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Zero-momentum cyclotron spin-flip mode in a spin-unpolarized quantum Hall system

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We report on a study of the zero-momentum cyclotron spin-flip excitation in the $\mathcal{V}=2$ quantum Hall regime. Using the excitonic representation the excitation energy is calculated up to the second-order Coulomb corrections. A considerable negative exchange shift relative to the cyclotron gap is established for cyclotron spin-flip excitations in the spin-unpolarized electronic system. Under these conditions this type of state presents the *lowest-energy* excitations. For a fixed filling factor ($\mathcal{V}=2$) the energy shift is independent of the magnetic field, which is in agreement with recent experimental observations.

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It is well known that in a translationally invariant two-dimensional (2D) electron system Kohn's theorem[1] prohibits coupling of a homogeneous external perturbation to collective excitations of the electrons. As a result, the energy of cyclotron excitations (CE) at zero momentum has no contribution from Coulomb interaction and the dispersion of CE starts from the cyclotron gap. In addition to inter-Landau-level cyclotron excitations (magnetoplasma (MP) mode) there are two other branches of collective excitations in the system of 2D electrons: intra-Landau-level spin-flip (SF) excitations (spin waves) and inter-Landau-level combined cyclotron spin-flip excitations (CSFE's). In the case of SF excitations, there exists Larmor's theorem which forbids any contribution from Coulomb interaction to the excitation energy at zero momentum in spin rotationally invariant systems (see, e.g., Ref. 2). However, in contrast to the CE and SF excitations, there are no symmetry reasons for the absence of many-body corrections to the zero-momentum energy of CSFE's. Moreover, it is well established now both theoretically and experimentally[3] that for the spin-polarized electron system ($\mathcal{V}=1$) the energy of cyclotron spin-flip excitations is strongly shifted to higher values relative to the cyclotron gap due to the exchange interaction. Therefore, the energy of combined cyclotron spin-flip excitations is a very convenient tool to probe many-body effects, for example, in the inelastic light scattering measurements performed at zero momentum.[3–5]

The sensitivity of CSFE energy at $\mathbf{q}=0$ to many-body effects strongly depends on the spin polarization of the electron system. For the spin-unpolarized electron system ($\mathcal{V}=2$), theory[2] developed within the *first-order* perturbation approach in terms of the parameter $r_C=E_C/\hbar\omega_c$ (E_C is the characteristic Coulomb energy, ω_c is the cyclotron frequency) predicts a zero many-body contribution to the zero-momentum energy of CSFE. We show below that calculation of the CSFE zero-momentum energy for the $\mathcal{V}=2$ system performed to within the *second-order* Coulomb corrections yields a considerable negative exchange shift relative to the cyclotron gap.

