Local dissipation effects in two-dimensional quantum Josephson junction arrays with a magnetic field

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We study the quantum phase transitions in two-dimensional arrays of Josephson-couples junctions with short range Josephson couplings (given by the Josephson energy E_J) and the charging energy E_C. We map the problem onto the solvable quantum generalization of the spherical model that improves over the mean-field theory method. The arrays are placed on the top of a two-dimensional electron gas separated by an insulator. We include effects of the local dissipation in the presence of an external magnetic flux $f=\Phi/\Phi_0$ in square lattice for several rational fluxes $f=0,\frac{1}{2},\frac{1}{3},\frac{1}{4}$, and $\frac{1}{6}$. We also have examined the $T=0$ superconducting-insulator phase boundary as a function of a dissipation α_0 for two different geometry of the lattice: square and triangular. We have found a critical value of the dissipation parameter independent on geometry of the lattice and presence magnetic field.

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I. INTRODUCTION

Over the past several years Josephson junction arrays (JJA) have gained increased interest since the display quantum mechanics on macroscopic scale. In the past years, due to development of the microfabrication techniques, it became possible to fabricate Josephson junction arrays with tailor specific properties. In these systems the superconductor-insulator (SI) transition can be driven by quantum fluctuations when the charging energy E_C becomes comparable to the Josephson coupling energy E_J.[1–8] It has been established that in arrays which are in the superconducting state, but with value E_C/E_J close to critical value, a magnetic field can be used to drive array into the insulating state.[9] There are experimental possibilities to fabricate different structures of the JJA like square[10–12] and triangular[12] networks which show that the critical value E_J/E_C can change its value depending on the geometry of the lattice. Zant *et al.* showed experimentally that not only geometry can influence on value E_C/E_J in JJA but also external magnetic field can lead system to the phase transition.[12,13] In a JJA an applied gate voltage relative to the ground plane V_g introduces a charge frustration.[14] The combination of charge frustration and Coulomb interaction results in the appearance of various Mott insulating phases separated by regions of phase coherent superconducting state.

It has been understood early that dissipation can be capable of driving an SI transitions. Problem how dissipation can be described on quantum mechanical level was firstly addressed by Caldeira and Leggett.[15] In this formalism dissipation introduces damping of phase fluctuations that is inversely proportional to the resistance of the environment. Quantum phase model of JJA was formulated in terms of an effective action.[16,17] Various theoretical methods have been applied in effects of Ohmic dissipation[18–21] such as coarse graining,[22,23] variational[18,23–27] and renormalization group approaches.[19,21,23,24] The dissipation due to quasi-particle tunneling in JJA[28] was investigated by means of the mean field calculations,[24] variational approaches[29] and Monte Carlo simulations.[30] Phase transitions of dissipative JJA's in magnetic field were analyzed by Kampf and Schön[31] and relied upon mean-field approximation mapped into tight-binding Schrödinger equation for Bloch electrons in magnetic field. It has been shown that at a fixed temperature we can observe a phase transition if we vary the magnetic field. The similar problem was solved by Kim and Choi[32] based on effective Hamiltonian obtained by the Feynman path integral formalism. Authors claim that especially in low temperatures variational method is not precise enough to perceive such a subtle transition.

Dissipation may arise from coupling with the substrate by means of the local damping model.[33] These studies indeed revealed the existence of a critical value of the sheet resistance which seems to agree with experimental results in JJA[34] and thin films.[35] On the other hand, some experimental studies discloses that the values of the critical resistance can show wide variations quite the contrary to the predicted universal value close to $h/4e^2$.[36] Furthermore, previous theoretical calculations relied upon variational and mean field approximations, which usually are not expected to be reliable at $T=0$ and be capable to handle spatial and quantum fluctuations effects properly, especially in two dimensions.

It has been shown[37] that there is a possibility of existence four phases in superconducting junction arrays with dissipation and the phase diagram depends on ratio E_J/E_C and dissipation parameter α_0. One is insulating, when both Cooper pairs and single electrons are frozen (E_J/E_C and α_0 small), mixed, when superconducting long-range coherence and single electrons tunneling coexist (E_J/E_C and α_0 large), and for intermediate values of the parameters we can obtain (E_J/E_C small and α_0 large) Cooper pairs remain frozen, but single electrons are free. In the opposite limit (E_J/E_C large and α_0 small) we have superconducting long-range phase order and there is not any dynamics of the single electrons.

The fact, that dissipation could play an important role in solid state physics appeared recently from the high-T_C point of view. The relation obtained by Homes *et al.*[38] proves that the characteristic time scale for dissipation could not be shorter than in the high-T_C superconductors. Relaxation time $\tau_{\text{diss}} \sim \hbar / k_B T_C$ (Planckian dissipation) is an essence of the Home's law and there is evidence that quantum-critical nature of the system could be present even in the normal state. It means that below this time scale we have purely quantum mechanical behavior and energy could not be turned out into heat. Conductivity in the normal state is tied with the relaxation time relation $\sigma_{\text{normal}} \sim \tau_{\text{diss}}$ and it indicates that σ_{normal} should be as small as it is allowed by Planck's constant.

