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Spin relaxation in the presence of crossed electric and magnetic fields: A quasiclassical approach

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A simple method for the investigation of spin relaxation phenomena in systems with Rashba and Dresselhaus spin-orbit interactions is developed. The method is applied to the investigation of the impact of external fields on the relaxation process. The calculation shows that the spin relaxation is strongly affected by a lateral electric field. The field enhances the lifetime of the magnetization and leads to an additional rotation of the magnetization. This field-induced rotation can be affected further by means of magnetic fields. We study the dependence of the field-induced rotation on the structure of the spin-orbit scattering and on the strength of the applied magnetic field.

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I. INTRODUCTION

Investigations of spin transport and spin relaxation phenomena are at present of much interest. This interest is stimulated by the notion that the electron spin can also be utilized in electrical devices. A couple of suggestions have been published in the literature, which focus on possible applications of the spin degree of freedom (see, e.g., Refs. 1–5). All of them are based on the observation that the electron spin can also be manipulated electrically. The coupling between the electric field and the electron spin is provided by the spin-orbit interaction, which is realized by the Rashba interaction in the simplest situation.[6]

However, a disadvantage of the spin degree of freedom compared to the charge is the fact that the magnetization is not a conserved quantity. Therefore, every initial magnetization decays with time. The decay is caused by the coupling of the spin and momentum, which is provided by the spin-orbit interaction (see, e.g., Ref. 7). Due to this coupling, the spin of every particle precesses around a different axis. Consequently, an ensemble of spins dephases in a short amount of time. In systems with strong spin-orbit interactions, in which the spins precess many times before the axis of the precession changes due to collisions, the dephasing proceeds on a time scale governed by the width of the initial spin packet.[7] The spin transport proceeds ballistically in this case. The magnetization is lost after the momentum relaxation time. Really long relaxation times can only be expected in systems with weak spin-orbit interaction[8] or near-degenerate points.[5] In such systems the axis of the precession is changed before the spin can appreciably rotate.[8] Therefore, the spin transport proceeds diffusely in this case.

The relaxation of the magnetization can be investigated experimentally by means of time-resolved Faraday or Kerr effect measurements (see, e.g., Refs. 9–11) or by means of time-resolved photoluminescence spectroscopy (see, e.g., Refs. 12 and 13). In such experiments the limit of weak Rashba interactions can easily be distinguished from the limit of strong Rashba interactions. Whereas in the first situation the magnetization decays simply exponentially at large times, in the second situation also oscillations of the magnetization can be observed. Both regimes have been investigated experimentally in Ref. 13. Additional information on the spin dynamics can be obtained by applying external fields. In particular the situation in a strong magnetic field has received much attention. Spin relaxation problems in such fields have been investigated in a number of papers both in the semiclassical limit (see, e.g., Refs. 14–21) and for quantizing fields (see, e.g., Refs. 22–24). These investigations show that a strong magnetic field has two effects. First, it deceases the relaxation rate and second it opens the route to new a mechanism for the spin relaxation, which takes explicitly advantage of a momentum-dependent g factor.[15–17] The reduction of the relaxation rate becomes in particular striking in the quantum-Hall limit, in which the simple exponential decay of the magnetization is replaced by an algebraic decay.[23,24]

An electric field leads to an additional rotation of the magnetization,[25–30] which also affects the decay of the magnetization directly. The electric-field-induced rotation of the magnetization can be observed if the electric field exceeds a critical field.[26,27] The critical field depends on material parameters like the Rashba interaction constant, the effective mass, and the ratio between the diffusion coefficient and the mobility. Studies of the impact of crossed electric and magnetic fields have not been published so far.

In investigations of the impact of an electric field on the spin relaxation mainly numerical methods and quantum kinetics have been used. Doing so, the coupling between spin and charge has been ignored. Despite this fact the application of these methods to spin relaxation problems has turned out to be difficult since the investigation of spin relaxation processes requires finding solutions to coupled systems of integral equations. On the other hand, since the momentum relaxation is the source of spin relaxation, it should be possible to find a representation of the spin transport coefficients and the Bloch equations in terms of momentum correlation functions, in particular in situations in which quantum interferences do not matter. From this point of view the application of the above-mentioned methods to the spin relaxation problem seems to be unnecessary complicated.

