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Commensurability oscillations in the surface-acoustic-wave-induced acoustoelectric effect in a two-dimensional electron gas

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We study the acoustoelectric effect generated by surface acoustic waves (SAW) in a high-mobility two-dimensional electron gas with isotropic and especially small-angle impurity scattering. In both cases the acoustoelectric effect exhibits Weiss oscillations periodic in B^{-1} due to the commensurability of the SAW period with the size of the cyclotron orbit and resonances at the SAW frequency $\omega = k\omega_c$ multiple of the cyclotron frequency. We describe how oscillations in the acoustoelectric effect are damped in low fields where $\omega_c \tau_* \lesssim 1$ (with the time scale τ_* dependent on the type of scattering) and find its nonoscillatory part, which remains finite to the lowest fields.

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Due to a finite wave number carried by surface acoustic waves (SAWs) their application enables one to access the properties of low-dimensional electron systems[1] that cannot be studied using the standard microwave absorption techniques. The observation of magneto-oscillations in the shift of SAW velocity caused by its interaction with the two-dimensional (2D) electrons in the vicinity of filling factor $\nu = 1/2$ and its comparison with Weiss geometrical oscillations[2–4] for electrons has enabled Willett *et al.*[5] to establish the existence of a composite fermion Fermi surface. Also, the additional length scale in the system permits transitions otherwise forbidden by Kohn's theorem,[6] thus making possible the detection of cyclotron transitions in a gas of composite quasiparticles.[7,8]

In this Communication we extend the analysis of the phenomenon of geometrical commensurability onto the acoustoelectric (AE) (drag) effect,[9–12] which has been studied experimentally in semiconductor structures in various regimes.[13,14] We show that by measuring magneto-oscillations and studying the frequency dependence of the dc electric field induced in a two-dimensional electron gas (2DEG) by a propagating SAW, one can access the same information about resonant and the Fermi surface effects in a 2DEG in the Boltzman transport regime as was previously studied in absorption and SAW propagation experiments.[1,8] To be able to describe the AE effect in high-mobility heterostructures, such as investigated experimentally in Ref. 14, we develop a theory for two types of structures: with isotropic and with small-angle impurity scattering.

Here we use an approach recently applied to the studies of another dc effect produced by a dynamical acoustic wave field, the SAW-induced magnetoresistance.[15] We investigate the AE effect in the linear order in the SAW power and find the parametric dependences of the steady-state electric field $E_{\rm AE}$ generated by the SAW in the direction of its propagation, $\mathbf{E}_{\rm AE} = \hat{\mathbf{q}} E_{\rm AE}\ (qR_c, \omega_c\tau_*, \omega/\omega_c)$, where $\mathbf{q}$ is the SAW wave vector, R_c and ω_c the electron cyclotron radius and frequency, ω the SAW frequency, and τ_* the effective electron-scattering time crucially dependent on the type of impurity scattering.[17,18] In the high-field limit, $\omega_c\tau_* \gg 1$, we describe Weiss oscillations of the AE effect as a function of qR_c. In addition, for low-density structures with heavy-mass carriers we predict resonances in $E_{\rm AE}$ at frequencies $\omega = k\omega_c$. For a small magnetic field (though large enough to ensure that $v_F B > E_{\omega q}$), we show that while commensurability oscillations are damped, there is a finite field-independent contribution to the AE drag.

Our theory consists of the analysis of the Boltzmann equation,

$$\hat{\mathcal{L}}[f(\mathbf{p},\mathbf{x},t)] = \hat{\mathcal{C}}[f(\mathbf{p},\mathbf{x},t)],$$

$$\hat{\mathcal{L}} = \partial_t + \omega_c R_c \cos\varphi \partial_x + \omega_c \partial_\varphi + eE\hat{\mathcal{P}},$$

$$\hat{\mathcal{P}} = v \cos\varphi \partial_\epsilon - \frac{\sin\varphi}{p}\partial_\varphi, \tag{1}$$

where $\hat{\mathcal{C}}$ is the collision integral, and the momentum-dependent part in $\hat{\mathcal{L}}$ and $\hat{\mathcal{P}}$ is written in terms of the electron kinetic energy $\epsilon = p^2/2m$ and angle φ, characterizing the direction of electron propagation with respect to the direction of propagation of the SAW. Here, v is the electron velocity, and $\mathbf{E} = \mathbf{l}_x E e^{i(\omega t - qx)}$ is the longitudinally polarized SAW field, screened by the 2DEG.

