Optical spectrum of a spin-split two-dimensional electron gas

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In this paper, we examine how spin-orbit interaction induced by the Rashba effect affects the optical spectrum of a two-dimensional electron gas (2DEG). The calculation is carried out on the basis of a standard random-phase approximation for electron-electron interaction. It is found that for a spin-split 2DEG, the spectrum of optical absorption depends strongly on two opticlike plasmon modes caused by inter-spin-orbit excitation. More interestingly, we find that the position and the width of the spectrum relate directly to important spintronic properties. From these theoretical findings, we propose that the Rashba spin splitting can be identified optically and important spintronic coefficients of a 2DEG can be measured via optical experiments.

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In recent years, the investigation of spin-polarized electron systems has become a fast-growing research area in condensed matter physics and electronics, owing to potential applications in future quantum computation and quantum communication.[1] It has been realized that unlike diluted magnetic semiconductors for which an external magnetic field B is required to achieve spin splitting of the carriers, the spin polarization of electrons or holes can also be observed in narrow-gap semiconductor quantum wells even at $B=0$.[2] In such systems, the spontaneous spin splitting of the two-dimensional electron gas (2DEG) can occur via inversion asymmetry of the microscopic confining potential due to the presence of the heterojunction.[3] This effect corresponds to an inhomogeneous surface electric field and, hence, is electrically equivalent to the Rashba spin splitting or Rashba effect.[4] Thus, narrow-gap quantum-well structures (such as InAs- and $In_{1-x}Ga_xAs$-based 2DEG systems) can serve as advanced spin-electronic (or spintronic) devices such as spin-based transistors,[5] waveguides,[6] filters,[7] etc.

At present, one of the most popularly used experimental techniques to identify the Rashba spin splitting (RSS) in a 2DEG is magnetotransport measurements under the condition where the Shubnikov–de Haas (SdH) oscillations are observable. From the periodicity of the SdH oscillations, the electron density in different spin branches can be measured and, from this result, the Rashba parameter can be determined.[8–11] From a technical point of view, there are several drawbacks in using the magnetotransport experiments for the measurement of the spintronic properties. First, relatively high B fields are required, especially for high-density samples, in order to observe the SdH oscillations. Second, other spin effects occurring at $B\neq 0$ (e.g., Zeeman splitting) may mix with the RSS. Third, Ohmic contacts have to be made on the sample to conduct the transport measurements. Hence, in order to carry out sample characterization more accurately and easily, it is more favorable to use optical experiments for the measurement of the spintronic properties. Recently, Xu found that in a spin-splitting 2DEG, two opticlike plasmon modes $\omega_\pm$ can be excited via inter-spin-orbit electronic transition and the frequencies of such collective excitations relate directly to the spintronic coefficients such as electron density in different spin branches and the Rashba parameter.[12] It is well known that in an electron gas system, the electronic transitions through charge- and spin-density oscillations can result in optical absorption and/or emission. As a result, opticlike plasmon modes can also be excited and detected optically from a spin-split 2DEG. Since the publication of Ref. 12, a serious question has been asked about how spin-orbit interaction (SOI) can affect the optical absorption and whether the optical spectrum induced by elementary electronic excitation can be used to determine the spintronic properties of a 2DEG. In this paper, we attempt to answer these questions by examining the effect of electron-electron (e-e) interaction on fast-electron optical spectrum of a spin-split 2DEG.

For a 2DEG formed in the xy plane, the Schrödinger equation including SOI can be solved analytically.[6] Applying the electron wave functions to the e-e interaction induced by the Coulomb potential, we can calculate the bare e-e interaction. We know that for a spin-degenerate 2DEG, the e-e interaction conserves both the linear and spin momentum during an e-e scattering event.[13] For a 2DEG with the RSS, the linear-momentum conservation still holds because the momentum flowing into the interaction should equal that flowing out. However, for a 2DEG in which the RSS depends explicitly on the linear momentum, the requirement of linear-momentum conservation can flip spins of the two colliding electrons. Hence, spin flip may occur in a spin-split 2DEG via e-e interaction. Taking these points into account and considering a case where only the lowest electronic subband is occupied, we can derive the Fourier transform of the matrix element for a bare e-e interaction.[12] Moreover, using the energy spectrum of a spin-split 2DEG, the electron density-density (d-d) correlation function can be obtained, in the absence of e-e screening, as[14,15]

$$\Pi^0_\beta(\Omega,q) = \sum_{\mathbf{k}} \frac{A^\beta_{\mathbf{kq}}[f(E_{\sigma'}(\mathbf{k}+\mathbf{q})) - f(E_\sigma(\mathbf{k}))]}{2[\hbar\Omega + E_{\sigma'}(\mathbf{k}+\mathbf{q}) - E_\sigma(\mathbf{k}) + i\delta]}. \quad (1)$$