The studied system is characterized by exact quantum numbers S, S_z, and $\mathbf{q}$ and by a nonexact but "good" quantum number δn corresponding to the change of the single-electron energy $\hbar\omega_c\delta n$ with an excitation. The relevant excitations with $\mathbf{q}=0$ and $\delta n=1$ may be presented in the form $\hat{K}^\dagger_{S,S_z}|\mathbf{0}\rangle$, where $|\mathbf{0}\rangle$ is the ground state and $\hat{K}^\dagger_{S,S_z}$ are "raising" operators: $\hat{K}^\dagger_{0,0}=\Sigma_{np\sigma}\sqrt{n+1}c^\dagger_{n+1,p,\sigma}c_{n,p,\sigma}$, $\hat{K}^\dagger_{1,0}=\Sigma_{np\sigma}\sqrt{n+1}(-1)^\sigma c^\dagger_{n+1,p,\sigma}c_{n,p,\sigma}$, and $\hat{K}^\dagger_{1,+/-}=\Sigma_{np}\sqrt{n+1}c^\dagger_{n+1,p,\uparrow/\downarrow}c_{n,p,\downarrow/\uparrow}$, [$c_{n,p,\sigma}$ is the Fermi annihilation operator corresponding to the Landau-gauge state (n,p) and spin index $\sigma=\uparrow,\downarrow$]. The commutators with the kinetic-energy operator $\hat{H}_1$ are $[\hat{H}_1,\hat{K}^\dagger_{S,S_z}]\equiv\hbar\omega_c\hat{K}^\dagger_{S,S_z}$. (The total Hamiltonian is $\hat{H}_{tot}=\hat{H}_1+\hat{H}_{int}$, where $\hat{H}_{int}$ is the exact Coulomb-interaction Hamiltonian.) If $|\mathbf{0}\rangle$ is unpolarized, we have $\hat{\mathbf{S}}^2\hat{K}^\dagger_{S,S_z}|\mathbf{0}\rangle\equiv S(S+1)\hat{K}^\dagger_{S,S_z}|\mathbf{0}\rangle$, $\hat{S}_z\hat{K}^\dagger_{S,S_z}|0\rangle\equiv S_z\hat{K}^\dagger_{S,S_z}|\mathbf{0}\rangle$, and besides get the identity $\langle 0|\hat{K}_{S,S_z}[\hat{H}_{int},\hat{K}^\dagger_{S,S_z}]|0\rangle\equiv 0$ ($|0\rangle$, to describe the zeroth-order ground state). The latter determines the first-order Coulomb corrections vanishing both for the $S=0$ MP mode and for the $S=1$ triplet states corresponding to the combined spin-cyclotron excitation. At the same time $[\hat{H}_{int},\hat{K}^\dagger_{0,0}]\equiv 0$[1] but $[\hat{H}_{int},\hat{K}^\dagger_{1,S_z}]\neq 0$, which means that the MP mode indeed has no exchange energy calculated to *any order* in r_C, whereas the triplet states have the exchange correction even in terms of r_C^2.

The second-order correction, $\Delta E_{SF}\sim\hbar\omega_c r_C^2$, does not depend on the magnetic field since $E_C=\alpha e^2/\varepsilon l_B$. The renormalization factor, α, is determined by the size-quantized wave function of electrons confined to the quantum well (QW). In the ideal 2D case $\alpha=1$. However, in experiments with comparatively wide QW's we expect a well-reduced value of α. Our *analytical* calculation of the second-order correction to the CSFE energy is performed in terms of r_C assumed to be small.

All three triplet states have certainly the same exchange energy, and it is sufficient to calculate this, e.g., for the CSFE with $S=1$ and $S_z=-1$. The obtained result confirms experimental observations.

In the homogeneous system under integer-quantum-Hall conditions there are no excitations of quasielectron or quasihole type but all excitations are reduced to excitonlike or many-exciton states. (E.g., a pair of separated electron and hole is a particular case of the $q\to\infty$ exciton.[2,6]) These are split off from the ground state by energy gaps. In this situation the adequate approach to the microscopic calculations is based on the *excitonic representation* (ER) technique.[7–9] The latter means that instead of single-electron states belonging to a continuously degenerate Landau level (LL) we employ *as the basic set the exciton states* $\mathcal{Q}^\dagger_{ab\ \mathbf{q}}|0\rangle$, where *the degen-*

eration becomes well lifted. The exciton creation operator is defined as[7–10]

$$\mathcal{Q}^{\dagger}_{ab\ \mathbf{q}} = \frac{1}{\sqrt{N_\phi}} \sum_p e^{-iq_x p} b^{\dagger}_{p+q_y/2} a_{p-q_y/2}. \tag{1}$$

$N_\phi = A/2\pi l_B^2$ stands for the number of magnetic-flux quanta, $\mathbf{q}=(q_x,q_y)$ is the 2D wave vector in units of $1/l_B$. Binary indexes a and b present both the LL number and spin index. [That is, $a=(n_a,\sigma_a)$, and a_p in Eq. (1) stands for the corresponding annihilation operator; when exploiting below the notation $a=n$ or $a=\overline{n}$ as sublevel indexes, this means that $a=(n,\uparrow)$ or $a=(n,\downarrow)$, respectively.] The annihilation exciton operator is $\mathcal{Q}_{ab\ \mathbf{q}} \equiv \mathcal{Q}^{\dagger}_{ba\ -\mathbf{q}}$. The commutation rules define a special Lie algebra,[7,9,10]