Realization of a quantum computer crucially depends on our ability to create and hold coherent superposition states, so decoherence presents the most fundamental trammel. Especially coupling between different devices and environment achieves dissipation, and hence decoherence which leads to exponential decay of superposition states into incoherent mixtures.[39] Both in superconducting qubits, based on superconducting interference devices and in single-pair tunneling devices the Josephson junction is a key element and it is the dissipation of the junction that sets the limit on decoherence time.[40]

The purpose of this paper is to study local dissipation effects in JJA in an analytical way to refine the phase diagram of the system for different geometry of the lattices and in the presence of the external magnetic field. We consider a network of the Josephson junction arrays with tunable dissipation which is placed on the top of a two-dimensional electron gas (2DEG) separated by an insulator. We drop nonlocal charging and dissipative terms. The problem we would like to address is then: What is the effect of the competition between magnetic, geometric and quantum effects on the ground state ordering in the two-dimensional Josephson arrays? To analyze 2D JJA beyond mean field approximation we employ the path-integral formulation of quantum mechanics and a quantum spherical model approach accurately tailored for the JJA systems. This formalism allows then for explicit implementation of the local dissipation effects and magnetic field into our considerations. Most theoretical studies investigated the simple square lattice geometry of JJA. Other structures were analyzed by Monte Carlo simulations and mean field calculations in magnetic field in the context of phase transitions.[41] On the other hand, Granato and Kosterlitz claim that transition in 2D array with differential geometry can be described by classical Ginzburg-Landau-Wilson effective free energy with a two complex field.[42] We analyze the quantitative changes in the phase diagram due to two different geometrical JJA structures without external magnetic field. We consider influence of the magnetic field on the square lattice because many different properties of an array depend on flux parameter $f=p/q$.[12,43]

The paper is organized as follows. In the next section we introduce the model. In Sec. IIC we formulate the problem in terms of the effective action of the quantum spherical model. The various zero temperature phase diagrams are then studied in Sec. III. Finally, in Sec. IV we summarize.

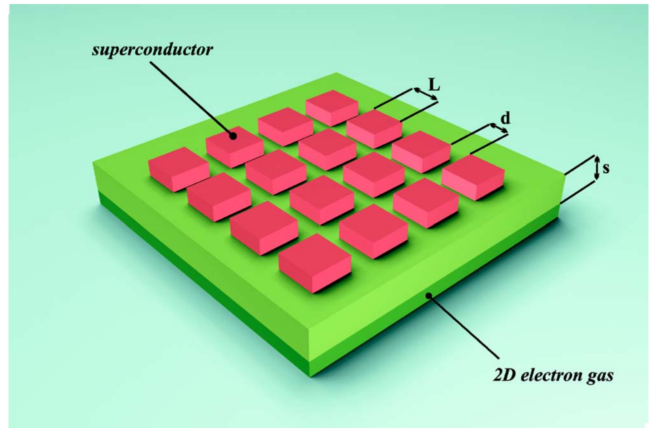

FIG. 1. (Color online) Schematic view of the 2D JJA in dissipative environment.

II. MODEL

We consider a two-dimensional Josephson junction array with lattice sites i, characterized by superconducting phase ϕ_i in dissipative environment. Possible experimental realization of the 2D JJA is shown in Fig. 1. The array can be formed by thin square superconducting islands of size L. The separation d between islands must be small enough to allow for the Josephson interactions. The variable dissipation is introduced by coupling with two-dimensional electron gas (Ohmic bath) which is localized within distance s. The corresponding Euclidean action in the Matsubara imaginary time τ formulation $(0 \leqslant \tau \leqslant 1/k_B T \equiv \beta)$, with T being temperature and k_B the Boltzmann constant $(\hbar = 1)$

$$\mathcal{S} = \mathcal{S}_C + \mathcal{S}_J + \mathcal{S}_D, \tag{1}$$

where

$$\mathcal{S}_C = \frac{1}{16E_C}\sum_i \int_0^\beta d\tau \left(\frac{d\phi_i}{d\tau}\right)^2,$$

$$\mathcal{S}_J = \sum_{\langle i,j\rangle} \int_0^\beta d\tau\, J_{ij}\{1 - \cos[\phi_i(\tau) - \phi_j(\tau)]\}, \tag{2}$$

$$\mathcal{S}_D = \frac{1}{2}\sum_i \int_0^\beta d\tau\, d\tau'\, \alpha(\tau - \tau')[\phi_i(\tau) - \phi_i(\tau')]^2.$$

The first part of the action Eq. (2) defines the electrostatic energy, where the self-charging energy parameter is

$$E_C = \frac{e^2}{2C_0}. \tag{3}$$

The second term is the Josephson energy E_J ($J_{ij} \equiv E_J$ for $|i-j| = |d|$ and zero otherwise). The third part of the action $\mathcal{S}_D$ describes the local dissipation where $\alpha(\tau - \tau')$ is a dissipative kernel. For local dissipation effects Fourier transform (with respect to imaginary times) of the kernel Eq. (2) is[17,28]

$$\alpha(\omega_n) = \frac{\alpha_0}{\pi}|\omega_n|, \tag{4}$$

where dimensionless parameter

$$\alpha_0 = \frac{R_Q}{R_0}, \quad R_Q = \frac{1}{4e^2} \tag{5}$$

describes strength of the Ohmic dissipation. Here the R_0 is the shunt to the ground and is interpreted as the shunt resistance present in many experiments. As we can see the dissipation part of the action Eq. (2) breaks the 2π periodicity in the phase variables since it allows for continuous charge fluctuations. In proximity-coupled arrays, dissipation can correlate the phase of a single island in different time.