It is the purpose of the present paper to derive a simple representation of the Bloch equations for systems with Rashba and Dresselhaus spin-orbit scattering in terms of momentum correlation functions. Doing so, we focus on the decay of a nonequilibrium magnetization on the Fermi sur-

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face. This situation can also be realized and be investigated experimentally (see, e.g., Ref. 13). Apart from being simple our approach has the merit of elucidating the underlying physical picture. We apply our approach to the investigation of spin relaxation in crossed electric and magnetic fields. Doing so, we produce new results on the impact of the electric field on the spin relaxation rate, on the field-induced rotation and the critical field in systems with combined Rashba and Dresselhaus interactions, on the impact of the magnetic field on the electric-field-induced spin precession, and on the g factor in such systems.

II. SPIN RELAXATION IN THE PRESENCE OF AN ELECTRIC FIELD

A. Rashba semiconductor

Our derivation of the generalized Bloch equations in the presence of an electric field uses the observation that the spin relaxation in a Rashba semiconductor with weak spin-orbit scattering is the result of the momentum relaxation. Therefore, it should be possible to find a representation of the transport coefficients governing the spin relaxation in terms of momentum correlation functions. To find such a representation we focus on a noninteracting two-dimensional electron gas subjected to Rashba interactions. The Hamilton operator is given by

$$H = \frac{\hat{p}^2}{2m} - \boldsymbol{\sigma}\cdot(\boldsymbol{N}\times\hat{\boldsymbol{p}}) + \boldsymbol{F}\cdot\hat{\boldsymbol{x}} + V(\hat{\boldsymbol{x}},t), \tag{1}$$

where $\boldsymbol{F}=F\boldsymbol{e}_x$ is the lateral field, $\boldsymbol{N}=N\boldsymbol{e}_z$ is a vector transverse to the two-dimensional plan, and $V(\hat{\boldsymbol{x}},t)$ is a time-dependent potential, which is responsible for momentum and energy relaxation. $\boldsymbol{\sigma}=\sigma_x\boldsymbol{e}_x+\sigma_y\boldsymbol{e}_y+\sigma_z\boldsymbol{e}_z$ is a vector composed of Pauli matrices.

The Heisenberg equations for the spin operator $\hat{\boldsymbol{S}}=\hbar\boldsymbol{\sigma}/2$, the momentum $\hat{\boldsymbol{p}}$, and the position operator $\hat{\boldsymbol{x}}$ take the form

$$\frac{d\hat{\boldsymbol{S}}}{dt} = \frac{2}{\hbar}\hat{\boldsymbol{S}}\times(\boldsymbol{N}\times\hat{\boldsymbol{p}}), \tag{2}$$

$$\frac{d\hat{\boldsymbol{p}}}{dt} = -\nabla V(\hat{\boldsymbol{x}},t) - \boldsymbol{F}, \tag{3}$$

and

$$\frac{d\hat{\boldsymbol{x}}}{dt} = \frac{\hat{\boldsymbol{p}}}{m} - \frac{2}{\hbar}\hat{\boldsymbol{S}}\times\boldsymbol{N}. \tag{4}$$

In order to obtain a quasiclassical description we use the Ehrenfest theorem. To this end, we write the operators in the form $\hat{\boldsymbol{p}}=\boldsymbol{p}+\delta\hat{\boldsymbol{p}}$, $\hat{\boldsymbol{S}}=\boldsymbol{S}+\delta\hat{\boldsymbol{S}}$ and $\hat{\boldsymbol{x}}=\boldsymbol{x}+\delta\hat{\boldsymbol{x}}$, where $\boldsymbol{p}$, $\boldsymbol{S}$, and $\boldsymbol{x}$ are the expection values, and take the expection values of the Heisenberg equations. Following Ehrenfest we ignore the quantum corrections to the equations of motion. Doing so, we find that the expection values satisfy again Eqs. (2)–(4).