Using the (ϵ, φ) parametrization of momentum space, we expand the distribution function $f(\mathbf{p}, \mathbf{x}, t)$ into

$$f(\epsilon,\varphi,x,t) = f_T(\epsilon) + \sum_m \sum_{\Omega Q} f^m_{\Omega Q}(\epsilon) e^{-i\Omega t + iQx} e^{im\varphi}, \tag{2}$$

where $f_T(\epsilon)$ is the equilibrium Fermi function and $f^m_{\omega q}$ characterize the nonequilibrium state caused by the SAW. We determine $g^m = \int_0^\infty d\epsilon f^m$, so that g^0 would characterize the electron density and $g^{\pm 1}$ combine into electric current. We separate the collision integral

$$\hat{\mathcal{C}}[f(\epsilon,\varphi)] = \hat{\mathcal{C}}_{\sigma/\eta}[f(\epsilon,\varphi)] - \frac{f^0(\epsilon) + (\partial_\epsilon f_T) g^0}{\tau_{in}} \tag{3}$$

into the elastic and inelastic parts. Relaxation of the nonequilibrium part of the distribution function towards an isotropic distribution is described by the term $\hat{\mathcal{C}}_{\sigma/\eta}$,

$$\hat{\mathcal{C}}_\eta[f] = \frac{f^0 - f}{\tau}, \quad \text{and } \hat{\mathcal{C}}_\sigma[f] = \frac{1}{\tau}\partial_\varphi^2 f. \tag{4}$$

We adopt the subscripts η for isotropic scattering and σ for small-angle scattering, approximated by diffusion along the Fermi surface. In Eq. (4), τ^{-1} is the momentum relaxation rate (thus, τ is the transport time which determines the time-scale upon which the nonequilibrium harmonics $f^{\pm 1}$ decay). Energy relaxation, with $\tau_{in}^{-1} \ll \tau^{-1}$, is taken into account by the last term in Eq. (3) using the relaxation time approximation.

The rectified (acoustoelectric) current can be described using

$$\mathcal{J} = j_x - i j_y = e\gamma \int_0^\infty d\epsilon v f_{00}^1(\epsilon), \tag{5}$$

where γ is the 2D density of states and $f_{00}(\epsilon,\varphi)$ is the steady-state homogeneous part of the nonequilibrium distribution. Below, we restrict the analysis to effects linear in the SAW power and perform a perturbative analysis. We assume that the force from the SAW field is much less than the Lorentz force, $E_{\omega q} \ll v_F B$, whereby electron cyclotron orbits are not destroyed by the SAW and no channeling of electron trajectories occurs. To describe the AE effect we relate the steady-state term $f_{00}(\epsilon,\varphi)$ to the SAW field and $f_{\omega q}(\epsilon,\varphi)$ at the SAW frequency by taking the $Q=0$, $\Omega=0$ harmonics of Eq. (1),

$$\partial_\varphi f_{00}(\epsilon,\varphi) = \frac{\hat{\mathcal{C}} f_{00}(\epsilon,\varphi)}{\omega_c} - \sum_{\pm\omega q} \frac{eE_{-\omega-q}}{\omega_c} \hat{\mathcal{P}} f_{\omega q}(\epsilon,\varphi).$$

We then evaluate the complex current, $\mathcal{J}$,

$$\mathcal{J} = -\sum_{\pm\omega q} \frac{e^2 E_{-\omega-q}}{\omega_c} \int_0^\infty d\epsilon \frac{\gamma\omega_c\tau}{[1 + i\omega_c\tau]} \times \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \left[\frac{2\epsilon}{m}\cos\varphi\,\partial_\epsilon - \frac{\sin\varphi}{m}\partial_\varphi\right] f_{\omega q}(\epsilon,\varphi). \tag{6}$$

Assuming energy independence of τ and density of states, γ, we arrive at

$$\mathcal{J} = \frac{e^2\gamma\tau/m}{[1 + i\omega_c\tau]} \sum_{\pm\omega q} E_{-\omega-q} g_{\omega q}^0,$$

thus reducing the problem to that of finding the ac density modulation $n_{\omega q} = \gamma_F g_{\omega q}^0$ excited by the SAW. The above intermediate result is characteristic for the classical Boltzmann regime of transport. In the above formula for $\mathcal{J}$, $(g_{\omega q}^0)^* = g_{-\omega-q}^0$, while summation over the SAW harmonics satisfies $\omega = sq$, where s is the SAW velocity.