A. Effect of the applied magnetic field

The perpendicular magnetic field B given by vector potential $\mathbf{A}$ enters action Eq. (2) through a Peierls factor according to

$$J_{ij} \to J_{ij}\exp\left(\frac{2\pi i}{\Phi_0}\int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathbf{A}\cdot d\mathbf{l}\right), \tag{6}$$

where $\Phi_0 = 2\pi c/e$ is a quantum of magnetic flux piercing a 2D lattice plaquette and integral runs between center of grains $\mathbf{r}_i$ and $\mathbf{r}_j$. Thus, the phase shift on each junction is determined by the vector potential $\mathbf{A}$ and in typical experimental situation it can be entirely ascribed to the external field B. The presence of B induces vortices in the system described by the flux parameter f ($\equiv \Phi/\Phi_0 = Ba^2/\Phi_0$ where a is area of an elementary plaquette). Our special interest are the values of the magnetic field when flux parameter $f = p/q$ represents rational values.

B. Method

To study the JJA model we can use a description in terms of an effective Ginzburg-Landau functional derived from the microscopic model Eq. (2). Several studies of JJA have followed this way, also known as the coarse grained approach first developed by Doniach.[4] Most of the existing analytical works on quantum JJA have employed different kinds of meanfield-like approximations which are not reliable for treatment spatial and temporal quantum phase fluctuations which quantum spherical approach captures. Formulation of the problem in terms of the spherical model[45–47] which was initiated by Kopeć and José[49] leads us to introduce the auxiliary complex field ψ_i whose expectation value is proportional to the $\mathbf{S}_i(\phi)$ vector defined by

$$\mathbf{S}_i(\phi) = [S_i^x(\phi), S_i^y(\phi)] \equiv [\cos(\phi_i), \sin(\phi_i)]. \tag{7}$$

Natural consequence definition $\mathbf{S}_i(\phi)$ is the following rigid constraint

$$|\mathbf{S}_i(\phi)|^2 = [S_i^x(\phi)]^2 + [S_i^y(\phi)]^2 = \cos^2(\phi_i) + \sin^2(\phi_i) \equiv 1. \tag{8}$$

The relation in Eq. (8) implies that a weaker condition also applies, namely

$$\sum_i |\mathbf{S}_i(\phi)|^2 = N. \tag{9}$$

Using the Fadeev-Popov method with the Dirac δ functional we obtain

$$\begin{aligned}\mathcal{Z} = &\int \left[\prod_i \mathcal{D}\psi_i \mathcal{D}\psi_j^*\right] \delta\left(\sum_i |\psi(\tau)|^2 - N\right) e^{-\mathcal{S}_J[\psi]} \\ &\times \int [\mathcal{D}\phi_i] e^{-\mathcal{S}_{C+D}[\phi]} \prod_i \delta[\mathrm{Re}\,\psi_i(\tau) - S_i^x(\phi(\tau))] \\ &\times \delta[\mathrm{Im}\,\psi_i(\tau) - S_i^y(\phi(\tau))].\end{aligned} \tag{10}$$

It is convenient to employ the functional Fourier representation of the δ functional to enforce the spherical constraint Eq. (9):

$$\delta[x(\tau)] = \int_{-i\infty}^{+i\infty} \left[\frac{\mathcal{D}\lambda}{2\pi i}\right] \exp\left[\int_0^\beta d\tau\, \lambda(\tau) x(\tau)\right], \tag{11}$$

which introduces the Lagrange multiplier $\lambda(\tau)$ thus adding a quadratic term (in ψ field) to the action Eq. (2). Evaluation of the effective action in terms of the ψ to second order in $\psi_i(\tau)$ gives the partition function of the quantum spherical model

$$\mathcal{Z}_{\mathrm{QSM}} = \int \left[\prod_i \mathcal{D}\psi_i \mathcal{D}\psi_j^*\right] \delta\left(\sum_i |\psi(\tau)|^2 - N\right) e^{-\mathcal{S}_{\mathrm{QSA}}[\psi]}, \tag{12}$$

where the action of the effective nonlinear σ model reads

$$\begin{aligned}\mathcal{S}_{\mathrm{QSA}}[\psi,\lambda] = &\sum_{\langle i,j\rangle} \int_0^\beta d\tau\, d\tau' \{[J_{ij}(\tau)\delta(\tau-\tau') + \mathcal{W}_{ij}^{-1}(\tau,\tau') \\ &- \lambda(\tau)\delta_{ij}\delta(\tau-\tau')]\psi_i\psi_j^* + N\lambda(\tau)\delta(\tau-\tau')\}.\end{aligned} \tag{13}$$

Furthermore

$$\begin{aligned}\mathcal{W}_{ij}(\tau,\tau') = &\frac{\delta_{ij}}{\mathcal{Z}_0} \int \left[\prod_i \mathcal{D}\phi_i\right] e^{i[\phi_i(\tau)-\phi_j(\tau')]} e^{-\mathcal{S}_{C+D}[\phi]} \\ &\equiv \mathcal{W}(\tau,\tau')\delta_{ij},\end{aligned} \tag{14}$$

is the phase-phase correlation function with statistical sum

$$\mathcal{Z}_0 = \int \left[\prod_i \mathcal{D}\phi_i\right] e^{-\mathcal{S}_{C+D}[\phi]}, \tag{15}$$

where action $\mathcal{S}_{C+D}[\phi]$ is just a sum of the electrostatic and dissipative term in Eq. (2). After introducing the Fourier transform of the field

