Spin injection across magnetic/nonmagnetic interfaces with finite magnetic layers

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We have reconsidered the problem of spin injection across ferromagnet/nonmagnetic-semiconductor (FM/NMS) and dilute-magnetic-semiconductor/nonmagnetic-semiconductor (DMS/NMS) interfaces, for structures with finite width d of the magnetic layer (FM or DMS). By using appropriate physical boundary conditions, we find expressions for the resistances of these structures which are in general different from previous results in the literature. When the magnetoresistance of the contacts is negligible, we find that the spin-accumulation effect alone cannot account for the d dependence observed in recent magnetoresistance data. In a limited parameter range, our formulas predict a strong d dependence arising from the magnetic contacts in systems where their magnetoresistances are sizable.

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I. INTRODUCTION

Spin injection across interfaces is one of the crucial ingredients for the successful implementation of spintronic devices.[1–3] For instance, the spin-transistor proposal[4] relies on subjecting injected spin-polarized electrons to a controllable spin precession between the ferromagnetic source and drain. Hybrid ferromagnetic/semiconductor (FM/NMS) and dilute-magnetic-semicondutor/nonmagnetic semiconductor junctions constitute basic interfaces in which to investigate spin-polarized transport. However, the efficiency of spin injection through ideal FM/NMS interfaces turns out to be disappointingly small due to the large conductivity mismatch[5] between the metallic ferromagnet and the semiconductor. The use of spin-selective interfaces can significantly enhance injection efficiencies.[6] This can be accomplished by inserting spin-dependent tunnel barriers between the FM and NMS layers.[7]

Particularly promising is spin injection from a dilute magnetic semiconductor (DMS) into a nonmagnetic semiconductor. DMS/NMS junctions (i) minimize the conductivity mismatch problem and (ii) naturally incorporate spin dependency in the transmission process.[8] As recently demonstrated, substantial spin injection can be achieved in these Mn-based heterostructures.[9] More recently, a novel large magnetoresistance effect has been observed in DMS/NMS/DMS geometries.[10] An interesting observation of Ref. 10 is the dependence of the magnetoresistance effect on the thickness of the DMS layers: the MR effect doubles with increasing DMS thickness.

Available formulas describing GMR-type effects in magnetic/nonmagnetic junctions assume one-dimensional (1D) geometries with semi-infinite magnetic layers.[11] In this case, the expressions for the electrochemical potentials for the spin-up and spin-down electrons contain only decaying exponentials in the magnetic regions.[12] The resistances in the parallel and antiparallel configurations are calculated assuming that the device extends for a spin-flip length into the magnetic contacts. In such an approach it is not clear how to properly take into account the voltage drop across the sample.

In this work we consider DMS/NMS/DMS and FM/NMS/FM 1D systems with *finite* magnetic layers, Fig. 1. As detailed below, we describe diffusive transport in these structures via the usual diffusion theory of van Son *et al.*[13] with proper physical boundary conditions between the several magnetic/nonmagnetic interfaces in the system. The formulas for the magnetoresistance obtained within our treatment are in general different from the ones previously obtained in literature, even in the experimentally relevant regime $d \gg \lambda_D$ and $x_0 \leq \lambda_N$; λ_D and λ_N denote the spin-flip lengths in the DMS and NMS layers, respectively, d is the length of each DMS layer and $2x_0$ is the length of the NMS layer. However, in the case of a FM/NMS/FM system where the conductivity mismatch is large, this difference is not important since the correction we find is very small in this case. For

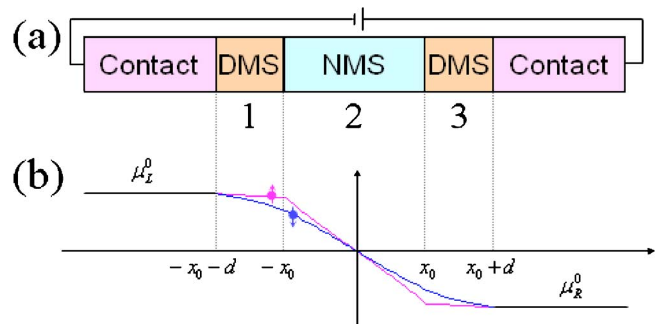

FIG. 1. (Color online) Schematic of the magnetic/nonmagnetic structure investigated here (a) and the corresponding spin-resolved electrochemical potentials (b) versus the position across the layers. Note that we consider magnetic layers (DMS) of finite widths d symmetrically placed around the nonmagnetic layer (NMS). The spin splitting of the electrochemical potentials across the magnetic/nonmagnetic interfaces is larger for a higher "conductivity mismatch" between these layers.