Since the structure of $\mathcal{J}$ repeats that of the Drude conductivity tensor, it is natural to work with the dc acoustoelectric field generated by the SAW in the direction of its propagation,

$$\mathbf{E}_{\text{AE}} = \hat{\mathbf{q}} \frac{2}{\epsilon_F} \text{Re}\{E_{-\omega-q} g_{\omega q}^0\}. \tag{7}$$

The following calculation aims to determine how the quantity of interest, E_{AE}, depends on a particular scattering mechanism [isotropic (η) and small angle (σ)].

The dynamical perturbation $g_{\omega q}(\varphi)$ can be found by taking Fourier harmonics of Eq. (1) at the frequency and wave number of the SAW,

$$\left[\partial_\varphi - \frac{\hat{\mathcal{C}}_{\sigma/\eta}}{\omega_c} - i\frac{\omega}{\omega_c} + iqR_c\cos\varphi\right] f_{\omega q}(\epsilon,\varphi) = -\frac{(\partial_\epsilon f_T) g_{\omega q}^0 + f_{\omega q}^0(\epsilon)}{\omega_c \tau_{in}} - \frac{eE_{\omega q}}{\omega_c} \hat{\mathcal{P}}[f_{00}(\epsilon,\varphi) + f_T(\epsilon)].$$

Here we neglect the term $f_{00} \propto |E_{\omega q}|^2$ since any resulting corrections in $f_{\omega q}(\epsilon,\varphi)$ would be nonlinear in the SAW power. Assuming a low-temperature regime, $kT \ll \omega_c p_F/q$, we integrate the above over energy approximating all energy-dependent parameters (such as R_c) by their respective values at the Fermi level, $\partial_\epsilon f_T \approx -\delta(\epsilon - \epsilon_F)$, and arrive at

$$\left[\partial_\varphi - \frac{\hat{\mathcal{C}}_{\sigma/\eta}}{\omega_c} - i\frac{\omega}{\omega_c} + iqR_c\cos\varphi\right] g_{\omega q}(\varphi) = \frac{ev_F E_{\omega q}}{\omega_c}\cos\varphi. \tag{8}$$

In the limit of $qR_c \gg 1$, the solution to Eq. (8) displays a fast-oscillating angular dependence, $e^{-iqR_c \sin\varphi}$, caused by the last term in brackets on the left-hand side (LHS) of Eq. (8). To take those fast oscillations into account[16,17] we write $g_{\omega q}(\varphi) = h_{\omega q}(\varphi) e^{-iqR_c\sin\varphi}$. Using angular Fourier harmonics of $h_{\omega q}(\varphi)$, this reads

$$g_{\omega q}^h = \sum_{k=-\infty}^{\infty} h_{\omega q}^k J_{k-n}(qR_c). \tag{9}$$

Having multiplied Eq. (8) by $e^{iqR_c\sin\varphi - ik\varphi}$ and integrated over angle φ, we arrived at the system of coupled equations for Fourier coefficients $h_{\omega q}^k$,

$$\left(ik - i\frac{\omega}{\omega_c}\right) h_{\omega q}^k - \frac{1}{\omega_c}\langle e^{iqR_c\sin\varphi - ik\varphi} \hat{\mathcal{C}}_{\sigma/\eta} g_{\omega q}(\varphi)\rangle = \frac{ekE_{\omega q}}{q} J_k(qR_c), \tag{10}$$

where $\langle\cdots\rangle = \int_0^{2\pi} d\varphi/2\pi$ stands for averaging over the angle φ.