$$[\mathcal{Q}^{\dagger}_{cd\ \mathbf{q}_1}, \mathcal{Q}^{\dagger}_{ab\ \mathbf{q}_2}] \equiv N_\phi^{-1/2} [e^{-i(\mathbf{q}_1 \times \mathbf{q}_2)_z/2} \delta_{b,c} Q^{\dagger}_{ad\ \mathbf{q}_1+\mathbf{q}_2} - e^{i(\mathbf{q}_1 \times \mathbf{q}_2)_z/2} \delta_{a,d} Q^{\dagger}_{cb\ \mathbf{q}_1+\mathbf{q}_2}], \tag{2}$$

where $\delta_{a,b} = \delta_{n_a,n_b}\delta_{\sigma_a,\sigma_b}$ is the Kronecker symbol. In the $\mathcal{V}=2$ case we get the following identity: $N_\phi^{-1/2}\mathcal{Q}^{\dagger}_{aa\ \mathbf{q}}|0\rangle \equiv \delta_{\mathbf{q},0}(\delta_{a,0}+\delta_{a,\overline{0}})|0\rangle$.

The advantage of the exciton states lies in the fact that an essential part of the Coulomb interaction Hamiltonian may be diagonalized in this basis. In the perturbative approach the excitonically diagonalized part $\hat{H}_{\mathrm{ED}}$ should be included into the unperturbed Hamiltonian $\hat{H}_0 = \hat{H}_1 + \hat{H}_{\mathrm{ED}}$ and only the off-diagonal part $\hat{\mathcal{H}}_{\mathrm{int}} = \hat{H}_{\mathrm{int}} - \hat{H}_{\mathrm{ED}}$ is considered as a perturbation.[7] Now *even within the zero-order approximation in terms of* $\hat{\mathcal{H}}_{\mathrm{int}}$ there are Coulomb corrections (depending on the $\mathbf{q}$ modulus) to the energies of basis excitons.[11] It is useful to take into account that all terms of the relevant $\hat{\mathcal{H}}_{\mathrm{int}}$ part may be presented in the form (cf. Ref. 7)

$$\hat{\mathcal{H}}_{\mathrm{int}} = \frac{e^2}{2\varepsilon l_B} \sum_{\mathbf{q},a,b,c,d} V(\mathbf{q}) [h_{n_a n_b}(\mathbf{q}) \delta_{\sigma_a,\sigma_b} \mathcal{Q}^{\dagger}_{ab\ \mathbf{q}}] \times [h_{n_c n_d}(-\mathbf{q}) \delta_{\sigma_c,\sigma_d} \mathcal{Q}^{\dagger}_{cd\ -\mathbf{q}}]. \tag{3}$$

Here $2\pi V(\mathbf{q})$ is the dimensionless 2D Fourier component of the averaged Coulomb potential (in the ideal 2D case $V=1/q$), and $h_{kn}(\mathbf{q}) = (k!/n!)^{1/2} e^{-q^2/4} (q_-)^{n-k} L_k^{n-k}(q^2/2)$ are the ER "building-block" functions (L_k^n is the Laguerre polynomial, $q_\pm = \mp i/\sqrt{2}(q_x \pm i q_y)$; cf. also Refs. 2, 7, and 9). The functions h_{kn} satisfy the identity: $h^*_{kn}(\mathbf{q}) \equiv h_{nk}(-\mathbf{q})$.