DMS/NMS/DMS systems, on the other hand, the conductivity mismatch is small and the magnetoresistance effect can be large. Our formulas significantly deviate from the earlier ones in this case, especially in the regime where λ_D, which can be magnetic-field dependent, is comparable to d.

II. SYSTEM AND APPROACH

Let us denote by 1, 2, and 3 the regions corresponding to the DMS (or FM), NMS, DMS (or FM) layers, respectively. Note that in Fig. 1 $x=0$ corresponds to the center of the structure (NMS layer), $x=\pm x_0$ correspond to the DMS/NMS interfaces, and $x=\pm(x_0+d)$ to the metal contact/DMS interfaces. Consider first the DMS/NMS/DMS case or, equivalently, a parallel configuration FM/NMS/FM. Our task is to find the resistance of the structure $R=V/j$, where $j=j_\uparrow+j_\downarrow$ is the total current through the structure and V is an applied bias ($+V/2$ is the potential of the left metal contact and $-V/2$ is the potential of the right contact).

A. Diffusive transport equations

Our starting point is the diffusive transport approach of Ref. 13, with the basic equations relating the spin-dependent electrochemical potentials and current densities

$$j_{\uparrow,\downarrow} = (\sigma_{\uparrow,\downarrow}/e) d\mu_{\uparrow,\downarrow}/dx, \tag{1}$$

which hold in all three regions 1, 2, and 3 (see Fig. 1) of our system; $\sigma_{\uparrow,\downarrow}$ denotes the conductivity of the spin-up (or -down) electrons in the corresponding layer. The difference in the spin-dependent electrochemical potential is[13]

$$\mu_\uparrow - \mu_\downarrow = \lambda^2 \frac{d^2(\mu_\uparrow - \mu_\downarrow)}{dx^2}, \tag{2}$$

where λ is the spin-flip length (λ_D or λ_N) in the corresponding layer. As usual, to solve the problem we first assume a general solution in each of the three regions and then match them at the many interfaces via proper boundary conditions. Below we outline this procedure, clearly stating the physical boundary conditions we use and the simplifying symmetries of the problem.

B. General solution for $\mu_{\uparrow,\downarrow}$

Consistent with Eq. (2), we use the general expression for the electrochemical potentials in region 1 (DMS)

$$\frac{1}{-e}\begin{pmatrix}\mu_\uparrow\\ \mu_\downarrow\end{pmatrix} = (A_1 x + B_1)\begin{pmatrix}1\\1\end{pmatrix} + [C_1\lambda_D \exp(x/\lambda_D) + D_1\lambda_D\exp(-x/\lambda_D)]\begin{pmatrix}1/\sigma_\uparrow^D\\ -1/\sigma_\downarrow^D\end{pmatrix}. \tag{3}$$

Here $\lambda=\lambda_D$ is the spin-flip length in the DMS layer. Note the presence of the exponentially growing term in Eq. (3), since we consider a *finite* DMS layer. The corresponding expressions for the electrochemical potentials in the regions 2 and 3 are obtained from Eq. (3) by the replacements $A_1\to A_{2,3}$, $B_1\to B_{2,3}$, $C_1\to C_{2,3}$, and $D_1\to D_{2,3}$; in addition we should make $\sigma^D_{\uparrow,\downarrow}\to\sigma=\sigma_N/2$ and $\lambda_D\to\lambda_N$ in the NMS region 2. The chosen form of the solution guarantees conservation of the total current everywhere in the sample. From the symmetry of the problem in the parallel configuration considered here, it follows that the electrochemical potentials $\mu_\uparrow$ and $\mu_\downarrow$ should be odd functions of the coordinate x [see Fig. 1(b)]. In terms of the coefficients this means that $A_3=A_1$, $B_3=-B_1$, $C_3=-D_1$, $D_3=-C_1$, $B_2=0$, and $C_2=-D_2$. Hence we can consider only the boundary conditions at the left metal contact/DMS interface and left DMS/NMS interface.

