Study of the resonance modes of a magnetic tunnel junction-like system

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The ferromagnetic resonance (FMR) modes of a magnetic tunnel junction-like system are investigated. Such a system consists of an interfacial (F/AF) interaction described by an exchange anisotropy field H_E and a magnetic coupling of two ferromagnetic layers separated by a nonmagnetic interlayer. The latter interaction is accounted for by bilinear J_1 and biquadratic J_2 coupling strengths. The dispersion relation, the resonant frequency, f, as well as the corresponding mode intensity, I, versus applied field H curves, have been studied. Analytical formulas for the resonance condition and intensity have been derived for the low magnetic coupling/high exchange anisotropy case. In this situation, the system is found to behave as two *uncoupled* layers with magnetic characteristics different from those of the initial layers; the effect of the low coupling is to modify the different anisotropies: J_1 contributes to the exchange anisotropy while J_2 modifies the magnetocrystalline anisotropies. For very strong coupling, the system behaves as a single (F/AF) system with effective exchange and magnetocrystalline anisotropy fields; these fields have been derived as a function of the individual layer magnetic parameters.

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I. INTRODUCTION

Ferromagnetic resonance (FMR) is a powerful method which has been widely used to investigate a variety of magnetic system.[1–9] The present work deals with the ferromagnetic resonance modes (position and intensity) of a magnetic tunnel junction-like system. Such a system may consist of a stacking of an antiferromagnetic layer (AF) and two ferromagnetic layers (A and B) separated by a nonmagnetic interlayer.[10–12] The interaction at the interface of layer A with AF gives rise to a unidirectionnal anisotropy called exchange anisotropy.[13–21] This anisotropy can be modeled as a magnetic field $\mathbf{H}_E$, the exchange anisotropy field. The layer A magnetization $\mathbf{M}_A$ is subjected to $\mathbf{H}_E$ and is thus pinned. Layers A and B are magnetically coupled. The magnetic coupling can be described by the bilinear J_1 and the biquadratic J_2 coupling parameters.[22–27] The former may favor a parallel alignement (ferromagnetic coupling) or an an antiparallel alignement (antiferromagnetic coupling) of $\mathbf{M}_A$ and $\mathbf{M}_B$ while the latter one (J_2) may lead to a perpendicular configuration of the magnetizations.

The geometry of the system and the energy will be displayed in Sec. II. The relations giving the mode positions (resonance frequency) and the FMR intensity will be derived (Sec. III) and discussed as a function of the coupling strengths J_1 and J_2 and of the exchange anisotropy field H_E: for zero coupling (Sec. IV), for low magnetic coupling/high exchange anisotropy (Sec. V), and for very strong magnetic coupling (Sec. VI).

II. THE MAGNETIC SYSTEM

The magnetic tunnel junction, the system under consideration here, is shown in Fig. 1. It consists of an antiferromagnetic layer (noted AF) on top of which is deposited a first ferromagnetic layer (labeled A). Layer A is separated from a second ferromagnetic layer B by a nonmagnetic interlayer. All the thin film layers are assumed to lie in the x-y plane, with the z axis normal to the film planes. The magnetization $\mathbf{M}_A$ of layer A is defined, in spherical coordinates, by the angles θ_A and ϕ_A; and similarly $\mathbf{M}_B$ (layer B) by the angles θ_B and ϕ_B. Two magnetic phenomenon arise in this system. First, the exchange anisotropy at the interface AF-A, is modeled as a magnetic field $\mathbf{H}_E$, the exchange anisotropy field. The field $\mathbf{H}_E$ is taken to be along the x axis (see Fig. 1). Second, we have the magnetic coupling between two ferromagnetic layers separated by a nonmagnetic interlayer. This interaction will be described by the bilinear J_1 and biquadratic J_2 coupling parameters. Moreover, layer A is supposed to have an in-plane uniaxial magnetocrystalline anisotropy, with the easy axis taken to be along the x axis. On the other hand, no in-plane magnetocrystalline anisotropy is included

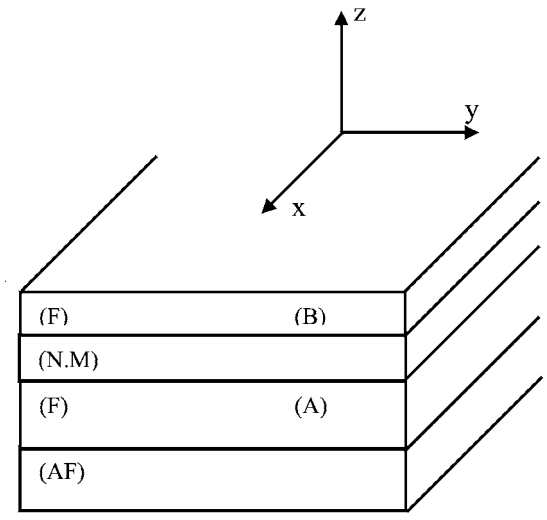

FIG. 1. The magnetic tunnel junction-like system. AF: antiferromagnetic layer. F: ferromagnetic layers (A and B). (N.M): nonmagnetic interlayer.

