

Flux flow properties of niobium thin films in clean and dirty superconducting limits

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Flux flow properties for clean and dirty superconducting limits in strong pinning niobium films are studied. Measurements of electric field vs current density characteristics at high J values ($J \approx 10^6$ A/cm²) are successfully compared to theoretical models of flux flow and of flux flow instability in a large range of temperatures and magnetic fields. The nonlinear regime at high dissipation is analysed in the frame of a modified Larkin Ovchinnikov model that takes into account a quasiparticles heating effect. This model elaborated by Bezuglyj and Shklovskij (BS) defines a transition magnetic field B_t above which the quasiparticles distribution became nonuniform due to a finite heat removal from the substrate. From the BS model, we can deduce values of the nonequilibrium lifetime of quasiparticles τ_{qp} , which are 10 to 100 times shorter in the dirty sample compared to the clean one and whose temperature dependance is specific to the electronic nature of the Nb film. The study of the nonlinear regime provides also a quantitative determination of the thermal transparency of the film-substrate interface.

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I. INTRODUCTION

Magnetic vortices in the mixed state of a superconductor can move under the action of the Lorentz driving force \vec{F}_L proportional to a transport current density $J - J_c$, where J_c is the pinning threshold. Vortex dynamics near J_c has been recently widely explored by direct methods such as magneto-optics.¹ Inversely, the regime at high vortex velocities v_v is only deduced from electric field vs current density characteristics $E(J)$ in accordance with the Josephson² equation $\vec{E} = -\vec{v}_v \times \vec{B}_a$, where \vec{B}_a is the magnetic field inside the superconductor. A first experiment³ in 1964 has interpreted a linear dependence between E and J as a steady flux flow (FF) of vortices. The FF regime is defined by the balance between F_L and a viscous drag force $F_\eta = -\eta_{FF} v_v$, where η_{FF} is the viscosity associated to vortex motion. A linear regime $J = \sigma_{FF} E + J_c$ for which the viscous coefficient η_{FF} and the conductivity σ_{FF} ($\sigma_{FF} = \eta_{FF} / B \Phi_0$) are constant is thus expected. The dissipation in a moderate “clean” superconductor at low magnetic fields and temperatures is essentially due to currents crossing vortex cores^{4,5}

$$\frac{\sigma_{FF}}{\sigma_n(T=0)} \simeq \frac{B_{c2}(T)}{B_a} - k(T), \quad (1)$$

with $\sigma_n(T=0)$ the conductivity of the normal state, $B_{c2}(T) = \Phi_0 / 2\pi\xi^2$ and $k(T)$ a temperature-dependent positive constant representing the dissipation due to Cooper pairs around vortex cores. This equation can be simplified in

$$\frac{\sigma_{FF}}{\sigma_n(T=0)} = A(T)/B_a - k(T), \quad (2)$$

where $A(T)$ is a temperature-dependent coefficient. The expression of the conductivity σ_{FF} for a “dirty” system adds the contribution of thermal effects⁶ in the vicinity of the vortex core

$$\frac{\sigma_{FF}}{\sigma_n(0)} \simeq \frac{B_{c2}(T) + \text{Im}(t) \times B_{c2}(0)}{B_a} - k(T), \quad (3)$$

where $\text{Im}(t)$ is a positive function with a maxima at $t = T/T_c \approx 0.5$.

At higher velocities v_v , Larkin and Ovchinnikov⁷ (LO) have calculated a nonlinearity of the conductivity $\sigma_{FF}(v_v)$ for temperatures T close to T_c and $B_a \ll B_{c2}$. High electric fields in the vicinity of the vortex core can change the electronic distribution and the electronic temperature T_{qp} leading to a decrease of η_{FF} when v_v increases. $E(J)$ characteristics become nonlinear and end with a jump into the normal state before the depairing current density is reached. In the absence of thermal runaway, the flux flow instability occurs at a critical vortex velocity v_v^* . To come to this critical point (J^*, E^*), the quasiparticles inside the driven vortex cores have gained enough energy from the electric field to escape from these normal regions and relax their energy into the condensate. This process is controlled by a nonequilibrium lifetime τ_{qp} of the quasiparticles. This electronic leakage leads to a continuous shrinkage of the vortex core radius ξ and to a decrease of σ_{FF} as a function of v_v .

