Critical dynamics, duality, and the exact dynamic exponent in extreme type-II superconductors

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The critical dynamics of superconductors is studied using renormalization group and duality arguments. We show that in extreme type-II superconductors the dynamic critical exponent is given exactly by $z=3/2$. This result does not rely on the widely used models of critical dynamics. Instead, it is shown that $z=3/2$ follows from the duality between the extreme type-II superconductor and a model with a critically fluctuating gauge field. Our result is in agreement with Monte Carlo simulations and at least one experiment.

I. INTRODUCTION

The high-T_c cuprate superconductors have very large values of the Ginzburg parameter κ. In a material like YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) at optimal doping, we have $\kappa \sim 100$. It is therefore a good approximation to assume that these materials are extreme type-II superconductors. By extreme type-II superconductor we mean $\kappa \rightarrow \infty$. In such a regime it is expected that the static critical properties at zero external field are the same as in superfluid ^{4}He.[1] This expectation is confirmed by experiments measuring the specific heat[2] and penetration depth[3] in bulk samples of YBCO. Thus, as far as static critical phenomena are concerned, there is no doubt that the critical behavior of bulk YBCO is governed by the three-dimensional *XY* universality class. On the basis of such a consensus, we might also expect that the dynamical universality class is the same as in superfluid ^{4}He. Unfortunately, we are far from reaching a consensus on the dynamical universality class of YBCO or, more generally, of any high-T_c cuprate. What is interesting here is that the lack of consensus comes both from the theoretical and experimental sides. The theoretical debate tries to establish whether the dynamical universality class corresponds to model-A or model-F dynamics.[4] Model-A dynamics is purely relaxational and gives the value $z \approx 2.015$ for the dynamic critical exponent in three dimensions. Model F, on the other hand, features a conserved density coupled to a spin-wave mode and gives the *exact* value $z=d/2$ for $d \in (2,4]$. For the dynamical universality class of extreme type-II superconductors, Monte Carlo simulations give $z \approx 3/2$,[5,6] which would be consistent with model-F dynamics and therefore with superfluid ^{4}He dynamical universality class. However, in a recent Letter, Agi and Goldenfeld[7] claim that the correct result from a lattice model should be instead $z \approx 2$—i.e., consistent with model-A dynamics. Experimentally, either large values for z are found, typically in the range $z \sim 2.3-3.0$,[8] or values consistent with model-F dynamics.[9,10] The arguments of Ref. 7 were further discussed in two recent Comments.[11]

In principle, model-F dynamics is not compatible with superconductor critical dynamics, since screening effects tend to suppress the spin-wave mode.[12,13] However, model A is also inappropriate to study the critical dynamics in superconductors. Indeed, model A does not give a gauge-independent result for z in the magnetic critical fluctuation (MCF) regime. A technically correct analysis should consider the extreme type-II regime as a limit of the full MCF regime which has a genuine local gauge symmetry. It turns out that in a gauge-invariant theory the only operators with a nonzero expectation value are the gauge-invariant ones.[14] For instance, the ac conductivity is such a gauge invariant quantity. Therefore, its scaling behavior is necessarily gauge independent. Since evaluation of z through model A in the MCF regime gives a gauge-dependent result, we conclude that model A is also not the right option.

In this paper we use renormalization group (RG) and duality arguments to determine the dynamical universality class of extreme type-II superconductors. We will establish that in three dimensions $z=3/2$ *exactly*. This result will be obtained by combining exact scaling arguments with exact duality results. While the scaling arguments are generally valid in $d \in (2,4]$, the duality arguments will be only valid at $d=3$. The result will be obtained through the following strategy. In Sec. II we use the RG to obtain exact scaling relations for the penetration depth and for the ac conductivity. This will be done in both the *XY* (extreme type-II limit) and MCF regimes. While much of the steps in this part of derivation are known, it is important to review this approach here to emphasize the intimate relationship between the scaling of the penetration depth and the one of the ac conductivity. The important point is that scaling behavior in the *XY* regime is different from the one in the MCF regime. In Sec. III we discuss the critical dynamics from the perspective of the exact duality between an extreme type-II superconductor at zero field and a model exactly equivalent to the Ginzburg-Landau (GL) model in the MCF regime. Indeed, the dual model features a fluctuating vector potential coupled to a complex field, dubbed the disorder parameter field as opposed to the order parameter field of the original model. The derived results for the GL model in the MCF regime will then be applied to the dual model, and duality relations between the currents will be used to establish that $z=3/2$.

