Effect of nonmagnetic disorder on criticality in the dirty U(1) spin liquid

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We investigate the effect of nonmagnetic disorder on the stability of the algebraic spin liquid (ASL) by deriving an effective field theory, nonlinear σ model (NLσM). We find that the anomalous critical exponent characterizing the criticality of the ASL causes an anomalous gradient in the NLσM. We show that the sign of the anomalous gradient exponent or the critical exponent of the ASL determines the stability of the "dirty" ASL. A positive exponent results in an unstable fixed point separating delocalized and localized phases, which is consistent with our previous study [Phys. Rev. B **70**, 140405 (2004)]. We find power-law suppression for the density of spinon states in contrast to the logarithmic correction in the free Dirac theory. On the other hand, a negative exponent destabilizes the ASL, causing the Anderson localization. We discuss the implication of our study in the pseudogap phase of high T_c cuprates.

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The effect of nonmagnetic disorder in the presence of strong correlations between electrons is one of the central interests in modern condensed matter physics. This situation is realized when nonmagnetic impurities are doped in the Mott insulator. Zn-doped high T_c cuprates are examples. In order to solve this problem, it should be clarified how to describe the Mott insulator. Here we concentrate on a paramagnetic Mott insulator (PMMI). According to one possible scenario, hole doping to a PMMI results in high T_c superconductivity.[1] Understanding the nature of the PMMI may be crucial for the mechanism of high T_c superconductivity. Recently, the PMMI was proposed to be the U(1) spin liquid [U(1)SL].[2] The U(1)SL is the state described by QED_3 in terms of massless Dirac spinons strongly interacting via noncompact U(1) gauge fields.[2] The key feature of the U(1)SL lies in the criticality characterized by anomalous critical exponents originating from long-range gauge interactions.[3–10] All correlation functions show algebraic power-law decay with anomalous critical exponents, resulting in no well-defined quasiparticles (spinons).[3–10] Owing to the power-law behavior the U(1)SL is sometimes called the algebraic spin liquid (ASL) more appropriately.

Recently, the role of nonmagnetic impurities in the ASL was investigated by the present author.[11] In contrast to the free Dirac theory in two spacial dimensions[12,13] long-range gauge interactions can induce a delocalized state at zero temperature.[11] The presence of nonmagnetic disorder destabilizes the free Dirac fixed point. The renormalization group (RG) flow goes away from the fixed point, indicating localization.[12,13] On the other hand, the ASL fixed point in QED_3 (Ref. 2) remains stable at least against weak disorder.[11] An unstable fixed point separating delocalized and localized phases is found.[11] The RG flow shows that the effect of disorder potential vanishes if we start from sufficiently weak disorder.

In the present paper we reexamine the effect of nonmagnetic disorder on the ASL by deriving an effective field theory, nonlinear σ model (NLσM). The main idea is firstly to integrate out gauge fluctuations. This gives the single particle propagator Eq. (2) with the anomalous exponent η_ψ, the hallmark of the ASL. Based on this ASL propagator we obtain the NLσM Eq. (8) by usual treatment. The core in the NLσM is the anomalous gradient exponent η_σ originating from the critical exponent η_ψ of the ASL. We find that the sign of η_σ or η_ψ determines the stability of the ASL against nonmagnetic impurities. In the case of $\eta_\sigma > 0$, an unstable fixed point separating delocalized and localized phases is found, which is consistent with our previous study.[11] The density of spinon states and the conductance of a spinon number (internal gauge charge) show power-law corrections in contrast to the logarithmic suppression in the free Dirac theory.[13] In the case of $\eta_\sigma < 0$ the ASL becomes unstable. The Anderson localization is expected. We discuss the implication of our study in the pseudogap phase of high T_c cuprates.