In the original LO theory, the nonlinear flux flow behavior is due only to the field-induced change in the quasiparticle distribution function and not Joule heating: the system is in thermal equilibrium with the bath, characterized by a temperature T_0 . Several experimental works in low-⁸⁻¹⁰ and high- T_c ^{11,12} superconductors have confirmed the validity of the LO model near the critical temperature, with, however, some discrepancies.

It comes out that the sample heating during the dissipative flux flow is not negligible and can yield a thermal runaway¹³ before the occurrence of the FF instability. Bezuglyj and Shklovskij¹⁴ (BS) have extended the LO theory in the thin film configuration by taking into account heating effects. In their model, the quasiparticles distribution function depends

TABLE I. Example of fundamental characteristics for “dirty” and “clean” Nb films.

Sample	substrate	$T_{c_{H=0}}$ (K)	$H_{c2}(0)$ (mT)	$\rho_{n_{9,2}K}$ ($\mu\Omega$ cm)	$\xi_{0_{BCS}}$ (nm)	l_{free} (nm)
Nb dirty	Al ₂ O ₃	8.92	4600	12.12	36	3.19
Nb clean	Al ₂ O ₃	9.13	1010	0.59	33	65
Nb dirty	Si	8.6	4430	9.9	35	3.9

on the vortex density and on the rate of heat removal from the film through the substrate. BS defines a transition magnetic field B_t , dependent on the thermal exchange coefficient h between the film and the substrate, under which the hypothesis of a uniform quasiparticles distribution (LO model) is still valid

$$B_t = \frac{0.374eh\tau_{qp}}{k_B\sigma_n d}, \quad (4)$$

where e is the electronic charge, k_B the Boltzman constant and d the film thickness. For $B_a \gg B_t$, dissipation during the FF regime raises the electronic temperature T_{qp} and thermal effects govern the FF instability. This effect can be understood from a simple dynamical picture where the intervortex spacing becomes small enough to allow an influence of a vortex core on the other. In other words, the condensate keeps a memory in terms of quasiparticles energy (and temperature) from vortex passing. In contrast to the B -independent v_v^* of the LO model, a $v_v^*(B_a)$ variation is now expected¹⁴ and takes the form

$$v_v^* \propto h(1-t)^{1/4} B_a^n, \quad (5)$$

with $n=-0.5$. BS proposed a scaling law between critical parameters J^* and E^* for the heated quasiparticles at $T_{qp} \gg T_0$:

$$\frac{E^*}{E_{LO}^*} = [1 - z(b_t)] \left(\frac{J^*}{J_{LO}^*} \right)^{-1}, \quad (6)$$

where E_{LO}^* and J_{LO}^* are critical parameters of the pure magnetic (LO) theory and $z(b_t)$ is a function of $b_t = B_a/B_t$.

Recent works^{15,16} at low temperatures ($T \leq 0.4T_c$) suggest that thermal effects can diminish the superconducting order parameter and lead to an expansion of vortex cores rather than to a shrinkage. The quasiparticles heating reduces critical field $B_{c2}(T)$ to $B_{c2}(T_{qp}) = B_a$ where the transition to the normal state occurs above v_v^* . The B_a dependence of v_v^* is the same as the one given by the BS theory.

In this paper, we explore and compare the dynamic of a vortex lattice at high velocities in niobium microbridges for “clean” and “dirty” superconducting limits in a range of temperature above $0.6T_c$. We reveal the influence of the electronic nature of superconductors on flux flow properties in the framework of the BS model. Values of h and the temperature behavior of τ_{qp} , which are important intrinsic

parameters for the development of electronic devices such as hot electrons bolometers, are also deduced.

II. EXPERIMENTAL DETAILS

Niobium thin films with thickness $d \approx 100$ nm are prepared by ion beam technique. Depositions are done either at ambient temperature (cool sample in dirty limit) or at 780 °C (warm sample in clean limit) on Si and Al₂O₃ substrates. Films are protected by a silicon thin layer of 5 nm thickness. Microbridges of 8.5 to 10 μ m width (w) are patterned to achieve a four points configuration measurement. Bridge lengths between voltage contacts are included between $l = 800$ μ m and $l = 3$ mm. Pinning is strong in niobium films: values of critical current densities J_c are typically a few 10^6 A/cm² at $0.7T_c$. These sample parameters are within the range of expected values for Nb films (see Table I). In all the experiments presented here, the magnetic field is applied perpendicular to the samples surface. More details are given in Ref. 17.