for layer B, i.e., the layer B magnetization is free to rotate within the plane in the absence of magnetic coupling. The external applied magnetic field $\mathbf{H}$ is taken to be in the plane of the films, making an α angle with the x axis. The microwave field $\mathbf{h}$ is along the y axis. With all these considerations, the total free energy of the system per unit area can be explicitly written as

$$\begin{aligned} E = t_A[&- M_A H \sin\theta_A \cos(\alpha - \phi_A) + K_{ueffA} \sin^2\theta_A \\ &+ K_A \sin^2\theta_A \sin^2\phi_A - M_A H_E \sin\theta_A \cos\phi_A] \\ &+ t_B[- M_B H \sin\theta_B \cos(\alpha - \phi_B) + K_{ueffB} \sin^2\theta_B] \\ &- J_1[\sin\theta_A \sin\theta_B \cos(\phi_A - \phi_B) + \cos\theta_A \cos\theta_B] \\ &- J_2[\sin\theta_A \sin\theta_B \cos(\phi_A - \phi_B) + \cos\theta_A \cos\theta_B]^2. \end{aligned} \tag{1}$$

In the two first terms of Eq. (1), t_A and t_B are the thicknesses of layers A and B, respectively. The total energy E consists for layer A (the first term) of the Zeemann energy (interaction of the external magnetic field $\mathbf{H}$ with the magnetizations), $-\mathbf{M}_A\mathbf{H}$, the shape and any out-of-plane uniaxial anisotropy with effective constant K_{ueffA} ($K_{ueffA}=K_u-2\pi M_A^2$, K_u being the uniaxial magnetocrystalline constant), the in-plane magnetocrystalline anisotropy with constant K_A, and the exchange anisotropy with exchange anisotropy field H_E. For layer B [the second term in Eq. (1)], the Zeeman energy and the effective uniaxial anisoptropy term (shape and magnetocrystalline) are displayed. The interlayer coupling energy is given by the two last terms. The nature and the strength of the coupling are described by the sign and the magnitude of J_1 and J_2. When J_1 dominates and if it is positive the energy is minimal when $\mathbf{M}_A$ and $\mathbf{M}_B$ are parallel (ferromagnetic coupling), while if it is negative, then the lowest energy is achieved when $\mathbf{M}_A$ and $\mathbf{M}_B$ are antiparallel (antiferromagnetic coupling). If, on the other hand, J_2 dominates and is negative (which was experimentally observed), then the minimum energy occurs when the magnetizations are oriented perpendicularly to each other (the 90°-type coupling).

At equilibrium, the magnetizations $\mathbf{M}_A$ and $\mathbf{M}_B$ must lie in the film plane, i.e., $\theta_A=\theta_B=90°$, due to the strong demagnetizing field of the thin films and to the fact that the applied magnetic field is in-plane. Then, the angles $\phi_{A,B}$ are given by the following two coupled equations (the equilibrium conditions):

$$H\sin(\alpha-\phi_A) = \frac{H_A}{2}\sin 2\phi_A + H_E \sin\phi_A + \frac{J_1}{a}\sin(\phi_A-\phi_B) + \frac{J_2}{a}\sin 2(\phi_A-\phi_B) \tag{2a}$$

and

$$H\sin(\alpha-\phi_B) = -\frac{J_1}{b}\sin(\phi_A-\phi_B) - \frac{J_2}{b}\sin 2(\phi_A-\phi_B), \tag{2b}$$

where $H_A=2K_A/M_A$ is the planar anisotropy field for layer A, $a=t_AM_A$ and $b=t_BM_B$.

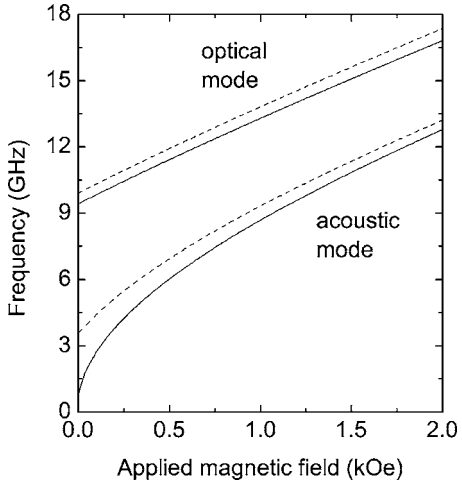

FIG. 2. Resonant frequency vs. applied field H. Ferromagnetic coupling: $J_1=0.5$ erg cm^{-2}, $J_2=0$. $H_E=0$ (solid line), $H_E=250$ Oe (dashed line). Layer A: $4\pi M_A=10$ kG, $H_{keffA}=-10$ kOe, $H_A=10$ Oe, $t_A=200$ Å, $\gamma_A/2\pi=2.8$ GHz kOe^{-1}. Layer B: $4\pi M_B=6$ kG, $H_{keffB}=-6$ kOe, $H_{inB}=0$ Oe, $t_B=100$ Å, $\gamma_B/2\pi=2.8$ GHz kOe^{-1}.

III. RESONANT MODE POSITION AND INTENSITY

A. The mode position

The normal modes of the system can be found by the use of the method based on the energy, in this case the equations coupling $\Delta\theta_i$, $\Delta\phi_i$ ($i=A,B$); the excursions during oscillations about the equilibrium position can be written in a matrix form.[28–30] The matrix elements consist of the second derivatives of the energy E with respect to θ_i and ϕ_i ($i=A,B$). A solution of the form $\exp(i\omega t)$ will be taken; ω is the (angular) frequency of precession. The solutions (normal modes) of the system will be found by setting the determinant of the matrix to zero. If one is using a variable frequency setup (frequency sweeper) and a fixed dc field, then one may solve for the frequency. One will then find a fourth-order equation in ω (the resonant frequency) with at most two meaningful solutions (real and positive numbers) for given coupling strength parameters and dc field intensity and direction values.[28–30] Thus computing the second derivatives of the energy (evaluated at the equilibrium positions), setting the determinant to zero, and rearranging the terms, one will obtain the following fourth-order equation in ω:

$$\begin{aligned} \omega^4 - \omega^2\Bigg[&\gamma_A^2 H_1^A H_2^A + \gamma_B^2 H_1^B H_2^B + c_1\left(\frac{\gamma_B^2 H_2^B}{b} + \frac{\gamma_A^2 H_2^A}{a}\right) \\ &+ c_2\left(\frac{\gamma_B^2 H_1^B}{b} + \frac{\gamma_A^2 H_1^A}{a}\right) + c_1c_2\left(\frac{\gamma_A^2}{a^2} + \frac{\gamma_B^2}{b^2}\right) + \frac{2c_0c_2\gamma_A\gamma_B}{ab}\Bigg] \\ &+ \gamma_A^2\gamma_B^2\left[H_2^A H_2^B + c_2\left(\frac{H_2^B}{a} + \frac{H_2^A}{b}\right)\right]\left[H_1^A H_1^B + c_1\left(\frac{H_1^B}{a} + \frac{H_1^A}{b}\right) + \frac{c_1^2 - c_0^2}{ab}\right] = 0, \end{aligned} \tag{3}$$

where γ_A and γ_B denote the gyromagnetic ratios of layers A and B, respectively. The parameters c_j contain the coupling

strength: $c_0=J_1+2J_2\cos(\phi_A-\phi_B)$, $c_1=J_1\cos(\phi_A-\phi_B)+2J_2\cos^2(\phi_A-\phi_B)$ and $c_2=J_1\cos(\phi_A-\phi_B)+2J_2\cos2(\phi_A-\phi_B)$, and

$$H_1^A=H\cos(\alpha-\phi_A)-H_{KeffA}-H_A\sin^2\phi_A+H_E\cos\phi_A, \tag{4a}$$

$$H_2^A=H\cos(\alpha-\phi_A)+H_A\cos 2\phi_A+H_E\cos\phi_A, \tag{4b}$$

$$H_1^B=H\cos(\alpha-\phi_B)-H_{KeffB}, \tag{4c}$$

$$H_2^B=H\cos(\alpha-\phi_B), \tag{4d}$$

where $H_{KeffA}=2K_{ueffA}/M_A$ and $H_{KeffB}=2K_{ueffB}/M_B$.

B. The FMR intensity

The FMR mode intensity is defined as corresponding to the area under the absorption line. In the present study, the applied magnetic field **H** is parallel to the film plane, along the x direction, and the microwave **h** field is along the y direction, the magnetizations precess in an elliptical orbit; the precession orbit is the y-z plane. The intensity is thus given by[30,31] (taking into account the fact that, in the more general case, the magnetization components m_i are complex quantities)

$$I=\frac{(t_Am_{Ay}+t_Bm_{By})(t_Am_{Ay}+t_Bm_{By})^*}{t_A(m_{Ay}m_{Ay}^*+m_{Az}m_{Az}^*)/2M_A+t_B(m_{By}m_{By}^*+m_{Bz}m_{Bz}^*)/2M_B}. \tag{5}$$

Here m denotes the time-dependent magnetization component for A and B (the asterisk denotes the complex conjugate expressions). The magnetization components, m, are assumed to be uniform throughout each individual layer. One can compute all the four rf magnetization components by the use of the 4×4 matrix and through the derivation of $\Delta\theta_A$, $\Delta\phi_A$, $\Delta\theta_B$, $\Delta\phi_B$. After some algebraic manipulations, the rf magnetizations are computed and substituted into the intensity I [Eq. (5)]. When the two magnetizations are pointing along the x axis (the H direction), the intensity can then be put into the following form:

$$I=\frac{2ab\omega^2(aq+b)^2}{ab\omega^2(aq^2+b)+b\gamma_A^2[qaH_2^A+c_2(q-1)]^2+a\gamma_B^2[bH_2^B-c_2(q-1)]^2}, \tag{6a}$$

where

$$q=\frac{\gamma_A^2c_2b(aH_1^A+c_1)+\gamma_A\gamma_Bc_0a(bH_2^B+c_2)}{\gamma_A^2b(aH_1^A+c_1)(aH_2^A+c_2)+\gamma_A\gamma_Bc_0c_2a-a^2b\omega^2}. \tag{6b}$$

Equation (3) has been numerically solved to study the resonant frequency versus applied field H; for **H** in the forward direction, the direction of $\mathbf{H}_E(\alpha=0°)$.

For the ferromagnetic coupling case ($J_1>0$), $\mathbf{M}_A$ and $\mathbf{M}_B$ are expected to be along **H** for all H values, $\phi_A=\phi_B=0°$. From the resolution of Eq. (3), two solutions ω_1 and ω_2 are expected, corresponding to the acoustic mode (where the magnetizations precess in phase) and to the optical mode (out of phase). The optical mode intensity will decrease with increasing coupling and would disappear for very strong coupling, in this case only the acoustic mode would remain (see Sec. VI). For ferromagnetic coupling, the high-frequency mode is the optical mode while the low frequency mode corresponds to the acoustic mode (note that this is the opposite to a fixed frequency analysis where the lower resonant field corresponds to the optical mode for ferromagnetic coupling). Examples of dispersion relation curves are shown in Fig. 2 for $J_1=+0.5$ erg/cm^{-2}, for $H_E=0$ (solid line) and $H_E=250$ Oe (dashed line); for other parameters used in the computation, see the caption of Fig. 2. The frequency of both modes increases with increasing field H. One can see that H_E induces a shift to higher frequency values for the two modes. For the low H field values, the shift in the acoustic mode is more important than that of the optical mode. However, as H increases, the shift in both modes is of the same order of magnitude.

