Conductance fluctuations and single-parameter scaling in two-dimensional disordered systems

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We have studied numerically the fluctuations of the conductance, g, and the applicability of single parameter scaling in two-dimensional disordered noninteracting systems. We have checked that the variance of $\ln g$ varies with the lateral sample size as $L^{2/3}$. In agreement with this, we have introduced a parameter to establish the applicability of the single-parameter scaling (SPS) hypothesis. We have found that SPS is satisfied in two-dimensional systems, except for the fluctuation states in the band tails, where a second parameter is needed.

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I. INTRODUCTION

The hypothesis of single-parameter scaling (SPS), motivated by the work of Thouless[1] and introduced by Abrahams *et al.*,[2] constitutes the main foundation of our understanding of localization in disordered systems. According to the SPS hypothesis the full conductance distribution function is governed by a single parameter, the ratio of the system size L to the localization length ξ. The localization length ξ is obtained from the decay of the average of the logarithm of the conductance, $\ln g$, as a function of the lateral system size L

$$\frac{1}{\xi} = -\lim_{L\to\infty} \frac{1}{2L}\langle \ln g\rangle. \tag{1}$$

The conductance of a finite size sample depends on the properties of the system and also on the leads used to measure it, which must be taken into account in any finite size analysis based on it.

The validity of the SPS hypothesis has been thoroughly checked in one-dimensional (1D) systems. In this case, it has been shown that, all the cumulants of $\ln g$ scale linearly with system size.[3] Thus, the distribution function of $\ln g$ approaches a Gaussian form for asymptotically long systems. In this limit, it is fully characterized by two parameters, the mean $\langle \ln g\rangle$ and the variance of $\ln g$

$$\sigma^2 = \langle \ln^2 g\rangle - \langle \ln g\rangle^2. \tag{2}$$

Both parameters are related to each other through a universal law

$$\frac{\sigma^2 L}{\gamma} = 1, \tag{3}$$

where γ is the average Lyapunov exponent, which in 1D systems is also the inverse of the localization length. Equation (3) was first derived in Ref. 4 within the so-called random phase hypothesis, which assumes that there exists a microscopic length scale over which phases of complex transmission and reflection coefficients become completely randomized. With this relation we reduce the two parameters of the distribution to only one and provides, therefore, a justification and interpretation for SPS in 1D systems.

Equation (3) has been proven to hold for many models under most situations. However, it has been shown[5–7] that in the tails of the spectrum, where states are much further apart from each other than the localization length, 1D systems do not obey SPS, as a consequence of the existence of a characteristic length related to the distance between states. For energies with a very small integrated density of states $N(E)$ we can define this new length as $l=1/N(E)$. In the region of fluctuation states, when the integrated density of states is very small, $N(E)\ll 1$, and so the distance between states is much larger than the localization length, $l\gg\xi$, the SPS hypothesis is not valid. For similar reasons, the SPS hypothesis does not apply just at the center of the band.[8]

The situation in two-dimensional (2D) and three-dimensional systems is not as clear as in 1D systems. In those dimensions is far more difficult to do analytical calculations and numerical simulations have been limited until recently to small sample sizes. Most numerical simulations in 2D systems have produced results in agreement with SPS,[9,10] although some recent disagreements have also been reported.[11,12] In the strong localization regime, $\ln g$ was claimed to be normally distributed,[13,14] but attempts to verify a relation between the average conductance and its variance, similar to the one in 1D systems, Eq. (3), have failed so far.

Recently, Slevin, Asada, and Deych[15] have studied numerically the behavior of the Liapunov exponent for 2D $L\times L$ systems. In the strongly localized regime, $\ln g$ is determined by this exponent, but in general it also depends on other transfer matrices exponents.[16] These authors found that γ is normally distributed, unlike $\ln g$ for these systems. Although the mean and the variance of the Liapunov exponent in 2D systems do not obey an equation similar to Eq. (3), their relation still can be characterized by a single parameter, namely the ratio of the system size to the localization length. So they concluded that the SPS hypothesis is verified in 2D systems.