Current-voltage characteristics were measured through fast current sweeps ($v_I \in [1; 250]$ A/s). An example is reported in Fig. 1. The electric field E and the current density J were determined from relations $E = V/l$ and $J = I/S$, where l and $S = d \times w$ are, respectively, length and section of microbridges. Parasitic thermal effects coming from contact resistance and classical Joule dissipation due to vortex motion are identified by performing experiments at different current sweep rates. To retain only nonequilibrium effects pertinent for the BS model, a high enough current rate v_I where the parameters J^* and E^* are constant (see Fig. 2) is applied.

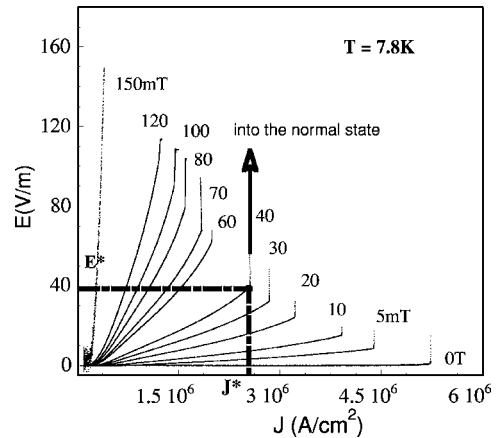


FIG. 1. $E(J)$ curves at $T=7.8$ K ($t=0.9$) for $0 < B_a < 150$ mT [$B_{c2}(T) = 420 \pm 8$ mT] in a “dirty” niobium film.

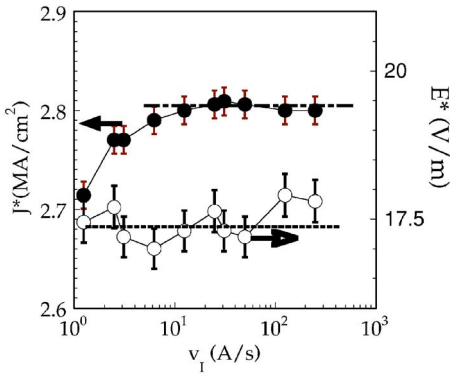


FIG. 2. (Color online) Variations of J^* and E^* vs v_I at $T=6$ K and $B_a=20$ mT in a “dirty” niobium film. See Fig. 1 for the experimental determination of J^* and E^* . These critical parameters are independent of the sweep rate above 20 A/s.

III. RESULTS AND DISCUSSION

Figure 1 depicts a series of typical (E, J) curves in a “dirty” niobium sample at $t \approx 0.9$ for $0 < b(t) = B_a/B_{c2}(T) < 0.35$. For current densities $J > J_c$, a linear regime $E \propto J$ occurs, which corresponds to a constant viscosity η_{FF} [see also Fig. 3(a)]. At higher currents, the response becomes nonlinear (decreasing η_{FF}) and finally ends by a flux flow jump at low magnetic fields. This general behavior is reported for both kinds of superconducting films. The flux flow can be characterised by two thresholds of vortex speed, one defining the onset (v_{min}) of the linear FF regime, the other its end (v_{max}). It is interesting to note that these FF velocity thresholds are independent of the applied field, at least in the intermediate field range 10–70 mT as shown in Fig. 3(b). A similar behaviour is found in the clean limit film with, however, much lower v_{max} (about 110 m/s).

The constant value of the onset of the linear regime can be interpreted by a dynamic phase transition^{18,19} under current from the motion of the amorphous vortex configuration at $J < J_c$ ($v = v_{min}$) (plastic flow) to the motion of the vortex crystal (moving crystal) at $J > J_c$ ($v = v_{min}$). The crystallization current is expected to exceed depinning current J_{dep} and is related to lattice defect concentration. Alternatively, a transition from vortex creep to flux flow with increasing Lorentz force can be considered. The existence and the value of the maximum vortex speed ending the linear FF regime will be