The corresponding intensities are plotted in Fig. 3. The effect of $\mathbf{H}_E$ is to increase (decrease) the intensity of the optical (acoustic) mode by practically the same amount.

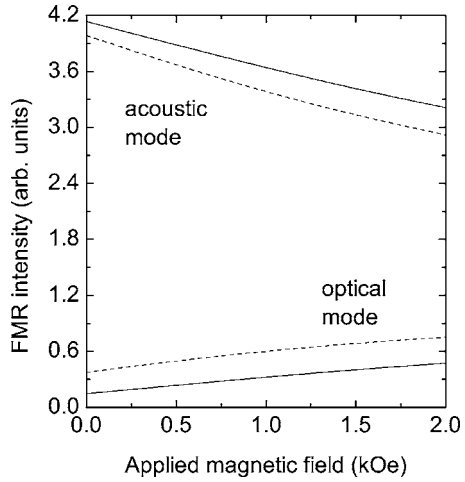

FIG. 3. FMR intensity vs. applied field H. Ferromagnetic coupling: $J_1=0.5$ erg cm^{-2}, $J_2=0$. $H_E=0$ (solid line), $H_E=250$ Oe (dashed line). Other parameters as for Fig. 2.

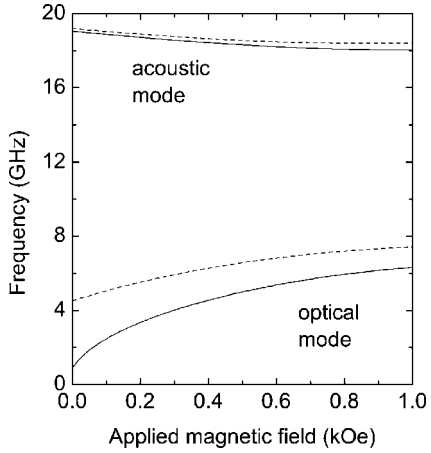

FIG. 4. Resonant frequency vs. applied field H. Antiferromagnetic coupling: $J_1=-2$ erg cm^{-2}, $J_2=0$. $H_E=0$ (solid line), $H_E=250$ Oe (dashed line). Other parameters as for Fig. 2.

When the coupling is antiferromagnetic ($J_1<0$) and if the coupling is strong enough to overcome the effect of the applied field H, then $\mathbf{M}_A$ and $\mathbf{M}_B$ are antiparallel, i.e., $\phi_A=0$, $\phi_B=\pi$ (assuming $a>b$). This case is displayed in Figs. 4 and 5 for $J_1=-2$ erg/cm^{-2} and H between 0 and 1 kOe. Note that, for the antiferromagnetic coupling, the lower frequency mode (the higher magnetic field) is the optical mode (this is the reversed situation compared to the ferromagnetic coupling). The mode behavior is different, here, from that observed in the ferromagnetic case. The acoustic mode (the higher frequency mode) slightly decreases with increasing applied field H while the optical mode frequency increases with H. The exchange anisotropy field H_E seems to affect more the optical than the acoustic modes, i.e., the shift to higher frequency value is more important for the optical than the acoustic modes. The FMR intensity is shown in Fig. 5; the increase of H_E leads to an increase (decrease) of the intensity of the acoustic (optical) mode which is the opposite of what was seen in the ferromagnetic coupling case.

In the following sections, some cases of interest will be studied. In particular, it will be shown that, when the magnetic coupling is zero, very low and strong compared to the exchange anisotropy effect, the resonance modes (frequency and intensity) can be described by analytical formula.

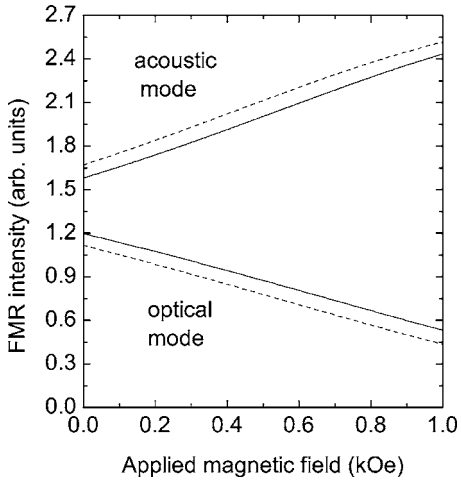

FIG. 5. FMR intensity vs. applied field H. Antiferromagnetic coupling: $J_1=-2$ erg cm^{-2}, $J_2=0$. $H_E=0$ (solid line), $H_E=250$ Oe (dashed line). Other parameters as for Fig. 2.

IV. THE ZERO COUPLING CASE

When the layers are not coupled ($J_1=J_2=0$) then $c_j=0$ and Eq. (3) will reduce to $(\omega^2-\gamma_A^2H_1^AH_2^A)(\omega^2-\gamma_B^2H_1^BH_2^B)=0$, which, upon substituting the H_j^i by their expressions [Eqs. (4a)–(4d)] will lead to the following two uncoupled equations:

$$\frac{\omega_A^2}{\gamma_A^2}=[H\cos(\alpha-\phi_A)-H_{KeffA}-H_A\sin^2\phi_A+H_E\cos\phi_A]\times[H\cos(\alpha-\phi_A)+H_A\cos 2\phi_A+H_E\cos\phi_A] \quad (7)$$

and

$$\frac{\omega_B^2}{\gamma_B^2}=H(H-H_{KeffB}). \quad (8)$$

Equation (7) is the resonance condition for a single layer with in-plane and exchange anisotropies,[20,21] while Eq. (8) is the well-known resonance condition for a ferromagnetic thin film with an in-plane applied field and where all the directions within the plane are equivalent (no in-plane anisotropy, i.e., $\alpha=\phi_B$). Thus in this uncoupled case, two peaks are expected to be observed in the FMR spectra whose positions are given by the resonance conditions [Eqs. (7) and (8)].

