize an Wilsonian approach. We first divide the  $\theta_{\sigma\Lambda}$  field defined on the momentum cutoff  $\Lambda$  into low and

high energy degrees of freedom,  $\theta_{\sigma\Lambda'}$  and  $h_{\sigma}$ , respectively,

$$\begin{aligned}\theta_{\sigma\Lambda}(x) &= \theta_{\sigma\Lambda'}(x) + h_{\sigma}(x), \\ \theta_{\sigma\Lambda'}(x) &= \int_{0 < p < \Lambda'} \frac{d^D p}{(2\pi)^D} e^{ip \cdot x} \theta_{\sigma\Lambda}(p), \\ h_{\sigma}(x) &= \int_{\Lambda' < p < \Lambda} \frac{d^D p}{(2\pi)^D} e^{ip \cdot x} \theta_{\sigma\Lambda}(p).\end{aligned}\quad (\text{A2})$$

Inserting these low and high energy degrees of freedom into the above Eq. (A1) and integrating over the high energy field variables  $h_{\sigma}$ , we obtain the following expression of a partition function

$$\begin{aligned}Z_{\Lambda} &= \int D\theta_{1\Lambda} D\theta_{2\Lambda} e^{-S_{\Lambda}[\theta_{1\Lambda}, \theta_{2\Lambda}]} \\ &= \int D\theta_{1\Lambda'} D h_1 D\theta_{2\Lambda'} D h_2 e^{-S_{\Lambda}[\theta_{1\Lambda'} + h_1, \theta_{2\Lambda'} + h_2]} \\ &= \int D\theta_{1\Lambda'} D\theta_{2\Lambda'} e^{-\tilde{S}_{\Lambda}[\theta_{1\Lambda'}, \theta_{2\Lambda'}]}.\end{aligned}\quad (\text{A3})$$

Here the effective action  $\tilde{S}_{\Lambda}[\theta_{1\Lambda'}, \theta_{2\Lambda'}]$  defined on the momentum cutoff  $\Lambda$  is given by

$$\begin{aligned}e^{-\tilde{S}_{\Lambda}[\theta_{1\Lambda'}, \theta_{2\Lambda'}]} &= \int D h_1 D h_2 \exp \left[ - \int d^D x \left( \frac{\kappa}{2} (\partial_{\mu} \theta_{1\Lambda'})^2 \right. \right. \\ &\quad \left. \left. + \frac{\kappa}{2} (\partial_{\mu} \theta_{2\Lambda'})^2 + \frac{\kappa}{2} (\partial_{\mu} h_1)^2 + \frac{\kappa}{2} (\partial_{\mu} h_2)^2 \right. \right. \\ &\quad \left. \left. - y_m \cos(\theta_{1\Lambda'} + \theta_{2\Lambda'} + h_1 + h_2) \right) \right] \\ &\equiv N \exp \left[ - \int d^D x \left( \frac{\kappa}{2} (\partial_{\mu} \theta_{1\Lambda'})^2 + \frac{\kappa}{2} (\partial_{\mu} \theta_{2\Lambda'})^2 \right) \right] \\ &\quad \times \langle e^{\int d^D x y_m \cos(\theta_{1\Lambda'} + \theta_{2\Lambda'} + h_1 + h_2)} \rangle_{h_1, h_2},\end{aligned}\quad (\text{A4})$$

where  $\langle \dots \rangle_{h_1, h_2}$  represents averaging over the Gaussian action of the high energy fields,  $S_h[h_1, h_2] = \int d^D x ((\kappa/2)(\partial_{\mu} h_1)^2 + (\kappa/2)(\partial_{\mu} h_2)^2)$ , and the constant  $N$  is given by  $N = \int D h_1 D h_2 e^{-S_h[h_1, h_2]}$ . Expanding the exponential to the second order in the fugacity ( $y_m$ ) expansion, we obtain the following expression of the effective action  $\tilde{S}_{\Lambda}[\theta_{1\Lambda'}, \theta_{2\Lambda'}]$ ,

$$\begin{aligned}\tilde{S}_{\Lambda}[\theta_{1\Lambda'}, \theta_{2\Lambda'}] &= \int d^D x \left( \frac{\kappa}{2} (\partial_{\mu} \theta_{1\Lambda'}(x))^2 + \frac{\kappa}{2} (\partial_{\mu} \theta_{2\Lambda'}(x))^2 - y_m \langle \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x) + h_1(x) + h_2(x)) \rangle_{h_1, h_2} \right) \\ &\quad - \int d^D x \int d^D x' \frac{y_m^2}{2} (\langle \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x) + h_1(x) + h_2(x)) \cos(\theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x') + h_1(x') + h_2(x')) \rangle_{h_1, h_2} \\ &\quad - \langle \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x) + h_1(x) + h_2(x)) \rangle_{h_1, h_2} \langle \cos(\theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x') + h_1(x') + h_2(x')) \rangle_{h_1, h_2}).\end{aligned}\quad (\text{A5})$$