At the $\mathcal{V}=2$ filling the CSFE state calculated to zeroth order in $\hat{\mathcal{H}}_{\mathrm{int}}$ is simply $|SF\rangle = \hat{Q}^{\dagger}_{01}|0\rangle$ (the notation $\hat{\mathcal{Q}}^{\dagger}_{ab\ \mathbf{0}} = \hat{Q}^{\dagger}_{ab}$ is employed). We thus have $[\hat{H}_{\mathrm{ED}}, Q^{\dagger}_{01}]|0\rangle = \langle 0|\mathcal{Q}_{ab\ \mathbf{q}}[\hat{\mathcal{H}}_{\mathrm{int}}, Q^{\dagger}_{01}]|0\rangle = 0$ for any indexes $\mathbf{q}$ and ab (one could check it directly using the ER approach; see also Refs. 2, 7, and 9). Action of $\hat{\mathcal{H}}_{\mathrm{int}}$ on the $|SF\rangle$ state does not generate one-exciton states but leads to two- and even three-exciton states. Therefore the excitonic basis should be extended.

Below we calculate *exactly* the second-order correction to the CSFE energy. Within the Green's-functions formalism[2] this would correspond to summation of *all* diagrams contributing to the spin-density response function up to second order in r_{C}.

It is well known that in a magnetized electron system the single LL approach inevitably fails if one studies the *second-order* Coulomb corrections. In other words, the LL mixing (LLM), even being small due to the $r_{\mathrm{C}} < 1$ condition, has to be properly considered. The LLM is not reduced to an "interaction" of nearest LL's but the mixing of occupied levels with all other levels must be taken into account. In terms of the two-exciton basis this means that at $\mathcal{V}=2$ the lower indexes of appropriate exciton operators are 0 or $\overline{0}$ but the upper ones run over all numbers of higher sublevels. There are eight different kinds of such two-exciton states. In our case the relevant ones are those corresponding to spin numbers $S_z=-1$ and $S_z=0$, namely: $|\nu,1\rangle = \mathcal{Q}^{\dagger}_{0n_2\ -\mathbf{q}_\nu}\mathcal{Q}^{\dagger}_{\overline{0}n_1\ \mathbf{q}_\nu}|0\rangle$, $|\nu,2\rangle = \mathcal{Q}^{\dagger}_{\overline{0}n_2\ -\mathbf{q}_\nu}\mathcal{Q}^{\dagger}_{\overline{0}n_1\ \mathbf{q}_\nu}|0\rangle$, $|\nu,3\rangle = \frac{1}{2}\mathcal{Q}^{\dagger}_{0n_2\ -\mathbf{q}_\nu}\mathcal{Q}^{\dagger}_{0n_1\ \mathbf{q}_\nu}|0\rangle$, $|\nu,4\rangle = \mathcal{Q}^{\dagger}_{0n_2\ -\mathbf{q}_\nu}\mathcal{Q}^{\dagger}_{\overline{0}n_1\ \mathbf{q}_\nu}|0\rangle$, and $|\nu,5\rangle = \frac{1}{2}\mathcal{Q}^{\dagger}_{\overline{0}n_2\ -\mathbf{q}_\nu}\mathcal{Q}^{\dagger}_{\overline{0}n_1\ \mathbf{q}_\nu}|0\rangle$ (certainly only the states with the $S=1$ momentum should be considered). We have used here ν as a composite index corresponding to the set $(n_1,n_2,\mathbf{q}_\nu)$. The two-exciton states of different types are orthogonal, i.e., $\langle I,\nu|\mu,J\rangle = 0$ if $I \neq J$ [μ is the set $(m_1,m_2,\mathbf{q}_\mu)$, below $\lambda=(l_1,l_2,\mathbf{q}_\lambda),\ldots$]. However, within the same type their orthogonalization rules should be defined in a special way.

First, let us consider a combination

$$\sum_\nu f_\nu |\nu,I\rangle \tag{4}$$

(summation is performed over all components of the composite index). However, only a certain transform of the function $f_\nu = f(n_1,n_2,\mathbf{q}_\nu)$ has a physical meaning. Indeed, actually only a projection of the sum (4) onto a certain two-exciton state $|\mu,J\rangle$ would be of any sense. With the help of the commutation rules (2) we obtain

$$\sum_\nu f_\nu \langle J,\mu|\nu,I\rangle \equiv \delta_{I,J}\{f_\mu\}_I \tag{5}$$