First we briefly review how the effective QED_3 Lagrangian is derived from the antiferromagnetic Heisenberg model, $H = J\Sigma_{\langle i,j\rangle}\vec{S}_i\cdot\vec{S}_j$ with $J > 0$. Inserting the fermionic representation of spin $\vec{S}_i = \frac{1}{2}f_{i\alpha}^{\dagger}\vec{\tau}_{\alpha\beta}f_{i\beta}$ into the Heisenberg model, and performing the standard Hubbard-Stratonovich transformation for an exchange hopping channel, we obtain an effective one-body Hamiltonian for the fermions coupled to an order parameter, $H_{eff} = -J\Sigma_{\langle i,j\rangle}f_{i\alpha}^{\dagger}\chi_{ij}f_{j\alpha} - \text{H.c.}$ Here $f_{i\alpha}$ is a fermionic spinon with spin $\alpha = \uparrow,\downarrow$, and χ_{ij} is an auxiliary field called a hopping order parameter. Notice that the hopping order parameter χ_{ij} is a complex number defined on links ij. Thus, it can be decomposed into $\chi_{ij} = |\chi_{ij}|e^{i\theta_{ij}}$, where $|\chi_{ij}|$ and θ_{ij} are the amplitude and phase of the hopping order parameter, respectively. Inserting this representation of χ_{ij} into the effective Hamiltonian, we obtain $H_{eff} = -J\Sigma_{\langle i,j\rangle}|\chi_{ij}|[f_{i\alpha}^{\dagger}e^{i\theta_{ij}}f_{j\alpha} - \text{H.c.}]$. Then, we can easily see that this effective Hamiltonian has internal U(1) gauge symmetry $H'_{eff}[f'_{i\alpha},\theta'_{ij}] = H_{eff}[f_{i\alpha},\theta_{ij}]$ under the following U(1) phase transformations, $f'_{i\alpha} = e^{i\phi_i}f_{i\alpha}$ and $\theta'_{ij} = \theta_{ij} + \phi_i - \phi_j$. This implies that the phase θ_{ij} of the hopping order parameter plays the same role as the U(1) gauge field a_{ij}. When a spinon hops on lattices, it obtains the Aharnov-Bohm phase owing to the U(1) gauge field a_{ij}. It is well known that the stable mean-field phase is a π flux state at half filling.[5] This means that a spinon gains the phase of π when it turns around one plaquette. In the π flux phase low-energy elementary excitations are massless Dirac spinons near nodal points showing gapless Dirac spectrum and U(1) gauge fluctuations.[5] In the low-energy limit the amplitude $|\chi_{ij}|$ is frozen to be $|\chi_{ij}|=J|\langle f_{j\alpha}^{\dagger}f_{i\alpha}\rangle|\equiv\chi_0$. As a result we obtain the following low-energy effective Lagrangian in terms of massless Dirac fermions near the nodal points interacting via compact U(1) gauge fields[2]

$$Z=\int D\psi_{n\sigma}Da_{\mu}e^{-\int d^3x\mathcal{L}},$$

$$\mathcal{L}=\sum_{\sigma=\uparrow,\downarrow}\sum_{n=1}^{2}\bar{\psi}_{n\sigma}\gamma_{\mu}(\partial_{\mu}+ia_{\mu})\psi_{n\sigma}+\frac{1}{2e^2}|\partial\times a|^2. \quad (1)$$

Here $\psi_{n\sigma}$ is the two-component massless Dirac spinon, where $n=1,2$ represent the nodal points, $(\pi/2,\pi/2)$ and $(\pi/2,-\pi/2)$, and $\sigma=\uparrow,\downarrow$, SU(2) spin. They are given by

$$\psi_{1\sigma}=\binom{f_{1e\sigma}}{f_{1o\sigma}}$$

and

$$\psi_{2\sigma}=\binom{f_{2o\sigma}}{f_{2e\sigma}},$$

respectively. In the spinon field $f_{nl\sigma}$ $n=1,2$ represent the nodal points, $l=e,o$, even and odd sites, and $\sigma=\uparrow,\downarrow$, its spin, respectively.[5] The Dirac matrices γ_{μ} are given by the Pauli matrices $\gamma_{\mu}=(\sigma_3,\sigma_2,\sigma_1)$ where they satisfy the Clifford algebra $[\gamma_{\mu},\gamma_{\nu}]_{+}=2\delta_{\mu\nu}$.[5] a_{μ} is the U(1) gauge field. The kinetic energy of the gauge field results from particle-hole excitations of high-energy spinons. e is an effective internal charge, not a real electric charge.