Nguyen *et al.*[17] proposed a model to account for quantum interference effects in the localized regime, where the different possible paths between two points decay exponentially with distance. The overall tunneling amplitude between two sites is then dominated by the shortest or forward-scattering paths. Medina and Kardar[18,19] studied in detail the model. They computed numerically the probability distribution for tunneling and found that is approximately log normal, with its variance increasing with distance as $r^{2/3}$ for 2D systems. This is in contrast with the 1D case, where the variance grows linearly with distance, and with the implicit assumptions of some works on 2D systems.

Following the work of Medina and Kardar,[18,19] we expect the variance of $\ln g$ in 2D systems to vary with system size as $L^{2/3}$, at least in the strongly localized regime. If this is so, a proper checking of SPS in this dimensionality should take this into account.

Our first goal consists in checking numerically that in 2D systems the size dependence of the variance of $\ln g$ is similar to the separation dependence of the variance of the tunneling amplitude between two sites.[17–19] Once this is established, we will check the validity of SPS hypothesis taking into account the implications of the previous size dependence, as we will explain later on.

A second aim of our work is to see if, in analogy with 1D systems, the SPS hypothesis is violated in the tails of the spectrum of 2D systems, where a characteristic length related to the separation between states exists. We will show that this is indeed the case.

II. MODEL

We consider 2D squares samples of size $L \times L$ described by the standard Anderson Hamiltonian

$$H = \sum_i \epsilon_i a_i^\dagger a_i + t \sum_{i,j} a_j^\dagger a_i + \text{h . c.}, \tag{4}$$

where the operator $a_i^\dagger (a_i)$ creates (destroys) an electron at site i of a regular lattice and ϵ_i is the energy of this site chosen randomly between $(-W/2, W/2)$ with uniform probability. The double sum runs over nearest neighbors. The hopping matrix element t is taken equal to -1, which set the energy scale, and the lattice constant equal to 1, setting the length scale. In this model, the energies are measured with respect to the center of the band. In 2D systems the band extends from $-4t-W/2$ to $4t+W/2$.

We have calculated the zero temperature resistance and the density of states of the systems from the Green functions. The resistance $R(0)$ is inversely proportional to the transmission coefficient T between two semi-infinite leads attached at opposite edges of the sample

$$R(0)^{-1} = 2\frac{e^2}{h}T, \tag{5}$$

where the factor of 2 comes from spin. From now on, we will measure the resistance in units of $h/(2e^2)$. The transmission coefficient can be obtained from the Green function $G(E)$ through the expression[21]

$$T = \text{Tr}[(i\hbar v_x)\text{Im}\, G(E)(i\hbar v_x)\text{Im}\, G(E)]. \tag{6}$$

v_x is the velocity operator in the x direction, where the leads are. The Green function can be calculated propagating strip by strip.[20,21] This drastically reduced the computational effort. Instead of inverting an $L^2 \times L^2$ matrix, we just have to invert L times $L \times L$ matrices. The leads serve to obtain the conductivity from the transmission formula in a way well controlled theoretically and close to the experimental situation. The iterative method is also useful for the calculation of the densities of states. This magnitude is obtained for several sample sizes L and then extrapolated to macroscopic sizes by plotting it as a function of $1/L$. With the iterative method we can easily solve square samples with lateral dimension up to $L=400$. We have considered ranges of disorder W from 4 to 12. The number of samples employed depends on the property calculated and will be specified later on.