II. SCALING AND DYNAMICS IN THE GINZBURG-LANDAU MODEL

In order to make the paper self-contained, we review in this section the basic scaling properties of the GL model in both the static and dynamic critical regimes.

A. Static scaling behavior

Let us consider the *bare* Hamiltonian of the GL model,

$$\mathcal{H} = \frac{1}{2}(\nabla \times \mathbf{A}_0)^2 + |(\nabla - iq_0\mathbf{A}_0)\psi_0|^2 + \mu_0^2|\psi_0|^2 + \frac{u_0}{2}|\psi_0|^4, \tag{1}$$

where $q_0 = 2e_0$ is the charge of the Cooper pair and $\mu_0^2 \propto \tau$, with $\tau = (T - T_c)/T_c$. In our notation the zero subindex denotes bare quantities while in renormalized quantities the zero subindices are absent. The bare Ginzburg parameter is $\kappa_0 = \lambda_0/\xi_0 = (u_0/2q_0^2)^{1/2}$, where λ_0 and $\xi_0 \equiv \mu_0^{-1}$ are the bare penetration depth and correlation length, respectively.[10] We can rewrite the above Hamiltonian in terms of renormalized quantities as follows:

$$\mathcal{H} = \frac{Z_A}{2}(\nabla \times \mathbf{A})^2 + Z_\psi|(\nabla - iq\mathbf{A})\psi|^2 + Z_\mu\mu^2|\psi|^2 + \frac{Z_u u}{2}|\psi|^4. \tag{2}$$

The renormalized correlation length is given by $\xi = \mu^{-1}$. From the Ward identities it follows that the renormalized charge squared is given by $q^2 = Z_A q_0^2$. The dimensionless couplings are $f \equiv \mu^{d-4}q^2$ and $g \equiv \mu^{d-4}Z_\psi^2 u_0/Z_u$. The fixed-point structure is well known but cannot be completely obtained by perturbative means. Fixed points associated with nonzero charge $f_* \neq 0$ are nonperturbative but their existence in the flow diagram is now well established.[15–24] The infrared-stable charged fixed point governs the MCF regime while the extreme type-II or *XY* regime is governed by the *uncharged XY* fixed point.[25]

The renormalized Ginzburg parameter is given by $\kappa = \mu/\mu_A$, where $\mu_A = \lambda^{-1}$ is the renormalized vector potential mass generated by the Anderson-Higgs mechanism. Due to the Ward identities, this can also be written as $\kappa = (g/2f)^{1/2}$, and therefore the renormalized Ginzburg parameter has the same form as the bare one, with the bare coupling constants replaced by the renormalized ones. From this it can be shown that the following *exact* evolution equation for the renormalized vector potential mass holds:[22]

$$\mu\frac{\partial\mu_A^2}{\partial\mu} = \left(d - 2 + \gamma_A - \frac{\beta_g}{g}\right)\mu_A^2, \tag{3}$$

where $\gamma_A(f,g) \equiv \mu\partial \ln Z_A/\partial\mu$ and $\beta_g \equiv \mu\partial g/\partial\mu$. Equation (3) implies that near the phase transition,

$$\mu_A \sim \mu^{(d-2+\eta_A)/2}, \tag{4}$$

where $\eta_A = \gamma_A^*$ is the anomalous dimension of the vector potential. In the MCF regime $\eta_A = 4-d$,[18,20,23] and we obtain that $\nu' = \nu$,[20] where ν' and ν are the penetration depth and correlation length exponents, respectively. This result was confirmed by Monte Carlo simulations.[21] In the *XY* regime, on the other hand, the penetration depth exponent is given by[12]

$$\nu' = \frac{\nu(d-2)}{2}, \tag{5}$$

where ν is the correlation length exponent. At $d=3$ we have $\nu \approx 2/3$ and $\nu' \approx 1/3$. The result $\nu' \approx 1/3$ is confirmed experimentally in high-quality single crystals of YBCO.[3] Note that in the *XY* regime $\kappa_0 \to \infty$ while $\kappa \to 0$.[12]