The following analysis of Eq. (10) depends on the form of the collision integral. For isotropic scattering,

$$\langle e^{iqR_c\sin\varphi - ik\varphi} \hat{\mathcal{C}}_\eta g_{\omega q}(\varphi)\rangle = \frac{J_k(qR_c) g_{\omega q}^0}{\tau} - \frac{h_{\omega q}^k}{\tau}, \tag{11}$$

and the elements $h_{\omega q}^k$ in equation Eq. (10) decouple. One therefore finds the mth angular harmonic of $g_{\omega q}(\varphi)$ as

$$g_{\omega q}^m = \sum_{k=-\infty}^{\infty} \frac{J_k(qR_c) J_{k-m}(qR_c)}{ik - i\frac{\omega}{\omega_c} + \frac{1}{\omega_c\tau}} \left\{\frac{g_{\omega q}^0}{\omega_c\tau} + \frac{ekE_{\omega q}}{q}\right\}.$$

Setting $m=0$, we find

$$g_{\omega q}^0 = \frac{eE_{\omega q}}{q} \sum_{k=-\infty}^{\infty} \frac{kJ_k^2(qR_c)}{ik - i\frac{\omega}{\omega_c} + \frac{1}{\omega_c\tau}} \frac{1}{[1-K]}, \tag{12}$$

$$K=\frac{1}{\omega_c\tau}\sum_{k=-\infty}^{\infty}\frac{J_k^2(qR_c)}{ik-i\frac{\omega}{\omega_c}+\frac{1}{\omega_c\tau}}. \tag{13}$$

In the limit of $qR_c \gg 1$, then $K \ll 1$ since $J_k^2(qR_c \gg 1) \sim 1/qR_c \to 0$. Additionally, the linear k dependence in Eq. (12) may be manipulated to read $k=-i(ik-i\omega/\omega_c+1/\omega_c\tau)+i(-i\omega/\omega_c+1/\omega_c\tau)$—the first term exactly cancels the resonance denominator, and application of the identity $\Sigma_k J_k^2(x)=1$ yields an approximate form of $g^0_{\omega q}$,

$$g^0_{\omega q}\approx\frac{eE_{\omega q}}{q}\left\{\frac{1}{i}+\frac{\omega}{\omega_c}\sum_{k=-\infty}^{\infty}\frac{J_k^2(qR_c)}{ik-i\frac{\omega}{\omega_c}+\frac{1}{\omega_c\tau}}\right\}. \tag{14}$$

For small-angle scattering the angle average in Eq. (10) takes the form[17,19]

$$\langle e^{iqR_c\sin\varphi-ik\varphi}\hat{\mathcal{C}}_\sigma g_{\omega q}(\varphi)\rangle=-\frac{(qR_c)^2}{2\tau}h^k_{\omega q}-\frac{\mathcal{G}_k}{\tau},$$

$$\mathcal{G}_{k,n}=\left(\frac{qR_c}{2}\right)^2[h^{k-2}_{\omega q}+h^{k+2}_{\omega q}]+k^2h^k_{\omega q}-\frac{qR_c}{2}[(2k+1)h^{k+1}_{\omega q}+(2k-1)h^{k-1}_{\omega q}]. \tag{15}$$

Coupling between different elements $h^k_{\omega q}$ in Eqs. (10) and (15) occurs with multipliers of $(qR_c)^2/\tau \lesssim \omega_c$ and $kqR_c/\tau \lesssim \omega_c$, and is now much weaker than the coupling one would obtain from a direct Fourier transform of Eq. (8). This enables us to solve[17,18] Eq. (10) perturbatively in $\mathcal{G}_k$. Note that we also attribute the term $k^2h^k_{\omega q}$ to the perturbative correction $\mathcal{G}_k$, since $k \lesssim qR_c$, and inclusion of this term in the leading approximation would exceed the chosen accuracy. Thus, we write

$$g^m_{\omega q}=\frac{eE_{\omega q}}{q}\sum_{k=-\infty}^{\infty}\frac{kJ_k(qR_c)J_{k-m}(qR_c)}{ik-i\frac{\omega}{\omega_c}+\frac{(qR_c)^2}{2\omega_c\tau}}-\sum_{k=-\infty}^{\infty}\frac{J_{k-m}(qR_c)}{ik-i\frac{\omega}{\omega_c}+\frac{(qR_c)^2}{2\omega_c\tau}}\frac{\mathcal{G}_k}{\omega_c\tau}. \tag{16}$$