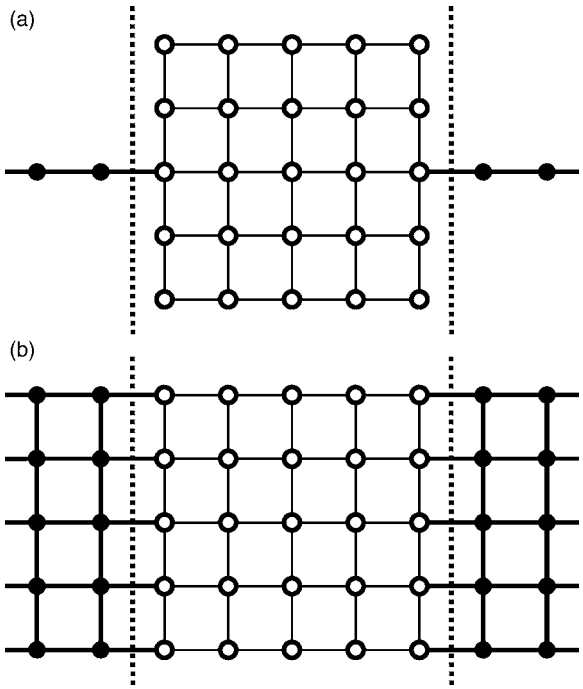

FIG. 1. Schematic picture of the sample and leads considered: (a) narrow leads and (b) wide leads. The open circles represent sites in the system and the solid dots sites in the leads. The lines represent the hopping between sites which are different in the system (thin lines) and in the leads (thick lines). The dashed lines are just a guide to the eye.

The geometry of the leads is known to affect the scaling behavior in the diffusive regime, and is not clear to what extent it affects the behavior in the localized regime. For this reason, we have used two types of leads: wide leads with the same width as the lateral dimension of the samples and narrow (one-dimensional) leads. These are attached to the sample at the centers of opposite edges, as shown in Fig. 1(a). The scheme of the wide leads is shown in Fig. 1(b). In both cases the leads are represented by the same Hamiltonian as the system, Eq. (4), without diagonal disorder. As we want to study the conductance in the tails of the density of states of the system, we have chosen the hopping matrix elements of the leads equal to 10 to ensure a broad energy band in the leads. We have checked that the results do not depend strongly on this choice. The narrow leads can be viewed as a simplified model of a point contact, while the wide leads should roughly correspond to electrodes in contact with the whole edge of the sample. We use cyclic periodic boundary conditions in the direction perpendicular to the leads.

In the model considered by Nguyen *et al.*[17] and Medina and Kardar,[18,19] the random potential could take only two values and the spectrum could be divided into three regions with qualitatively different spectral and transport properties.[8] In our model, with diagonal disorder uniformly distributed, the situation is more complex, but we still can divide the

whole spectrum in a band corresponding to energies $|E|<2dt+W$, where d is the dimensionality of the system, and two tails outside this region. In the band the SPS hypothesis is satisfied, while in the tails we expect that this is not satisfied, in analogy with 1D systems. In the tails, the density of states is very low, so an additional relevant length is present and we may need more than one parameter to describe the behavior of the system.

III. ZERO TEMPERATURE CONDUCTANCE FLUCTUATIONS

In 1D systems the variance of the logarithm of the zero temperature conductance is proportional to the length of the system. In 2D systems, the situation is more complicated. Scaling theory tell us that the variance of $\ln R(0)$ should only depend on the same parameter as $\langle \ln R(0)\rangle$. We have checked that, in the localized regime, the average of $\ln R(0)$ follows, to a good approximation, a law of the form

$$\langle \ln R(0)\rangle = \frac{2L}{\xi} + c, \tag{7}$$

where L is the lateral size of the samples, ξ is the localization length, and c is a small constant that only depends on the type of leads, but not on the disorder or the Fermi energy as long as we are not in the tails of the spectra. According to SPS, the ratio L/ξ fully determines $\langle \ln R(0)\rangle$ and the other moments of the distribution, in particular the variance. Similarly, the variance is a universal function of the average. We expect the variance of $\ln R(0)$ to be proportional to a power law of L/ξ with a dimensionality dependent exponent,[18] not necessarily equal to unity. The aim of this section is to determine numerically this exponent and compare it with previous predictions. At the same time, it will provide us with a useful tool to properly check the validity of SPS in 2D systems.

