

Slow microwaves in left-handed materials

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Remarkably slow propagation of microwaves in two different classes of left-handed materials (LHM's) is reported from microwave-pulse and continuous-wave transmission measurements. Microwave dispersion in a composite LHM made of split-ring resonators and wire strips reveals group velocity $v_g \sim c/50$, where c is the free-space light velocity. Photonic crystals (PhC's) made of dielectric Al_2O_3 rods reveal $v_g \sim c/10$. Group delay dispersion of both the composite LHM and PhC's determined from the experiment is in complete agreement with that obtained from theory. The slow group velocities are quantitatively described by the strong dispersion observed in these materials.

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Recent observation of left-handed electromagnetism in a composite metamaterial¹ (CMM) made of split-ring resonators and wire strips and photonic crystals (PhC's) comprised of periodic arrangements of metallic or dielectric elements has revealed the unique properties of these left-handed materials (LHM's). Negative refraction¹⁻⁴ and imaging by flat lens,^{5,6} which are consequences of left-handed electromagnetism, are well established in both CMM and PhC's. The CMM simultaneously possesses negative permittivity $\epsilon' < 0$ and permeability $\mu' < 0$ in a small frequency window, resulting in a refractive index $n' < 0$, since $\tilde{n} = \sqrt{\tilde{\mu}} \times \sqrt{\tilde{\epsilon}} < 0$ with $\tilde{n}(\omega) = n' + in''$, $\tilde{\mu}(\omega) = \mu' + i\mu''$, and $\tilde{\epsilon}(\omega) = \epsilon' + i\epsilon''$. The PhC, although locally has positive $\mu'(\omega)$ and $\epsilon'(\omega)$, possesses effective negative refractive index $n'(\omega) < 0$ in certain frequency ranges due to its dispersion characteristics.

The electromagnetic (EM) waves are expected to experience delay in the LHM due to the strong dispersion, first noted by Veselago.⁷ Another peculiar EM property of the LHM is backward wave propagation with opposing phase and group velocity directions. Fundamentally, investigation of group velocity is interesting as the wave propagation direction and speed are determined by the group velocity rather than the phase velocity. Also, low group velocities would allow control of light propagation and could lead to several applications including delay line filters and phase shifters, from microwave to optical frequencies. Photonic dispersion determined previously for multilayer of Fibonacci quasi crystals^{8,9} revealed a slowing down of the wave packets close to the photonic band edge with $v_g \sim c/5$. Slow v_g of the order of order of $c/3$ was observed in Si/SiO₂ photonic crystals,¹⁰ and similar values are obtained on AlGaAs-based photonic crystal slabs.¹¹ Besides, a recent theoretical investigation¹² has proposed subluminal propagation in the LHM.

In this report we present the results of pulse propagation in two different classes of left-handed materials: the CMM and PhC's. The results show that the group velocity is as low as $c/50$ in the CMM and $c/10$ in PhC's. The experimental results of group velocity dispersion are in good agreement with the theory.

The CMM was made by interleaving parallel arrays of split-ring resonators (SRR's) and wire strips (WS's) etched out on a circuit board using photolithography. The circuit board material is made of Taconic FR-35 with $\epsilon' = 4.7$ in the microwave region. The cell size is 0.5 cm, the distance between the interleaved arrays. An array of SRR's and WS's should result in an effective-negative-permeability and negative-permittivity material. Therefore a lattice of such material produces a negative-index metamaterial (NIM) that has effective $n' < 0$. Further details of the fabrication of the CMM are given in Refs 1, 13, and 15.

The microwave PhC's consist of an array of cylindrical Al_2O_3 rods, having dielectric constant $\epsilon' = 9$, radius 0.316 cm, and height 1.25 cm arranged on a square lattice. Two different PhC's having different r/a ratios, r being the radius and a lattice constant, were fabricated. For PhC1, $r/a = 0.175$, and for PhC2, it was 0.35. These left-handed PhC's manifest tailor-made refractive index¹⁴ with high $dn/d\omega$ which is well established from continuous-wave (cw) microwave experiments.¹⁵