B. Dynamic scaling behavior

The ac conductivity is given by

$$\sigma(\omega) = \frac{q^2 K(-i\omega)}{-i\omega}, \tag{6}$$

where $K(-i\omega)$ is obtained from the current-current correlation function at zero momentum[26]—i.e., $K(-i\omega) = \lim_{|\mathbf{p}|\to 0} K(-i\omega, \mathbf{p})$ where $K(-i\omega, \mathbf{p}) = \sum_\mu K_{\mu\mu}(-i\omega, \mathbf{p})$ with

$$K_{\mu\nu}(-i\omega, \mathbf{p}) = \langle|\psi|^2\rangle\delta_{\mu\nu} - \frac{1}{q^2}\langle\hat{J}_\mu(\omega, \mathbf{p})\cdot\hat{J}_\nu(-\omega, \mathbf{p})\rangle, \tag{7}$$

and $\hat{J}_\mu(\omega, \mathbf{p})$ is the Fourier transform of the superconducting current:

$$J_\mu = -iq(\psi^*\partial_\mu\psi - \psi\partial_\mu\psi^*) - 2q^2|\psi|^2 A_\mu. \tag{8}$$

The function $K_{\mu\nu}(-i\omega, \mathbf{p})$ is purely transverse.[26] The superfluid density ρ_s is given by $\lim_{\omega\to 0} K(-i\omega)$ and thus, by virtue of the Josephson relation, we obtain

$$\lim_{\mu\to 0} K(-i\omega) \sim (-i\omega)^{(d-2)/z}. \tag{9}$$

Since $q^2 \sim \mu^{\eta_A}$, we obtain from Eq. (6) the behavior

$$\sigma(\omega)|_{T=T_c} \sim (-i\omega)^{(d-2-z+\eta_A)/z}. \tag{10}$$

Therefore, in the *XY* universality class we have[12,13]

$$\sigma(\omega)|_{T=T_c} \sim (-i\omega)^{(d-2-z)/z}, \tag{11}$$

while in the MCF regime we obtain

$$\sigma(\omega)_{T=T_c} \sim (-i\omega)^{(2-z)/z}. \tag{12}$$

From Eq. (6) we obtain that below T_c the low-frequency behavior of the ac conductivity is

$$\sigma(\omega) \approx \frac{\mu_A^2}{-i\omega}. \tag{13}$$

In Ref. 12 this is written symply as $\sigma(\omega) \approx \rho_s/(-i\omega)$; i.e., the charge is not shown explicitly since only the *XY* regime was considered and in this case the charge does not fluctuate. Taking the low-frequency behavior (13) into account, we obtain, from Eqs. (3) and (6),

$$\mu\frac{\partial\sigma(\omega)}{\partial\mu} \approx \left(d - 2 - z + \gamma_A - \frac{\beta_g}{g}\right)\sigma(\omega), \tag{14}$$

which implies that near the phase transition,

$$\sigma(\omega) \sim \mu^{d-2-z+\eta_A}. \tag{15}$$

Since in the *XY* universality class $\eta_A = 0$, we recover from Eq. (15) the well-known scaling[12]

$$\sigma(\omega) \sim \mu^{d-2-z}. \tag{16}$$

Note that Fisher *et al.*[12] need to assume the Josephson relation $\rho_s \sim \mu^{d-2}$ to derive the *XY* scaling of the ac conductivity. Within our approach the more general scaling relation (15) follows from Eq. (3) and the *XY* scaling emerges as a particular case. In the MCF regime we obtain[27–29]