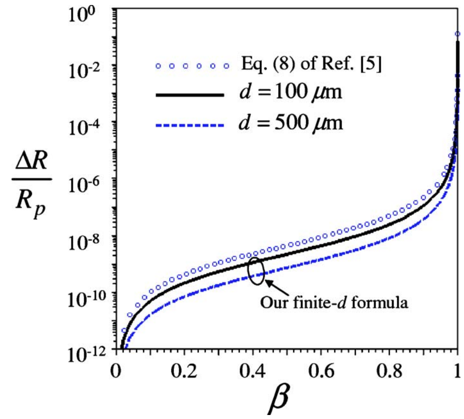

FIG. 2. (Color online) $\Delta R/R_p$ as a function of β for FM/NMS/FM structures with finite FM layers d. The solid and dashed curves correspond to the formula derived in the present paper [Eqs. (10) and (11)], while the curve with circles is obtained from Eq. (8) of Ref. 5. Here we use $x_0=1\ \mu$m, $\lambda_D=10$ nm, $\lambda_N\to\infty$, $d=100\ \mu$m, and $\sigma^D=100\sigma^N$. The three curves do not differ significantly for the parameters used.

C. Boundary conditions

At the metal contact/DMS interface we use the continuity of the *total* current and the equality of the electrochemical potentials: $\mu_\uparrow=\mu_\downarrow=-eV/2$. The condition $\mu_\uparrow=\mu_\downarrow$ is consistent with assuming a metal contact with infinite conductivity. Thus we obtain the following set of equations:

$$C_1 + D_1 \exp[2(x_0+d)/\lambda_D] = 0, \tag{4}$$

$$-A_1(x_0+d) + B_1 = V/2, \tag{5}$$

and

$$j = -A_1(\sigma_\uparrow^D + \sigma_\downarrow^D). \tag{6}$$

At the DMS/NMS interface we use the continuity of the electrochemical potentials, $\mu_\uparrow(-x_0^-)=\mu_\uparrow(-x_0^+)$, $\mu_\downarrow(-x_0^-)=\mu_\downarrow(-x_0^+)$, the conservation of the total current $j=j_\uparrow+j_\downarrow$, and conservation of the $j_\uparrow$ component across the interface. Then we have the following additional set of equations:

$$-A_1x_0 + B_1 + \begin{pmatrix}\lambda_D/\sigma_\uparrow^D\\ -\lambda_D/\sigma_\downarrow^D\end{pmatrix}[C_1\exp(-x_0/\lambda_D) + D_1\exp(x_0/\lambda_D)] = -A_2x_0 \mp 2C_2\lambda_N \sinh(x_0/\lambda_N)/\sigma, \tag{7}$$

$$-A_1\sigma_\uparrow^D-[C_1\exp(-x_0/\lambda_D)-D_1\exp(x_0/\lambda_D)]$$
$$=-A_2\sigma-2C_2\cosh(x_0/\lambda_N), \quad (8)$$

and

$$j=-2\sigma A_2. \quad (9)$$

Hence we have seven equations, Eqs. (4)–(9), for seven unknown quantities A_1, A_2, B_1, C_1, C_2, D_1, and j. By solving these equations we can determine the resistance of the structure. This we do next.

III. RESULTS

For a FM/NMS/FM and a DMS/NMS/DMS system ("parallel" configuration), we find for $R_p=V/j$

$$R_p=\frac{2x_0}{\sigma^N}+\frac{2d}{\sigma^D}+\frac{2\beta^2}{(1-\beta^2)(\sigma^D/\lambda_D)\coth(d/\lambda_D)+(\sigma^N/\lambda_N)\coth(x_0/\lambda_N)}, \quad (10)$$

where $\beta=(\sigma_\uparrow^D-\sigma_\downarrow^D)/(\sigma_\uparrow^D+\sigma_\downarrow^D)$, $\sigma^N=2\sigma$ is the total conductivity in the NMS layer, and $\sigma^D=\sigma_\uparrow^D+\sigma_\downarrow^D$ is the conductivity in the DMS layer; d is the length of each DMS layer and $2x_0$ is the length of the NMS layer. The term proportional to the magnetic-layer width d in Eq. (10) is not present in earlier formulas in the literature. Note that the term proportional to β^2 in Eq. (10) differs from that in Eq. (1) of Ref. 10 because it is d dependent and also because of its distinctive x_0 dependence.[14]

For the antiparallel configuration (opposite FM layers), we can similarly obtain an expression for the resistance R_{ap}. Interestingly, R_{ap} can be obtained from R_p in Eq. (10) by changing $\coth(x_0/\lambda_N)$ to $\tanh(x_0/\lambda_N)$

$$R_{ap}=\frac{2x_0}{\sigma^N}+\frac{2d}{\sigma^D}+\frac{2\beta^2}{(1-\beta^2)(\sigma^D/\lambda_D)\coth(d/\lambda_D)+(\sigma^N/\lambda_N)\tanh(x_0/\lambda_N)}. \quad (11)$$

In obtaining Eq. (11) we have also assumed that the spin-up and the spin-down conductivities of region 3 are $\sigma_\downarrow^D$ and $\sigma_\uparrow^D$, respectively. We emphasize that the term proportional to β^2 [Eqs. (10) and (11)] does not reduce to that in Ref. 5 for $d\to\infty$ since the boundary conditions used here are different.