The corresponding intensities of these peaks have been evaluated. When the layers are uncoupled then $c_j=0$. For the intensity of layer B, when $c_j=0$, then $q=0$. Substuting in Eq. (6a) $q=0$ and replacing ω by ω_B [Eq. (8)], one will find

$$I_B=\frac{2bH_1^B}{H_1^B+H_2^B}. \quad (9)$$

Since $\alpha=0$, then $\phi_B=0$ for all H and Eq. (9) becomes

$$I_B=\frac{2t_BM_B(H-H_{KeffB})}{2H-H_{KeffB}}. \quad (10)$$

For the intensity, I_A, of layer A, it is easy to see that both numerator and denominator in q are equal to zero (the first and last terms in the denominator of q [Eq. (6b)] cancel because of the resonance condition [Eq. (7)], recall in this case that $\omega^2=\gamma_A^2H_1^AH_2^A$). Due to this indetermination, the intensity of each layer is usually found separately. Alternatively, one can compute q and I for low coupling strengths, then let c_j go to zero [see Eq. (17) in Sec. V, below] to find the intensity of the uncoupled layers; this way will be shown later in the following section. Both methods will lead to the same result, for layer A:

$$I_A=\frac{2aH_1^A}{H_1^A+H_2^A}, \quad (11)$$

where H_1^A and H_2^A are given by Eqs. (4a) and (4b). Recall that in the present case $\alpha=0$, therefore $\mathbf{M}_A$ is expected to be along the x axis for all H, i.e., $\phi_A=0$. Thus substuting H_1^A and H_2^A, one will find

$$I_A = \frac{2t_A M_A (H - H_{KeffA} + H_E)}{2H - H_{KeffA} + H_A + 2H_E}. \quad (12)$$

Equations (10) and (12) giving the intensities of single layers with different anisotropies are quite known relations.[31,32] The terms multiplying $2a$ and $2b$ in Eqs. (9) and (11) are referred to as the "elliptic terms."[33] Note that the ellipticity factor in Eq. (12) explicitly depends on the in-plane anisotropy field H_A and the exchange bias H_E.

In the case where, for layer A, the exchange and in-plane fields, H_E and H_A, are very small compared to $(H–H_{Keff})$ [this assumption is often true, for instance, in permalloy, $(-H_{Keff})$, equal to $4\pi M_S$, is about 10 kOe, while H_E and H_A are around 20 and 5 Oe], the intensity I_A may be written as the sum of different contributions to the intensity. Making the above approximations, the intensity I_A will be

$$I_A = I_0 + I_{ex} + I_{in}, \quad (13a)$$

where

$$I_0 = \frac{2t_A M_A (H - H_{KeffA})}{2H - H_{KeffA}} \quad (13b)$$

would be the intensity of layer A if it did not have in-plane and exchange anisotropies [similar to Eq. (9) for layer B],

$$I_{ex} = \frac{2t_A M_A H_{KeffA}}{(2H - H_{KeffA})^2} H_E \quad (13c)$$

is the contribution of the exchange anisotropy to the intensity of the layer, and

$$I_{in} = \frac{2t_A M_A (H_{KeffA} - H)}{(2H - H_{KeffA})^2} H_A \quad (13d)$$

is the intensity due to the in-plane anisotropy.

Note that, since H_{KeffA} is negative, both the exchange and the in-plane anisotropies will tend to decrease the intensity I_A. For a very strong applied field, it is easy to see that both I_{ex} and I_{in} vanish and the intensity I_A will be equal simply to the known result, $I_A = t_A M_A$. (This is in fact the intensity per unit area; experimentally, however, the total intensity is $M \times V$, where V is the volume of the sample.)

V. LOW MAGNETIC COUPLING

For an arbitrary magnetic coupling, there is no analytical formula for the resonance modes. For the low magnetic coupling case, it will be shown in this section that the resonance modes, frequency, and intensity, found numerically from Eqs. (3) and (6a), can be described by analytical formula. Indeed, neglecting second-order terms of the magnetic coupling parameters $(c_i c_k)$ in Eq. (3) and solving for ω, one will obtain the following two solutions:

$$\omega_1^2 = \omega_A^2 + \frac{c_1}{a}\gamma_A^2 H_2^A + \frac{c_2}{a}\gamma_A^2 H_1^A \quad (14a)$$

and

$$\omega_2^2 = \omega_B^2 + \frac{c_1}{b}\gamma_B^2 H_2^B + \frac{c_2}{b}\gamma_B^2 H_1^B, \quad (14b)$$

where ω_A and ω_B are given by the expressions in Eqs. (7) and (8) i.e., the uncoupled layer case; H_1^A, H_2^A, H_1^B, and H_2^B have been defined earlier in Eqs. (4a)–(4d). Note that in the limit where there is no coupling $(c_1 = c_2 = 0)$, then ω_1 and ω_2 correctly reduce to ω_A and ω_B as expected.

Furthermore, Eqs (14a) and (14b) can also be written as

$$\omega_1^2 = \gamma_A^2 \left(H_1^A + \frac{c_1}{a}\right)\left(H_2^A + \frac{c_2}{a}\right) \quad (15a)$$

and

$$\omega_2^2 = \gamma_B^2 \left(H_1^B + \frac{c_1}{b}\right)\left(H_2^B + \frac{c_2}{b}\right), \quad (15b)$$

i.e., the above equations will lead to the previous ones [Eqs. (14a) and (14b)], if the second-order terms $(c_1 c_2)$ are neglected as was assumed in the beginning.