$$\sigma(\omega) \sim \mu^{2-z}. \tag{17}$$

III. DUALITY AND CRITICAL DYNAMICS

A. Duality and disorder field theory

In the extreme type-II limit $\kappa_0 \to \infty$ and we have essentially a superfluid model at zero field; i.e., the corresponding Hamiltonian is the same as in Eq. (1) with $\mathbf{A}_0 = 0$. The lattice version of this model in the London limit is exactly dual to the so-called "frozen" superconductor.[30,31] Starting from the Villain form of the *XY* model we obtain, after dualizing it, the following lattice model Hamiltonian:

$$H = \sum_l \left[\frac{1}{2K} (\boldsymbol{\nabla} \times \mathbf{h}_l)^2 - 2\pi i \mathbf{M}_l \cdot \mathbf{h}_l \right], \tag{18}$$

where K is the bare superfluid stiffness, $a_{l\mu} \in (-\infty, \infty)$, and $M_{l\mu}$ is an integer link variable satisfying the constraint $\boldsymbol{\nabla} \cdot \mathbf{M}_l = 0$. The lattice derivative is defined as usual, $\nabla_\mu f_l \equiv f_{l+\hat{\mu}} - f_l$. The link variables play the role of vortex currents, and the zero lattice divergence constraint means that only closed vortex loops should be taken into account. Integration over $\mathbf{a}_l$ gives a long-range interaction between the link variables. The link variables will interact through a potential $V(\mathbf{r}_l - \mathbf{r}_m)$ behaving at large distances like $V(\mathbf{r}_l - \mathbf{r}_m) \sim 1/|\mathbf{r}_l - \mathbf{r}_m|$. At short distances the potential is divergent. This short-distance divergence can be regularized by adding to the Hamiltonian (18) a core energy term $(\varepsilon_0/2)\Sigma_l \mathbf{M}_l^2$. Writing the constraint $\boldsymbol{\nabla} \cdot \mathbf{M}_l = 0$ using the integral representation of the Kronecker delta and performing the sum over $\mathbf{M}_l$ using the Poisson formula we arrive at the Hamiltonian

$$H = \sum_l \left[\frac{1}{2\varepsilon_0} (\boldsymbol{\nabla} \theta_l - 2\pi \mathbf{N}_l - 2\pi \sqrt{K} \mathbf{h}_l)^2 + \frac{1}{2} (\boldsymbol{\nabla} \times \mathbf{h}_l)^2 \right], \tag{19}$$

where we have rescaled $\mathbf{h}_l$. The dual-lattice Hamiltonian (19) has exactly the same form as the Hamiltonian for a Villain lattice superconductor. Note that $2\pi\sqrt{K}$ plays the role of the charge. The sum over the integers $\mathbf{N}_l$ can be converted into a disorder field theory (DFT),[32] which has precisely the same form as the original GL model in Eq. (1), except that the physical properties of the fields have changed. The electromagnetic vector potential $\mathbf{A}_0$ is replaced by the gauge field $\mathbf{h}_0$ describing vortices, and the charge q_0 becomes the Biot-Savart-type coupling strength between vortices $2\pi\sqrt{\rho_s^0}$, where ρ_s^0 is the bare superfluid density. An important aspect

of duality is that the disorder field theory that comes out of it has an "inverted" temperature axis;[15] i.e., the broken symmetry phase of the disorder field theory corresponds to the symmetric phase of the original theory and vice versa. This is actually the meaning of the expression "disorder field" used in this paper: instead of having an *order parameter* as in the original model, the dual model has a *disorder parameter*. Both models describe the same physics and have of course the *same* critical temperature T_c. Since the Ward identities for either theory imply that the critical singularities are the same irrespective of whether T_c is approached from above or from below, we can use the *same* scale μ to study the scaling behavior in both models.[22,33] The critical exponents α and ν of the dual model are the same as in the original model. This is because the dual model gives the *same* free energy of the original model, up to nonsingular terms. Therefore, the exponent appearing in the scaling of the singular part of the free energy, α, is the same in both models. The hyperscaling relation then implies that ν is also the same in both models.