Here, $\beta=(\sigma'\sigma)$ is defined for electronic transition from the σ' branch to the σ branch with $\sigma=\pm 1$ referring to different spin branches, $\hbar\Omega$ is the excitation energy, $E_\sigma(\mathbf{k}) = \hbar^2k^2/2m^* + \sigma\alpha k$ is the energy spectrum of a spin-split 2DEG with $\mathbf{k}=(k_x,k_y)$ being the electron wave vector, m^* is the electron effective mass, α is the Rashba parameter which measures the strength of SOI, $\mathbf{q}=(q_x,q_y)$ is the change of electron wave vector during an *e-e* scattering event, and $f(x)$ is the Fermi-Dirac function. Furthermore, $A^\beta_{\mathbf{kq}} = 1+\beta(k + q\cos\theta)/|\mathbf{k}+\mathbf{q}|$ with θ being the angle between $\mathbf{k}$ and $\mathbf{q}$. Thus, under the random-phase approximation, the dynamical dielectric function matrix becomes[12]

$$\epsilon(\Omega,q) = \begin{bmatrix} 1+a_1 & 0 & 0 & a_4 \\ 0 & 1+a_2 & a_3 & 0 \\ 0 & a_2 & 1+a_3 & 0 \\ a_1 & 0 & 0 & 1+a_4 \end{bmatrix}. \quad (2)$$

Here, $\beta=1=(++)$, $2=(+-)$, $3=(-+)$, and $4=(--)$ are defined for different transition channels, $j=1$ and 4 (2 and 3) are for intra-SO (inter-SO) transition, $a_j = -V_qG_q\Pi^0_j(\Omega,q)$, $V_q = 2\pi e^2/\kappa q$ with κ being the dielectric constant of the material, and $G_q = \int dz' \int dz |\psi_0(z')|^2|\psi_0(z)|^2 e^{-q|z'-z|}$ with $\psi_0(z)$ being the ground-state electron wave function along the z direction. In this study, we use a matrix to represent the dielectric function. For a spin-split 2DEG which is essentially a two-level system when only the lowest subband is included, there are four channels for electronic transition (i.e., $j=1$, 2, 3, and 4 defined here) induced by *e-e* interaction. Hence, the dielectric function is a 4×4 matrix. This is similar to a spin-degenerate 2DEG when two electronic subbands are taken into consideration.[16] The inverse dielectric function matrix then takes the form

$$\epsilon^{-1}(\Omega,q) = \begin{bmatrix} 1-a_1^* & 0 & 0 & -a_4^* \\ 0 & 1-a_2^* & -a_3^* & 0 \\ 0 & -a_2^* & 1-a_3^* & 0 \\ -a_1^* & 0 & 0 & 1-a_4^* \end{bmatrix}, \quad (3)$$

where $a_1^* = a_1/a_{14}$, $a_2^* = a_2/a_{23}$, $a_3^* = a_3/a_{23}$, $a_4^* = a_4/a_{14}$, $a_{14} = 1+a_1+a_4$, and $a_{23} = 1+a_2+a_3$. Using Eq. (2), the plasmon modes can be determined by the determinant of the dielectric function matrix $|\epsilon(\Omega,q)| = a_{14}a_{23}$ via $|\epsilon(\Omega,q)| \to 0$.[12] This approach is similar to those dealing with a spin-degenerate 2DEG where the two- or multilevel structures are caused by the presence of more than one electronic subband.[16] In the presence of *e-e* screening the electron *d-d* correlation function becomes

$$\Pi_\beta(\Omega;q) = \sum_\gamma \epsilon^{-1}_{\beta\gamma}(\Omega;q)\Pi^0_\gamma(\Omega;q). \quad (4)$$

Using the Kubo formula in the absence of electronic scattering centers (such as impurities and phonons), the optical spectrum or conductivity for a spin-split 2DEG can be calculated through[15]

$$\sigma(\Omega) = -e^2\Omega \lim_{q\to 0}(1/q^2) \sum_{\sigma',\sigma} \mathrm{Im}\,\Pi_{\sigma'\sigma}(\Omega,q). \quad (5)$$