We have difficulty in solving Eq. (1) owing to instanton excitations originating from the compactness of the U(1) gauge field. Instantons represent tunneling events between energetically (nearly) degenerate but topologically inequivalent vacua. In the U(1) gauge theory the instanton has a magnetic monopole configuration. Magnetic monopoles are known to play a crucial role in confinement physics.[2,14] Recently, it was shown that the instanton effect can be suppressed at least in the large N limit ($\sigma=\uparrow,\downarrow\to\sigma=1,2,\ldots,N$) where N is the flavor number of massless Dirac fermions.[2] Existence of quantum criticality in the QED_3 with noncompact U(1) gauge fields is the key mechanism of suppression of instanton excitations. It is well known that the QED_3 has a stable charged fixed point in the large N limit. Hermele *et al.* examined the stability of the charged fixed point against instanton excitations.[2] At the fixed point charges can be sufficiently screened by critical fluctuations of Dirac fermions. The larger the flavors N of the Dirac fermions, the smaller the electric charges, thus resulting in huge magnetic charges. This can cause a negative scaling dimension to instanton fugacity.[2] As a result, instanton excitations can be suppressed at the charged fixed point in the large N limit. Deconfinement of charged matter fields, here the massless Dirac spinons can be achieved at the quantum critical point. The QED_3 with noncompact gauge fields appears as the critical field theory at the charged fixed point. All correlation functions exhibit power-law behaviors with anomalous critical exponents generated from long-range gauge interactions. In this respect, the state described by the QED_3 is called the ASL and the charged fixed point, the ASL critical point.

As discussed in the above, the quantum criticality of the effective QED_3, Eq. (1) is the crucial point for deconfinement of the Dirac spinons. We should remember that the ASL criticality can survive only in the limit of large flavors N corresponding to the SU(N) quantum antiferromagnet ($\sigma=1,2,\ldots,N$). In the case of the physical SU(2) antiferromagnet ($\sigma=\uparrow,\downarrow$) it is not clear if the ASL criticality remains owing to spontaneous chiral symmetry breaking (SχSB) causing antiferromagnetism. It is believed that there exists the critical flavor number N_c associated with the SχSB in the QED_3.[2,5,6,15,16] The precise value of the critical number is far from consensus.[16] If the critical value is larger than 2, the SχSB is expected to occur for the physical $N=2$ case. Then, the Dirac fermions become massive. In this case, the massive Dirac spinons are confined to form spin 1 excitations, antiferromagnons.[5] On the other hand, in the case of $N_c<2$ the ASL criticality remains stable against the SχSB. In the present paper we assume that the chiral symmetry is preserved even in the SU(2) quantum antiferromagnet. There is a supporting argument for existence of the ASL even in the case of $N_c>2$.[1] If there are additional massless fluctuations carrying internal U(1) gauge charges, the SχSB can be forbidden. These additional gapless fluctuations can arise from quantum critical points in association with antiferromagnetism or superconductivity. For example, critical fluctuations of doped holes near the superconducting transition quantum critical point are expected to increase the flavor number of massless fluctuations.[17] If the total flavor number of massless Dirac spinons and doped holes exceeds the critical value N_c, the SχSB is not expected to occur.[17,18] In this case, the quantum critical point would be described by the ASL for spin degrees of freedom.[1] The role of doped holes in the ASL will be discussed later in more detail.

The criticality of the ASL is characterized by the critical exponents of correlation functions. The single particle propagator $G_{ASL}(k)=\langle\psi_{n\sigma}(k)\bar{\psi}_{n\sigma}(k)\rangle$ is given by[3,6,8–10]