We notice that the momentum $\boldsymbol{p}$ is coupled to the spin by the equation of motion for the position vector $\boldsymbol{x}$. The latter has the form

$$\frac{d\boldsymbol{x}}{dt} = \frac{\boldsymbol{p}}{m} - \frac{2}{\hbar}\boldsymbol{S}\times\boldsymbol{N}. \tag{5}$$

The first term in this equation is of the order of p/m, the second of the order of N. The splitting of the energy levels due to the Rashba interaction is $\Delta_p=|N|p$. Therefore, the second term on the right-hand side (RHS) of Eq. (5) is small compared to the first term if the Rashba level splitting is small compared to the Fermi energy. This is the case in most systems. Accordingly, the second term can be ignored in a first approximation. In this case the equations of motion for $\boldsymbol{x}$ and $\boldsymbol{p}$ are closed. Therefore, the spin dynamic depends on the random field $V(\boldsymbol{x},t)$ only via $\boldsymbol{p}$. Thus, we can consider the momentum as a random variable, which is characterized by the correlation functions

$$\langle\boldsymbol{p}\rangle = -m\mu_{xx}\boldsymbol{F} \tag{6}$$

and

$$\langle\langle p_i(0)p_j(t)\rangle\rangle = \frac{m^2D}{\tau}\delta_{ij}e^{-t/\tau} \tag{7}$$

for $t>\tau$, where D is the (spectral) diffusion coefficient and τ is the momentum relaxation time.

The magnetization satisfies the equation

$$S_k(t) = G_{kl}(t,0)S_l(0), \tag{8}$$

where

$$\left(\frac{d}{dt}\delta_{ik} + \frac{2}{\hbar}(\boldsymbol{N}\times\boldsymbol{p})_j\epsilon_{ijk}\right)G_{kl}(t,t') = \delta_{il}\delta(t-t'), \tag{9}$$

and a summation with respect to double indices is performed. To calculate the configuration average we split the momentum into two parts according to the equation $\boldsymbol{p}=-m\mu_{xx}\boldsymbol{F}+\delta\boldsymbol{p}$, where $\boldsymbol{\omega}=\boldsymbol{N}\times\delta\boldsymbol{p}$. Doing so, we obtain the Dyson equation

$$G_{kl}(t,t') = G^0_{kl}(t,t') - \frac{2}{\hbar}\int_0^\infty dt_1 G^0_{ki}(t,t_1)\epsilon_{imn}\omega_m(t_1)G_{nl}(t_1,t'). \tag{10}$$

Here G^0 is the retarded solution to the equation

$$\left(\frac{d}{dt}\delta_{ik} - \frac{2m\mu_{xx}F}{\hbar}\epsilon_{i2k}\right)G^0_{kl} = \delta_{il}\delta(t-t'). \tag{11}$$

To calculate the configuration average we use the Born approximation. In this approximation the equation for the calculation of the configuration averaged Green function takes the form

$$\begin{aligned}\left(\frac{d}{dt}\delta_{ik} - \frac{2m\mu_{xx}F}{\hbar}\epsilon_{i2k}\right)\overline{G_{kl}}(t,t') &= \delta_{il}\delta(t-t') + \frac{4}{\hbar^2}\int_0^\infty dt_2\epsilon_{kmn}G^0_{np}(t,t_2)\\ &\times\epsilon_{prs}\langle\langle\omega_m(t)\omega_r(t_2)\rangle\rangle\overline{G_{sl}}(t_2,t').\end{aligned} \tag{12}$$

To simplify this equation further we use the observation that the decay of the correlation function $\langle\langle\omega_m(t)\omega_r(t_2)\rangle\rangle$ is governed by the time scale τ. If we restrict the consideration to the limit of weak spin-orbit scattering, we expect that the variation of the function $\overline{G}$ is negligible on this time scale. Accordingly,

$$\left(\frac{d}{dt}\delta_{ik} - \frac{2m\mu_{xx}F}{\hbar}\epsilon_{i2k}\right)\overline{G}_{kl}(t,t') = \delta_{il}\delta(t-t') + \frac{4}{\hbar^2}\epsilon_{kmn}G^0_{np}(1/\tau) \times \epsilon_{prs}\langle\langle\omega_m(0)\omega_r(0)\rangle\rangle\overline{G}_{sl}(t,t'), \tag{13}$$

where $G^0(1/\tau)$ is the Laplace transform of the function $G^0(t,0)$ at the Laplace frequency $s=1/\tau$.