$$\phi_i(\tau) = \frac{1}{N\beta}\sum_{\mathbf{k}} \sum_{n=-\infty}^{+\infty} \phi_{\mathbf{k},n} \exp[-(i\omega_n\tau - \mathbf{k}\cdot\mathbf{r}_i)] \tag{16}$$

with $\omega_n = 2\pi n/\beta$ ($n = 0, \pm 1, \pm 2, \ldots$) being the Bose Matsubara frequencies, we can write expression Eq. (14) in the form

$$\mathcal{W}(\tau,\tau') = \frac{1}{\mathcal{Z}_0}\int\left[\prod_{\mathbf{k}}\mathcal{D}\phi_{\mathbf{k},n}\phi^*_{\mathbf{k},n}\right]\exp\left\{-\frac{1}{4N\beta}\sum_{n=-\infty}^{+\infty}\left[\frac{1}{4E_C}\omega_n^2 + \frac{\alpha_0}{\pi}|\omega_n|\right]\phi_{\mathbf{k},n}\phi^*_{\mathbf{k},n} + \frac{1}{\beta N}\sum_{n=-\infty}^{+\infty}[\phi_{\mathbf{k},n}(e^{i\omega_n\tau}-e^{-i\omega_n\tau'}) + \phi^*_{\mathbf{k},n}(e^{-i\omega_n\tau}-e^{-i\omega_n\tau'})]\right\}. \quad (17)$$

Using the Hubbard-Stratonovich transformation the phase-phase correlation function reads

$$\mathcal{W}(\tau,\tau') = \exp\left\{-\frac{1}{\beta}\sum_{n\neq 0}\frac{1-\cos[\omega_n(\tau-\tau')]}{\frac{1}{8E_C}\omega_n^2 + \frac{\alpha_0}{2\pi}|\omega_n|}\right\}. \quad (18)$$

The low temperature properties of the expression $\mathcal{W}^{-1}(\omega_n)$ lead to critical value $\alpha_0 = 2$ (see Appendix A). Finally, for small frequencies and $\alpha_0 \leqslant 2$ inverse of the correlation function Eq. (18) becomes

$$\mathcal{W}^{-1}(\omega_n) = \frac{1}{8E_C}\omega_n^2 + \frac{\alpha_0}{2\pi}|\omega_n| \quad (19)$$

for $\omega_n \neq 0$ and $\mathcal{W}^{-1}(\omega_n) = 0$ otherwise. In the thermodynamic limit $(N\to\infty)$ the steepest descent methods become exact; the condition that the integrand in Eq. (12) has a saddle point $\lambda(\tau) = \lambda_0$ leads to an implicit equation for λ_0:

$$1 = \frac{1}{\beta N}\sum_{\mathbf{k}}\sum_{n\neq 0} G(\mathbf{k},\omega_n), \quad (20)$$

where

$$G^{-1}(\mathbf{k},\omega_n) = \lambda_0 - J_{\mathbf{k}} + \frac{1}{8E_C}\omega_n^2 + \frac{\alpha_0}{2\pi}|\omega_n|, \quad (21)$$

with $J_{\mathbf{k}}$ as Fourier transform of the Josephson couplings J_{ij}.

III. PHASE DIAGRAMS

A Fourier transform of Eq. (13) in momentum and frequency space enables one to write the spherical constraint Eq. (20) explicitly as

$$1 = \frac{1}{\beta}\int_{-\infty}^{+\infty} d\xi \sum_{n\neq 0}\frac{\rho(\xi)}{\lambda - \xi E_J + \frac{1}{8E_C}\omega_n^2 + \frac{\alpha_0}{2\pi}|\omega_n|}. \quad (22)$$

As is usual in a spherical model, the critical behavior and the phase transitions boundary crucially depends on the spectrum given by density of states $\rho(\xi)$ and is determined by the denominator of the summand in the spherical constraint Eq. (22). To proceed, it is desirable to introduce density of states for a two dimensional lattice in form

$$\rho(\xi) = \frac{1}{N}\sum_{\mathbf{k}}\delta\left(\xi - \frac{J_{\mathbf{k}}}{E_J}\right), \quad (23)$$

where $J_{\mathbf{k}}$ is energy dispersion. The problem of computing the density of states $\rho^{\square}_{p/q}(\xi)$ for a two-dimensional square lattice with uniform magnetic flux per unit plaquette reduces to the solution of Harper's equation,[50] e.g., relevant for tight-binding electrons on 2D lattice.[51] Analytical results for the density of states for a square lattice were presented recently,[9] and based on analytically solving Harper's equation and receiving dispersion relation $J^{\square}_{\mathbf{k},p/q}$. Influence of the local dissipation effects will be considered on a triangular lattice without magnetic field. In that case we can use definition Eq. (23) but the dependence on the wave vector is different and could be described by expression

$$J^{\Delta}_{\mathbf{k}} = E_J\left[\cos(k_x) + 2\cos\left(\frac{1}{2}k_x\right)\cos\left(\frac{\sqrt{3}}{2}k_y\right)\right]. \quad (24)$$