Here $\sigma(\Omega)$ is induced by current-current correlation via electron interactions with the external electromagnetic field,[17] which normally does not change the wave vector of an electron, and that is why q has to be small in Eq. (5). This is in contrast to conventional dynamic conductivity induced by electronic scattering mechanisms.[18] At the long-wavelength (i.e., $q\to 0$) limit, one finds $\lim_{q\to 0} q^{-2}\,\mathrm{Im}\,\Pi_{++}(\Omega,q) = \lim_{q\to 0} q^{-2}\,\mathrm{Im}\,\Pi_{--}(\Omega,q) = 0$ so that the intra-SO transition does not contribute to the optical spectrum. Moreover, we find that a strong optical absorption can occur via inter-SO transition, especially for transition from a lower $-$ spin branch to a higher $+$ branch. When $q\to 0$ and for low temperatures (i.e., $T\to 0$), considering only the processes for optical absorption (i.e., $\Omega > 0$), the optical conductivity induced by inter-SO transition is given by

$$\sigma(\Omega) = (e^2/16\hbar)\Theta(\omega_- - \Omega)\Theta(\Omega - \omega_+) \times \left[\frac{2[1 + 2\,\mathrm{Re}\,a_2 + 2(\mathrm{Re}\,a_2)^2]}{(1 + \mathrm{Re}\,a_2 + \mathrm{Re}\,a_3)^2 + (\mathrm{Im}\,a_3)^2} - 1 \right], \quad (6)$$

with $\mathrm{Re}\,a_2 = -(\omega_p^2/\omega_0\Omega)\ln[(\omega_+/\omega_-)(\Omega+\omega_-)/(\Omega+\omega_+)]$, $\mathrm{Re}\,a_3 = -(\omega_p^2/\omega_0\Omega)\ln|(\omega_-/\omega_+)(\Omega-\omega_+)/(\Omega-\omega_-)|$, and $\mathrm{Im}\,a_3 = (\pi\omega_p^2/\omega_0\Omega)\Theta(\omega_- - \Omega)\Theta(\Omega-\omega_+)$. Here, the $[\cdots]$ part comes from *e-e* screening, $\Theta(x)$ is a unit-step function, $\omega_p^2 = 2\pi e^2 n_e q/\kappa m^*$ is the plasmon frequency of a spin-degenerate 2DEG, $\omega_0 = 16\pi n_e\hbar/m^*$, $\omega_\pm = 4\alpha\sqrt{\pi n_\pm}/\hbar$, and $n_e = n_+ + n_-$ is the total density of the 2DEG. For $T\to 0$ the electron density in the $\pm$ spin channel is[12] $n_\pm = (n_e/2) \mp (k_\alpha/2\pi)\sqrt{2\pi n_e - k_\alpha^2}$ with $k_\alpha = m^*\alpha/\hbar^2$, which results in

$$\omega_- - \omega_+ = 4m^*\alpha^2/\hbar^3. \quad (7)$$

From Eq. (6), we see immediately that the edges of the spectrum are respectively at $\Omega \sim \omega_\pm = 4\alpha\sqrt{\pi n_\pm}/\hbar$ and the width of the spectrum is $\omega_- - \omega_+ = 4m^*\alpha^2/\hbar^3$. It has been demonstrated[12] that $\omega_\pm$ are frequencies of two opticlike plasmon modes induced by inter-SO electronic transition. Thus, the shape and profile of the optical spectrum of a spin-split 2DEG are determined mainly by inter-SO plasmon excitation.

In this paper, we consider an InGaAs/InAlAs-based 2DEG in which strong RSS has been observed experimentally.[8,9,11,19] The electron effective mass for InGaAs is $m^* = 0.042m_e$ with m_e being the electron rest mass. We take the typical sample parameters for a heterojunction in the calculations. When the total electron density n_e is about 10^{11} cm^{-2}, the Rashba parameter α for the heterojunction can reach up to $(3-4)\times 10^{-11}$ eV m.[8,19] Using these sample parameters, the opticlike plasmon frequency induced by inter-SO transition is of the order $\omega_\pm \sim 5$ THz. In the calculations, the plasmon frequency $\omega_p \sim q^{1/2}$ is taken as 0.5 THz, which is much smaller than $\omega_\pm$.