Now we evaluate the average of the cos potentials over the Gaussian action  $S_h[h_1, h_2]$  of the high energy fields  $h_1, h_2$ . The term of the first order in the fugacity  $y_m$  is obtained to be

$$\begin{aligned}\langle \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x) + h_1(x) + h_2(x)) \rangle_{h_1, h_2} &= \frac{1}{2} (e^{i\theta_{1\Lambda'}(x) + i\theta_{2\Lambda'}(x)} \langle e^{ih_1(x) + ih_2(x)} \rangle_{h_1, h_2} + \text{H.c.}) \\ &= \exp \left[ - \frac{1}{2} G_{h_1}(0) - \frac{1}{2} G_{h_2}(0) \right] \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x)) \\ &= B_1(0) B_2(0) \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x)).\end{aligned}\quad (\text{A6})$$

Here  $G_{h_{\sigma}}(x)$  is the propagator of the high energy fields, given by  $G_{h_{\sigma}}(x) = (1/\kappa) \int_{\Lambda' < p < \Lambda} [d^D p / (2\pi)^D] e^{ip \cdot x} (1/p^2)$ , and its associated factor  $B_{\sigma}$ ,  $B_{\sigma} = \exp[-\frac{1}{2} G_{h_{\sigma}}(0)]$ .  $G_{h_1} = G_{h_2}$  is trivially shown, resulting in  $B_1 = B_2$ . This is the reason why the factor 2 appears in Eq. (10). Notice from the momentum integral that the quantities,  $G_{\sigma}$  and  $B_{\sigma}$  depend on the momentum cutoff. The terms of the second order in the fugacity  $y_m$  can be calculated in the same way

$$\begin{aligned}&\langle \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x) + h_1(x) + h_2(x)) \cos(\theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x') + h_1(x') + h_2(x')) \rangle_{h_1, h_2} - \langle \cos(\theta_{1\Lambda'}(x) + \theta_{2\Lambda'}(x) + h_1(x) \\ &\quad + h_2(x)) \rangle_{h_1, h_2} \langle \cos(\theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x') + h_1(x') + h_2(x')) \rangle_{h_1, h_2} \\ &= \frac{1}{4} B_1^2(0) [B_1^2(x-x') - 1] B_2^2(0) [B_2^2(x-x') - 1] \cos(\theta_{1\Lambda'}(x) + \theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x) + \theta_{2\Lambda'}(x')) + \frac{1}{4} B_1^2(0) [B_1^{-2}(x-x') - 1] B_2^2(0) \\ &\quad \times [B_2^{-2}(x-x') - 1] \cos(\theta_{1\Lambda'}(x) - \theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x) - \theta_{2\Lambda'}(x')) \approx \frac{1}{4} B_1^2(0) [B_1^2(\xi) - 1] B_2^2(0) [B_2^2(\xi) - 1] \cos(2\theta_{1\Lambda'}(z) \\ &\quad + 2\theta_{2\Lambda'}(z)) + \frac{1}{4} B_1^2(0) [B_1^{-2}(\xi) - 1] B_2^2(0) [B_2^{-2}(\xi) - 1] \left[ 1 - \frac{1}{2} (\xi \cdot \partial \theta_{1\Lambda'}(z) + \xi \cdot \partial \theta_{2\Lambda'}(z))^2 \right],\end{aligned}\quad (\text{A7})$$

where  $z \equiv \frac{1}{2}(x+x')$  and  $\xi \equiv x-x'$ .



The last step in the Wilsonian RG approach is the rescaling in the coordinates  $x$  and the momentum cutoff  $\Lambda'$ . Inserting Eq. (A6) and (A7) into Eq. (A5), and performing the rescaling  $x \rightarrow e^l x'$  in the resulting effective action Eq. (5), we obtain the following expression of the effective action

$$\begin{aligned} S_{\Lambda'}[\theta_{1\Lambda'}, \theta_{2\Lambda'}] &= \int d^D x' e^{Dl} \left[ \frac{\kappa}{2} \left( 1 + \frac{y_m^2}{8\kappa} B_1^2(0) B_2^2(0) A \right) e^{-2l} (\partial_{\mu'} \theta_{1\Lambda'}(x'))^2 + \frac{\kappa}{2} \left( 1 + \frac{y_m^2}{8\kappa} B_1^2(0) B_2^2(0) A \right) e^{-2l} (\partial_{\mu'} \theta_{2\Lambda'}(x'))^2 \right. \\ &\quad \left. - y_m B_1(0) B_2(0) \cos(\theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x')) \right] \\ &= \int d^D x' \left[ \frac{\kappa'}{2} (\partial_{\mu'} \theta_{1\Lambda'}(x'))^2 + \frac{\kappa'}{2} (\partial_{\mu'} \theta_{2\Lambda'}(x'))^2 - y'_m \cos(\theta_{1\Lambda'}(x') + \theta_{2\Lambda'}(x')) \right] \end{aligned} \quad (\text{A8})$$