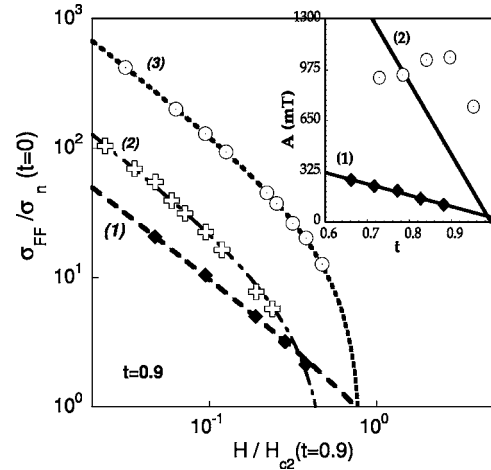


FIG. 4. Ratio σ_{FF}/σ_n as function of $b(t)$ at $t \approx 0.9$. Samples (1) and (2) are, respectively, “clean” and “dirty” films on sapphire substrate, and sample (3) is a “dirty” film on a Si substrate. Dashed lines represent fits to equation $A(t)/B_a - k(t)$ with $A(t)$ and $k(t)$ as fitting parameters. Inset: Comparison of $A(t)$ points and $H_{c2}(t)$ curves (solid lines). For sample (1), $H_{c2}(t)$ is corrected by a factor 1.035.

discussed below in the section dedicated to the nonlinear behaviour at high dissipation.

The conductivity σ_{FF} in the linear regime is treated within the ranges $0.02 < b < 0.5$ and $0.65 < t < 0.95$ in the framework of the flux flow model. Figure 4 analyzes the B_a dependence of the ratio σ_{FF}/σ_n and shows the good accordance of experimental data with the $A(T)/B_a - k(T)$ formula for both kind of samples. For “clean” systems, data on almost two decades of dissipation agree with Eq. (2), giving $A(T)$ absolute values shifted by only 3.5% from the $H_{c2}(T)$ curve deduced from ac resistivity measurements (see inset of Fig. 4). This result validates the approximation of a normal core with a radius of the order of $\xi(T)$ inside which the dissipation occurs. On the contrary, the coefficient $A(T)$ for “dirty” films does not follow the temperature variation given by Eq. (2) and displays a maximum at $t \approx 0.85$. For temperatures close to T_c , $A(T)$ decreases toward the normal conductivity but remains above the B_{c2} values, meaning that the ratio σ_{FF}/σ_N is higher than expected, i.e., the dissipation inside the core is lower than what is found in clean samples. Larkin and Ovchinnikov²⁰ have calculated σ_{FF} in the theoretical frame

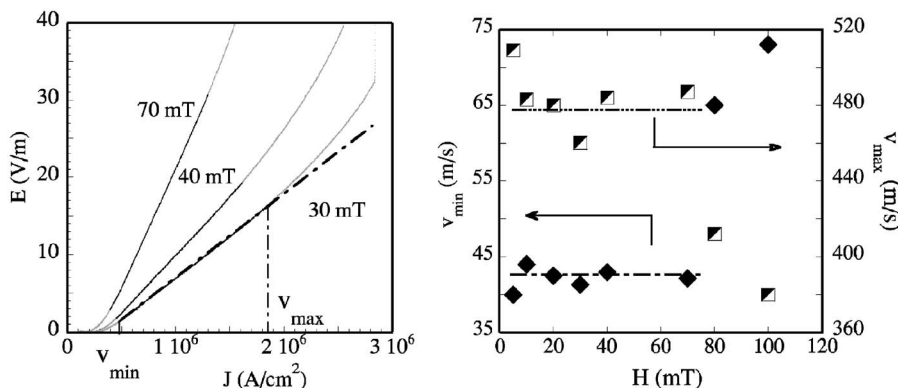


FIG. 3. Left: $E(J)$ curves at $T = 7.8$ K ($t \approx 0.9$) for $B_a = 30, 40, 70$ mT in a “dirty” niobium film. Right: Magnetic field dependence of the thresholds of the vortex velocity v_{min} and v_{max} .