The dependence of the optical conductivity on total electron density n_e and the Rashba parameter α is shown, respectively, in Figs. 1 and 2. Because the optical absorption coefficient is proportional to optical conductivity,[20] $\sigma(\Omega)$ can

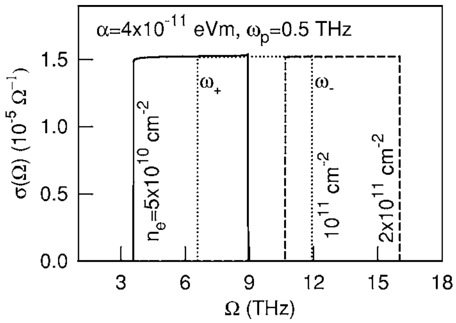

FIG. 1. Optical conductivity $\sigma(\Omega)$ as a function of excitation frequency Ω at a fixed Rashba parameter α for different total electron density n_e as indicated. Here, $\omega_p=(2\pi e^2 n_e q/\kappa m^*)^{1/2}$ and $\omega_\pm=4\alpha\sqrt{\pi n_\pm}/\hbar$ with $n_\pm$ being the electron density in the $\pm$ spin branch.

represent basic features of the optical absorption spectrum. From Figs. 1 and 2, we see that for a spin-split 2DEG the optical spectrum induced by *e-e* interaction via plasmon excitation takes roughly a rectangular shape. Because optical absorption is achieved mainly via inter-SO electronic transition, the position and the width of the spectrum are determined by the frequencies of two opticlike plasmon modes $\omega_\pm$. When $q\to 0$, $\omega_p^2 \sim q \ll \Omega\omega_0$ so the influence of the *e-e* screening on the shape of the spectrum is rather weak except at the absorption edges. From Fig. 1, we note that at a fixed value of α, the blueshift of the spectrum can be achieved via increasing the total electron density, because $\omega_\pm$ increases with n_e. However, varying the electron density of a sample does not change the width of the spectrum. It should be noted that although $\omega_\pm=4\alpha\sqrt{\pi n_\pm}/\hbar$ is a functional form of n_e, $\omega_- - \omega_+$ does not depend on n_e [see Eq. (7)]. From Fig. 2, we find that the strength of SOI affects strongly both the position and the width of the optical spectrum. At a fixed n_e, the increase in α leads to a blueshift and to a broadening of the optical spectrum, because the plasmon frequencies $\omega_\pm$ increase with α and their difference depends on α^2 [see Eq. (7)]. Our numerical results indicate that when $\omega_p < 1$ THz, $\sigma(\Omega)$ depends very little on the value of ω_p, similar to the dispersion relation of the inter-SO plasmon modes $\Omega_\pm$ shown in Ref. 12.

An important conclusion we can draw from these theoretical results is that if we can measure the optical spectrum of a spin-split 2DEG, important spintronic coefficients can be obtained optically. From the width of the spectrum, we can determine the value of the Rashba parameter using Eq. (7). From the position or edges of the spectrum, we can obtain two plasmon frequencies $\omega_\pm=4\alpha\sqrt{\pi n_\pm}/\hbar$ and then the electron densities in the $\pm$ spin branches $n_\pm$. Thus, the total electron density $n_e=n_+ + n_-$ and the spin polarization $P=(n_- - n_+)/n_e$ can be determined straight away. The results shown in the paper indicate that at a $q\to 0$ and $T\to 0$ limit, the amplitude of $\sigma(\Omega)$ induced by elementary electronic excitation is roughly a universal value $\sigma(\Omega)\simeq e^2/16\hbar$. From Eq. (6), we see that $\mathrm{Re}\, a_2 \sim \mathrm{Re}\, a_3 \sim \mathrm{Im}\, a_3 \sim \omega_p^2 \sim q$ so that $\mathrm{Re}\, a_2 \sim \mathrm{Re}\, a_3 \sim \mathrm{Im}\, a_3 \ll 1$ while $q\to 0$. This implies that the *e-e* screening affects weakly the shape and amplitude of $\sigma(\Omega)$. We note that the present study deals with an ideal case where the electronic scattering mechanisms (e.g., impurities and phonons) are not taken into consideration. These scattering centers can shift and broaden the electron density of states via self-energies[21] and, therefore, can alter the *d-d* correlation function and the current-current correlation. As a result, the shape and the amplitude of $\sigma(\Omega)$ may be affected by impurity and phonon scattering. However, it has been demonstrated theoretically[22] that at the long-wavelength limit, the phonon scattering does not affect the plasmon modes of a spin-split 2DEG. Together with the fact that the impurity scattering affects also weakly the plasmon modes of an electronic system,[23] we believe that for a high-mobility sample (i.e., for weak impurity scattering) at low temperatures (i.e., for weak phonon scattering), the basic features of the optical spectrum are mainly induced by *e-e* interaction via collective excitation. Namely, the width of the optical spectrum for a spin-split 2DEG should depend very little on electronic scattering mechanisms. Furthermore, it is known that for a spin-degenerate electron gas, the plasmon frequency normally lead to an absorption peak in optical conductivity. For a spin-split 2DEG, we see here that two optical-like plasmon frequencies correspond to two absorption edges. The reason behind this interesting feature is that, in contrast to fully quantized electronic states in energy space in a spin-degenerate 2DEG, the strength of SOI and the separation of the $\pm$ spin branches in a 2DEG with SOI depend on the electron wave vector $\mathbf{k}$ in k space. In such a case, the *e-e* interaction, especially for inter-SO transition, is achieved mainly through varying $\mathbf{k}$. Consequently, an elementary electronic excitation such as a plasmon depends on the changes of $\mathbf{k}$, which should obey the momentum and energy conservation laws. Our results indicate that when $\omega_+ \leq \Omega \leq \omega_-$, the scattering channels open up for inter-SO electronic transition accompanied by the absorption of photons. Hence, the plasmon frequencies $\omega_\pm$ are at the edges of absorption spectrum.