Closed formulas for the density of states are placed in Appendix B. The systems display a critical point at $\lambda_0 = G^{-1}(\mathbf{k}=0,\omega_n=0) \equiv \max[J(\mathbf{k}=0)]$. This fixes the saddle point value of the Lagrange multiplier; λ sticks to that value at criticality $(\lambda=\lambda_0)$ and stays constant in the whole low temperature phase. By substituting the value of λ_0 to Eq. (22) and after performing the summation over Matsubara frequencies, in $T\to 0$ limit we obtain the following result:

$$1 = \frac{1}{\pi}\int_{-\infty}^{+\infty} d\xi \frac{\rho(\xi)}{\sqrt{\left(\frac{\alpha_0}{2\pi}\right)^2 - \frac{J_0-\xi E_J}{2E_C}}} \times \ln\left[\frac{\frac{\alpha_0}{2\pi} + \sqrt{\left(\frac{\alpha_0}{2\pi}\right)^2 - \frac{J_0-\xi E_J}{2E_C}}}{\frac{\alpha_0}{2\pi} - \sqrt{\left(\frac{\alpha_0}{2\pi}\right)^2 - \frac{J_0-\xi E_J}{2E_C}}}\right]. \quad (25)$$

It is easy to see that by specifying density of states $\rho(\xi)$ Eq. (23) with the Coulomb energy E_C Eq. (3) for a given superconducting network without and in the presence of the external magnetic field, we are able to study the zero temperature JJA phase diagram. The solutions and boundary cases above the equation we will examine in the next subsections.

A. Small α_0 limit

In the limit $\alpha_0\to 0$ expression (25) reduces to a zero-temperature critical line in the absence of dissipation effects:

$$1 = \int_{-\infty}^{+\infty} d\xi\, \rho(\xi)\sqrt{\frac{2E_C}{J_0 - \xi E_J}}, \quad (26)$$

which is in accordance with previous calculations.[7] When α_0 is nonzero, but still small, the situation changes due to a dissipation that is a factor which produces additional quantum fluctuations. In consequence for small α_0 the critical value E_J/E_C decreases as

$$E_J/E_C = A_0^2 - \left(\frac{2}{\pi^2}\frac{A_1}{A_0}\right)^2\alpha^2 + \frac{2}{A_0}\left(\frac{2}{\pi^2}\frac{A_1}{A_0}\right)^3\alpha^3 - \frac{5}{A_0^2}\left(\frac{2}{\pi^2}\frac{A_1}{A_0}\right)^4\alpha^4 + O(\alpha^5), \quad (27)$$

where corresponding coefficients read

$$A_0 = \sqrt{2}\int_{-\infty}^{+\infty} d\xi \frac{\rho(\xi)}{\sqrt{J_0/E_J - \xi}}, \quad (28)$$

TABLE I. Factors of the expansion critical values E_J/E_C for small dissipation parameter α_0 [Eqs. (28) and (29)].

DOS	ρ^{Δ}	$\rho_0^{\square}$	$\rho_{1/2}^{\square}$	$\rho_{1/3}^{\square}$	$\rho_{1/4}^{\square}$	$\rho_{1/6}^{\square}$
A_0	1.01087	1.28576	1.34085	1.65397	2.06193	3.78672
A_1	1.92619	5.82281	4.83852	11.8065	19.1268	175.717

$$A_1 = \int_{-\infty}^{+\infty} d\xi \frac{\rho(\xi)}{J_0/E_J - \xi}. \tag{29}$$

The numerical values of the factors A_0 and A_1 are classified in Table I. For small ratio E_J/E_C there is no change to mobility of both Cooper's pairs and single electrons. If we increase the Josephson energy (or decrease Coulomb energy) we induce phase transitions between the insulating and superconducting phase. We have a global coherence state but in which there is no single electron dynamics. The critical values of the inverse Coulomb energy E_J/E_C^{crit} for $\alpha_0=0$ are depicted in Fig. 2(b) for several values of the magnetic field and are simply equal to the first coefficient of the expansion Eq. (27): $E_J/E_C^{\mathrm{crit}}|_{\alpha_0=0}=A_0^2$. We note the nonmonotonic dependence of the Coulomb energy on the magnetic flux ratio p/q.

B. Critical value α_0

At zero temperature dissipation suppresses quantum fluctuations entirely and drives the system to a global coherent state. Due to the fact that the correlation function Eq. (18) for low temperatures, $\mathcal{W}^{-1}(\omega_n)\sim|\omega_n|^{2/\alpha_0-1}$, becomes divergent for $\alpha_0\geqslant 2$ (see Appendix A), the critical lines are cut at this point which is depicted on phase diagrams in Fig. 2(a) and Fig. 3. This boundary value α_0 does not depend on magnetic field and on the geometry of the network. It seems to be universal for different types of lattices without and in presence of the external magnetic field. The system behaves as if it were classical because of the big contribution to the action Eq. (2) which is generated by large values of the dissipation parameter α_0. Dissipation can correlate the phase on a single island in different time and this correlation has the biggest impact on a global coherent state when α_0 is greater then critical value. Similar behavior of the phase diagrams with critical value of the dissipation parameter was predicted by several authors.[18,21,48] Theory developed by Chakravaraty *et al.*[18] reveals that phase transition takes place at point $\alpha_{\mathrm{crit}}=1/d$ where d is the dimension of the system. The insulating phase disappears for $\alpha_0>2$ in the model considered by Wagenblast *et al.*,[48] the authors claim that dissipative processes completely suppress the phase fluctuations. Kampf and Schön[29] using variational procedure showed that different mechanisms, Ohmic and quasiparticle damping, lead to different critical values of α_0. In magnetic field the dependence of critical temperature on several ratios of the magnetic fluxes is periodic with period 1 and symmetric around $p/q=1/2$.[31] Note that phase diagrams for different lattice geometry and in the presence of a magnetic field with effects of the local dissipation has not been presented.