Figure 1 shows a schematic diagram of the experimental setup used for the pulse measurements carried out in the time domain. The setup consists of an HP 70820A microwave transition analyzer (MTA) connected to an HP 8341B synthesized sweeper (SS). A Princeton pulse generator FG100 is employed to get a pulse of width 35 ns. The pulse thus generated with a carrier frequency in the microwave region is made to pass through two transmission lines (A and B) using a splitter. Two isolators are connected to the beam splitter to prevent reflected signal interfering with the signals fed to the reference and sample transmission lines. The signals are analyzed by the two MTA input channels. To enhance the sensitivity of the measurement, the difference in the time delay between the reference and transmission lines, with no sample loaded, is set to zero. An X-band rectangular waveguide transmission line is used for measurements on the CMM. The PhC was assembled in a parallel plate waveguide in which the propagation mode is TM with electric field parallel to the Al_2O_3 rods. In all the measurements, the background

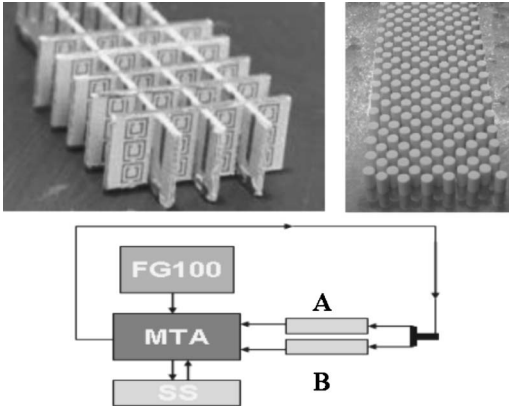


FIG. 1. Photographs of composite metamaterial (a) and photonic crystal (b). Schematic diagram of the pulse delay measurement setup (c).

time delay caused by the transmission lines and rectangular waveguide is well accounted for to obtain the time delay due to the sample alone. The experiment is validated by measuring pulse delay in an empty waveguide, which matches with the predicted value as shown in Fig. 2.

Previous cw measurements¹⁵ on the CMM indicate that the LH transmission band lies between 9.8 and 10.4 GHz. Therefore pulse measurements were carried out in this region on CMM samples of two different lengths: $l=1.5$ cm (four slabs) and $l=1$ cm (three slabs). Figure 2 shows the group velocity dispersion in the CMM of two different lengths determined from pulse delay using $v_g=l/t$, where t is the time delay. A similar group velocity dispersion in both samples indicates that there is no dependence on the sample thickness. The v_g is very low and varies from 0.08 to 0.01 between 9.8 and 10.3 GHz. v_g appears to diverge near the lowest frequencies. These frequencies correspond to the edge of the CMM passband in which the left-handed nature is not well established. Consequently only the results in a frequency window 9.9–10.3 GHz are considered representative of the CMM.

Pulse delay in homogenous and inhomogeneous material media can occur due to scattering of waves. Also, at the interface of vacuum and the material medium a pulse suffers from a short delay due to the transient time needed for the transmitted wave to reorganize and eventually pass through the PhC. However, the time delay caused by these two mechanisms is short and cannot account for the long delay times observed. The present results can be understood from the dispersion characteristics of the waves in the CMM.

Analytic calculations suggest that effective $\tilde{\epsilon}(\omega)$ of WS (Ref. 16) and $\tilde{\mu}(\omega)$ of SRR (Ref. 17) can be described by

$$\tilde{\epsilon}(\omega), \tilde{\mu}(\omega) = 1 - \frac{\omega_{e,mp}^2 - \omega_{e,m0}^2}{\omega^2 - \omega_{e,m0}^2 + i\omega\gamma_{e,m}},$$

where $\omega=2\pi f$, ω_{e0} (ω_{m0}) is the low-frequency edge of the electric (magnetic) forbidden bands, ω_{ep} (ω_{mp}) is electric (magnetic) plasma frequencies, and γ_e (γ_m) is the corresponding damping factor. The index of refraction $n'(\omega) < 0$

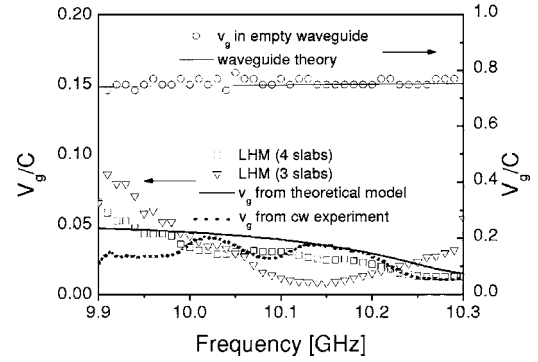


FIG. 2. Group velocity dispersion of CMM in a waveguide. Slowest v_g of $c/50$ is obtained at 10.3 GHz in this configuration. Solid line is a fit to the analytical model.