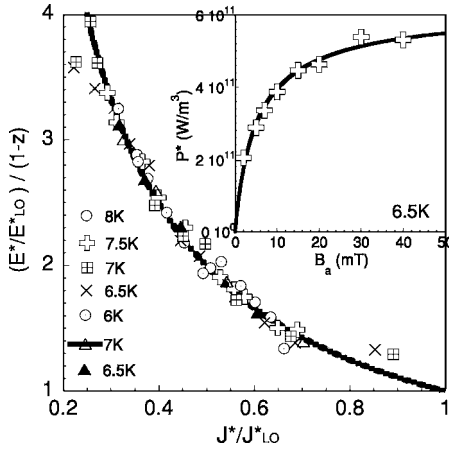


FIG. 5. Comparison between (E^*, J^*) data and universal scaling law $y=1/x$. Open symbols are for the “dirty” film at $T=6, 6.5, 7, 7.5, 8$ K. Triangle symbols are for the “clean” sample at $T=6.5, 7$ K. Inset: $P^*=P_{LO}^*[1-z(B_a/B_t)]$ variation as a function of B_a at $T=6.5$ K for the “dirty” film. The two parameters P_{LO}^* and B_t are deduced from this fit.

of the Ginzburg-Landau theory. Their formula depending on T and B_a does not fit our present data for “dirty” samples. A complementary theoretical work in thin film configuration where the thickness is of the same order as the magnetic penetration length λ is needed. At high magnetic fields, a recent theoretical work⁵ has successfully fitted the experimental curves of Fig. 1 with a modified time-dependent Ginzburg-Landau equation. In summary, dissipative phenomena governing the FF regime are very different between “clean” and “dirty” films although a clear linear $E(J)$ regime is observed in both cases.

Looking now at higher dissipation, properties of the jump into the normal state are discussed within the BS model for which the finite thermal exchange h between the superconducting film and the substrate plays a role. Values of critical coordinates (J^*, E^*) are confronted with the scaling law defined by Eq. (6). First, the field B_t is directly deduced from the $z(b_t)$ dependence of the dissipative power $P^*=E^* \times J^*$. Inset of Fig. 5 shows good accordance between experimental data and theoretical curves with two adjustable parameters B_t and $P_{LO}^*=E_{LO}^* \times J_{LO}^*$. In a second step, E_{LO}^* and J_{LO}^* are extracted from the B_t [i.e., $z(b_t)$] dependence of J^* and E^* according to the BS model. The curves $J^*(z)$ and $E^*(z)$ follow the dependence predicted by Eqs. (33) and (35) of Ref. 14 with only one adjustable parameter per equation, respectively J_{LO}^* and E_{LO}^* . This analysis is performed in a large range of T and B_a as seen in Fig. 5, which displays the expected scaling law predicted by Eq. (6). The extracted values of B_t are about 5–10 mT in our experimental range for all samples, which corresponds to vortex densities around 2 to 5 vortex/ μm^2 . These low densities tend to show that the regime of quasiparticles heating occurs in a large field domain even for low temperatures. The field dependence of v^* in the form B^n in both superconducting limits (Fig. 6), which is not predicted in the frame of the LO model, gives another proof of the quasiparticles heating mechanism in our samples. The experimental n values are close to 0.4, which is

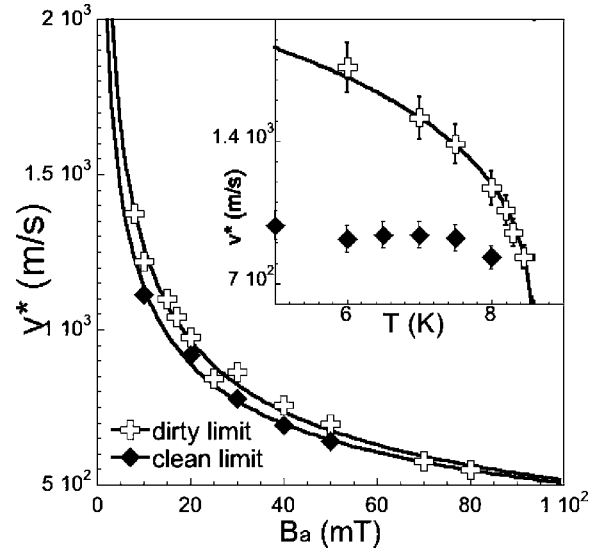


FIG. 6. Extracted $v^*(B_a)$ values for “clean” and “dirty” films at $T=7.5$ K. Solid lines fit data with B_a^n , $n=0.35$ and $n=0.38$, respectively, for “clean” and “dirty” film. Inset: corresponding $v^*(T)$ for $B_a=20$ mT. The solid line represents the $(1-T/T_c)^{0.25}$ fit curve.

in a reasonable agreement with the BS model, although some refinements of the theory are needed here. The highest velocities v^* are found in the dirty limit. For “dirty” films, the theoretical relation fits exactly the experimental $v^*(T)$ variation (inset of Fig. 6) in the range $B_a > B_t$ by taking a constant heat exchange coefficient h versus temperature. In contrast, $v^*(T)$ for “clean” niobium shows that h will vary a lot with temperature.