Medina and Kardar[18] checked that the variance of the tunneling amplitude between two points separated a distance r goes as $r^{2/3}$ in the strongly localized regime of a model with random diagonal energies that can only take two values. We want to check if their results can be extended in a double way: to the more standard model with a random uniform distribution of diagonal energies and to the conductance of the system, measured through the transmission between two leads attach to the sample. First of all, we consider 1D semiinfinite leads, since the transmission between their points of contact should be similar to the tunneling amplitude between them. For this case, we have calculated the variance σ^2 of $\ln R(0)$ for square samples as a function of the lateral size L of the samples and the range of the disorder W. In Fig. 2 we show the variance of $\ln R(0)$ as a function of $\langle \ln R(0)\rangle^{2/3}$ for different values of the disorder and the Fermi energy. We have used as independent variable in this figure $\langle \ln R(0)\rangle^{2/3}$, instead of $(L/\xi)^{2/3}$, because in this way the results are more easily verified experimentally. In any case, both procedures give very similar results. Inside the figure, we specify for each set of points the value of the disorder W and of the Fermi energy E. Each point of the graph represents an average over at least 2000 samples. The data that lie on the same line are from the bulk of the spectra, while the few data that do not overlap with the rest correspond to energies on the tails of the spectra. The line fitting the complete set of overlapping points in Fig. 2 is $\sigma^2=(6.18\pm0.04)\langle \ln R(0)\rangle^{2/3} -13.8\pm0.5$.

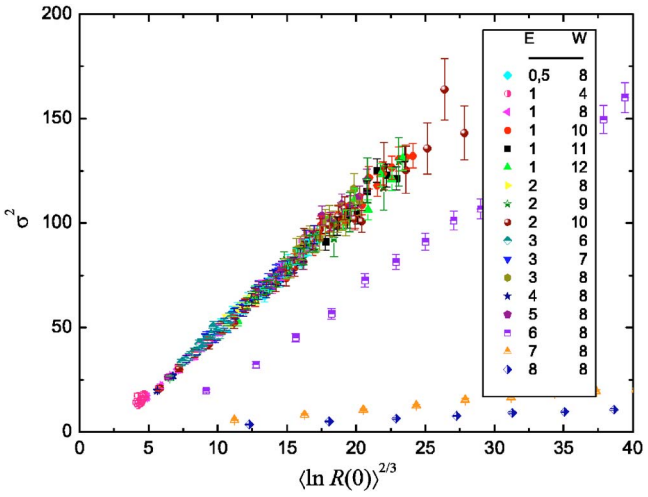

FIG. 2. (Color online) σ^2 obtained with 1D leads as a function of $\langle \ln R(0)\rangle^{2/3}$ for the values of the disorder and the Fermi energy shown in the graph.

As regards the data from the bulk, we can extract the following conclusions from Fig. 2. Since the data are fitted very well by a straight line, the variance of $\ln R(0)$ scale with sample size with an exponent of 2/3, as predicted by Medina and Kardar[18,19] for the fluctuations of the tunneling amplitude as a function of distance. The good overlap of the data points also indicates the validity of the SPS hypothesis, since the variance σ^2 only depends on the same parameter as the mean of $\ln R(0)$. In disagreement with assumptions of previous works, SPS does not imply a linear dependence of the variance with the scaling parameter, L/ξ. This implication is only true in 1D systems.

We now turn our attention to the points coming from the tails. We see in Fig. 2 that the last three sets in the legend are fitted fairly well by straight lines, when plotted versus $\langle \ln R(0)\rangle^{2/3}$, but their slopes and their constant terms depend now on the specific values of the disorder and the Fermi energy. To determine the variance in the tails, we need at least one more parameter to specify the corresponding slope. This slope provides us with an excellent tool to check the applicability of SPS hypothesis in 2D systems, what we will do in the next section.

Since in many cases the resistance is studied with wide leads attached to the samples, it is interesting to know whether the previous results also apply to this very different geometry. Thus, in Fig. 3 we have plotted the variance of $\ln R(0)$ as a function of $\langle \ln R(0)\rangle^{2/3}$ for the case of wide leads. The different values of the disorder and the Fermi energy employed are specified inside the figure. The data that lie on the same line are from the bulk of the spectra, while the few data that do not overlap with the rest correspond to energies on the tails of the spectra. Again, the data are fitted