A. Magnetoresistances

For metallic FM/NMS/FM systems we define the resistance change as usual, i.e., $\Delta R=R_{ap}-R_p$. The full expression is too lengthy to be shown here. However, in the physically relevant limit $\lambda_D\ll d$ and $x_0\ll\lambda_N$ we find from Eqs. (10) and (11)

$$\frac{\Delta R}{R_p}=\frac{\beta^2(\sigma^N\lambda_D/x_0\sigma^D)^2}{(1-\beta^2)[(1-\beta^2)+\sigma^N\lambda_D/x_0\sigma^D+(\sigma^N\lambda_D/\lambda_N\sigma^D)^2 1/(1-\beta^2)]}. \quad (12)$$

In deriving the above equation we have approximated R_p in the denominator by its dominant contribution $2x_0/\sigma^N$ from the central NMS layer. Again, our Eq. (12) is different from the corresponding one [Eq. (8)] in Ref. 5. However, as can be seen from comparison of Eq. (12) and Eq. (8) of Ref. 5, this deviation is not really important for *metallic* ferromagnets. Indeed, $\Delta R/R_p$ which follows from our equations practically coincides with the quantity in Eq. (8) of Ref. 5 for realistic values of the parameters when $\sigma^N\lambda_D/\sigma^D\lambda_N\ll 1$, $\sigma^N\lambda_D/\sigma^D x_0\ll 1$, and for β's not anomalously close to unity. Figure 2 shows our calculated magnetoresistance $\Delta R/R_p$, with R_p and R_{ap} defined in Eqs. (10) and (11) for a FM/NMS/FM structure with finite FM layers. Note that for the parameters used in Fig. 2, our results do not significantly differ from those of Ref. 5.

In DMS/NMS/DMS systems, on the other hand, only the parallel configuration is possible. In this case, we define the relevant resistance change in the system with respect to the zero magnetic-field resistance R_0, i.e., $\Delta R^D=R_p-R_0$. We find

$$\frac{\Delta R^D}{R_0}=\frac{\beta^2}{(1-\beta^2)(\sigma^D/\lambda_D)\coth(d/\lambda_D)+(\sigma^N/\lambda_N)\coth(x_0/\lambda_N)}\times\frac{1}{(x_0/\sigma^N+d/\sigma^D)}+\frac{d[1/\sigma^D(H)-1/\sigma^D(0)]}{(x_0/\sigma^N+d/\sigma^D)}. \quad (13)$$

Note that R_0 is simply the combined NMS/DMS layer resistance $2x_0/\sigma^N+2d/\sigma^D$, since no spin accumulation occurs for zero magnetic field. In Eq. (13) we have indicated explicitly the additional magnetoresistance arising from the DMS (or FM) regions through $\sigma^D(H)$.

Note that $\Delta R^D/R_0$ goes to zero for either $d\to\infty$ or $x_0\to\infty$. The exact expression for $\Delta R/R_p$ obtained from Eqs. (10) and (11) [not the approximate one in Eq. (12)] also vanishes in these limits. Note that this vanishing of the resistance change is symmetrical in d and x_0 as expected. Earlier formulas in the literature do not show this symmetry since they assume different boundary conditions.

In contrast to metallic FM/NMS/FM systems, DMS/

NMS/DMS structures can have conductivities of the DMS and NMS regions which are comparable. In addition, λ_D can be of the same order as d in some experiments. Experimental results in this case, e.g., the d dependence of the magnetoresistance effect, should be interpreted with the use of Eq. (13). As we mentioned earlier, the magnetoresistance effect observed in Ref. 10 doubles with the DMS thickness. Let us find out whether our 1D formula can explain this observed thickness dependence. Note that the d dependence in Eq. (13) appears through the $\coth(d/\lambda^D)$ term (spin-accumulation effect). Since $\sigma^D=\sigma_\uparrow^D+\sigma_\downarrow^D$ can in general be magnetic-field dependent, there is also a possible d dependence arising from the second term which describes the resistances of the DMS leads.