Taking into account the good accordance between experimental data and theory, $\tau_{qp}(T)$ is extracted from the product $P_{LO}^*=J_{LO}^* \times E_{LO}^*$ and the normal conductivity σ_n [Eqs. (34) and (36) of Ref. 14 giving $\tau_{qp}=2.67(B_t/P_{LO}^*)(\sigma_n/e)k_B T_c(1-t)$]. This τ_{qp} calculation has the advantage to not depend on Fermi velocity v_F , which is difficult to know precisely. Two very different microscopic mechanics are revealed with the $\tau_{qp}(T)$ dependence for both superconducting limits. For “dirty” niobium films, quasiparticles lifetime doesn’t change much with temperature and lies in the range 30–50 ps, this order of magnitude being in accordance with data obtained from magnetoconductance measurements²¹ and photonic Nb detectors²² (Fig. 7).

This order of magnitude is also compatible with a simple calculation considering that vortex motion is characterized by pair breaking upstream from the core and by quasiparticles recombination downstream. Quasiparticles inside the core can relax their extra energy given by the electric field only if their characteristic time of presence in this normal region is larger than τ_{qp} . When the vortex motion becomes too fast, this energy relaxation cannot occur anymore and an effective increase in quasiparticle energy that will ultimately lead to the instability, takes place. In this description, τ_{qp} obeys the simple law $\xi/v_{\max}=\tau_{qp}$. As $\xi \approx 10$ nm in the dirty sample at 7.8 K, we obtain with $v_{\max}=480$ m/s [see Fig. 3(b)], a characteristic time $\tau_{qp}=6.10^{-11}$ s, in perfect agreement with the value obtained from the BS model.

In the clean sample the same analysis yields $\tau_{qp}=5.10^{-10}$ s for this reduced temperature $t=0.9$, which is

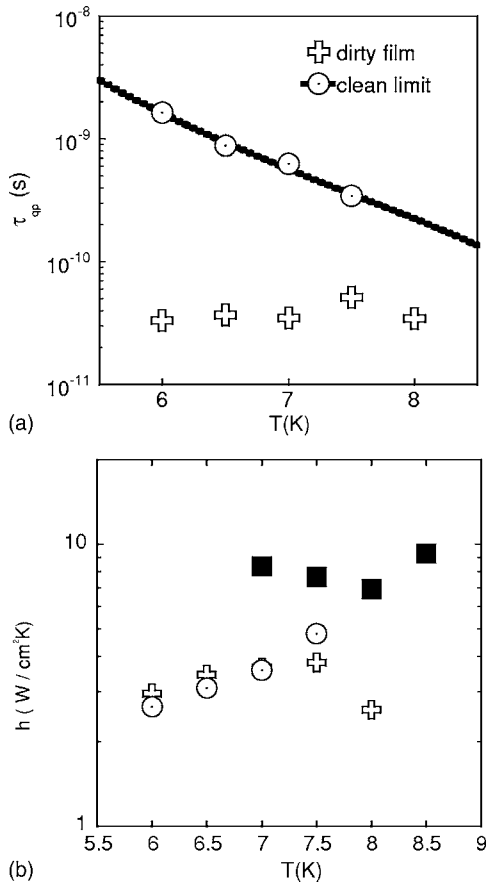


FIG. 7. Variations of duration $\tau_{qp}(T)$ (a) and coefficient $h(T)$ (b). The dotted line in (a) fits experimental data with recombination model $e^{2\Delta(T)/k_B T}$. Squares symbols are data for Au/Nb/Au on sapphire substrate.

again perfectly compatible with our experimental data ($\xi_{T=7.5\text{ K}}=58\text{ nm}$, $v_{\max}=110\text{ m/s}$). The quasiparticle energy relaxation process is, however, ruled by different mechanisms in the two samples. When the mean free path is shorter than the coherence length (dirty sample), the energy relaxation occurs within the vortex core. In the opposite situation (clean sample), the quasiparticles can relax their energy given by the electric field only after covering a distance within the core equivalent to several ξ . In the microscopic picture where the quasiparticle energy rise comes from Andreev reflections at the vortex core walls, one can see that an increase of the quasiparticles energy can occur below v_{\max} in clean samples but it is only above v_{\max} that quasiparticles can escape from the core, leading to its shrinkage. This efficient process of quasiparticles energy rise, even at low v_v in clean samples, is coherent with the fact that v_{\max} is here lower than in dirty systems.