The continuum dual model for superconductors was introduced in Ref. 17 and further discussed in Ref. 34 (see also Ref. 35). Such a continuum dual theory represents a generalization of the London model. In the extreme type-II limit and zero external magnetic field, it is obtained from the continuum limit of Eq. (19),

$$\tilde{\mathcal{H}} = \frac{1}{2} (\boldsymbol{\nabla} \times \mathbf{h}_0)^2 + |(\boldsymbol{\nabla} - i\tilde{q}_0 \mathbf{h}_0)\phi_0|^2 + m_0^2 |\phi_0|^2 + \frac{v_0}{2} |\phi_0|^4, \tag{20}$$

where the *dual bare charge* is given by

$$\tilde{q}_0 = \frac{2\pi\mu_{A,0}}{q} = 2\pi\sqrt{\rho_s^0}. \tag{21}$$

The Hamiltonian (20) has the same form as the GL Hamiltonian in Eq. (1). In Eq. (20) the gauge field $\mathbf{h}_0$ is minimally coupled to the already mentioned *disorder field* ϕ_0. We should note that this disorder GL-like theory is valid strictly in $d = 3$. Indeed, if we try to extrapolate to the range of dimensionalities as in scaling relations of Sec. II, we would obtain that the renormalized superfluid density, as the "charge" of the DFT, scales like $\rho_s \sim \mu^{4-d}$, which would agree with Josephson's relation *only* for $d = 3$.

B. Critical dynamics of the dual model

In the duality transformations in the lattice only the phase of the order paramerer plays a role.[26,36] Actually, often duality arguments are worked out directly in the continuum by relating the gradient of the phase to a "magnetic field" that couples to the vortex loops within the dual model.[35,37] In the case of critical dynamics it is simpler to follow a similar approach in order to derive the main results.

The superfluid velocity $\mathbf{v}_s$ satisfies the dynamical equation[26]

$$\frac{\partial \mathbf{v}_s}{\partial t} = \Gamma_0 \rho_s^0 \nabla^2 \mathbf{v}_s + q\mathbf{E}, \tag{22}$$

where Γ_0 is the bare kinetic coefficient, $\mathbf{E}$ is the electric field, and we have neglected the noise for simplicity. The super-

fluid velocity is related as usual to the phase of the order parameter—i.e., $\mathbf{v}_s = \nabla \theta$. Because of the vortices, $\nabla \times \mathbf{v}_s \neq 0$. The vortex current is given by

$$\mathbf{J}_v = \frac{\sqrt{\rho_s^0}}{2\pi} \nabla \times \mathbf{v}_s. \tag{23}$$

Thus, by taking the curl of Eq. (22), we obtain a dynamical equation for the vortex current:

$$\frac{\partial \mathbf{J}_v}{\partial t} = \Gamma_0 \rho_s^0 \nabla^2 \mathbf{J}_v - \frac{\mu_{A,0}}{2\pi} \frac{\partial \mathbf{B}}{\partial t}, \tag{24}$$

where $\mathbf{B}$ is the macroscopic magnetic induction field. In Fourier space and at zero momentum we obtain

$$\mathbf{J}_v(\omega) = -\frac{\mu_{A,0}}{2\pi} \mathbf{B}(\omega). \tag{25}$$

Thus, the above simple analysis already shows us that the introduction of dynamics in the duality approach leads to interesting consequences. In the original model linear response theory relates the current to the electric field in Fourier space as

$$\mathbf{J}(\omega, \mathbf{p}) = \sigma(\omega, \mathbf{p})\mathbf{E}(\omega, \mathbf{p}). \tag{26}$$

With respect to the dynamics of the system, the statistical mechanics duality implies also *electric-magnetic duality*, in which case the electric field is replaced by the (true) magnetic field in a linear response theory for the vortex current.