The correlation function in the second term of the RHS of Eq. (13) can easily be calculated. The calculation yields

$$\langle\langle\omega_m(0)\omega_r(0)\rangle\rangle = N_z^2\delta_{mr}(1-\delta_{m3})\frac{m^2D}{\tau}. \tag{14}$$

Thus, we finally obtain the equation

$$\left(\frac{d}{dt}\delta_{ik} + \delta_{ik}\Omega(g_F + \delta_{k3}) - \frac{2m\mu_{xx}}{\hbar}(\boldsymbol{N}\times\boldsymbol{F})_j\epsilon_{ijk}\right)S_k(t) = 0 \tag{15}$$

in the limit $\Omega\tau \ll 1$. Here

$$g_F = \frac{1}{1+\omega_F^2\tau^2}, \tag{16}$$

$$\Omega = \frac{4m^2DN^2}{\hbar^2}, \tag{17}$$

and

$$\omega_F = \frac{2m\mu_{xx}FN}{\hbar}. \tag{18}$$

According to Eqs. (15)–(18) the electric field affects the relaxation in two ways. First, it leads to an additional rotation of the magnetization in the plane transverse to $\boldsymbol{N}\times\boldsymbol{F}$. Second, it decreases the spin relaxation rate. The field-induced rotation has recently been discussed in Refs. 26–28. In Refs. 26 and 27 it has been shown that the rotation is observable if the electric field exceeds a critical field F_c. To investigate the impact of the new factor g_F on the relaxation process we solve Eq. (13). Doing so, we take advantage of the fact that the matrix is block diagonal. For the component parallel to $\boldsymbol{N}\times\boldsymbol{F}$ we obtain the result

$$S_y(t) = \exp(-g_F\Omega t)S_y(0). \tag{19}$$

It shows that a large field increases considerably the lifetime of the magnetization in the direction of $\boldsymbol{N}\times\boldsymbol{F}$. For the components transverse to $\boldsymbol{N}\times\boldsymbol{F}$ we obtain the equations

$$S_x(t) = \exp\left(-\frac{\Omega t}{2} - g_F\Omega t\right)\left[\left(\frac{\Omega}{2\kappa_F}\sin(\kappa_F t) + \cos(\kappa_F t)\right)S_x(0) + \frac{\omega_F}{\kappa_F}\sin(\kappa_F t)S_z(0)\right], \tag{20}$$

$$S_z(t) = \exp\left(-\frac{\Omega t}{2} - g_F\Omega t\right)\left[\left(-\frac{\Omega}{2\kappa_F}\sin(\kappa_F t) + \cos(\kappa_F t)\right)S_z(0) - \frac{\omega_F}{\kappa_F}\sin(\kappa_F t)S_x(0)\right], \tag{21}$$

for $|F| > F_c$, where

$$\kappa_F = \sqrt{\omega_F^2 - \frac{\Omega^2}{4}} \tag{22}$$

and

$$F_c = \frac{m}{\hbar}|N|\frac{D}{\mu_{xx}}. \tag{23}$$

These equations reduce to those discussed in the Refs. 26 and 27 in the limit $\omega_F\tau \ll 1$. In the limit $\omega_F\tau \gg 1$ they contain rapid oscillating functions, which invalidate our assumption that the change of the magnetization during the time τ is negligible. Therefore, the range of applicability of Eqs. (20) and (19) is restricted to the limit $\omega_F\tau \ll 1$. Accordingly, we can conclude that a large electric field enhances strongly the lifetime of the magnetization in the direction of $\boldsymbol{N}\times\boldsymbol{F}$, but do not obtain new results for the transverse components in the same limit.