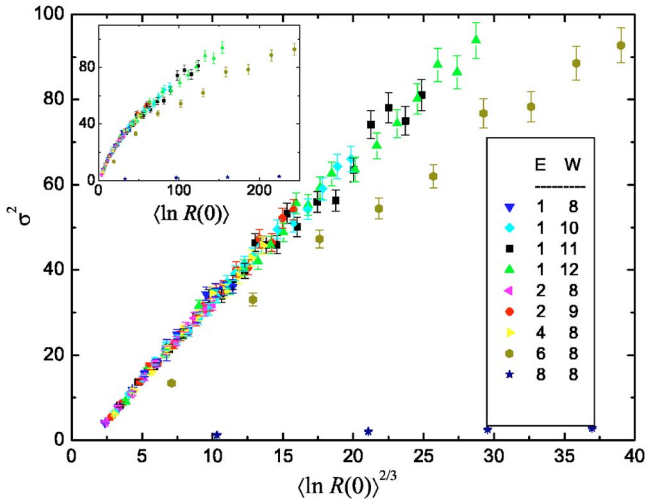

FIG. 3. (Color online) σ^2 obtained with wide leads as a function of $\langle \ln R(0) \rangle$ to the power of 2/3 for the values of the disorder and the Fermi energy shown in the graph. The inset shows the same data as the main part as a function of $\langle \ln R(0) \rangle$.

well by straight lines proving that the predictions of Medina and Kardar[18,19] have a very general applicability to the conductance of 2D systems, and do not restrict to the model and to the magnitude considered by these authors. The results do not restrict to the strongly localized regime, being valid even when we approach the diffusive regime, which corresponds to small values of Figs. 2 and 3 and is better appreciated in a logarithmic scale (not shown). Of course, this behavior must eventually change at small values of $\langle \ln R(0) \rangle$, since the variance must be positive. In fact, we know that the variance of $\ln R(0)$ should tend to zero in the limit of large conductances as a consequence of the universal conductance fluctuations. The mean and the variance of the Liapunov exponent do appreciably change behavior much before we enter the diffusive regime, while $\langle \ln R(0) \rangle$ and σ^2 do not present any significant variation when we approach this regime. The $(L/\xi)^{2/3}$ (or alternatively $\langle \ln R(0) \rangle^{2/3}$) behavior of the variance of $\ln R(0)$ has not been recognized by many authors, what has complicated their analysis of the data.[8,11,12] In some cases, even it has been assumed that SPS requires a linear dependence of σ^2 with distance.[11,12] The results coming from the bulk of the spectra overlap in a single curve, while those from the tails present different slopes. The line fitting the set of all overlapping points in Fig. 3 is $\sigma^2 = (3.66 \pm 0.02) \times \langle \ln R(0) \rangle^{2/3} - 4.40 \pm 0.12$. The slope of the overlapping points depends on the type of leads used. In the inset of Fig. 3 we show the same data as in the main part as a function of $\langle \ln R(0) \rangle$ to clearly prove that the variance of $\ln R(0)$ is not proportional to distance in 2D systems.

If the data in Figs. 2 and 3 were plotted on a double logarithmic scale, we would see a straight line with a small curvature due to the constant term present in these two figures. We note that care must be taken when extracting the power law behavior in this way due to the presence of this constant term.

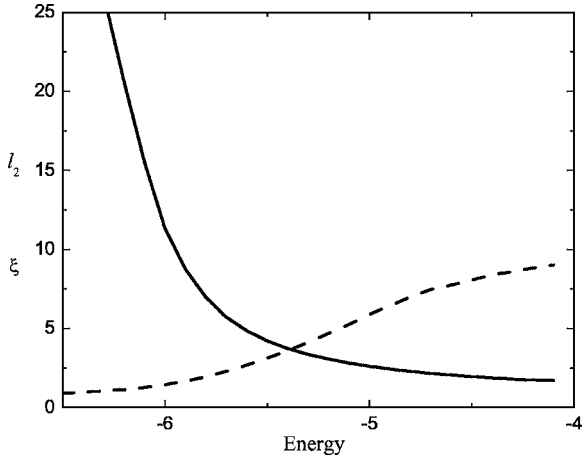

FIG. 4. Characteristic distance between states l_2 (thick line) and localization length (dashed line) as a function of Fermi energy for a disorder $W=8$.