in the range $\max(\omega_{m0}, \omega_{e0}) < \omega < \min(\omega_{mp}, \omega_{ep})$. Due to the resonance effect just below the magnetic and electric plasma frequencies, both permittivity and permeability functions undergo large changes with frequency, which results in unusual negative values for these two material parameters. Consequently a negative value can be assigned to the refractive index $n'(\omega)$ when both $\mu'(\omega)$ and $\epsilon'(\omega)$ are negative, as well as a strong dispersion results. A notable feature of the group velocity dispersion is the decrease in v_g with the increase in frequency, which validates the nature of the analytical expression in accordance with the causality requirement. The present v_g dispersion is robust and compares well with that determined from the cw technique.¹⁵ The excellent agreement between these two values is evident from Fig. 2. Also, a fit to the experimental data using the analytical model and $v_g = \tilde{\beta} / [k(\tilde{n} + \omega d\tilde{n}/d\omega)]$ is shown in the figure, where $\tilde{k} = \tilde{n}\omega/c$, $\tilde{\beta} = \sqrt{\tilde{k}^2 - (\pi/a)^2}$, and $a=2.3$ cm, the width of the x -band waveguide. It can be seen that the v_g determined from the model follows the experimental data very well for the fit parameters $f_{e0}=5$ GHz, $f_{m0}=8$ GHz, $f_{ep}=25$ GHz, $f_{mp}=10.9$ GHz, $\gamma_e=1.3$ GHz, and $\gamma_m=0.043$ GHz. These values are consistent with that obtained from the cw measurement.¹⁵ The free-space group velocity determined using the model varies from 0.052 to 0.034 (in the units of $1/c$) in the region 9.8–10.3 GHz.

In Fig. 3, v_g of PhC1 of thickness 10 layers for the pulse propagation along the ΓX direction of the first Brillouin zone is shown. The slow wave propagation with v_g varying from 0.2 to 0.35 in the region 7.73–9.4 GHz is evident. Figure 4 shows the group delay dispersion for three different samples of thickness 8, 12, and 16 layers of PhC2. For all the three measurements the pulse propagation is along the ΓM direction of the first Brillouin zone. Note that the v_g is very low and varies from 0.45 to 0.1 between 7 and 8.3 GHz. The results also indicate that there is no dependence of v_g on sample thickness. As in the case of PhC1, PhC2 also indicates slow group velocities.

The low v_g obtained on all the samples of PhC1, PhC2, and CMM is striking and forms a salient feature of this communication. In the case of PhC's, the slow v_g can be attributed to the dispersion characteristics of the waves, which can be determined from the band structure.

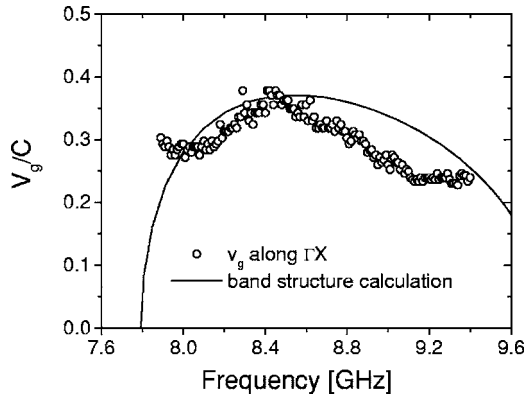


FIG. 3. Group velocity dispersion of left-handed PhC1 with $r/a=0.175$. Note the low v_g with the lowest value of $c/5$ at 9.3 GHz.

The band structures of the PhC1 and PhC2 are shown in Fig. 5. We have investigated the band structure of both the PhC's for left-handed or right-handed behavior of microwaves following Refs. 2, 4, and 18. The strong modulation in the PhC would result in guided modes with waves having different phase velocities at different frequencies. The phase and group velocities in a medium are given by $\vec{v}_p = (c/n_p)\hat{k}_f$ with $\hat{k}_f = \vec{k}_p/|\vec{k}_p|$ and $\vec{v}_g = \nabla_{\vec{k}}\omega$. An effective refractive index can be defined, $n_p = \text{sgn}(\vec{v}_g \cdot \vec{k}_f)c|\vec{k}_f|/\omega$, and calculated from the band structure. The sign of n_p is determined from the behavior of the EFS,^{2,19,20} which are drawn from the dispersion curve of the band structure.