B. Systems with Dresselhaus interactions

The method presented above can easily be extended to systems with Dresselhaus interactions. In this case the spin-orbit contribution to the Hamilton operator is given by

$$H_D = \beta(\hat{p}_x\sigma_x - \hat{p}_y\sigma_y). \tag{24}$$

The equation of motion for the spin can again be written in the form

$$\frac{d\hat{\boldsymbol{S}}}{dt} = \frac{2}{\hbar}\hat{\boldsymbol{S}}\times\hat{\boldsymbol{\omega}}, \tag{25}$$

where

$$\hat{\boldsymbol{\omega}} = \beta(-\hat{p}_x, \hat{p}_y, 0). \tag{26}$$

Following the same steps as before we obtain the result

$$\left[\frac{d}{dt}\delta_{ik} + \delta_{ik}\Omega\left(\frac{1}{1+\omega_F^2\tau^2} + \delta_{k3}\right) - \frac{2m\mu_{xx}\beta}{\hbar}F_x\epsilon_{ixk}\right]S_k(t) = 0 \tag{27}$$

for $\Omega\tau \ll 1$, where

$$\Omega = \frac{4m^2D\beta^2}{\hbar^2}. \tag{28}$$

ω_F is given here by the same formula as Eq. (18), except that N has to be replaced by β.

Equation (27) shows that the spin relaxation in a Dresselhaus semiconductor proceeds nearly in the same way as in a Rashba semiconductor. The main difference between both systems is that the electric-field-induced rotation in the Rashba semiconductor proceeds in the plane transverse to

$\boldsymbol{N}\times\boldsymbol{F}$—in a Dresselhaus semiconductor, however, in the plane transverse to $\boldsymbol{F}$.

Of particular interest are systems in which both the Rashba interaction and the Dresselhaus interaction are present.[5] The spectrum of such systems has the property that the spin does not couple to the momentum at degenerate points.[5] Therefore, particular long spin relaxation times can be achieved in such systems by tuning the Rashba interaction properly. For such systems our method yields the result

$$\left(\frac{d}{dt}\delta_{ik}+\Omega_{ik}-\frac{2m\mu_{xx}\beta}{\hbar}F_x\epsilon_{ixk}-\frac{2m\mu_{xx}}{\hbar}(\boldsymbol{N}\times\boldsymbol{F})_j\epsilon_{ijk}\right)S_k(t)=0 \tag{29}$$

in the limit $\omega_F\tau\ll 1$, where

$$\Omega_{ik}=\frac{4m^2D}{\hbar^2}\begin{pmatrix} N^2+\beta^2 & 2N\beta & 0\\ 2N\beta & N^2+\beta^2 & 0\\ 0 & 0 & 2(N^2+\beta^2)\end{pmatrix}. \tag{30}$$

To diagonalize the tensor Ω_{ik} we turn the coordinate system by the angle $\pi/4$ around the z axis. In the new frame the tensor Ω takes the form

$$\Omega'=\frac{4m^2D}{\hbar}{}_2\begin{pmatrix} N(N-\beta) & 0 & 0\\ 0 & N(N+\beta) & 0\\ 0 & 0 & 2(N^2+\beta^2)\end{pmatrix}. \tag{31}$$

Here the prime indicates that the system is turned by $\pi/4$. The components of the magnetization in the new frame are referred to as S_x', S_y', and S_z'.

Equation (31) shows clearly that the tensor Ω has zero modes at the points of degeneracy $N=\pm\beta$. To investigate the impact of the electric field on the spin relaxation at such a point we focus on the situation at $N=\beta$. Doing so, we find that S_x' is conserved even in the presence of the electric field. If we switch on the electric field, the rotation of the magnetization proceeds in the plane transverse to $\boldsymbol{e}_x'$ exclusively. For the components of the magnetization in this plane we obtain the equation

$$\begin{pmatrix} S_y'(t)\\ S_z'(t)\end{pmatrix}=e^{-\Omega_{zz}t}\begin{pmatrix}\cos(\omega t) & \sin(\omega t)\\ -\sin(\omega t) & \cos(\omega t)\end{pmatrix}\begin{pmatrix} S_x'(0)\\ S_y'(0)\end{pmatrix}, \tag{32}$$

where

$$\omega=\sqrt{2}\frac{2m\mu_{xx}NF}{\hbar}. \tag{33}$$

These equations show that the critical field F_c in such systems is equal to zero. Thus such systems are also particularly convenient for the investigation of electric field effects, since they permit the investigation of field effects already in the presence of very weak electric fields.