with  $A = \int d^D \xi [B_1^{-2}(\xi) - 1][B_2^{-2}(\xi) - 1] \xi^2$ . As a result we find the scaling relations between the renormalized and bare couplings

$$\begin{aligned} \kappa' &= e^{(D-2)l} \left( 1 + \frac{y_m^2}{8\kappa} B_1^2(0) B_2^2(0) A \right) \kappa, \\ y'_m &= e^{Dl} B_1(0) B_2(0) y_m. \end{aligned} \quad (\text{A9})$$

The above expressions completely coincide with those in Ref. 13 when the two flavors are reduced to one flavor. Using the cutoff dependent green function  $G_{h_\sigma}(0) \sim (1/\kappa)l$  in  $\Lambda' = e^{-l}\Lambda$ , we obtain the cutoff dependent values,  $B_\sigma(0) = e^{-\alpha(1/\kappa)l}$  and  $A = 8\beta(2/\kappa)l$ , where  $\alpha$  and  $\beta$  are positive numerical constants. Inserting these into Eq. (A9) and expanding the exponentials in the limit of  $l \rightarrow 0$ , we obtain the RG equations, Eq. (10) for the stiffness  $\kappa$  and instanton fugacity  $y_m$ . Notice that the two flavors  $\sigma=1,2$  lead to the numerical factor 2 in Eq. (10).

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<sup>15</sup>From the relation of  $\rho_R = |\psi_R|^2 = Z_\psi^{-1} |\psi_B|^2 = Z_\psi^{-1} \rho_B$  it is necessary to know the wave function renormalization constant  $Z_\psi$ . Here  $\rho$  is a phase stiffness and  $\psi$ , a Higgs field.  $R$  and  $B$  represent *renormalized* and *bare*, respectively. The renormalization factor  $Z_\psi$  can be easily obtained from the self-energy calculation in the first diagram of Fig. 2. The self-energy  $\Sigma(p)$  of the Higgs field is given by  $\Sigma(p) = e^2 \int [d^3 k / (2\pi)^3] (1/|p-k|^2) (2p-k)_\mu D_{\mu\nu}(k) (2p-k)_\nu$ , where  $D_{\mu\nu}(k) = (1/k^2) (\delta_{\mu\nu} - k_\mu k_\nu / k^2)$  is the propagator of gauge fields in the Landau gauge. The reference of H. Kleinert and F. S. Nogueira, Nucl. Phys. B **651**, 361 (2003) shows  $Z_\psi^{-1} = 1 - (2/3\pi) e^2$  leading to  $\gamma$  in Eq. (5). In the same way we can obtain the RG equation for the internal electric charge  $e^2$ . From the relation of  $e_R^2 = Z_a e_B^2$ , we try to find the renormalization factor  $Z_a$  of the U(1) gauge field  $a_\mu$ . It can be derived from the polarization function  $\Pi_{\mu\nu}(q)$  in the second diagram of Fig. 2. It is given by  $\Pi_{\mu\nu}(q) = e^2 \int [d^3 k / (2\pi)^3] [(2q-k)_\mu (2q-k)_\nu / |q-k|^2 |k|^2]$ . The same reference gives  $Z_a = 1 - (1/24\pi) e^2$  which leads to  $\lambda$  in Eq. (5).

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<sup>19</sup>A. M. Polyakov, *Gauge Fields and Strings* (Harwood Academic, New York, 1987), Chap. 4.

<sup>20</sup>In the argument of Ref. 2 the dual vortex action in the presence of instanton excitations is given by  $\mathcal{L}_{dual} = |\partial_\mu \Phi_v|^2 + m_v^2 |\Phi_v|^2 + (u_v/2) |\Phi_v|^4 - y_m (\Phi_v^n + \Phi_v^{Tn})$ . Here  $\Phi_v$  is the vortex field and  $m_v$ ,  $u_v$ , its corresponding mass and self-interaction strength, respec-

tively. The last term is due to instanton excitations, where  $y_m$  is the instanton fugacity. When there is no Berry phase contribution,  $n$  is given by 1. This instanton induced effective potential is known to be relevant at the critical point  $m_v^*=0$  (Ref. 17). On the other hand, the contribution of Berry phase results in  $n=4$  owing to destructive interference of instanton events as discussed in the text. This effective potential is irrelevant at the critical point (Ref. 2).