In order to investigate in more detail the d dependence in $\Delta R^D/R_0$, we need to know the magnetic-field dependences of the quantities entering $\Delta R^D/R_0$ [Eq. (13)], particularly the magnetic-field dependence of the spin-dependent conductivities $\sigma_{\uparrow,\downarrow}^D$. Unfortunately, these dependences are not known. However, we can still gain some insight into the problem by neglecting the magnetic-field dependence of the conductivities. In this case, the second term in Eq. (13) is zero and we can investigate the expected d dependence arising from the spin-accumulation effect alone. Note that the quantity β is null in the absence of a magnetic field and increases as a function of it. Hence, we plot $\Delta R^D/R_0$ as a function of degree of spin polarization β for different values of d. Figures 3(a) and 3(b) show plots of $\Delta R^D/R_0$ vs β for different parameters. We have chosen the values of d, x_0, and $\sigma^D/\sigma^N=1/3$ corresponding to the real experimental situation of Ref. 10. From these plots we can see that even though the absolute value of the magnetoresistance is comparable to the experimental value, the d dependence is not as pronounced as in the experiment of Ref. 10. It is indeed true that the magnetoresistance effect is larger for increasing d. However, this behavior only appears at values of λ_D comparable to the thickness d and even for $\lambda_D=0.5$ $\mu m>d$, Fig. 3(b), the effect is much too small to explain the experimental data. We conclude that the spin-accumulation effect by itself does not explain—with realistic parameters—the observed d dependence of the data in Ref. 10.

Let us mention here the possibility of explaining the experiment by the magnetoresistance of the leads [see the second term in Eq. (13)]. Control experiments on a layer of the DMS injector material patterned into a Hall geometry showed ~1% negative MR.[10] However, the transport problem we deal with here can be shown to reduce to that of a 1D system.[11] Note that in the case when the magnetic field dependence of σ^D is well pronounced and for realistic values $x_0 \gg d \gg \lambda_D$ and for $\beta \simeq 1$ the first term in Eq. (13) is estimated as λ_D/σ^D while the second term is of the order of d/σ^D. Thus in the case when the magnetoresistance of the leads is not zero the d dependence associated with it can be more pronounced than the one associated with the spin-accumulation effect.

B. Some limiting cases

From the general equations we derived above, we can obtain expressions for some interesting limiting cases. If $\sigma_\downarrow^D \to 0$, i.e., $\beta \to 1$, we find from Eq. (10)

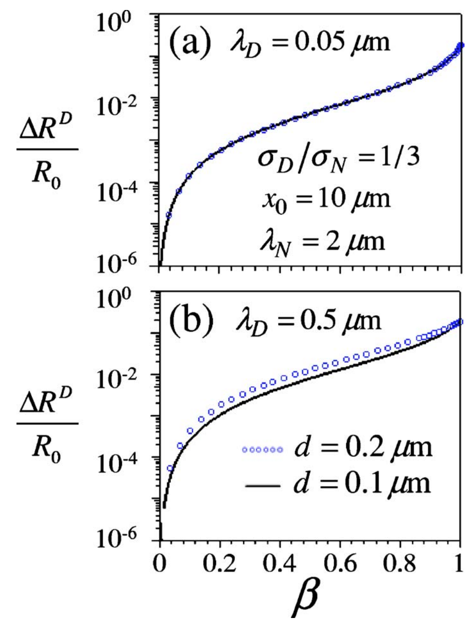

FIG. 3. (Color online) $\Delta R^D/R_0$ vs β for two distinct DMS thicknesses and spin-flip lengths. Only the spin-accumulation part of the magnetoresistance, i.e., the first term in Eq. (13), is considered here. For $d \gg \lambda_D$ the magnetoresistance ΔR^D does not show any dependence on d (a) [in (a), the two curves corresponding to $d=0.2$ and 0.1 μm lie on top of each other]. On the other hand, for λ_D comparable to d the magnetoresistance (b) increases slightly with d; this effect is too small to explain the d dependence of the magnetoresistance in Ref. 10. Note that for the parameters used the part $2d/\sigma^D$ of the resistance R_0 is much smaller than $2x_0/\sigma^N$.

$$R_p = \frac{2x_0}{\sigma^N} + \frac{2d}{\sigma_\uparrow^D} + 2\frac{\lambda_N}{\sigma^N}\tanh(x_0/\lambda_N). \tag{14}$$

If, in addition, $\lambda_N \to \infty$, then

$$R_p = \frac{2x_0}{\sigma^N} + \frac{2d}{\sigma_\uparrow^D} + \frac{2x_0}{\sigma^N}. \tag{15}$$

Hence the resistance of the central part (NMS) doubles as expected. Note that the resistance of the DMS part does not necessarily double.