(cf. Ref. 7). Here the curly brackets mean the transform

$$\{f_\nu\}_I = f_\nu - N_\phi^{-1} \sum_\lambda \mathcal{F}^{(I)}_{\nu\lambda} f_\lambda, \quad \text{if } I = 1,\ 2,\ \text{or } 4; \quad \text{and}$$

$$\{f_\nu\}_I = \frac{1}{2}\left(f_\nu - N_\phi^{-1} \sum_\lambda \mathcal{F}^{(I)}_{\nu\lambda} f_\lambda \right), \quad \text{if } I = 3 \text{ or } 5. \tag{6}$$

The definition of the kernels $\mathcal{F}^{(I)}_{\nu\lambda} f_\lambda$ is also parametrized by the kind I of the state, namely,

$$\mathcal{F}^{(1)}_{\nu\lambda} = \mathcal{F}^{(3)}_{\nu\lambda} = \mathcal{F}^{(5)}_{\nu\lambda} \equiv \delta_{n_1,l_1}\delta_{n_2,l_2} e^{i(\mathbf{q}_\nu \times \mathbf{q}_\lambda)_z},$$

$$\mathcal{F}^{(2)}_{\nu\lambda} \equiv \delta_{n_1,l_2}\delta_{n_2,l_1} e^{-i(\mathbf{q}_\nu \times \mathbf{q}_\lambda)_z}, \quad \text{and } \mathcal{F}^{(4)}_{\nu\lambda} \equiv 0. \tag{7}$$

Note that the transform $\{\cdots\}_I$ is to within a factor equivalent to its double application: $\{\{f\}_I\}_I = K_I\{f\}_I$, where $K_1=K_2=2$ and $K_3=K_4=K_5=1$. Therefore, if we replace, e.g., $f_\nu \to f_\nu + K_I\varphi_\nu - \{\varphi_\lambda\}_I$ (φ_ν is an arbitrary function), then this operation does not affect the combinations (4) and (5). So, only the "anti-

symmetrized" part $\{f_\nu\}_I$ contributes to the matrix-element calculations. The origin of this feature of the two-exciton states is related to the permutation antisymmetry of the total wave function describing the electron system studied (cf., e.g., Refs. 6 and 7). There is also a useful identity,

$$\sum_\nu w(n_1,n_2) f_\nu^* \{g_\nu\}_I \equiv \sum_\nu w(n_1,n_2) \{f_\nu\}_I^* g_\nu, \tag{8}$$

which is valid for any kinds of the transforms $\{\cdots\}_I$ if the function w in Eq. (5) is assumed to be such that $w(n_1,n_2) \equiv w(n_2,n_1)$. In particular, Eq. (5) gives the equations

$$\langle I,\nu|\mu,J\rangle \equiv \delta_{I,J}\{\delta_{\nu\mu}^{(I)}\}_I, \tag{9}$$

where

$$\delta_{\nu\mu}^{(1)} = \delta_{\nu\mu}^{(2)} = \delta_{\nu\mu}^{(4)} \equiv \delta_{n_1,m_1}\delta_{n_2,m_2}\delta_{\mathbf{q}_\nu,\mathbf{q}_\mu}$$

and

$$\delta_{\nu\mu}^{(3)} = \delta_{\nu\mu}^{(5)} \equiv \frac{1}{2}(\delta_{n_1,m_1}\delta_{n_2,m_2}\delta_{\mathbf{q}_\nu,\mathbf{q}_\mu} + \delta_{n_1,m_2}\delta_{n_2,m_1}\delta_{\mathbf{q}_\nu,-\mathbf{q}_\mu}).$$

Summation in the $\{\delta_{\nu\mu}^{(I)}\}_I$ transform is performed over the first index: e.g., $\{\delta_{\nu\mu}^{(1)}\}_1 \equiv \delta_{\nu\mu}^{(1)} - \mathcal{F}_{\nu\mu}^{(1)}/N_\phi$, and so on.