In the case of PhC1 with $r/a=0.175$ careful observation of the band structure (solid line in Fig. 5) reveals that in the second band near the band edges where the light line crosses the bands, the group velocity is antiparallel to the propagation vector \vec{k} and the refractive index is negative. This is a characteristic of the left-handed behavior in the PhC. The group velocity dispersion determined from the band structure (solid line) is shown in Fig. 3. The experimental result strikingly matches with the theory. A key feature of the band structure is the flattening of the bands at the band edges. The v_g in these regions is expected to be low. Below the band edge 7.7 GHz, $v_g=0$ and no propagation takes place. It is interesting to note that as frequency decreases from 8.5 to 7.7 GHz, v_g decreases and both v_g and the refractive index approach zero.² The discrepancy between the experimental and theoretical v_g near the band edge 7.7 GHz is due to a drastic reduction in signal strength at the band edges, which results in larger error in the measurement of v_g .

Following a similar band structure analysis for PhC2 it can be deduced that in the region 7–8.5 GHz the electromagnetism is righthanded. In this region the directions of \vec{k} and \vec{v}_g such that $\vec{v}_g \cdot \vec{k}_f > 0$. Figure 4 shows the group velocity dispersion determined from band structure. Evidently, as in the case of PhC1, excellent agreement between the theoretical and experimental results is achieved in the entire frequency range. It is important to note that maximum slowing down of the v_g is obtained close to the band edge 8.3 GHz. A

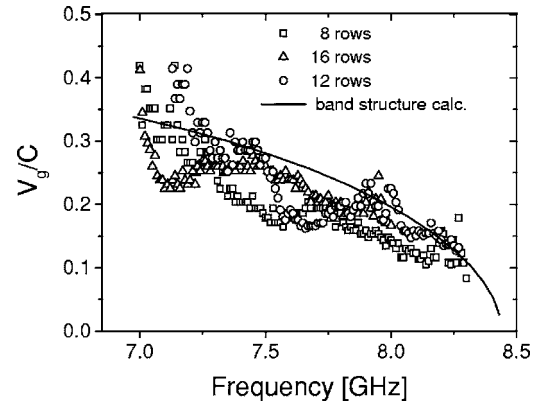


FIG. 4. Group velocity dispersion of right-handed PhC2 with $r/a=0.35$. Note the very low v_g with the lowest value of $c/10$ at 8.3 GHz.

similar low v_g value for PhC1 could have been obtained but is prevented by the very weak signal strength.

In conclusion, low group velocities are observed in left-handed metamaterial and photonic crystals. These results validate the behavior of the waves expected from the dispersion characteristics in these media. A quantitative description of the experimental results is obtained using the analytical model in the case of the CMM and band structure in the case of PhC's. An important feature is a strong slowing down of the waves at the band edges of the PhC's. The group velocity reduction observed in the photonic crystals is more than 3 times larger than that of Fibonacci quasicrystals and 9 times that of colloidal photonic crystals of polystyrene.²¹ Such low v_g are due to strong modulation (with a dielectric contrast 9–1) in the present PhC's by way of weak modulation in the colloidal polystyrene (contrast 2.5–1) PhC. The present investigation indicates that very slow group velocities, lower than that presented here, are possible in metamaterials and photonic crystals having low transmission loss. These materials can be tailor made to have a strong dispersion with very high $dn/d\omega$ to result in lower group velocities. The availability of high-permittivity materials with minimal losses facilitates the design of tailor made photonic crystals with slow v_g . The ability to mold and slow light leads to several appli-

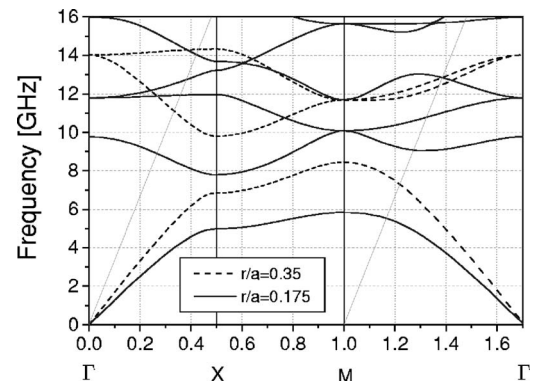


FIG. 5. TM-mode band structure of square lattices PhC1 and PhC2. Note the flattening of the bands at the band edges which results in slow group velocities.

cations of metamaterials and photonic crystals which are not possible with conventional materials. Important of them are delay line filters and phase shifters. The slow group velocities obtained at microwave frequencies can also be obtained

at optical frequencies by scaling the material parameters to micron sizes.

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