IV. VIOLATION OF SINGLE-PARAMETER SCALING IN THE TAILS OF THE SPECTRUM

From Figs. 2 and 3, we expect SPS to be valid for the data that overlap into a single curve and not to apply to the data that fall away from this main curve. In order to check this quantitatively, we have to characterize the variance of $\ln R(0)$ trough one (or a few) parameter that we can later relate to the characteristic lengths of the problem. For each range of the disorder and Fermi energy, we fit σ^2 as a function of $(L/\xi)^{2/3}$ to a straight line

$$\sigma^2 = A\left(\frac{L}{\xi}\right)^{2/3} + B. \tag{8}$$

This behavior is satisfied pretty well in all cases, even for the energies in the tail of the spectra whose data do not overlap with the rest, as we have already mentioned. The slope A of the fitted line for each disorder and each Fermi energy is the most convenient parameter to characterize the behavior of the variance. For the cases where A is constant, the SPS hypothesis is valid, while if A varies this is not the case anymore and we have to try to relate A to other relevant parameters of the problem. A plays here the same role as the parameter $\tau = \sigma^2 L/\gamma$ plays in 1D systems.

Similarly to the situation in 1D systems, we can try to relate our slope A to the length associated with the distance between states in the tails. This characteristic length is obtained from the density of states in the following way. At the same time as we calculate the transmission amplitude from the Green functions, we obtain the local density of states and sum it over the sites of the sample. We do this for different sample sizes and extrapolate the results to macroscopic sizes. We then get the integrated density of states $N(E)$ and from it the characteristic distance between states, which in two dimensions is defined as

$$l_2 = [\pi N(E)]^{-1/2}. \tag{9}$$

In Fig. 4 we represent the characteristic length l_2 (thick

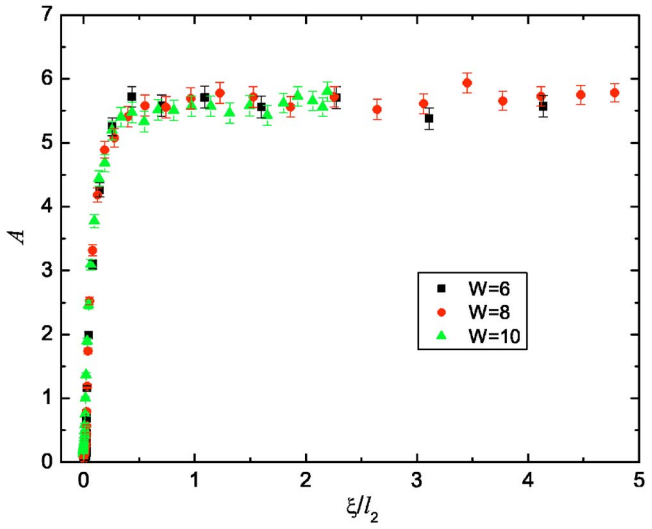

FIG. 5. (Color online) Slope A of the variance σ^2 vs $(L/\xi)^{2/3}$ as a function of the second characteristic parameter ξ/l_2. The values of the disorder are specified in the graph and for each of them we consider several values of the Fermi energy.

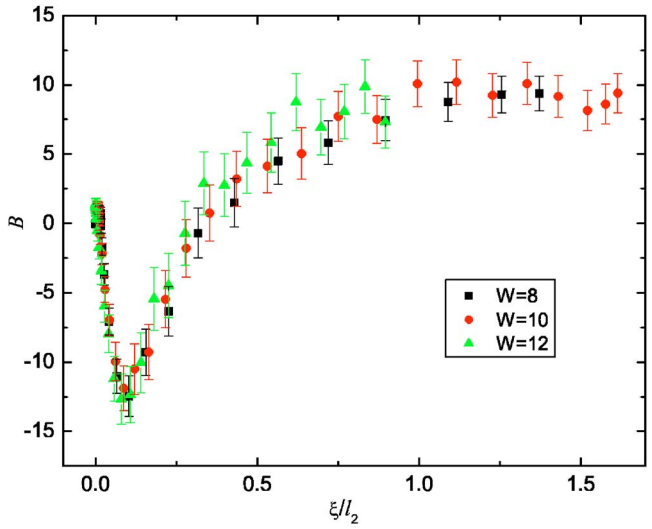

FIG. 6. (Color online) Constant term B of the variance σ^2 vs $(L/\xi)^{2/3}$ as a function of ξ/l_2 for the values of the disorder shown in the graph and several Fermi energies for each of them.

line) and the localization length ξ (dashed line), extrapolated to macroscopic sizes, as a function of energy for a disorder $W=8$. Far in the tails (small energies) the length l_2 becomes very large and is much larger than the localization length for the values of the disorder considered.