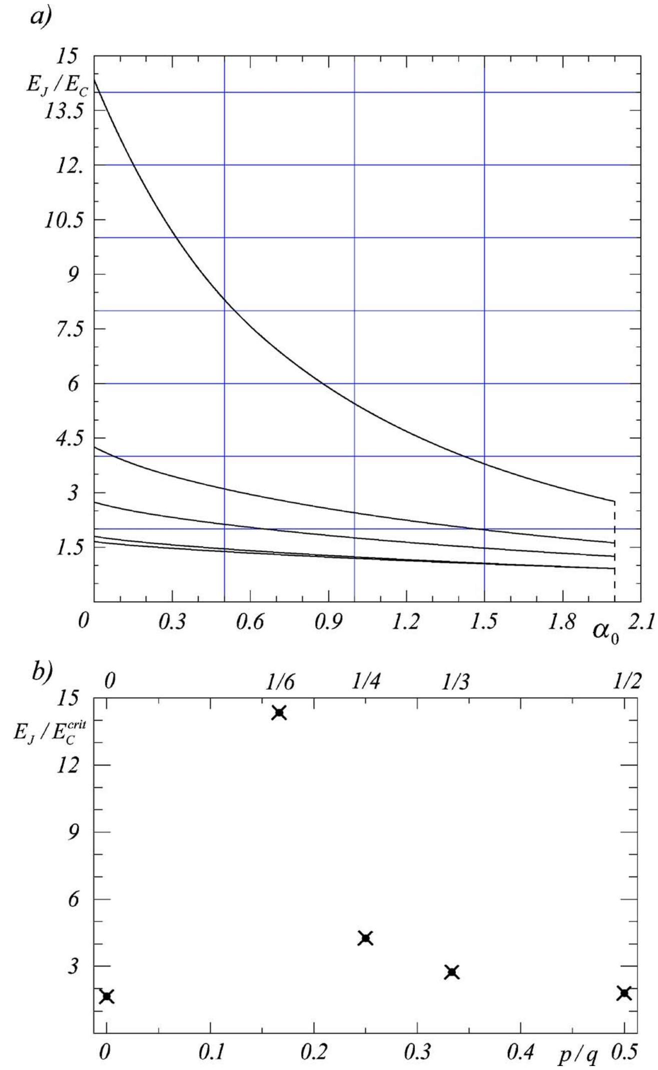

FIG. 2. (Color online) (a) Zero temperature phase diagram for the total charging energy E_J/E_C vs parameter of dissipation α_0 for square lattice. Insulating state is below the curve. From the top we have $p/q=\frac{1}{6},\frac{1}{4},\frac{1}{3},\frac{1}{2}$, and 0. (b) Nonmonotonical dependence the critical value of the inverse Coulomb energy E_J/E_C^{crit} for several ratio of the magnetic fluxes $p/q=0,\frac{1}{2},\frac{1}{3},\frac{1}{4}$, and $\frac{1}{6}$.

IV. SUMMARY

We have studied the ground phase diagram two-dimensional Josephson junction arrays in quantum regime by analytical methods using the quantum-spherical approach with exactly evaluated density of states for square and triangular lattice. For square lattice we could take into consideration perpendicular magnetic field which was described by rational magnetic flux $f=p/q$ for a number of values $1/q$. Zero temperatures phase diagram indicates that for small values α_0 quantum fluctuations destroy the long-range phase coherence. The arrays can be in two phases: Mott-insulator phase and global coherent state. We can travel between phases varying Coulomb energy or dissipation parameter. When α_0 is greater than critical value, the dissipation suppresses quantum fluctuations and the array orders even for small ratio E_J/E_C. Why can dissipation restore the global

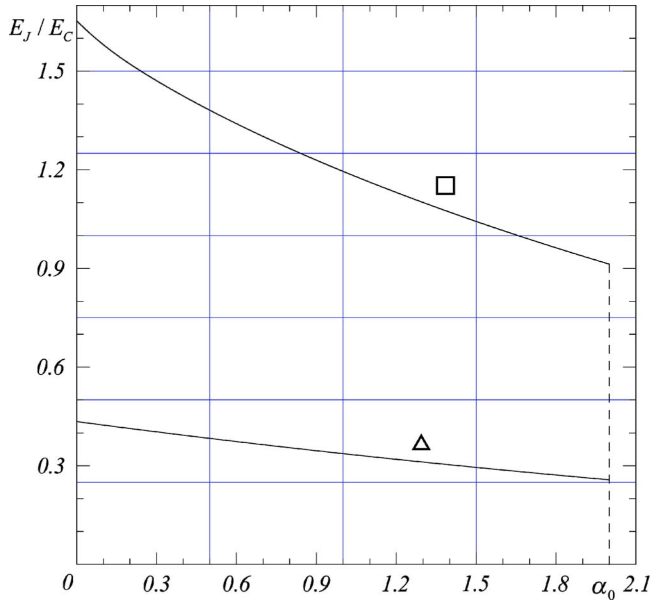

FIG. 3. (Color online) Zero temperature phase diagram for the normalized Josephson energy E_J/E_C vs parameter of dissipation α_0 for square and triangular lattice. Insulating state is below the curves.