$$G_{ASL}(k)\approx-i\frac{\gamma_{\mu}k_{\mu}}{k^{2-\eta_{\psi}}}, \quad (2)$$

where η_{ψ} is the anomalous critical exponent of the ASL propagator. Here we briefly sketch how the ASL propagator Eq. (2) is derived from the effective QED_3, Eq. (1). The single particle propagator is generally given by $G_{ASL}^{-1}(k)=G_0^{-1}(k)+\Sigma(k)$, where $G_0^{-1}(k)=i\gamma_{\mu}k_{\mu}$ is the inverse of the bare spinon propagator, and $\Sigma(k)$, the spinon self-energy resulting from long-range gauge interactions. In the usual $1/N$ expansion[3,5,6] the self-energy is represented by $\Sigma(k)=\int d^3q/(2\pi)^3\gamma_{\mu}G_0(k+q)\gamma_{\nu}D_{\mu\nu}(q)$, where $D_{\mu\nu}(q)$ is the renormalized propagator of the U(1) gauge field due to particle-hole excitations (polarization) of massless Dirac fermions. The gauge propagator is obtained to be $D_{\mu\nu}(q)\approx\Pi_{\mu\nu}^{-1}(q)=8/Nq(\delta_{\mu\nu}-q_{\mu}q_{\nu}/q^2)$ in the Lorentz gauge, where $\Pi_{\mu\nu}(q)=N\int d^3k/(2\pi)^3\mathrm{Tr}[G_0(k)\gamma_{\mu}G_0(k+q)\gamma_{\nu}]$ is the polarization function of the Dirac fermions. Inserting this gauge propagator into the expression of the self-energy, we obtain the spinon self-energy of logarithmic momentum dependence $\Sigma(k)=i\eta_\psi\gamma_\mu k_\mu \ln(\Lambda/k)$, where η_ψ is the anomalous exponent and Λ, the momentum cutoff. The absolute value of the exponent η_ψ is proportional to the inverse of the flavor number, i.e., $|\eta_\psi|\sim N^{-1}$.[3,6,8–10] Since the QED_3 is the critical field theory at the charged critical point, all correlators should exhibit power-law behaviors. In this respect the logarithmic momentum dependence should be considered to be the lowest order in the power-law behavior. As a result we can obtain the ASL single particle propagator Eq. (2) from the following nonperturbative consideration[6] $G_{ASL}^{-1}(k)=i\gamma_\mu k_\mu[1+\eta_\psi \ln(\Lambda/k)]\approx i\gamma_\mu k_\mu(\Lambda/k)^{\eta_\psi}$. Although the spinon propagator exhibits its algebraic form, it is difficult to give a definite physical meaning. This is because it is not gauge invariant. All physical observables should be gauge invariant. The critical exponent η_ψ should be evaluated in a gauge invariant way. The following gauge invariant green function is usually considered, $G_{ASL}(x)=\langle T_\tau[\psi_{n\sigma}(x)e^{i\int_0^x d\zeta_\mu a_\mu(\zeta)}\bar{\psi}_{n\sigma}(0)]\rangle$. It is not easy to obtain the critical exponent η_ψ by calculating this gauge invariant green function. Its precise value is far from consensus and under current debate. The crucial point in the following discussion is the sign of the exponent η_ψ. Most evaluations[3,8–10] suggest its negative sign, $\eta_\psi<0$. However, as argued in Ref. 6, its negative sign is unphysical in the sense that the ASL propagator is more coherent at long distances than the propagator of the free Dirac theory owing to long-range gauge interactions. This result is completely in contrast to the usual role of interactions. Interactions would make the propagator less coherent. This is indeed true in the critical field theories with local repulsive interactions (for example, ϕ^4 theory). Positive critical exponents are well known in these theories. If the critical exponent η_ψ is positive, the long-range gauge interactions destabilize the Fermi liquid pole (the pole of the single particle green function in the free Dirac theory). The renormalization factor $Z(p)$ representing weight of quasiparticles is given by $Z(p)\sim p^{\eta_\psi}$ with momentum p. In the case of $\eta_\psi>0$ it vanishes in the long wavelength and low energy limit, i.e., $Z(p\to 0)\to 0$, leading to Luttinger liquidlike power-law correlators. This is the ASL as a critical state. In this respect the ASL can be considered to be a two-dimensional realization of the one-dimensional Luttinger liquid.[6] In the present paper we do not determine its sign. Instead we use the exponent η_ψ as a phenomenological parameter. We consider both cases, $\eta_\psi<0$ and $\eta_\psi>0$. Here we assume that the ASL green function Eq. (2) is obtained in a gauge invariant way[3,6,8–10] and thus the critical exponent η_ψ is gauge invariant. We repeat that its absolute value is given by $|\eta_\psi|\sim N^{-1}$ in the $1/N$ approximation.[3,6,8–10]

The ASL propagator $G_{ASL}(k)=\langle\psi_{n\sigma}(k)\bar{\psi}_{n\sigma}(k)\rangle=-i(\gamma_\mu k_\mu/k^{2-\eta_\psi})$ can be easily obtained from the following Lagrangian

$$\mathcal{L}_{ASL}=\sum_{\sigma=\uparrow,\downarrow}\sum_{n=1}^{2}\bar{\psi}_{n\sigma}\gamma_\mu\partial_\mu\partial^{-\eta_\psi}\psi_{n\sigma}, \tag{3}$$