The first-order corrections (in terms of $\hat{\mathcal{H}}_{\rm int}$) to the CSFE state are presented as an expansion over the two-exciton states $|\nu,I\rangle$ and three-exciton states $Q_{01}^\dagger|\nu,I\rangle$, namely,

$$|SF\rangle = Q_{01}^\dagger|0\rangle + \sum_{I=1,2}\sum_\nu C_\nu^{(I)}|\nu,I\rangle + \sum_{I=3,4,5}\sum_\nu C_\nu^{(I)} Q_{01}^\dagger|\nu,I\rangle \tag{10}$$

(here the LLM manifests itself by the fact that ν runs over *all* unoccupied levels: $n_1+n_2\geq 2$). A regular application of the perturbative approach[12] leads to the following expression for the exchange correction to the energy: $\Delta E_{\rm SF} = \langle SF|\hat{\mathcal{H}}_{\rm int}Q_{01}^\dagger|0\rangle$. Substituting $|SF\rangle$ from Eq. (10) we see that the *contribution of the two-exciton states* to the energy arises only due to the terms of Eq. (3) which do not commute with $Q_{01}^\dagger$,

$$\Delta E_{1-2} = \sum_{I=1,2}\sum_\nu C_\nu^{(I)^*}\langle I,\nu|[\hat{\mathcal{H}}_{\rm int}, Q_{01}^\dagger]|0\rangle. \tag{11}$$

The coefficients $C_\nu^{(I)}$ are determined by the equations

$$\sum_\mu C_\mu^{(I)}\langle I,\nu|\mu,I\rangle = -\langle I,\nu|[\hat{\mathcal{H}}_{\rm int}, Q_{01}^\dagger]|0\rangle/\Delta_\nu \tag{12}$$

($I=1,2$), where $\Delta_\nu = \hbar\omega_c(n_1+n_2-1)$ stands for the difference of the cyclotron energies in the states $|\nu,I\rangle$ and $Q_{01}^\dagger|0\rangle$. Calculating the commutator in Eqs. (11) and (12) [employing the rules (2)], and then using the properties (5) and (8) of the summation over index, we obtain

$$\Delta E_{1-2} = -N_\phi^{-1}\sum_{I=1,2}\sum_\nu \{F_\nu\}_I F_\nu^*/(n_1+n_2-1) \tag{13}$$

[in units of 2 Ry$=(e^2/\varepsilon l_B)^2/\hbar\omega_c = m_e^* e^4/\varepsilon^2\hbar^2$], where

$$F_\nu = V(\mathbf{q}_\nu)[h_{1n_1}(\mathbf{q}_\nu) - \delta_{1,n_1}h_{00}(\mathbf{q}_\nu)]h_{0n_2}(-\mathbf{q}_\nu). \tag{14}$$

Now we calculate the contribution ΔE_{3-5} which is determined by the *three-exciton states* [see Eq. (10)]. This correction arises from the commuting part (with $Q_{01}^\dagger$) of $\hat{\mathcal{H}}_{\rm int}$ acting on the state $Q_{01}^\dagger|0\rangle$, i.e.,

$$\Delta E_{3-5} = \sum_{I=3,4,5}\sum_\nu C_\nu^{(I)^*}\langle I,\nu|Q_{01}Q_{01}^\dagger\hat{\mathcal{H}}_{\rm int}|0\rangle. \tag{15}$$

The equations for the coefficients are

$$\sum_\mu C_\mu^{(I)}\langle I,\nu|Q_{01}Q_{01}^\dagger|\mu,I\rangle = -\langle I,\nu|Q_{01}Q_{01}^\dagger\hat{\mathcal{H}}_{\rm int}|0\rangle/\tilde{\Delta}_\nu \tag{16}$$