The above discussion has to be linked to the study of the $\tau_{qp}(T)$ dependence, which is related to the physical mechanisms attached to this parameter. For example, quasiparticles recombination to Cooper pairs with emission of phonons follows a law in $e^{2\Delta(T)/k_B T}$ with the temperature variation of the superconductive gap being $\Delta(T) \approx \Delta(0)(1-t)^{0.5}$. Figure 7(a) shows that the $\tau_{qp}(T)$ variation in the clean sample conveniently fits with a recombination model.²³ The only adjust-

able parameter $\Delta(0)$ is found to be about $2k_B T_c$, higher than the BCS value $\Delta(0) \approx 1.76k_B T_c$. Such a deviation from the weak coupling BCS limit has already been reported in the case of Nb thin films.²⁴ On the contrary, a quasiconstant $\tau_{qp}(T)$ variation in dirty samples shows that scattering mechanisms dominate and that quasiparticles are ejected from the core at energies $\delta E \gg \Delta(0)$, as suggested by Kaplan *et al.*²⁵ The predominance of scattering mechanisms is coherent with the fact that quasiparticles relax their energy in the core on a distance $l_{\text{free}} < \xi$. The limiting mechanism of the instability is thus the inelastic scattering within the core for the dirty sample. Recombination process, on the other hand, describes the fact that quasiparticles scattering within the core is not the predominant phenomenon that restrains the electronic leakage. The limiting mechanism will take place outside the core, which is coherent with a recombination mechanism. Although an extensive microscopic calculation would be needed to describe the physics of a vortex core in motion, we can already see that the phenomenology of clean and dirty samples will be quite different and is indeed associated to distinct behaviors.

Knowing τ_{qp} and B_t , heat transfer coefficients h are extracted from Eq. (4). Values of h are of the same order of magnitude (around 2.7 to 4.7 W/cm² K) for thin films on Al₂O₃ substrates for both superconducting limits. These values are one to two orders of magnitude lower than h values for epitaxial BiSrCaCuC films on SrTiO₃ substrates²⁶ deduced from the first experimental analysis based on BS model, or for epitaxial YBaCuO films on sapphire substrates,²⁷ but are 10 to 100 times higher h found for YBaCuO monocrystals²⁸ only glued. Present values seem realistic although a higher heat transfer coefficient would have been expected in the case of the clean (oriented) thin film. We, however, find that h is independent of the electronic nature of Nb films and shows high values of thermal resistance between substrate and superconducting film. In this way, the thermal contact is improved with the insertion of a metallic layer between substrate and Nb film [Fig. 7(b)].

As a final remark, we can compare our results obtained in the present work to those reported in our previous paper¹⁷ considering only a nonthermal LO process. A significant difference is found for the clean samples where the $\tau_{qp}(T)$ variation extracted from the BS model now follows a law relative to a recombination process with a very reasonable $\Delta(0)$.

IV. CONCLUSION

We have compared flux flow properties in clean and dirty superconducting limits of niobium thin films. For “clean” films, dissipation is essentially due to currents crossing vortex core with a radius $\xi(T)$. On the contrary, FF properties are governed by thermal effects in the vicinity of the vortex core in “dirty” films. We have identified a nonlinear regime at high dissipation in the frame of a modified Larkin Ovchinnikov model that takes into account a quasiparticles heating effect. Nonequilibrium lifetime of quasiparticles τ_{qp} in “clean” films is interpreted as a recombination process. Values of τ_{qp} are about 10 to 100 times (few 10 ps) shorter in

“dirty” niobium films and reveal different microscopic mechanism for quasiparticles relaxation suggesting high performances for hot electrons bolometers. We hope that this study will motivate future experimental and theoretical works to understand in detail FF properties and nonlinear regime in “dirty” superconducting limit of thin films.