<sup>21</sup>E. Fradkin and S. H. Shenker, Phys. Rev. D **19**, 3682 (1979).

<sup>22</sup>The  $z$  component spin is given by  $n^z=|\langle z_1 \rangle|^2-|\langle z_2 \rangle|^2$ . The vortex  $\Phi_1$  leads to  $\langle z_1 \rangle=0$  in the core and thus it has down spin  $n^z=-|\langle z_2 \rangle|^2=-1/2$ . Owing to the similar reason the vortex  $\Phi_2$  has up spin  $n^z=|\langle z_1 \rangle|^2=1/2$  in the core.

<sup>23</sup>In the present paper the definition of  $\Phi_2$  differs from that in Ref. 2. If  $\Phi_2$  is transformed to  $\Phi_2^\dagger$ , Eq. (8) is rewritten to be

$$S_{dual} = \int d^3x \left[ |(\partial_\mu - ic_{1\mu})\Phi_1|^2 + |(\partial_\mu + ic_{2\mu})\Phi_2|^2 + m^2(|\Phi_1|^2 + |\Phi_2|^2) + \frac{u}{2}(|\Phi_1|^4 + |\Phi_2|^4) + \frac{1}{2\rho}|\partial \times c_1|^2 + \frac{1}{2\rho}|\partial \times c_2|^2 + \frac{e^2}{2}|c_{1\mu} + c_{2\mu}|^2 - z_m(\Phi_1^\dagger\Phi_2 + \Phi_1\Phi_2^\dagger) \right].$$

This dual vortex action is the same as that in Ref. 2 in the absence of Berry phase, if the vortex gauge field  $c_{2\mu}$  is replaced with  $c_{2\mu}=-c_{1\mu} \equiv -c_\mu$ . The instanton induced terms describe tunnelling events associated with spin flipping in vortex cores. In this respect instantons are nothing but Skyrmions.

<sup>24</sup>The dual vortex Lagrangian of the XY model [ $\mathcal{L}_{XY}=(\rho/2)|\partial_\mu\phi|^2$ ] is obtained to be in the low energy limit,

$$\mathcal{L}_{XY}^{dual} = \frac{\kappa}{2}|\partial_\mu\theta - c_\mu|^2 + \frac{1}{2\rho}|\partial \times c|^2,$$

where  $\theta$  is the phase of vortex fields and  $c_\mu$ , the vortex gauge

field. *This dual Lagrangian is basically the same as Eq. (9) in the absence of instanton excitations.* RG equations are easily obtained to be (Ref. 15)

$$\frac{d\kappa}{dl} = (D-2)\kappa - \gamma\rho\kappa,$$

$$\frac{d\rho}{dl} = (4-D)\rho - \lambda\rho^2.$$

The fixed point of  $\rho^*=0$  and  $\kappa^*=0$  is unstable against nonzero values of  $\rho$ . As a consequence these RG equations have the stable XY fixed point of  $\rho^*=1/\lambda$  and  $\kappa^*=0$ .

<sup>25</sup>Consider the electromagnetic duality of  $eg=1$ , where  $e$  is an internal electric charge and  $g$  a corresponding magnetic charge. If the electric charge becomes small owing to screening by massless charged particles, the magnetic charge gets large.

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<sup>27</sup>A. W. Sandvik, A. V. Chubukov, and S. Sachdev, Phys. Rev. B **51**, R16483 (1995).

<sup>28</sup>N. Nagaosa, A. Furusaki, M. Sigrist, and H. Fukuyama, J. Phys. Soc. Jpn. **65**, 3724 (1996).

<sup>29</sup>Contributions of random exchange couplings may be approximated to be random potentials or random masses of spinons in the  $CP^1$  representation. The total exchange coupling  $J_{ij}^t$  can be considered to be  $J_{ij}^t=J_{ij}+\delta J_{ij}$ , where  $J_{ij}$  is the normal exchange coupling without randomness between sites  $i, j$  and  $\delta J_{ij}$ , the random exchange coupling. The effective Lagrangian for the spinons is usually obtained to be  $\mathcal{L}_z=|(\partial_\mu - ia_\mu)z_\sigma|^2 + m_{rz}^2|z_\sigma|^2 + (u_z/2)(|z_\sigma|^2)^2$  in the continuum limit (Refs. 2 and 3). Here  $m_{rz}$  is the total mass of spinons and  $u_z$ , the strength of local interactions. The total mass is expected to be  $m_{rz}^2 \sim J^t - J_c = J - J_c + \delta J \equiv m_z^2 + V$ , where  $J_c$  is the critical coupling. The normal mass without disorders is given by  $m_z^2 \sim J - J_c$ .  $V \sim \delta J$  can be considered to be a random potential or random mass of spinons.