where η_ψ is the anomalous critical exponent as discussed above. Integration over U(1) gauge fluctuations in the QED_3 Eq. (1) results in the ASL green function Eq. (2) as explicitly demonstrated above. Equation (2) is easily derived from Eq. (3). Therefore, Eq. (3) can be considered to be an effective field theory resulting from integration over the U(1) gauge field in the QED_3 Eq. (1). In this respect the Dirac spinon field $\psi_{n\sigma}$ in Eq. (3) is clearly different from that in Eq. (1). It should be considered to be a renormalized field resulting from the gauge interactions. The renormalized Dirac spinon field arises from the self-energy correction. We view Eq. (3) as the prototype describing the ASL. More generally, one may think that Eq. (3) represents one class of critical field theories depending on the critical exponent η_ψ. This point of view is parallel to the standpoint that a free fermion theory is the foundation describing Fermi liquid. One cautious theorist can argue that the ASL Lagrangian Eq. (3) is not sufficient because Eq. (3) does not include appropriate vertex corrections, but this guess is not correct. Remember that the critical exponent η_ψ was obtained in a gauge-invariant way although the calculation of η_ψ in a gauge-invariant way was not explicitly demonstrated in the above owing to its complexity.[3,6,8–10] It is well known that we cannot satisfy the gauge invariance without vertex corrections.[7] Thus, the evaluation of η_ψ in a gauge-invariant way includes vertex corrections. This argument can be checked by considering a two-particle green function. The critical exponent of a spin-spin correlation function, obtained in a gauge-invariant way, including vertex corrections explicitly in Eq. (1), is given by twice the exponent of the single particle propagator in Eq. (3), i.e., $2\eta_\psi$ in the case of $\eta_\psi<0$.[4,7] This was also pointed out in Ref. 10. This implies that we can treat the ASL effective Lagrangian Eq. (3) as a "free" theory.

We introduce a random potential $V(\mathbf{r})$ in the ASL [Eq. (3)]

$$S=\int d^Dx\sum_{\sigma=\uparrow,\downarrow}\sum_{n=1}^{2}[\bar{\psi}_{n\sigma}\gamma_\mu\partial_\mu\partial^{-\eta_\psi}\psi_{n\sigma}+V(\mathbf{r})\bar{\psi}_{n\sigma}\psi_{n\sigma}]. \tag{4}$$

As the randomness is independent of time and the $\mathcal{L}_{ASL}$ is quadratic in $\psi_{n\sigma}$, each frequency sector in Eq. (4) is separated from each other. Thus it is sufficient to consider only the zero frequency sector. This motivates us to explore $D=2$. Here we assume that $V(\mathbf{r})$ is a Gaussian random potential of $\langle V(\mathbf{r})V(\mathbf{r}')\rangle=W\delta(\mathbf{r}-\mathbf{r}')$ with $\langle V(\mathbf{r})\rangle=0$. Using the standard replica trick to average over the Gaussian random potential, we obtain the following effective action in the presence of nonmagnetic disorder

$$S=\int d^2\mathbf{r}\sum_{\alpha=1}^{M}(\bar{\Psi}_\alpha\Gamma_i\partial_i\partial^{-\eta_\psi}\Psi_\alpha+i\epsilon\bar{\Psi}_\alpha\Psi_\alpha)-\frac{W}{2}\sum_{\alpha,\alpha'=1}^{M}\int d^2\mathbf{r}\bar{\Psi}_\alpha\Psi_\alpha\bar{\Psi}_{\alpha'}\Psi_{\alpha'}. \tag{5}$$

Here

$$\Psi_\alpha = \begin{pmatrix} \psi_{1\uparrow\alpha} \\ \psi_{2\uparrow\alpha} \\ \psi_{1\downarrow\alpha} \\ \psi_{2\downarrow\alpha} \end{pmatrix}$$

is the eight-component spinor where $\alpha=1,\ldots,M$ is an replica index and the limit $M\to 0$ is to be taken at the end. $\Gamma_\mu = I\otimes\sigma^3\otimes\gamma_\mu$ is the eight-by-eight gamma matrix. I acts on SU(2) spin space and σ^3 different nodal points. We have included an infinitesimal imaginary potential ϵ in order to generate correlation functions.[13] In Eq. (5) one can easily read the bare scaling dimension of the disorder strength W. It is given by $\dim[W]=-2\eta_\psi$. In the case of $\eta_\psi>0$ this is consistent with our previous RG study.[11] In the study the RG equation is found to be $dW/d\ln L \approx -\chi e^2 W$ to the first order in W. χ is a positive numerical constant. Considering the charged fixed point $e_c^2 \sim N^{-1}$ with the flavor N, the RG equation yields $\dim[W]=-\chi e_c^2 \sim -N^{-1} \sim -\eta_\psi$. But, in the opposite case of $\eta_\psi<0$ the scaling dimension of the disorder strength W is positive, indicating the instability of $W=0$.