Generally, the current of the original GL model is related to the vortex current in the dual model by the formula

$$\nabla \times \mathbf{J}(t,\mathbf{r}) = \int dt' \int d^3r' Q(t-t',\mathbf{r}-\mathbf{r}')\mathbf{J}_v(t',\mathbf{r}'), \tag{27}$$

generalizing the classical static duality relation

$$2\pi \mu_{A,0} \mathbf{J}_v = \nabla \times \mathbf{J}, \tag{28}$$

which is equivalent to Eq. (23), since $\mathbf{J} = q\rho_s^0 \mathbf{v}_s$. Note that the factor $2\pi\sqrt{\rho_s^0}$ accounts for an elementary flux quantum in the dual model corresponding to a unit charge.[35] Equation (28) also follows from the continuum limit of the exact duality transformation on the lattice.

The low-frequency limit of $Q(\omega) \equiv Q(\omega, \mathbf{p}=0)$ is given by

$$Q(\omega) \approx 2\pi q \sqrt{\rho_s} = 2\pi \mu_A. \tag{29}$$

Thus, when the fluctuations are taken into account, Eq. (28) will hold approximately in the low-frequency limit with $\mu_{A,0}$ replaced by μ_A.

The electric-magnetic duality in the response functions implies a linear response

$$\mathbf{J}_v(\omega, \mathbf{p}) = \tilde{\sigma}(\omega, \mathbf{p})\mathbf{B}(\omega, \mathbf{p}), \tag{30}$$

where $\tilde{\sigma}$ is the *dual ac conductivity*. The above equation is a generalization of Eq. (25). In order to check the validity of Eq. (30), we rewrite Eq. (26) in real space:

$$\mathbf{J}(t,\mathbf{r}) = \int dt' \int d^3r' \sigma(t-t',\mathbf{r}-\mathbf{r}')\mathbf{E}(t',\mathbf{r}'), \tag{31}$$

where $\mathbf{E} = -\partial \mathbf{A}/\partial t$. By taking the curl of Eq. (31) we obtain, after some trivial algebra, precisely Eq. (30), with the dual conductivity related to the conductivity of the original model through the formula

$$\tilde{\sigma}(\omega) = \frac{i\omega}{2\pi q \sqrt{\rho_s}}\sigma(\omega), \tag{32}$$

where we have assumed a low-frequency regime at zero momentum.

Since the DFT Hamiltonian (20) has the same form as the GL model in the MCF regime, we have that an anomalous dimension $\eta_h = 1$ is generated for the gauge field $\mathbf{h}$. Therefore, the scaling behavior of the ac conductivity in the MCF regime given by Eq. (17) also applies here and we obtain that $\tilde{\sigma}(\omega) \sim \mu^{2-\tilde{z}}$, where $\tilde{z}$ denotes the dynamic critical exponent of the DFT. Thus, simple dimensional analysis in Eq. (32), using the scaling relation (16) for $d=3$, yields

$$\tilde{z} = \frac{3}{2}. \tag{33}$$

Note that the dynamic exponent z drops out in the power counting. The above result would also follow by considering a renormalized version of Eq. (25), where $\mu_{A,0}$ is replaced by μ_A. In the extreme type-II limit $\mu_A \sim \mu^{1/2}$, which should have the same scaling as $\tilde{\sigma} \sim \mu^{2-\tilde{z}}$, implying once more that $\tilde{z} = 3/2$.

The question now is how the exponent z is related to the exponent $\tilde{z}$ of the dual model. We have seen that the static exponents coincide, for the obvious reason that both original and dual models lead to the same singular contribution for the free energy. The relation between $\tilde{z}$ and z is much less obvious. It can be obtained as follows. From Eq. (11) and the Josephson relation we obtain that the right-hand side of Eq. (32) scales for $d=3$ and $\mathbf{p}=0$ as

$$\tilde{\sigma}(\omega) \sim (-i\omega)^{1/2z}. \tag{34}$$

Since $\tilde{\sigma}(\omega)$ must scale like in the MCF regime, we can use Eq. (12) and write also

$$\tilde{\sigma}(\omega) \sim (-i\omega)^{(2-\tilde{z})/\tilde{z}}. \tag{35}$$

Comparison of Eqs. (34) and (35) leads to the scaling relation

$$2z = \frac{\tilde{z}}{2-\tilde{z}}. \tag{36}$$

Insertion of the exact result (33) in the above equation gives then the *exact* result

$$z = \frac{3}{2}. \tag{37}$$

It is interesting to note that the scaling relation (36) still gives a reasonable result for $\tilde{z}$ when the mean-field value $z = 2$ is used, leading to $\tilde{z} = 8/5 = 1.6$, which is not too discrepant with the exact value (33).