($I=3,4,5$), where $\tilde{\Delta}_\nu = \hbar\omega_c(n_1+n_2)\geq 2$. Substituting $Q_{01}Q_{01}^\dagger \equiv N_\phi^{-1/2}(Q_{00}-Q_{\bar{1}\bar{1}}) + Q_{01}^\dagger Q_{01}$ into Eqs. (15) and (16) we deduce that the operator $Q_{01}^\dagger Q_{01}$ gives no contribution, whereas action of the remaining terms reduces the convolutions in Eqs. (15) and (16) to the "bra-ket" products of two-exciton states. In so doing we find a huge contribution (eventually $\sim N_\phi$) into Eq. (15) due to the commuting part of $N_\phi^{-1/2}(Q_{00}-Q_{\bar{1}\bar{1}})$, which is actually nothing else but the second-order correction (in terms of r_C) to the ground state, namely: $\Delta E_0 = \Sigma_{\nu;I=3,4,5} C_\nu^{(I)}\langle 0|\hat{\mathcal{H}}_{\rm int}|\nu,I\rangle$. According to Eq. (16)

$$\{C_\nu^{(I)}\}_I = -[e^2/(\varepsilon l_B\hbar\omega_c)]\{G_\nu\}_I/(n_1+n_2) \tag{17}$$

($I=3,4,5$) with

$$G_\nu = V(\mathbf{q}_\nu)h_{0n_1}(\mathbf{q}_\nu)h_{0n_2}(-\mathbf{q}_\nu). \tag{18}$$

The noncommuting part determines the corrections to the bra vectors in Eq. (15). For the $I=3$ states we get $N_\phi^{-1/2}\left[Q_{00}-Q_{\bar{1}\bar{1}}, \frac{1}{2}\mathcal{Q}_{0n_2\,-\mathbf{q}_\nu}^\dagger \mathcal{Q}_{0n_1\,\mathbf{q}_\nu}^\dagger\right]|0\rangle = -2|\nu,3\rangle/N_\phi$, and at $I=4,5$ correspondingly $-(1+\delta_{n_1,1})|\nu,4\rangle/N_\phi$ and $-(\delta_{n_1,1}+\delta_{n_2,1})|\nu,5\rangle/N_\phi$ [the identities (2) have been used]. The similar corrections to the bra vectors in Eq. (16) do not affect the equation (17) for $C_\nu^{(I)}$.

The desirable exchange shift should be measured from corrected energy of the ground state. We keep thus in Eq. (15) only the contribution of the noncommuting part (i.e., considering $\Delta E_{3-5} \to \Delta E_{3-5} - \Delta E_0$). Then by substituting Eq. (3) for $\hat{\mathcal{H}}_{\rm int}$ into Eq. (15) and using again the summation rules (5) and (8) we find from Eqs. (15) and (17) the $I=3-5$ correction

$$\Delta E_{3-5} = \frac{1}{N_\phi}\sum_\nu [(2+\delta_{n_1,1}+\delta_{n_2,1})\{G_\nu\}_3 + (1+\delta_{n_1,1})G_\nu]G_\nu^*/(n_1+n_2) \tag{19}$$

(in units of 2 Ry). The combination with Eq. (13) yields

$$\Delta E_{\rm SF} = \Delta E_{1-2} + \Delta E_{3-5}. \tag{20}$$

The sum over ν in Eqs. (13) and (19) means summation over n_1 and n_2 and the integration over $\mathbf{q}_\nu$. This is a routine procedure and the suitable sequence of operations is as follows. First we perform the summation over all of $n_1\geq 1$ and $n_2\geq 1$ keeping the sum $n_\nu = n_1+n_2$ fixed. Then we make the integration over $\mathbf{q}_\nu$. According to the above definition, the transforms $\{F_\nu\}_I$ and $\{G_\nu\}_I$ already contain an integration, therefore some terms in Eq. (8) present twofold integration