coherent state? If we imagine that we allow electrons to tunnel between superconducting arrays and a metallic substrate, remaining Cooper's pairs will become more mobile too. Therefore we will not be equal to set a number pair on the island. Because the superconducting phase ϕ and the number of Cooper's pair $\hat{n}$ follows uncertainty relation $[\phi,\hat{n}]=2i$; then if we cannot say anything about the quantity of Cooper's pair on the island the phase is determined and in consequence we obtain a global coherent state. In the case when a system is in the presence of the magnetic field damping is stronger than in the absence. If we vary flux parameter $f=p/q$ we could drive the array into an insulating state. The magnetic field could affect the dissipation because it influences the resistance of at least the coherent parts and thus changes the effective shunting. For small values dissipation parameter α_0 Josephson energy in triangular lattice is damped less than for a square lattice even in the presence of the magnetic field. We can notice that if the quantum fluctuations of the phase superconducting order parameter are important, the dissipation plays a decisive role in constituting the onset of global superconductivity.

APPENDIX A: LOW TEMPERATURE PROPERTIES OF THE CORELLATOR

We write correlation function Eq. (18) in the form

$$\mathcal{W}(\tau)=\exp\left\{-\frac{1}{\beta}\sum_{n\neq 0}\frac{1-\cos(\omega_n\tau)}{\frac{1}{8E_C}\omega_n^2+\frac{\alpha_0}{2\pi}|\omega_n|}\right\} \tag{A1}$$

and observe that the sum over ω_n is symmetric when we change $\omega_n\rightarrow-\omega_n$. In that case (getting rid of abs) we can write

$$\mathcal{W}(\tau)=\exp\left\{-\frac{2}{\beta}\sum_{n\geqslant 1}\frac{1-\cos(\omega_n\tau)}{\frac{1}{8E_C}\omega_n^2+\frac{\alpha_0}{2\pi}\omega_n}\right\} \tag{A2}$$

noticing then sums over $n\geqslant 1$ and $n\leqslant 1$ are the same. Now we are splitting the above expression into two parts and neglecting the ω_n^2 in the second contribution (in low temperatures the more important part contains ω_n) we get

$$\mathcal{W}(\tau)=\exp\left[-\frac{2}{\beta}\sum_{n\geqslant 1}\frac{1}{\frac{1}{8E_C}\omega_n^2+\frac{\alpha_0}{2\pi}\omega_n}\right]\exp\left[\frac{2}{\beta}\sum_{n\geqslant 1}\frac{\cos(\omega_n\tau)}{\frac{\alpha_0}{2\pi}\omega_n}\right]. \tag{A3}$$

Evaluating the sums we can write

$$\mathcal{W}(\tau)=\exp\left[-\frac{1}{\alpha_0}\mathbf{H}\left(\frac{8\beta E_C}{\alpha_0}\right)\right]\exp\left\{\frac{2}{\alpha_0}\log\left[2\sin\left(\frac{\pi}{\beta}|\tau|\right)\right]\right\}, \tag{A4}$$

and using the asymptotic relation (valid for $\beta\rightarrow\infty$) of the harmonic number function $\mathbf{H}(n)=\Sigma_{i=1}^{n}i^{-1}$:

$$\mathbf{H}(a\beta)=\ln a\beta+\gamma+\frac{1}{2}a\beta-\frac{1}{12}\left(\frac{1}{a\beta}\right)^2+\frac{1}{120}\left(\frac{1}{a\beta}\right)^4+\cdots O\left(\frac{1}{\beta^6}\right), \tag{A5}$$

where γ is Euler's constant, finally we have found

$$\mathcal{W}^{-1}(\tau)=\exp\left(\frac{\gamma}{\alpha_0}\right)\left(2\frac{\pi}{\beta}|\tau|\right)^{-2/\alpha_0}. \tag{A6}$$

After Fourier transform the correlator becomes

$$\mathcal{W}^{-1}(\omega_n)=\Gamma\left(1-\frac{2}{\alpha_0}\right)\sin\left(\frac{\pi}{\alpha_0}\right)\exp\left(\frac{\gamma}{\alpha_0}\right)|\omega_n|^{2/\alpha_0-1}, \tag{A7}$$

besides the Euler gamma function $\Gamma(z)$ is defined by the integral[44]

$$\Gamma(z)=\int_0^\infty t^{z-1}e^{-t}dt \quad \text{for Re } z>0. \tag{A8}$$

and can be viewed as a generalization of the factorial function, valid for complex arguments. Finally we can see that correlator

$$\mathcal{W}^{-1}(\omega_n)\sim|\omega_n|^{2/\alpha_0-1} \tag{A9}$$

at zero temperature diverges for $\alpha_0\geqslant 2$.