The above results imply the following duality relation between the conductivities of the original and dual models,

$$\sigma(\omega)\tilde{\sigma}(\omega) \sim \text{const}, \tag{38}$$

as the critical point is approached. The above duality relation is analogous to the well-known Dirac relation between electric and magnetic charges.[30,36]

IV. DISCUSSION

In this paper we have combined RG and duality arguments in order to solve a controversial issue—namely, on the value of the dynamic critical exponent z in extreme type-II superconductors. Our analysis implies that $z=3/2$ exactly, therefore confirming the result given by the Monte Carlo simulations of Ref. 6 and the experimental value of Ref. 9. An interesting consequence of our analysis is that the original model and its dual share the same dynamic exponent. Thus, z, α, and ν are the same in both models, while η is not the same.[38]

It remains to discuss how our analysis fits in the classification of dynamic models of Hohenberg and Halperin.[4] We have obtained a value of z identical to the one of model E, which is a model critical dynamics for the spin-wave modes of a superfluid. Since the spin waves are decoupled from the vortex degrees of freedom[36] and the duality analysis of the critical dynamics takes precisely the latter into account, model E should not be expected to govern the critical dynamics of an extreme type-II superconductor at zero external field. The right model critical dynamics is actually given by a generalization of model A in which the dynamics of the vortex loops are taken into account. A simplified version of such a model was considered in the beginning of Sec. III B, in Eqs. (22) and (24). These equations describe respectively the dynamics of the supercurrent and the vortex current. An interesting physical consequence of such a dynamics is the electric-magnetic duality in the linear response of the supercurrent and the vortex current. Such a point of view leads to a modified dynamic London equation, which is derived from the two following Maxwell equations generalized in such a way to include also the vortex current:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - \mathbf{J}_v, \tag{39}$$

$$\nabla \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}. \tag{40}$$

The remaining Maxwell equations are not affected by the vortices, since $\nabla \cdot \mathbf{J}_v = 0$. Due to this, the electric-magnetic duality holds only in the above two Maxwell equations. It would only hold in all the four equations if open vortex lines were present, in which case magnetic monopoles would be attached to the vortex line ends. This is obviously not the case here since the U(1) group is not compact.[39]

The electric-magnetic duality of the above equations corresponds to $\mathbf{E} \to \mathbf{B}$, $\mathbf{B} \to -\mathbf{E}$, and $\mathbf{J} \to -\mathbf{J}_v$. We will derive the London equation using the approximation where $\mathbf{J}_v$ has a very weak dependence on $\mathbf{r}$—i.e., by using Eqs. (25) and (28). Thus, by taking the curl of Eq. (40) and taking into account Eqs. (25) and (28) and $\nabla \cdot \mathbf{B} = 0$, we obtain

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} + \mu_{A,0}^2 \mathbf{B} + \frac{\mu_{A,0}}{2\pi} \frac{\partial \mathbf{B}}{\partial t} = 0. \tag{41}$$

Note that the above dynamic London equation contains a damping term. The general case is of course more complicated and Eqs. (30) and (32) must be used instead. Remarkably, the duality approach allows us to derive an exact value for the dynamic exponent just by using scaling arguments. However, in order to derive the scaling functions for the ac conductivity a more explicit calculation is necessary. This will be the subject of a forthcoming work.[40]

We conclude by saying that although the dual dynamics studied here is different from the quantum case,[27,37] the result (38) applies also there. We should be note, however, that in the quantum case the dynamics can be derived from the quantum Lagrangian, which is not case for a classical system near the critical temperature.

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