Moreover, Eqs. (15a) and (15b) can be put in a very interesting form. For the case of a ferromagnetic coupling with $\mathbf{H}$ in the forward direction, i.e., $\alpha = \phi_A = \phi_B = 0$ (or even in the antiferromagnetic coupling but in the saturated state), and if one substitutes the H_j^i by their expressions and rearranges the terms, then Eq. (15a) can be written as

$$\frac{\omega_1^2}{\gamma_A^2} = \left[H - \left(H_{KeffA} - \frac{2J_2}{t_A M_A}\right) + \left(H_E + \frac{J_1}{t_A M_A}\right)\right] \times \left[H + \left(H_A + \frac{2J_2}{t_A M_A}\right) + \left(H_E + \frac{J_1}{t_A M_A}\right)\right]. \quad (16a)$$

Comparing Eq. (16a) with Eq. (7) (for $\alpha = \phi_A = 0$), one may regard the mode ω_1 as that of a single layer with an effective anisotropy field given by $(H_{keffA} - 2J_2/t_A M_A)$, an in-plane anisotropy field $(H_A + 2J_2/t_A M_A)$, and an exchange anisotropy field $(H_E + J_1/t_A M_A)$. Note that the magnetogyric ratio γ_A is the same as that of layer A.

In the same manner, Eq. (15b) can be written as

$$\frac{\omega_2^2}{\gamma_B^2} = \left[H - \left(H_{KeffB} - \frac{2J_2}{t_B M_B}\right) + \frac{J_1}{t_B M_B}\right]\left(H + \frac{2J_2}{t_B M_B} + \frac{J_1}{t_B M_B}\right). \quad (16b)$$

Similarly ω_2 can describe the resonant frequency of a layer with an effective anisotropy field given by $(H_{keffB} - 2J_2/t_B M_B)$, an in-plane anisotropy field $H_B = 2J_2/t_B M_B$, and an exchange anisotropy field $H_E^B = J_1/t_B M_B$.

Note that ω_1 [in Eqs. (14a), (15a), and (16a)] depends only on the layer A parameters, while ω_2 contains only the layer B parameters. Thus, when the magnetic coupling is low, the system behaves as two *uncoupled* layers with magnetic characteristics different from those of layers A and B. The effect of the low coupling is to modify the different anisotropies; the bilinear coupling, J_1, contributes to the exchange anisotropy while the biquadratic coupling, J_2, modifies the magnetocrystalline anisotropies (uniaxial and in-plane). Note that one cannot experimentally separate the contributions of J_1 and J_2; generally a value of $(J_1 + 2J_2)$

(when the magnetizations are parallel) is derived (see, for instance, Refs. 25 and 26). From the present analysis, it is inferred that physically J_1 seems to be equivalent to an exchange anisotropy while J_2 is equivalent to an in-plane anisotropy. In fact, the equivalence between J_2 and the in-plane magnetic anisotropy (a case similar to the latter one) was explicitly worked out in coupled layers with in-plane magnetic anisotropies; it was shown that a small deviation of the in-plane easy axis direction of one layer with respect to the one of the second layer may be interpreted as a small biquadratic coupling.[34] Also, Eqs. (16a) and (16b) are valid for ferromagnetic (F) ($J_1>0$) and antiferromagnetic (AF) ($J_1<0$) coupling. For F (AF), the exchange anisotropy field H_E increases (decreases) for layer A. For layer B, which initially does not have in-plane and exchange anisotropies, the effect of the low coupling is to give to the layer a small exchange bias and an in-plane anisotropy. Moreover, these additional anisotropies vary as $1/t$, giving them a surface anisotropy character.

One should, now, compute the intensities of these modes and show that they correspond to two uncoupled layers. Once again for ferromagnetic coupling, and **H** in the forward direction, $\alpha=0$, $\phi_A=\phi_B$, then $c_0=c_1=c_2=J_1+2J_2=J_{eff}$. For the mode corresponding to ω_1 the first and last terms of the denominator in q [Eq. (6b)] cancel because of the resonance condition [Eq. (15a)]; but it is easy to see that for low coupling (J_{eff} goes to zero), $1/q$ goes to zero. Using this fact in Eq. (6a) and replacing ω_1 by its expression [Eq. (15a)], the intesity, I_1, is found to be

$$I_1 = \frac{2a\left(H_1^A + \frac{J_{eff}}{a}\right)}{H_1^A + H_2^A + \frac{2J_{eff}}{a}}. \tag{17}$$

Note that if $J_{eff}=0$ (no coupling), I_1 reduces to I_A as expected [in fact, this is a good way to find the intensity of the uncoupled layers from Eqs. (6a) and (6b)]. Substituting the H_j^A by their expressions, the intensity of mode 1 [Eq. (17)] in the forward direction can be written as

$$I_1 = \frac{2t_A M_A\left[H - \left(H_{KeffA} - \frac{2J_2}{t_A M_A}\right) + \left(H_E + \frac{J_1}{t_A M_A}\right)\right]}{2H - \left(H_{KeffA} - \frac{2J_2}{t_A M_A}\right) + \left(H_A + \frac{2J_2}{t_A M_A}\right) + 2\left(H_E + \frac{J_1}{t_A M_A}\right)}. \tag{18}$$

Comparing Eq. (18) with the intensity of a single layer [Eq. (12)], one can see that Eq. (18) is indeed the intensity of a single layer with these characteristics: $(H_{keffA}-2J_2/t_A M_A)$, $(H_A+2J_2/t_A M_A)$, and $(H_E+J_1/t_A M_A)$. So, the analysis of the FMR intensity and the mode position are coherents.