III. SPIN RELAXATION IN THE PRESENCE OF CROSSED ELECTRIC AND MAGNETIC FIELDS

A. Rashba semiconductor

To take into account an external magnetic field we replace $\hat{\boldsymbol{p}}$ by $\hat{\boldsymbol{p}}-\hat{\boldsymbol{A}}$, where $\hat{\boldsymbol{A}}=B_z/2\boldsymbol{e}_z\times\hat{\boldsymbol{x}}$. In this case the Ehrenfest theorem yields the equations

$$\frac{d\boldsymbol{S}}{dt}=\frac{2m}{\hbar}\boldsymbol{S}\times(\boldsymbol{N}\times\boldsymbol{v}), \tag{34}$$

$$\frac{d\boldsymbol{x}}{dt}=\boldsymbol{v}-\frac{2}{\hbar}\boldsymbol{S}\times\boldsymbol{N}, \tag{35}$$

and

$$m\frac{d\boldsymbol{v}}{dt}=\boldsymbol{v}\times\boldsymbol{B}-\frac{2}{\hbar}(\boldsymbol{S}\times\boldsymbol{N})\times\boldsymbol{B}_z-\boldsymbol{\nabla}V(\boldsymbol{x},t)-\boldsymbol{F}, \tag{36}$$

where $\boldsymbol{v}$ is the velocity. The second term on the RHS of Eqs. (35) and (36) can be ignored under the same conditions as before. Doing so, we find again that the equations for the calculation of the particle velocity and the particle position are closed. Accordingly, investigation of Eq. (34) requires only specification of the velocity correlation functions. In the presence of a magnetic field they take the form

$$\langle v_x(t)\rangle=-\mu_{xx}F, \tag{37}$$

$$\langle v_y(t)\rangle=\mu_H FB, \tag{38}$$

$$\langle\langle v_x(t)v_x(t')\rangle\rangle=\langle\langle v_y(t)v_y(t')\rangle\rangle =\frac{D}{\tau}\cos[\omega_c(t-t')]\exp(-|t-t'|/\tau), \tag{39}$$

and

$$\langle\langle v_x(t)v_y(t')\rangle\rangle=-\langle\langle v_y(t)v_x(t')\rangle\rangle =\frac{D}{\tau}\sin[\omega_c(t-t')]\exp(-|t-t'|/\tau) \tag{40}$$

for $t\gg\tau$, where $\mu_H=\tau^2/m^2$ and $\omega_c=B_z/m$. If we use these correlation functions, we obtain the result

$$\left[\frac{d}{dt}\delta_{ik}+\tilde{\Omega}\delta_{ik}(1+\delta_{k3})-\mu_B g_{eff}B_z\epsilon_{i3k} -\frac{2m}{\hbar}\epsilon_{ijk}(\boldsymbol{N}\times\boldsymbol{\mu})_j\right]S_k(t)=0 \tag{41}$$

for $\omega_F\tau\ll 1$, where

$$\tilde{\Omega}=\frac{\Omega}{1+(\omega_c\tau)^2}, \tag{42}$$

$$g_{eff}=\tilde{\Omega}\omega_c\tau/(B_z\mu_B), \tag{43}$$

$$\boldsymbol{\mu}=\mu_{xx}\boldsymbol{F}+\boldsymbol{F}\times\boldsymbol{B}\mu_H, \tag{44}$$