Setting $m=0$, and solving up to second-order Eqs. (15) and (16) in $\mathcal{G}_k$ we find that in the leading order[18] of the parameters $\omega/qR_c\omega_c=s/v_F\ll 1$, $k/qR_c\lesssim 1$ ($qR_c \gg 1$ and $qR_c \ll \omega_c\tau$), the main contribution to the zeroth harmonic $g^0_{\omega q}$ is given by

$$g^0_{\omega q}\approx\frac{eE_{\omega q}}{q}\left\{\frac{1}{i}+\frac{\omega}{\omega_c}\sum_{k=-\infty}^{\infty}\frac{J_k^2(qR_c)}{ik-i\frac{\omega}{\omega_c}+\frac{(qR_c)^2}{2\omega_c\tau}}\right\}. \tag{17}$$

Using the similarity of Eqs. (14) and (17), we express the SAW-induced electric field, $\mathbf{E}_{\rm AE}=\hat{\mathbf{q}}E_{\rm AE}$, for both isotropic and small-angle scattering cases in terms of the effective scattering rate τ_*^{-1},

$$E_{\rm AE}=\frac{2es|E_{\omega q}|^2}{\epsilon_F}\sum_{k=-\infty}^{\infty}\frac{\tau_*J_k^2(qR_c)}{\tau_*{}^2(\omega-k\omega_c)^2+1}, \tag{18}$$

$$\tau_*^{-1}=\begin{cases}\tau^{-1}, & \text{for isotropic scattering},\\ \frac{(qR_c)^2}{2}\tau^{-1}, & \text{for small-angle scattering (Ref. 18)}.\end{cases} \tag{19}$$

Geometrical commensurability manifests itself in Eq. (18) through the appearance of the Bessel function $J_k(qR_c)$. Additionally, the presence of a finite wave vector lifts selection rules for electron transitions, allowing transitions otherwise forbidden by Kohn's theorem,[6] such as resonances at multiples of the cyclotron frequency.

The dynamical redistribution of electrons leads to screening of the external SAW field $E^{\rm SAW}_{\omega q}$ by the 2DEG. We relate $E_{\omega q}$ to the unscreened field by inclusion of the dielectric function, $E_{\omega q}=E^{\rm SAW}_{\omega q}/\kappa(\omega,q)$. In the Thomas-Fermi approximation, and in the limit of $qR_c>1$, we find that $\kappa^{-1}(\omega,q)\approx a_{\rm scr}q$, (Ref. 15), where $a_{\rm scr}=\chi/2\pi e^2\gamma$ is the donor-related Bohr radius (χ is the background dielectric constant), and introduce the dimensionless parameter which is a measure of the amplitude of the screened SAW field normalized by the Fermi energy,

$$\mathcal{E}=(ea_{\rm scr}E^{\rm SAW}_{\omega q}/\epsilon_F)^2. \tag{20}$$

In the limit of $\omega_c\tau_*\gg 1$, we discuss two extreme cases, $v_F \gg s$ and $v_F \lesssim s$, where s is the SAW speed[5] in GaAs, $s=2.8\times 10^5\ {\rm cm\,s^{-1}}$. At electron densities $n_e \sim 10^{10}\div 10^{12}\ {\rm cm^{-2}}$, $v_F \gg s$ and the relevant frequency regime in structures with realistic mobility appears to be $\omega/\omega_c=(s/v_F)qR_c\ll 1$. In this situation, the largest contribution to the dc field then comes from the term in Eq. (18) with $k=0$,

$$E_{\rm AE}\approx\frac{2\mathcal{E}\epsilon_F}{es\tau_*}\frac{J_0^2(qR_c)}{1+(\omega\tau_*)^{-2}}. \tag{21}$$

It is interesting to note that the magnetic-field dependence of the amplitude of geometrical oscillations described by Eqs. (21) strongly differs for the two limiting types of scattering considered above. In the case of isotropic (short-range) scatterers the oscillation amplitude decreases as N^{-1} with the oscillation number $N\sim qR_c/\pi$. In contrast, for low-angle scattering it is nonmonotonic. It increases linearly in N up to $N^\sigma\sim\sqrt{2\omega\tau}/\pi$ where the oscillations amplitude has a maximum followed by a gradual N^{-3} decrease.

The result in Eq. (18) also shows that in a low-density 2DEG such that $v_F\lesssim s$, resonances in the AE effect at $\omega=k\omega_c$ become possible. Due to the oscillatory behavior of Bessel functions $J_k(qR_c)$ at $qR_c\gg 1$, and since $\omega/\omega_c=(s/v_F)qR_c$, resonances would appear in the experiment as a sequence of Lorentzians of apparently random height. Similar behavior may be expected in a gas of "heavy" composite fermions,[5] though a rigorous analysis should require a self-consistent Chern-Simons field.