C. Current spin polarization

We can also easily determine some other relevant quantities in our system. For example, the degree of spin polarization $\alpha(x)$ of the current at the DMS/NMS (or FM/NMS) interface ($x=x_0$) is equal to

$$\alpha(x_0) = \frac{j_\uparrow - j_\downarrow}{j_\uparrow + j_\downarrow} = \frac{\beta}{1 + (\sigma^D/\sigma^N)(1-\beta^2)[\lambda_N \tanh(x_0/\lambda_N)]/[\lambda_D \tanh(d/\lambda_D)]}, \tag{16}$$

for parallel alignment. This result coincides exactly with Eq. (7) of Ref. 5 in the limit $x_0 \ll \lambda_N$, $\lambda_D \ll d$. For metallic ferromagnetic contacts we can have antiparallel alignment as well. Note that $\alpha(x)$ is an even (odd) function of x for parallel (antiparallel) alignment.

For completeness, we give below some other expressions for $\alpha(x)$. At $x=\pm(x_0+d)$ (even function) we have

$$\alpha(x=-x_0-d)=\beta-\frac{\beta}{\cosh(d/\lambda_D)\{1+(\sigma^N/\sigma^D)[1/(1-\beta^2)][\lambda_D\tanh(d/\lambda_D)]/[\lambda_N\tanh(x_0/\lambda_N)]\}}. \quad (17)$$

Note that for $d \gg \lambda_D \alpha(x=-x_0-d)$ approaches the maximal possible value β. In addition

$$\alpha(x=0)=\alpha(x=-x_0)\frac{1}{\cosh(x_0/\lambda_N)}. \quad (18)$$

Thus in the middle of the sample the spin current polarization is exponentially suppressed. In the case of antiparallel configuration the corresponding values of α at $x=-x_0$ and $x=-x_0-d$ can be obtained from formulas (16) and (17), respectively, by substituting $\tanh(x_0/\lambda_N)$ with $\coth(x_0/\lambda_N)$.

IV. CONCLUSION

In conclusion, we have derived the expressions for the resistances of FM/NMS/FM (or DMS/NMS/DMS) structures using physical boundary conditions at the corresponding interfaces. We have considered magnetic layers (DMS or FM) of finite length d. When the magnetoresistance of the contacts is negligible (i.e., when σ_D is not magnetic-field dependent), we find that the d dependence of the spin-accumulation contribution to the magnetoresistance is not enough to explain the experimental results of Ref. 10. On the other hand, in systems where σ_D is magnetic-field dependent our formulas allow for a sizable d dependence arising from the magnetoresistance of contacts. Interestingly, for realistic parameters $x_0 \gg d \gg \lambda_D$ and with $\beta \simeq 1$ the magnetoresistance of the contacts is larger than that from the spin-accumulation effect.

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[11] It should be mentioned that despite the 2D geometry of the real experiment (Ref. 10), the transport problem reduces to that of a 1D system. This result follows from the specific current density distribution established by minimizing the Joule heat [A. Khaetskii (unpublished)]. In a real experimental situation the DMS leads lie on the top of the NMS layer (at $|x|>x_0$); thus the current lines are bent and the current enters the metallic contacts in the direction perpendicular to the original x direction. An *exact* analytical treatment of this geometry shows that the current which enters or leaves the NMS layer flows only near the corners of the NMS/DMS interfaces. It turns out that the relevant width Δx of the DMS layer where transport occurs is of the order of the NMS layer height. The current density exponentially decays for $|x| \geq (x_0+\Delta x)$. Therefore we have essentially a "bent" 1D geometry.

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[14] Note that Eq. (1) of Ref. 10 contains a misprint. In the numerator of this equation, instead of the first power of β it should be β^2. The correct version of this formula can be found in G. Schmidt and L. W. Molenkamp, in *Semiconductor Spintronics and Quantum Computation* (Ref. 2). Note, however, that our Eq. (10) contains totally different d and x_0 dependences as compared to Eq. (1) of Ref. 10.