and μ_B is the Bohr magneton. These equations show that the orbital part of the magnetic field affects the spin relaxation in three ways. First, it turns the plane of the electric-field-induced rotation by the Hall angle $\omega_c\tau$. Therefore, the field-induced rotation proceeds actually in the plane transverse to $\boldsymbol{N}\times\boldsymbol{j}$ in a Rashba semiconductor, where $\boldsymbol{j}$ is the current density. Second, it reflects itself in an effective Zeeman field, which results from the interplay between the cyclotron motion and the spin-orbit scattering. Third, it reduces the relaxation rate. Equation (42), which describes the reduction of the relaxation rate, has recently been derived in Ref. 21 using quantum kinetics. Our derivation shows that the underlying physics is very simple and produces new information on the impact of the magnetic field on the field-induced rotation and on the effective g factor. The latter is quite large, in particular for small magnetic fields.

So far we have restricted the consideration to the investigation of the impact of the orbital part of the magnetic field. The consideration of an explicit Zeeman term in the Hamiltonian does not present any difficulty. If such a term is taken into account, it yields an additional contribution of the structure

$$\left.\frac{d\boldsymbol{S}}{dt}\right|_{Zeeman} = -\mu_B g \boldsymbol{S}\times\boldsymbol{B} \tag{45}$$

to Eq. (41), provided $|\mu_B g B\tau| \ll 1$. Thus g_{eff} is replaced by $g_{eff}\rightarrow g_{eff}+g$.

B. Systems with Dresselhaus interactions

For a system with Dresselhaus interactions we obtain the simple equation

$$\left[\frac{d}{dt}\delta_{ik} + \tilde{\Omega}\delta_{ik}(1+\delta_{iz}) + \mu_B g_{eff} B\epsilon_{i3k} + \frac{2m\beta}{\hbar}\epsilon_{ijk}\tilde{\mu}_j\right]S_k(t) = 0 \tag{46}$$

for $\omega_F\tau \ll 1$, where

$$\tilde{\boldsymbol{\mu}} = \mu_{xx}\boldsymbol{F} - \boldsymbol{F}\times\boldsymbol{B}\mu_H \tag{47}$$

and $\tilde{\Omega}$ is given by Eq. (42) with N replaced by β in Ω. This equation shows that the impact of the magnetic field on the spin relaxation in a system with Dresselhaus interactions is different from that on a Rashba semiconductor. The interplay between the spin-orbit interaction and the cyclotron motion leads in both systems to an effective Zeeman field. In a Dresselhaus semiconductor, however, the effective g factor takes the opposite sign. Therefore, the effective g factor can be varied in a wide range simply by tuning the the Rashba and Dresselhaus interaction constants N and β properly. This fact manifests itself also in a system in which both interactions are present. For such a system we obtain the equation

$$\left[\frac{d}{dt}\delta_{ik} + \frac{1}{1+(\omega_c\tau)^2}\Omega_{ik} - \frac{4m^2 D\omega_c\tau}{\hbar^2[1+(\omega_c\tau)^2]}(N^2-\beta^2)\epsilon_{i3s} - \frac{2m}{\hbar}\epsilon_{ijk}(\boldsymbol{N}\times\boldsymbol{\mu} - \beta\tilde{\boldsymbol{\mu}})_j\right]S_k(t) = 0, \tag{48}$$

where Ω_{ik} is given by Eq. (30). This equation shows that the g factor of the system can be tuned by means of the quantities N and β. It can be made positive as well as negative.

To investigate Eq. (48) further we focus on the situation $N=\beta$. In this case the effective g factor vanishes but the magnetization still performs a rotation due to the electric field. In this case the equation of motion for the magnetization takes the form

$$\begin{pmatrix} s & 0 & -\omega_- \\ 0 & s+\Omega_{xx}+\Omega_{xy} & -\omega_+ \\ \omega_- & \omega_+ & s+\Omega_{zz} \end{pmatrix} S'(s) = S'_0, \tag{49}$$

where

$$\omega_- = 2\sqrt{2}mFBN\mu_H/\hbar \tag{50}$$

and

$$\omega_+ = 2\sqrt{2}mFN\mu_{xx}/\hbar \tag{51}$$

in the primed frame discussed in Sec. II B. This equation shows that the precession of the magnetization in the presence of crossed electric and magnetic fields proceeds in both the y'-z' and x'-z' planes. This result has immediate consequences for the applications discussed in Ref. 5. According to Eq. (49) every initial magnetization in a system with combined Rashba and Dresselhaus interactions decays in the presence of crossed electric and magnetic fields even at the point of degeneracy. Therefore, the spin transistor of Ref. 5 becomes unstable in the presence of crossed fields.