In the tails, where SPS hypothesis fails, the next thing to try is a two parameter scaling and the natural candidate for our second parameter is ξ/l_2, the ratio of the localization length to the characteristic distance between states. In Fig. 5 we plot the slope A as a function of ξ/l_2 for different values of the disorder and the Fermi energy. The values of the disorder are specified in the graph and for each of them we consider several values of the Fermi energy. Each point corresponds to an average over 100,000 samples. We see that the data overlap fairly well into a universal curve in agreement with a two-parameter scaling hypothesis. For large values of ξ/l_2, the curve tends to a constant as we enter the region where SPS is valid. The data in Fig. 5 correspond to wide leads, but similar results are obtained for 1D leads. The limiting value of the data for large values of ξ/l_2 depends on the leads, and corresponds to the slope of the overlapping points in Figs. 2 and 3 for 1D and wide leads, respectively, when represented as a function of $(L/\xi)^{2/3}$. This slope is equal to 9.07 ± 0.05 for 1D leads and to 5.43 ± 0.03 for wide leads.

The parameter B in Eq. (8) should also be constant for the SPS hypothesis to apply or a function of ξ/l_2 for a two parameter scaling to hold. In Fig. 6 we show the dependence of the constant term B with our second characteristic parameter ξ/l_2 for the values of the disorder specify in the graph. For each disorder we have considered several values of the Fermi energy. The data collapse again into a universal curve, supporting the applicability of two parameter scaling in the tails of the spectra, that leads to SPS in the bands where the localization length becomes much larger than the separation between states. We note that the decreasing behavior of B for small values of ξ/l_2 may be a finite size effect due to the fact that the sample sizes employed are larger than the length l_2. In any case this does not affect any of our conclusions. Figure 6 corresponds to results for 1D leads. Similar ones are also obtained for wide leads, although the relative errors involved are larger because the value of B is much smaller in this case.

V. DISCUSSION AND CONCLUSIONS

Our results can be tested experimentally, in particular Figs. 2 and 3. The information required may be obtained from raw data of conductance as a function of gate voltage as measured in many experiments (see for example Refs. 22 and 23). Averaging over a small range of gate voltages one can obtain both $\langle \ln(g) \rangle$ and $\mathrm{Var}(\ln(g))$. As our results are for $T=0$, it would be necessary to verify that the temperature is low enough to be below the variable-range hopping regime. This is easy to achieve close to the crossover region, far from the strongly localized regime. It is worth mentioning that in Ref. 23 the authors claim that for $T<0.1$ K the conductance is already temperature independent, while they are able to measure good conductance fluctuations for different samples at $T=10$ mK. So it seems that an experimental verification of Figs. 2 or 3 is certainly possible. We note that we expect that for different sample geometries the slope of the curve of $\mathrm{Var}(\ln(g))$ as a function of $\langle \ln(g) \rangle^{2/3}$ may change, but the results should fit a straight line in all cases for two-dimensional samples. Our results might also be tested in an indirect way in the variable-range hopping regime, but this is outside the scope of this work.

We have shown that, in 2D disordered noninteracting systems, the variance of $\ln R(0)$ grows with system size proportionally to $(L/\xi)^{2/3}$. This behavior is independent of the leads attached to the sample and is similar to that of the tunneling amplitude between two sites as a function of distance. It is valid all the way from the strongly localized to the diffusive regime. The slope of the line of σ^2 vs $(L/\xi)^{2/3}$ constitutes an

excellent parameter to verify the applicability of the SPS hypothesis. We have checked that this hypothesis is valid for energies in the band, while a second parameter is needed for energies in the tail of the spectra. A good choice for this second parameter is the ratio between the localization length and the distance between the fluctuation states in these tails.

The distribution function of $\ln R(0)$ is not Gaussian and the values of the third moment can be measured reliably. So any scaling hypothesis has to be tested with this parameter also. Preliminary results indicate that the skewness only depends on L/ξ for energies in the band, corroborating the SPS hypothesis, while its behavior for energies in the tails of the spectra can be characterized by the same two parameters as the variance.

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