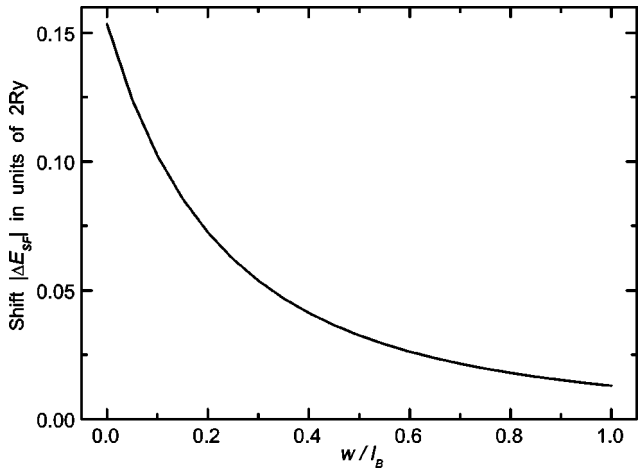

FIG. 1. The CSFE exchange shift is calculated from the formula of Eq. (21) with the modified Coulomb interaction $V(q)=q^{-1}e^{q^2w^2}\,\mathrm{erfc}(qw)$; the shift value absolute at $w=0$ is $(1-\ln 2)/2$.

over 2D vectors $\mathbf{q}_\lambda$ and $\mathbf{q}_\nu$. Really the latter, with the help of formula $(2\pi)^{-2}\int\int d\mathbf{q}_1 d\mathbf{q}_2 U(q_1,q_2)(q_{1+}q_{2-})^m e^{\pm i(\mathbf{q}_1\times\mathbf{q}_2)_z} \equiv 2^{-m}\int_0^\infty\int_0^\infty dq_1 dq_2 (q_1q_2)^{m+1}U(q_1,q_2)J_{\pm m}(q_1q_2)$ (J_m is the Bessel function, U is an arbitrary function), is reduced to integration over absolute values q_λ and q_ν. Finally the summation over n_ν is performed.

In so doing, a simplifying circumstance was found: all of the twofold-integration terms cancel each other in the final combination (20). (This feature is not a general one but only inherent in our specific case.[13]) All the rest of the terms result in the following expression:

$$\Delta E_{\mathrm{SF}} = -\sum_{n=2}^{\infty}\frac{2-2^{2-n}}{nn!(n^2-1)}\int_0^\infty dq q^{2n+3}V^2(q)e^{-q^2}. \quad (21)$$

Here, for the ideally 2D system the integration is calculated analytically. Then the summation is easily performed, yielding $\Delta E_{\mathrm{SF}}=(\ln 2-1)/2=-0.153\,4\ldots$ (in units of 2 Ry).

So, the exchange interaction lowers thereby the CSFE energy relative to the singlet MP mode. Another feature of the found shift is its independence of the magnetic field. Due to the $\mathbf{q}=0$ condition the studied state is optically active. In the recent work[5] the inelastic light scattering (ILS) was studied in a single 30-nm AlGaAs/GaAs QW in the situations where $\mathcal{V}=2;4$. The triplet and MP cyclotron excitations are manifested as peaks in the ILS spectra. The measurements were performed in magnetic fields varied in a wide range, but with the filling factor kept constant. The central triplet line is shifted downward from the cyclotron energy by 0.35 meV independently of the B magnitude. Thus, a qualitative agreement with our calculation is obvious.

Quantitative comparison should be done with taking into account of finite thickness of a two-dimensional electron gas. The calculation in Fig. 1 incorporates the effect of the finite width of the 2D layer. This is carried out by writing the Coulomb vertex as $V(q)=F(qw)/q$, where the form factor $F(qw)$ is parametrized by an effective thickness w. If the variational envelope function is chosen in the form $|\psi(z)|^2\sim\exp(-z^2/2w^2)$, then $F(qw)=e^{w^2q^2}\,\mathrm{erfc}(wq)$ (see Ref. 14). Exactly this form factor is employed in the calculation based on Eq. (21). Taking into account the value of Ry=5.67 meV in GaAs, we find from Fig. 1 that the agreement with the experiment is obtained at $w\approx 0.5l_B$. This is quite a reasonable value for the 30-nm GaAs structure.

As a concluding remark we notice that the triplet cyclotron excitation in spin-unpolarized electron system seems to have been observed earlier,[4] although in this paper experimental observations were related to the roton minimum and a different dependence of energy shift on magnetic field was detected.

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