IV. RESULTS

In this paper we have developed a simple method for the investigation of spin relaxation processes in systems with Rashba and Dresselhaus interactions in the presence of external fields. Our calculation reproduces the existing results on the impact of an electric field on systems with Rashba interactions[26,27,29] and extends them to systems in which both Rashba and Dresselhaus interactions are present to systems with large electric fields and to systems with crossed electric and magnetic fields. In line with the results of Refs. 26, 27, and 29 we find that a lateral electric field affects strongly the spin relaxation. However, the impact of the field on the relaxation process depends on the structure of the spin-orbit interaction. Although the field leads to an additional rotation of the magnetization in both the presence of the Rashba interaction and the Dresselhaus interaction, the plane of the field-induced precession is different in both cases. In the presence of the Rashba interaction the precession proceeds in the plane perpendicular to $\boldsymbol{N}\times\boldsymbol{F}$, in the presence of the Dresselhaus interaction in the plane perpendicular to $\boldsymbol{F}$. The plane can be turned continuously by tuning the Rashba interaction constant N and the Dresselhaus interaction constant β properly.

The field-induced rotation can be observed if the electric field exceeds a critical field. The critical field depends on the mobility, on the diffusion coefficient, on the effective mass, on the Rashba interaction constant, and on the Dresselhaus interaction constant. It vanishes at the points of degeneracy $|N|=|\beta|$. Therefore, the investigation of systems near degenerate points offers the opportunity to study nonlinear field effects already at vanishing small fields.

The rotation of the magnetization manifests also in the spin lifetime. Our calculation shows that the lifetime of the spin component transverse to the precession plane increases with increasing electric field. The equation, which describes the decrease of the relaxation rate in the presence of an electric field, has the same structure as that in the presence of a very strong magnetic field derived recently in Ref. 21. Thus our results show that a lateral electric field affects the spin relaxation qualitatively in the same way as a magnetic field. At present mainly magnetic fields are used in the investigation of spin relaxation phenomena. Our results show that every experiment which can be performed by means of magnetic fields can be performed completely electrically as well.

The field-induced rotation of the magnetization can be affected further by magnetic fields. A magnetic field turns the plane of the field-induced precession. In both a Rashba semiconductor and a Dresselhaus semiconductor the precession plane is turned by a Hall angle, however, in different ways. In a Rashba semiconductor the plane is turned in such a way that the field-induced precession proceeds always in the plane perpendicular to $\boldsymbol{N}\times\boldsymbol{j}$, where $\boldsymbol{j}$ is the current density. In a Dresselhaus semiconductor, however, the normal of precession plane, which is parallel to the current density vector in the absence of the magnetic field, acquires also a component transverse to $\boldsymbol{j}$. The differences between the impact of the magnetic field on the spin relaxation in both systems manifests itself in particular at a point of degeneracy. In the presence of crossed fields the magnetization decays even at a point of degeneracy, although the tensor Ω_{ik} is degenerate, since the field always turns the magnetization into a nonconserved direction. Therefore the spin transistor discussed in Ref. 5 becomes unstable at sufficiently large crossed fields.

The impact of a transverse magnetic field on the spin relaxation is not restricted to the electric-field-induced precession. The cyclic motion of the charge carriers in a magnetic field leads also to an additional rotation, which changes the effective g factor and reduces the relaxation rate. The sign and magnitude of the effective g factor depend on the magnitude of the spin-orbit interaction. The spin-orbit contribution to the effective g factor vanishes at a point of degeneracy and decreases with increasing magnetic field. The equations describing the reduction of the relaxation rate with increasing magnetic field obtained here agree with those of Ref. 21.

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