Performing the standard Hubbard-Stratonovich transformation and the Gaussian integration for the spinon field Ψ_α, we obtain the following effective action in terms of the order parameter field $\mathbf{Q}$

$$S_{eff} = \int d^2\mathbf{r}\left[-\frac{1}{2W}\mathrm{Tr}[\mathbf{Q}^2(\mathbf{r})] + \mathrm{Tr}\ln(\Gamma_i\partial_i\partial^{-\eta_\psi} + i\mathbf{Q} + i\epsilon)\right]. \tag{6}$$

Here $\mathbf{Q}$ is the $8M\times 8M$ matrix field. Its saddle point is given by $\mathbf{Q}_{\alpha\alpha'} = W\langle\bar{\Psi}_\alpha\Psi_{\alpha'}\rangle$. The saddle-point equation is obtained to be

$$1 = 8W\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{1}{|\mathbf{k}|^{2-2\eta_\psi}+Q^2} \equiv WF(Q^2). \tag{7}$$

Here we replaced $\mathbf{Q}_{\alpha\alpha'}$ with $\mathbf{Q}_{\alpha\alpha'}=\delta_{\alpha\alpha'}Q$. Since $F(Q^2)$ is a monotonically decreasing function from infinity to zero in the case of $\eta_\psi<1$, there exists only one nonzero solution for Q.[19] The relation of $|\eta_\psi|\sim N^{-1}$ guarantees this consideration. In the free Dirac theory ($\eta_\psi=0$) $Q=\Lambda e^{-\pi/4W}$ with the momentum cutoff Λ is obtained. In the ASL the meaning of nonzero Q is not clear. In Fermi liquid it corresponds to the imaginary part of the self-energy and thus the finite value of Q generates a finite density of quasiparticle states due to disorder, but in the present case there are no well-defined quasiparticles. Thus we cannot define the density of quasiparticle states. Mathematically, it gives an imaginary part to the single particle propagator. In this respect we can think that a diffusive behavior of critical spinons emerges by disorder.

Next we consider small fluctuations around this saddle point. Here we follow Ref. 20. Inserting the expression $\mathbf{Q}_{\alpha\alpha'}=Q\mathbf{U}_{\alpha\alpha'}$ into Eq. (6) and expanding the logarithmic term to the second order in $\mathbf{U}_{\alpha\alpha'}$, we obtain the NL$\sigma$M (Ref. 20)

$$S_{NL\sigma M} = \int d^2\mathbf{r}\left(\frac{1}{2g_\sigma}\mathrm{Tr}\left[\nabla\mathbf{U}^\dagger(\mathbf{r})\frac{1}{|\nabla|^{\eta_\sigma}}\nabla\mathbf{U}(\mathbf{r})\right] + \epsilon\mathrm{Tr}[\mathbf{U}(\mathbf{r})+\mathbf{U}^\dagger(\mathbf{r})]\right). \tag{8}$$

Here g_σ^{-1} is considered to be the conductance σ_{sn} of a spinon number (internal gauge charge). Its bare value $g_\sigma^{0^{-1}}$ is obtained to be $g_\sigma^{0^{-1}} = \frac{1}{2}\int d^2k/(2\pi)^2[Q/(k^{2-2\eta_\psi}+Q^2)]^2$. The anomalous gradient exponent η_σ is given by $\eta_\sigma=2\eta_\psi$. As mentioned in the introduction, the anomalous gradient is the crucial feature in the NLσM. Its existence can be interpreted to be the mirror of the criticality η_ψ in the original system, here the ASL. Generally speaking, we are considering the role of nonmagnetic disorder in the critical systems with anomalous critical exponents. The stability of the ASL critical point is determined by the anomalous gradient exponent η_σ or the critical exponent η_ψ of the ASL. In the case of $\eta_\sigma>0$ ($\eta_\psi>0$) the ASL critical point remains stable against weak randomness. But in the case of $\eta_\sigma<0$ ($\eta_\psi<0$), it becomes unstable and the Anderson localization occurs. We discuss this main issue by investigating the RG equation of the stiffness parameter g_σ.