To study the damping of geometrical oscillations at low magnetic fields $\omega_c\tau_*<1$, we use the method of residues, transforming the summation in Eq. (18) into the integral

$$E_{\rm AE}\approx\frac{\mathcal{E}\omega p_F/2\pi i}{e\omega_c\tau_*}\oint_C\frac{[1+\sin(2qR_c-z\pi)]\cot(\pi z)dz}{\left[z-\frac{\omega}{\omega_c}+\frac{i}{\omega_c\tau_*}\right]\left[z-\frac{\omega}{\omega_c}-\frac{i}{\omega_c\tau_*}\right]},$$

where for $qR_c \gg 1$, $J_k(qR_c) \approx \sqrt{2/\pi qR_c}\cos(qR_c - k\pi/2 - \pi/4)$, and the contour C consists of two parts: in the upper half-plane, $C_+ = x + i0$ and in the lower half plane, $C_- = x - i0$. Each contour, $C_\pm$ is then moved away from the real axis, $C_\pm \to C'_\pm = x \pm i|y|$, such that $e^{-2|y|} \ll 1$, when each contour picks up exactly one residue from the poles at $z_\pm = \omega/\omega_c \pm i/\omega_c\tau_*$ (here and below the subscript $\pm$ is determined by the subscript of the contour). The numerator of the integrands in the shifted line integrals $\int_{C_\pm} dz$ are approximated using $e^{-2|y|} \ll 1$, thus yielding $\cot(\pi z) \approx \mp i$ and $\sin(qR_c - z) \approx \pm e^{\pm(2iqR_c - iz)}/2i$. After this, each contour is then moved, $C'_\pm \to C''_\pm$, such that $\mathrm{Im}\, C''_\pm \to \mp\infty$, passing the real axis as they approach the opposite extremes of the complex plane. Thus, we arrive at

$$E_{\mathrm{AE}} = \frac{\mathcal{E}\omega p_F}{e}\left\{1 + \sin\left(2qR_c - \frac{\pi\omega}{\omega_c}\right)e^{-\pi/\omega_c\tau_*}\right\}.$$

The latter equation is typical for damped geometrical oscillations.[18] It shows how commensurability oscillations die away when $\pi/\omega_c\tau_* \gtrsim 1$ and that the onset of oscillations occurs at the magnetic-field value such that

$$R_c < R_c^* \sim \begin{cases} l/\pi & \text{for isotropic scattering,} \\ l\sqrt[3]{2/\pi(lq)^2} & \text{for small-angle scattering.} \end{cases}$$

Together with the result in Eq. (21), the latter offset condition shows that the number of B^{-1} oscillations, $N \sim qR_c^*/\pi$ detectable in a sample with the mean-free path $l = v_F\tau \gg 2\pi/q$, is larger when its mobility is limited by short-range scatterers ($N < N^\eta$) than when scattering is due to smooth disorder ($N < N^\sigma$), where

$$N^\eta \sim \frac{ql}{\pi}, \quad \text{versus } N^\sigma \sim \min\left\{(ql)^{1/3}, \sqrt{\frac{sql}{\pi v_F}}\right\}. \quad (22)$$

In the regime of damped oscillations a finite and apparently field-independent AE effect persists up to the field $v_F B > E_{\omega q}$ (when channeling takes it toll),

$$E_{\mathrm{AE}} \approx e\omega p_F(a_{\mathrm{scr}}E_{\omega q}^{\mathrm{SAW}}/\epsilon_F)^2. \quad (23)$$

The above presented analysis explains why the observed magnetic-field dependence of the AE effect by Shilton *et al.*[14] contained only one pronounced oscillatory feature associated with geometrical commensurability before the AE effect saturated at a finite value in low magnetic fields. This contrasted a comparison[14] made with the SAW absorption modeled in the τ approximation, which suggested that many ($N \sim 3 \div 10$) oscillations should become visible while varying the SAW wavelength from $\lambda \sim 10$ μm to $\lambda \sim 3$ μm against a mean-free path of $l = 30$ μm. In structures with low-angle scattering N^σ in Eq. (22) increases very slowly with the increase of SAW wave number and remains $N \sim 1 \div 2$ for all three SAW sources used in Ref. 14.