One can express I_1 in term of I_A; making the approximation that J_{eff} is small and using Eq. (12), Eq. (18) can be written as

$$I_1 = I_A\left[1 + \frac{J_1 + 2J_2}{t_A M_A} \times \frac{H_A + H_{KeffA}}{(H - H_{KeffA} + H_E)(2H - H_{KeffA} + 2H_E + H_A)}\right]. \tag{19}$$

Note that for very strong applied field H, I_1 will tend to I_A.

For the intensity of the second mode (corresponding to ω_2), it is easy to see that as J_{eff} goes to zero, q will go to zero. Using this limit in Eq. (6a) and replacing ω_2 by its expression [Eq. (15b)], the intensity I_2 will be

$$I_2 = \frac{2b\left(H_1^B + \frac{J_{eff}}{b}\right)}{H_1^B + H_2^B + \frac{2J_{eff}}{b}} \tag{20}$$

or after replacing H_j^B by their expressions

$$I_2 = \frac{2t_B M_B\left[H - \left(H_{KeffB} - \frac{2J_2}{t_B M_B}\right) + \frac{J_1}{t_B M_B}\right]}{2H - \left(H_{KeffB} - \frac{2J_2}{t_B M_B}\right) + \frac{2J_2}{t_B M_B} + \frac{2J_1}{t_B M_B}}. \tag{21}$$

The same remark can be made once again. Eq. (21) can be considered as the intensity of a layer (by comparison with the general formula) with an effective uniaxial anisotropy $(H_{keffB}-2J_2/t_B M_B)$, an in-plane anisotropy $H_B=2J_2/t_B M_B$, and an exchange bias $H_E^B=J_1/t_B M_B$.

In terms of I_B, the combination of Eqs. (10) and (21) would give

$$I_2 = I_B\left(1 + \frac{J_1 + 2J_2}{t_B M_B}\frac{H_{KeffB}}{(H - H_{KeffB})(2H - H_{KeffB})}\right). \tag{22}$$

VI. STRONG MAGNETIC COUPLING

When the magnetic coupling is ferromagnetic and very strong, one expects $\phi_A=\phi_B=\phi_F$ for all H values ($\phi_F=0$ for $\alpha=0$). In this case, $c_0=c_1=c_2=J_1+2J_2=J_{eff}$ and the terms in J_{eff}^2 dominate in Eq. (3). Keeping only the second-order terms in J_{eff} in Eq. (3), substituting the H_j^i by their expressions, and rearranging the terms will lead to the following resonance relation:

$$\frac{\omega^2}{\gamma_{eff}^{F2}}=[H\cos(\alpha-\phi_F)-H_{Keff}^F-H_A^{eqF}\sin^2\phi_F+H_E^{eqF}\cos\phi_F]\times[H\cos(\alpha-\phi_F)+H_A^{eqF}\cos 2\phi_F+H_E^{eqF}\cos\phi_F]. \quad (23a)$$

Equation (23a) is the resonance relation of a single layer [compare with Eq. (7)] with the following magnetic characteristics: the effective magnetogyric ratio, γ_{eff}^F, is given by

$$\gamma_{eff}^F=\frac{t_AM_A+t_BM_B}{\frac{t_AM_A}{\gamma_A}+\frac{t_BM_B}{\gamma_B}}, \quad (23b)$$

the effective uniaxial anisotropy field, H_{Keff}^F, is

$$H_{Keff}^F=\frac{t_AM_AH_{KeffA}+t_BM_BH_{KeffB}}{t_AM_A+t_BM_B}, \quad (23c)$$

and equivalent exchange anisotropy field, H_E^{eqF}, and in-plane anisotropy field, H_A^{eqF}, are given, respectively, by

$$H_E^{eqF}=\frac{t_AM_AH_E}{t_AM_A+t_BM_B} \quad (23d)$$

and

$$H_A^{eqF}=\frac{t_AM_AH_A}{t_AM_A+t_BM_B}. \quad (23e)$$

Thus, in the strong ferromagnetic coupling, only one mode (the acoustic mode) will be seen in the FMR spectrum. The whole system behaves as a AF/F bilayer system with exchange bias H_E^{eqF} (lower than H_E) and with effective magnetogyric ratio and effective anisotropy fields.

The intensity of this mode can be computed. For strong coupling (J_{eff} very large), it is easy to see that q, in Eq. (6b), will tend to 1, while $J_{eff}(q-1)$ appearing in the denominator of I [Eq. (6a)] will reach the limiting value $ab(\gamma_BH_2^B-\gamma_AH_2^A)/(a\gamma_B+b\gamma_A)$. Substituting these values into I and replacing ω by its expression for strong magnetic coupling [Eq. (23a)], one will find

$$I^F=\frac{2(a+b)(aH_1^A+bH_1^B)}{(aH_1^A+bH_1^B)+(aH_2^A+bH_2^B)}. \quad (24)$$

Once again, $\phi_F=0$ (i.e., $\alpha=0$) will be considered; after substituting the H_i^j by their expressions and rearraging the terms, Eq. (24) may be written as

$$I^F=\frac{2(t_AM_A+t_BM_B)(H+H_E^{eqF}-H_{Keff}^F)}{2H+2H_E^{eqF}+H_A^{eqF}-H_{Keff}^F}, \quad (25)$$

where H_{Keff}^F, H_E^{eqF} and H_A^{eqF} are given by Eqs. (23c)–(23e). Comparing Eqs. (25) and (12), one can see that Eq. (25) may indeed represent the intensity of a ferromagnetic layer with an exchange anisotropy (given by H_E^{eqF}) and in-plane magnetocrystalline anisotropy (H_A^{eqF}) and effective uniaxial anisotropy (H_{Keff}^F). The intensity formula is thus in agreement with the interpretation of the resonance relation.