One can easily derive the following RG equation in one loop order[14]

$$\frac{dg_\sigma}{d\ln L} = -(d-2+\eta_\sigma)g_\sigma + Ag_\sigma^2. \tag{9}$$

Here d is the spacial dimension and $d=2$ is the present case. A is a positive numerical constant. In the case of $\eta_\psi\to 0$ the above RG equation is reduced to that of the free Dirac theory.[13] Furthermore, this equation corresponds to Eq. (7) in our previous study.[11] Inserting $g_\sigma\to W$ and $\eta_\sigma\sim N^{-1}\to e^2$ into Eq. (9), we obtain the following RG equation of the disorder strength W, $dW/d\ln L=-e^2W+AW^2$. This completely coincides with Eq. (7) in the previous study.[11]

Now we discuss the phases from the RG Eq. (9). First we consider the case of a positive exponent, $\eta_\sigma>0$. Then, this RG equation shows an unstable fixed point $g_\sigma^*=\eta_\sigma/A$ consistent with our previous study.[11] In the case of weak randomness $g_\sigma<g_\sigma^*$ the g_σ goes to zero owing to the first term, implying the emergence of a delocalized spinon state. In the case of strong randomness $g_\sigma>g_\sigma^*$, the g_σ goes to infinity owing to the second term, indicating localization of the spinons. Solving the RG Eq. (9), we obtain the spinon conductance

$$\sigma_{sn} = \sigma_{sn}^0\left(\frac{L}{l_e}\right)^{\eta_\sigma} + \frac{A}{\eta_\sigma}\left[1-\left(\frac{L}{l_e}\right)^{\eta_\sigma}\right], \tag{10}$$

where $\sigma_{sn}^0 \sim g_\sigma^{0^{-1}}$ is the bare spinon number conductance and l_e, the elastic mean free path.[13] The key feature is the power-law correction resulting from the anomalous critical exponent η_ψ. This is in contrast to the free Dirac theory. In the limit $\eta_\sigma\to 0$, the present conductance formula is reduced to the logarithmic suppression, $\sigma_{sn}=\sigma_{sn}^0 - A\ln(L/l_e)$ consistent with that of the free Dirac theory.[13] The density of the spinon states can be easily calculated in the delocalized regime. Although it is not clearly defined in the ASL, the terminology is used below as it is. The formal expression of the density of states is given by $\rho = \lim_{M\to 0}\rho_0/16M\langle Tr[\mathbf{U}^\dagger + \mathbf{U}]\rangle$.[13] We find that the density of states also has the power-law correction

$$\frac{\rho-\rho_0}{\rho_0} = -Bg_\sigma \frac{1}{\eta_\sigma}(l_e^{-\eta_\sigma} - L^{-\eta_\sigma}), \tag{11}$$

where B is a positive numerical constant. This expression shows a finite density of spinon states in the limit of $L\to\infty$, given by $\rho=\rho_0(1-Bg_\sigma\eta_\sigma^{-1}l_e^{-\eta_\sigma})$. In the limit $\eta_\sigma\to 0$ we also recover the logarithmic suppression $(\rho-\rho_0)/\rho_0 = -Bg_\sigma \ln(L/l_e)$ in the free Dirac theory.[13] On the other hand, in the case of a negative exponent $\eta_\sigma<0$ the RG Eq. (9) shows a runaway characteristic for the g_σ. This implies the Anderson localization of spinons and thus the ASL disappears in the presence of nonmagnetic impurities.

Next we discuss how the delocalized state of spinons can emerge. In two spacial dimensions the NLσM based on the free Dirac theory does not have a stable fixed point[13] indicating a delocalized state. But, the presence of a topological term such as a Wess-Zumino-Witten (WZW) term or Berry phase (θ) term can result in a stable critical point.[21] This is well known in the disordered metal and antiferromagnetic spin chain with spin 1/2. On the other hand, the delocalization in the present study has nothing to do with such topological terms. It originates from the criticality of the ASL. It is well known that the NLσM has an unstable fixed point above two dimensions.[13] The criticality of $\eta_\psi>0$ leads the ASL to be in $(2+2\eta_\psi)$ dimensions effectively. As a result, the delocalization emerges against weak randomness. We would like to stress that the mechanism of delocalization in the critical phase (ASL) totally differs from that arising from the topological terms. This is reflected in the RG equation. In the delocalization induced by the topological terms the stable fixed point appears from the g_σ^2 term in the RG equation.[21] This is associated with the destructive interference effect of interactions. On the other hand, the delocalization driven by the criticality arises from the g_σ term [Eq. (9)]. This is due to increase of effective dimensionality, as discussed above.