APPENDIX B: DENSITY OF STATES

In the case of zero magnetic field the density of states for the square two-dimensional lattice reads simply

$$\rho_0^{\square}(\xi)=\frac{1}{\pi^2}\mathbf{K}\left(\sqrt{1-\left(\frac{\xi}{2}\right)^2}\right)\Theta\left(1-\left|\frac{\xi}{2}\right|\right), \tag{B1}$$

where

$$\mathbf{K}(x) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1-x^2\sin^2\phi}}, \tag{B2}$$

is the elliptic integral of the first kind[44] and the unit step function is defined by

$$\Theta(x) = \begin{cases} 1 & \text{for } x > 0, \\ 0 & \text{for } x \leqslant 0. \end{cases} \tag{B3}$$

The procedure obtains closed formulas for density of states in the presence of the magnetic field based on analytically solving Harper's equation. The Harper's equation was studied intensively[52–54] but only in a numerical way and missed analytic closed formulas for density of states in the presence of the magnetic field. Only an expression for case $p/q=1/2$ was received by Tan and Thouless and also was formulated in terms of the elliptic integrals.[55] Therefore, the density of states for a square lattice with the magnetic quantum flux per plaquette for value $p/q=\frac{1}{2}$ reads[9]

$$\rho_{1/2}^{\square}(\xi) = \frac{|\xi|}{2}\rho_2^{\square}\left(\frac{\xi^2-4}{2}\right). \tag{B4}$$

It is only one gapless case instead of $p/q=0$.

Obtaining closed formulas for the next case $p/q=1/3$ was more difficult, and we have the following expression:

$$\begin{aligned}\rho_{1/3}^{\square}(\xi) = {}& \frac{1}{4\sqrt{2}}|\xi^2-2|\sqrt{\xi^2-8}\rho_0^{\square}\left[\frac{1}{2}\xi(\xi^2-6)\right]\left\{\left|\sec\left(\alpha+\frac{\pi}{2}\right)\right|\right. \\ &\times[\Theta(\xi+1+\sqrt{3})-\Theta(6-\xi^2)-\Theta(\xi-1-\sqrt{3})] \\ &+\sec\left(\alpha+\frac{\pi}{6}\right)[\Theta(\xi+\sqrt{6})-\Theta(\xi+2)+\Theta(\xi) \\ &-\Theta(\xi+1-\sqrt{3})]+\sec\left(\alpha+-\frac{\pi}{6}\right)[\Theta(\xi-1+\sqrt{3}) \\ &\left.-\Theta(\xi)+\Theta(\xi-2)\Theta(\xi-\sqrt{6})]\right\},\end{aligned} \tag{B5}$$

where

$$\alpha = \frac{1}{3}\arctan\left(\frac{\sqrt{32-\xi^2(\xi^2-6)^2}}{\xi(\xi^2-6)}\right). \tag{B6}$$

For $p/q=1/4$ the expression for density of states is straightly given by

$$\begin{aligned}\rho_{1/4}^{\square}(\xi) = {}& \frac{1}{2}|\xi^2-4|\rho_0^{\square}\left[\frac{1}{2}(\xi^4-8\xi^2+4)\right] \\ &\times\{\sqrt{4+|\xi^2-4|}[\Theta(8-\xi^2)-\Theta(4+2\sqrt{2}-\xi^2)] \\ &+\sqrt{4+|\xi^2-4|}\Theta(4-2\sqrt{2}-\xi^2)\}.\end{aligned} \tag{B7}$$

Finally the most complicated case $p/q=1/6$ with six symmetric singularities divides symmetrically on the positive and negative part of the axis ξ:

$$\begin{aligned}\rho_{1/6}^{\square}(\xi) = {}& \frac{1}{4\sqrt[4]{2}}|\xi^4-8\xi^2+8|\sqrt{\xi^4-8\xi^2-16}\rho_0^{\square}\left[\frac{1}{2}(\xi^2-6)\right] \\ &\times\left\{\left|\sec\left(\alpha+\frac{\pi}{2}\right)\right|[\Theta(\xi+1+\sqrt{3})-\Theta(6-\xi^2)\right. \\ &-\Theta(\xi-1-\sqrt{3})]+\sec\left(\alpha+\frac{\pi}{6}\right)[\Theta(\xi+\sqrt{6}) \\ &-\Theta(\xi+2)+\Theta(\xi)-\Theta(\xi+1-\sqrt{3})]+\sec\left(\alpha+-\frac{\pi}{6}\right) \\ &\left.\times[\Theta(\xi-1+\sqrt{3})-\Theta(\xi)+\Theta(\xi-2)\Theta(\xi-\sqrt{6})]\right\},\end{aligned} \tag{B8}$$

where

$$\alpha = \frac{1}{3}\arctan\left(\frac{|(\xi^4-8\xi^2+8)\sqrt{16+8\xi^2-\xi^4}|}{\xi^6-12\xi^4+24\xi^2+32}\right). \tag{B9}$$

The density of states for a triangular lattice with six nearest neighbors we can write in the form

$$\rho^{\Delta}(\xi) = \frac{2}{\pi^2\sqrt{\kappa_0}}\mathbf{K}\left(\sqrt{\frac{\kappa_1}{\kappa_0}}\right)\left[\Theta\left(\xi+\frac{3}{2}\right)-\Theta(\xi-3)\right], \tag{B10}$$

where

$$\begin{aligned}\kappa_0 = {}& (3+2\sqrt{3+2\xi}-\xi^2)\left[\Theta\left(\xi+\frac{3}{2}\right)-\Theta(\xi+1)\right] \\ &+4\sqrt{3+2\xi}[\Theta(\xi+1)-\Theta(\xi-3)],\end{aligned} \tag{B11}$$

$$\begin{aligned}\kappa_1 = {}& 4\sqrt{3+2\xi}\left[\Theta\left(\xi+\frac{3}{2}\right)-\Theta(\xi+1)\right]+(3+2\sqrt{3+2\xi}-\xi^2) \\ &\times[\Theta(\xi+1)-\Theta(\xi-3)].\end{aligned} \tag{B12}$$

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