In the other AE experiment with surface acoustic waves[13] known to us, the mobility of samples was not sufficient to observe geometrical oscillations for the available range of the SAW wavelength, though the saturated nonoscillatory low-field AE was seen, as well as Shoubnikov–de Haas oscillations of the AE effect which crosses over into a rich structure in the quantum Hall effect (QHE) regime. To incorporate quantum effects into the drag current analysis, one would have to take into account the formation of Landau levels and to replace the classical Boltzmann equation by a quantum kinetic equation.[15,20] We left such an analysis, which is beyond the scope of this Communication, and refer an interested reader to earlier works on the AE effect in the QHE regime (Refs. 12 and 21). Formation of Landau levels makes the electron response to SAW field strongly dependent on the electron Fermi energy, and, therefore, local density of carriers, n_e. The latter feature has been used in Ref. 12 to relate the drag current in the QHE regime to the density derivative of 2DEG conductivity, $d\sigma(n_e)/dn_e$, thus explaining the experimental observations by Esslingen *et al.*[13]

In conclusion, we present a microscopic theory of geometrical oscillations of the acousto-electric effect caused by surface acoustic waves in a high-mobility 2D electron gas giving its comparative analysis in structures with either isotropic or small-angle impurity scattering.

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[1] R. L. Willett, Adv. Phys. **46**(5), 447 (1997).
[2] D. Weiss *et al.*, Europhys. Lett. **8**, 179 (1989).
[3] R. R. Gerhardts *et al.*, Phys. Rev. Lett. **62**, 1173 (1989).
[4] C. W. J. Beenakker, Phys. Rev. Lett. **62**, 2020 (1989).
[5] R. L. Willett *et al.*, Phys. Rev. Lett. **71**, 3846 (1993).
[6] W. Kohn, Phys. Rev. **123**, 1242 (1961).
[7] S. H. Simon and B. I. Halperin, Phys. Rev. B **48**, 17368 (1993).
[8] I. V. Kukushkin *et al.*, Nature (London) **415**, 409 (2002).
[9] A. L. Efros and Y. M. Galperin, Phys. Rev. Lett. **64**, 1959 (1990).
[10] J. Bergli and Y. Galperin, Europhys. Lett. **54**, 661 (2001).
[11] J. Bergli and Y. M. Galperin, Phys. Rev. B **64**, 035301 (2001).
[12] V. I. Fal'ko *et al.*, Phys. Rev. B **47**, 9910 (1993).
[13] A. Esslinger *et al.*, Solid State Commun. **84**, 939 (1992); A. Esslinger *et al.*, Surf. Sci. **305**, 83 (1994).
[14] J. M. Shilton *et al.*, Phys. Rev. B **51**, 14770 (1995).
[15] J. P. Robinson *et al.*, Phys. Rev. Lett. **93**, 036804 (2004); M. P. Kennett, *et al.*, Phys. Rev. B **71**, 195420 (2005).
[16] M. H. Cohen *et al.*, Phys. Rev. **117**, 937 (1960).
[17] A. M. Rudin *et al.*, Phys. Rev. B **58**, 15698 (1998).
[18] A. D. Mirlin *et al.*, Phys. Rev. B **64**, 125319 (2001); **63**, 245310 (2001); A. D. Mirlin and P. Wölfle, *ibid.* **58**, 12986 (1998).
[19] The average is calculated by turning the operator $\hat{C}_\sigma \propto \partial_\varphi^2$ onto the term $e^{iqR_c \sin\varphi - ik\varphi}$, which produces a constant and sinusoidal coefficients of single and double frequency. Turning these coefficients into complex exponentials and using the relation $J_{k-n}(qR_c) = \int_0^{2\pi} d\varphi e^{iqR_c \sin\varphi - i(k-n)\varphi}/2\pi$ gives the matrix elements $h_{\omega q}^k$, $h_{\omega q}^{k\pm1}$ and $h_{\omega q}^{k\pm2}$, respectively.
[20] I. A. Dmitriev *et al.*, Phys. Rev. Lett. **91**, 226802 (2003); I. A. Dmitriev *et al.*, Physica E (Amsterdam) **25**, 205 (2004).
[21] F. von Oppen *et al.*, Phys. Rev. Lett. **87**, 106803 (2001).