If the WZW term appears in the NLσM Eq. (8), a stable strong coupling fixed point is expected to exist[21] in both cases, $\eta_\psi>0$ and $\eta_\psi<0$. The density of states would exhibit a power-law behavior near the fixed point. Moreover, it is expected to vanish as energy decreases down to zero.[21] This is in contrast to the present case. However, it is not clear whether this new fixed point is stable against instanton excitations. The spinons would be renormalized by the disorder effect. It is difficult to determine how the renormalized spinons affect the internal gauge charge. This problem will be an important future work on the stability of the ASL against both instantons and disorder.

Our present study has important implications in the role of nonmagnetic disorder in high T_c cuprates. According to one scenario[1], the pseudogap (PG) state is proposed to be the ASL. Then, the ASL should be stable against disorder because all samples include disorder. The stability of the ASL against disorder depends on the sign of the critical exponent η_ψ. As pointed out earlier, most evaluations[3,8–10] support a negative critical exponent. This would destabilize the ASL against disorder, resulting in the Anderson localization. A positive critical exponent[6] is necessary for the stability of the ASL. Our investigation requires more careful determination of the critical exponent η_ψ. Furthermore, the present study tells us that the previous studies of disorder effects in the PG state[22] are difficult to be applied. This is because the studies[22] are based on the free fermion theory ignoring strong gauge fluctuations. The existence of the delocalized state in the impurity-doped ASL seems to be inconsistent with experiments.[22] In experiments a nonmagnetic impurity is believed to localize spin 1/2. This trapped spin acts as a free spin, showing the Curie-Weiss behavior in the spin susceptibility.[22] We expect that this inconsistency may be resolved by considering strong disorder. We note that strong disorder causes the localization of spinons. Zn impurity can be considered to be a strong scatterer owing to its compact electronic shell structure.

In the present paper we did not consider the effect of hole doping. Doped holes are represented by holons in the context of the slave boson theory. Holons can affect gauge fluctuations. For example, when holons are condensed, gauge fluctuations become massive via the Anderson-Higgs mechanism. As a result a free Dirac theory is obtained. The spinon can be localized even in weak disorder. The Curie-Weiss behavior can be easily understood based on the free Dirac theory in the superconducting state.[23] In the PG phase the holons are not condensed. Gauge fluctuations remain massless. In this case the damping effect in gauge fluctuations is expected to arise from holon contributions.[24,25] The role of disorder in the presence of damped gauge fluctuations would be an interesting future work.[25]

Lastly, we would like to comment about the present approach based on the NLσM. The NLσM approach has some advantages, compared with the previous access based on the fermionic action.[11] First, the NLσM approach is more efficient than the fermionic one in investigating physics of phase transitions. The NLσM assumes the presence of a local order parameter. On the other hand, the previous study[11] is based on the absence of local ordering. If we start from a disordered phase without local ordering and approach a critical point associated with a phase transition, a local order parameter is to emerge, strongly fluctuating. In order to describe critical fluctuations of the order parameter, we explicitly introduce the local order parameter and obtain an effective field theory of the order parameter field by integrating over the fermions. The resulting effective field theory is the Ginzburg-Landau free energy. As a matter of fact the NLσM is nothing but the Ginzburg-Landau free-energy formulation. If we start from the fermion action, we realize that it is not easy to describe phase transitions. This is because phase transitions are basically nonperturbative phenomena not captured by perturbative calculations based on the fermionic action. In order to describe the phase transitions it is necessary to sum infinitely many diagrams in the fermionic action. On the other hand, in the Ginzburg-Landau formulation the phase transitions are easily described. This is the reason why the NLσM is called an effective field theory for phase transitions. Second, it is also easy to calculate some physical quantities like conductance and density of states. These quantities directly appear in the NLσM. We can easily obtain the conductance and density of states as a function of energy and size of a system by the present RG calculation. Last, topologically nontrivial excitations in the NLσM are not captured in the fermionic action although these are not intensively discussed in the present paper.

In summary, we examined the effect of nonmagnetic disorder on the ASL criticality characterized by its anomalous critical exponent. In the case of a positive exponent the critical point remains stable at least against weak disorder. But, in the opposite case the critical point becomes unstable.

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