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- ¹P. E. Goa, H. Hauglin, M. Baziljevich, E. Ilyashenko, P. L. Gammel, and T. H. Johansen, *Semicond. Sci. Technol.* **14**, 729731 (2001).
- ²B. D. Josephson *et al.*, *Phys. Lett.* **16**, 242 (1965).
- ³Y. B. Kim and C. F. Hempstead, *Phys. Rev. Lett.* **12**, 145 (1964).
- ⁴J. Bardeen and M. J. Stephen, *Phys. Rev.* **140**, 1197 (1965).
- ⁵D. Li, A. M. Malkin, and B. Rosenstein, *Phys. Rev. B* **70**, 214529 (2004).
- ⁶J. R. Clem, *Phys. Rev. Lett.* **20**, 735 (1968); W. S. Chow, *Phys. Rev. B* **1**, 2130 (1970).
- ⁷A. I. Larkin and Yu. N. Ovchinnikov, *Zh. Eksp. Teor. Fiz.* **68**, 1915 (1975) [*Sov. Phys. JETP* **41**, 960 (1975)].
- ⁸L. E. Musienko, I. M. Dmitrenko, and V. G. Voloskaya, *JETP Lett.* **31**, 567 (1980).
- ⁹W. Klein, R. P. Huebener, S. Gauss, and J. Parisi, *J. Low Temp. Phys.* **61**, 413 (1985).
- ¹⁰C. Peroz, C. Villard, A. Sulpice, and P. Butaud, *Physica C* **369**, 222 (2002).
- ¹¹S. G. Doettinger, R. P. Huebener, R. Gerdemann, A. Kuhle, S. Anders, T. G. Trauble, and J. C. Villegier, *Phys. Rev. Lett.* **73**, 1691 (1994).
- ¹²Z. L. Xiao and P. Ziemann, *Phys. Rev. B* **53**, 15 265 (1996).
- ¹³M. T. Gonzalez, J. Vina, S. R. Curras, J. A. Veira, J. Maza, and F. Vidal, *Phys. Rev. B* **68**, 054514 (2003).
- ¹⁴A. I. Bezuglyj and V. A. Shklovskij, *Physica C* **202**, 234 (1992).
- ¹⁵M. N. Kunchur, *Phys. Rev. Lett.* **89**, 137005 (2002).
- ¹⁶D. Babic, J. Bentner, C. Surgers, and C. Strunk, *Phys. Rev. B* **69**, 092510 (2004).
- ¹⁷C. Villard, C. Peroz, and A. Sulpice, *J. Low Temp. Phys.* **131**, 957 (2003).
- ¹⁸A. E. Koshelev and V. M. Vinokur, *Phys. Rev. Lett.* **73**, 3580 (1994).
- ¹⁹N. Kokubo, R. Besseling, and P. H. Kes, *Phys. Rev. B* **69**, 064504 (2004).
- ²⁰A. I. Larkin and Yu. N. Ovchinnikov, *Nonequilibrium Superconductivity*, edited by D. N. Langenberg and A. I. Larkin (North-Holland, Amsterdam, 1986), p. 508.
- ²¹M. Hikita, Y. Tajima, T. Tamamura, and S. Kurihara, *Phys. Rev. B* **42**, 118 (1990).
- ²²A. V. Sergeev and M. Yu. Reizer, *Int. J. Mod. Phys. B* **10**, 635 (1996).
- ²³S. G. Doettinger, S. Kittelberger, R. P. Huebener, C. C. Tsuei, *Phys. Rev. B* **56**, 14 157 (1997).
- ²⁴A. V. Pronin, M. Dressel, A. Pimenov, A. Loidl, I. V. Roshchin, and L. H. Greene, *Phys. Rev. B* **57**, 14 416 (1998).
- ²⁵S. B. Kaplan, C. C. Chi, D. N. Langenberg, J. J. Chang, S. Jafarey, and D. J. Scalapino, *Phys. Rev. B* **14**, 4854 (1976).
- ²⁶Z. L. Xiao, P. Voss-de Haan, G. Jakob, and H. Adrian, *Phys. Rev. B* **57**, R736 (1998).
- ²⁷Z. L. Xiao, E. Y. Andrei, and P. Ziemann, *Phys. Rev. B* **58**, 11 185 (1998).
- ²⁸Z. L. Xiao, E. Y. Andrei, P. Shuk, and M. Greenblatt, *Phys. Rev. B* **64**, 094511 (2001).