Furthermore, if one defines an effective magnetization of the whole system, M_{eff}, as the total magnetic moment per unit volume, i.e., $M_{eff}=(t_AM_A+t_BM_B)/(t_A+t_B)$, then Eq. (25) will be

$$I^F=\frac{2(t_A+t_B)M_{eff}(H+H_E^{eqF}-H_{Keff}^F)}{2H+2H_E^{eqF}+H_A^{eqF}-H_{Keff}^F}. \quad (26)$$

The term multiplying the elliptic factor is indeed magnetization times thickness (the total magnetic thickness), as in Eq. (12) (or multiplying by the surface S, I will be proportional to $M\times V$).

One may also study the strong antiferromagnetic coupling. If the applied magnetic field H is strong enough to overcome the effect of the antiferromagnetic coupling, i.e., the two magnetizations are parallel (in the direction of **H**), then the resonance modes are governed by the same equations as for the ferromagnetic coupling [Eqs. (23a)–(23e)]. On the other hand, when the effect of the strong antiferromagnetic coupling is greater than the effect of H, the magnetization will be antiparallel, i.e., $\phi_A=\phi_{AF}$ and $\phi_B=\phi_{AF}+\pi$ (assuming $a>b$) ($\phi_{AF}=0$ for $\alpha=0$). In this case, $c_0=J_1-2J_2=J_{eff}^*$ and $c_1=c_2=-J_{eff}^*$. Following the same procedure as for the ferromagnetic coupling, i.e., keeping only the second-order terms in J_{eff}^*, and rearranging the terms, Eq. (3) will reduce to the following resonance condition:

$$\frac{\omega^2}{\gamma_{eff}^{AF2}}=[H\cos(\alpha-\phi_{AF})-H_{Keff}^{AF}-H_A^{eqAF}\sin^2\phi_{AF}+H_E^{eqAF}\cos\phi_{AF}][H\cos(\alpha-\phi_{AF})+H_A^{eqAF}\cos 2\phi_{AF}+H_E^{eqAF}\cos\phi_{AF}], \quad (27a)$$

where

$$\gamma_{eff}^{AF}=\frac{t_AM_A-t_BM_B}{\frac{t_AM_A}{\gamma_A}-\frac{t_BM_B}{\gamma_B}} \quad (27b)$$

is an effective magnetogyric ratio,

$$H_{Keff}^{AF}=\frac{t_AM_AH_{KeffA}+t_BM_BH_{KeffB}}{t_AM_A-t_BM_B} \quad (27c)$$

is the effective uniaxial anisotropy field,

$$H_E^{eqAF}=\frac{t_AM_AH_E}{t_AM_A-t_BM_B} \quad (27d)$$

is an equivalent exchange anisotropy field, and

$$H_A^{eqAF}=\frac{t_AM_AH_A}{t_AM_A-t_BM_B} \quad (27e)$$

is the in-plane anisotropy field of the whole system.

Once again the whole system behaves as a single ferromagnetic layer exchange coupled to an antiferromagnetic

layer F/AF with effective exchange anisotropy, in-plane, and uniaxial magnetocrystalline anisotropy fields and magnetogyric ratio.

The intensity of this mode for the strong antiferromagnetic coupling can be computed by the same manner. Using Eqs. (6a) and (6b) and taking into account the fact that the magnetizations are antiparrallel, the intensity is found to be

$$I^{AF} = \frac{2(t_A M_A - t_B M_B)^2 (H + H_E^{eqAF} - H_{Keff}^{AF})}{(t_A M_A + t_B M_B)(2H + 2H_E^{eqAF} + H_A^{eqAF} - H_{Keff}^{AF})}. \tag{28}$$

Once again, the intensity relation is in agreement with the resonance relation.

Note that in the strong ferromagnetic coupling case, the equivalent exchange and in-plane anistropy fields of the whole system are lower than the H_E and H_A of layer A. However, in the antiferromagnetic coupling, H_E^{eqAF} and H_A^{eqAF} are larger than H_E and H_A respectively. Hence, in the latter case, one can increase the exchange bias of the equivalent system to any value by the right choice of $(t_A M_A - t_B M_B)$, i.e., by the choice of the ferromagnetic layer thicknesses and nature (through the magnetizations).

VII. CONCLUSION

The ferromagnetic resonance (FMR) modes of a magnetic tunnel junction-like system are described. The dispersion relation, the resonant frequency f vs applied field, as well as the corresponding mode intensity, I, have been studied in different situations. The effect of the exchange anisotropy field H_E and magnetic coupling strengths (J_1 and J_2) on resonance mode behavior is investigated for arbitrary coupling (J_1 and J_2) parameters. Analytical formulas for the resonance condition and intensity have been derived for the low magnetic coupling/high exchange anisotropy case. In this situation, the system is found to behave as two *uncoupled* layers with magnetic characteristics different from those of the initial layers; the effect of the low coupling is to modify the different anisotropies, J_1 contributes to the exchange anisotropy while J_2 modifies the magnetocrystalline anisotropies. For very strong coupling, the system behaves as a single ferromagnetic layer exchange coupled to a antiferromagnetic one (F/AF system) with effective exchange and magnetocrystalline anisotropy fields; these fields have been derived as a function of the individual layer magnetic parameters. The exchange bias of this equivalent system is lower (higher) than H_E for ferromagnetic (antiferromagnetic) coupling.

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