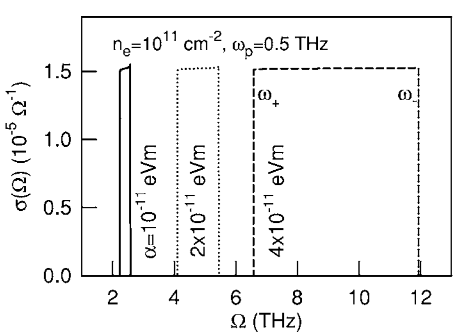

FIG. 2. Optical spectrum $\sigma(\Omega)$ as a function of excitation frequency Ω at a fixed total electron density n_e for different Rashba parameters α as indicated.

Recent photoconduction measurements on spin-split 2DEGs have shown that spin-induced optical absorption does occur in InGaAs-based quantum wells.[24] It should be noted that in these experiments, the response of spin-split electrons to the radiation fields is detected by transport measurements (i.e., the Ohmic contacts are still required and the magnetic fields are present to observe the cyclotron resonances). From a theoretical point of view, the photoconductivity is mainly induced by electron-photon-impurity (-phonon) scattering[25] and all possible values of q can make a contribution. Hence, the spectrum observed by photoconduction measurement differs from those obtained from optical experiments. Optical spectra of spin-degenerate electron gas systems have been observed through techniques such as optical absorption or transmission,[26] the Raman spectrum,[27] ultrafast pump-and-probe experiments,[28] etc. On the basis that the optical spectrum of a spin-split 2DEG is mainly induced by opticlike plasmon excitation via inter-SO transition, the optical spectrum of a spin-split 2DEG can also be measured using these state-of-the-art experimental techniques. In particular, it is known that the plasmon frequency of a spin-degenerate 2DEG depends on changes of electron wave vector via $\omega_p \sim q^{1/2}$. To measure the optical spectrum of such a system, techniques such as grating couplers are needed.[28] However, the plasmon frequencies $\omega_\pm$ induced by inter-SO transition in a 2DEG with RSS are opticlike and well defined. Hence, the optical spectrum of a spin-split 2DEG can be more easily measured optically.

In this work, we have found that the optical spectrum induced by electron-electron interaction in a spin-split 2DEG relates directly to important spintronic properties. Through examining the position and the width of the fast-electron optical spectrum, these spintronic properties can be measured easily and accurately. We have proposed a way to determine optically the spintronic coefficients such as the Rashba parameter, electron density in different spin orbits, spin polarization, etc. As a conclusion, we believe that the Rashba spin splitting in a narrow-gap 2DEG can be identified optically and the drawbacks of sample characterization using magnetotransport experiments can be overcome. Finally, we hope that the important and interesting theoretical predictions in this paper merit attempts at experimental verification.

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