Anomalous Hall effect in a two-dimensional electron gas with spin-orbit interaction

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We discuss the mechanism of anomalous Hall effect related to the contribution of electron states below the Fermi surface (induced by the Berry phase in momentum space). Our main calculations are made within a model of two-dimensional electron gas with spin-orbit interaction of the Rashba type, taking into account the scattering from impurities. We demonstrate that such an "intrinsic" mechanism can dominate but there is a competition with the impurity-scattering mechanism, related to the contribution of states in the vicinity of the Fermi surface. We also show that the contribution to the Hall conductivity from electron states close to the Fermi surface has the intrinsic properties as well.

I. INTRODUCTION

The measurement of the anomalous Hall effect[1] (AHE) in ferromagnetic metals and semiconductors is a powerful tool to characterize the magnetization of thin films and mesoscopic objects. That is why it recently attracted a lot of attention in the spintronics community.[2] On the other hand, the theoretical aspects and possible physical mechanisms of AHE are still not quite well understood, and the great efforts are directed now toward the physical explanation of AHE in magnetic semiconductors, pyrochlore crystals, and magnetic compounds with inhomogeneous magnetization. The state of art in the theory of AHE has been recently reviewed by Sinova *et al.*[3]

Relatively well-studied interpretations of AHE include the skew scattering[4,5] and side-jump[6] mechanisms, which have been recently revised and physically clarified by Crépieux and Bruno[7] and Dugaev *et al.*[8] These mechanisms are related to the scattering from impurities, with the spin-orbit (SO) interaction playing the leading role, so that the spin-flip interaction is included into the amplitude of scattering from impurities.

It was shown recently that the nonzero spin chirality in some inhomogeneous ferromagnets also leads to the nonvanishing AHE.[9–13] Such a *chirality* mechanism of AHE is related to the Berry phase[14] acquired by the wave function of electrons moving adiabatically in the inhomogeneous magnetization field. In principle, the SO interaction is not a necessary element of this mechanism. However, in earlier presented theoretical models, the nonvanishing average AHE was obtained only in the presence of SO interaction. Otherwise, the spatially averaged off-diagonal conductivity was found to be zero. A model of magnetic nanostructure without SO interaction, leading to the Berry-phase-induced AHE,[15] has been called the *topological* mechanism of AHE.

Now the main interest is directed toward another "*intrinsic*" mechanism of AHE recently proposed by Jungwirth,[16] Yao,[17] and Onoda and Nagaosa.[18] This mechanism uses the SO interaction included into the Hamiltonian of electrons in the crystal lattice. The key element of this theory is the Berry phase in the *momentum* space, so that a nontrivial topology of the electron energy bands is necessary for the intrinsic mechanism. A similar idea of the calculations of Hall conductivity has been used long ago in the context of the quantum Hall effect.[19,20]

The Berry-phase-induced intrinsic mechanism of AHE does not take into account scattering from impurities. It was suggested before that the impurities can be totally neglected in case of intrinsic SO interaction because the effect of impurities should be necessarily extrinsic. Correspondingly, the calculations of AHE have been performed in a zero density-of-impurity limit, $N_i \rightarrow 0$,[16,18] which was taken before the static limit of $\omega \rightarrow 0$ in the Kubo formula for conductivity $\sigma_{ij}(\omega)$.

In this paper we present the result of calculations taking into account the effect of impurity scattering. We demonstrate that there exists another intrinsic contribution to AHE related to the electron states near the Fermi surface. This contribution is not geometric, and we show that it can be of the same order of magnitude as the Berry-phase-induced one, which has been calculated before. To calculate properly the nongeometric contribution, we should take into account scattering from impurities leading to the finite relaxation time of electron states at the Fermi surface. Accordingly, we take the limit of $\omega \rightarrow 0$ *before* $N_i \rightarrow 0$, which ensures the transition between disordered and clean to be continuous. At the same time, the nongeometric contribution is intrinsic, and it does not vanish if we finally take the limit of $N_i \rightarrow 0$.

In our calculations, we use a model of two-dimensional electron gas (2DEG) with SO interaction in the form of Rashba term.[21] We show that, in accordance with the well-known result of Středa[22] and Crépieux and Bruno,[22] there are two terms in the off-diagonal conductivity, one of which σ_{xy}^I, is due to the electron states near the Fermi energy (it corresponds to the nongeometric contribution), and the other one σ_{xy}^{II}, is related to the contribution of all occupied electron states below the Fermi energy (identified as the Berry-phase-induced intrinsic mechanism, which has been calculated before).

In agreement with other works on different models of the electron energy spectrum, our calculation confirm that for the

2DEG with Rashba SO interaction, there is a nonvanishing contribution σ_{xy}^{II} from the states below the Fermi surface. This contribution is absent in the case of the side-jump and skew scattering of electrons within the model of a parabolic energy band. However, we emphasize that the contribution to σ_{xy} from the vicinity of the Fermi surface (σ_{xy}^{I}) cannot be totally neglected.

Recently, the role of impurity scattering in AHE has been analyzed within the double-exchange model with noncoplanar spin configuration.[23] It was shown that two different mechanisms can contribute to the AHE, one of which is related to the spin texture in the real space and the other to a momentum-space skyrmion density at the Fermi level. In the latter case, the impurity scattering was relevant. This conclusion is in agreement with our results on a different model.

II. MODEL AND THE KUBO FORMULA

We consider a model of 2DEG in the x-y plane under a homogeneous magnetization field $\mathbf{M}_0$ directed along the axis z, with the SO interaction described by the Rashba term.[21] The Hamiltonian of this model reads

$$H_0 = \varepsilon_{\mathbf{k}} + \alpha(\sigma_x k_y - \sigma_y k_x) - M\sigma_z, \tag{1}$$

where $\varepsilon_{\mathbf{k}} = k^2/2m$, α is the coupling constant of SO interaction, $M = g\mu_B M_0$, M_0 is the magnitude of magnetization, and we take units with $\hbar = 1$. Due to the SO interaction, the components of velocity operators, $v_i = \partial H/\partial k_i$, are matrices in the spin space

$$v_x = \frac{k_x}{m} - \alpha\,\sigma_y, \quad v_y = \frac{k_y}{m} + \alpha\,\sigma_x. \tag{2}$$

We also include into consideration the scattering from impurities, described by a disorder potential $V(\mathbf{r})$. We assume that the disorder potential is short range and weak, and we will treat it in the Born approximation of the impurity scattering. Thus, the Hamiltonian of our model is $H = H_0 + V(\mathbf{r})$, and any physical variables including the Hall conductivity should be calculated with a corresponding averaging over the random potential $V(\mathbf{r})$.

It should be noted that such a model does not take into account the SO interaction in the impurity potential $V(\mathbf{r})$. It is known that the SO interaction in the impurity scattering produces the AHE *via* well-studied side-jump and skew scattering mechanisms (see Ref. 7 and references therein). Here we concentrate on other possible mechanisms induced by the SO interaction in the potential of the crystal lattice.

The conductivity tensor can be calculated using the general Kubo formalism

$$\sigma_{ij}(\omega) = \frac{e^2}{\omega}\mathrm{Tr}\int\frac{d\varepsilon}{2\pi}\langle\hat{v}_i\hat{G}(\varepsilon+\omega)\hat{v}_j\hat{G}(\varepsilon)\rangle, \tag{3}$$

where $\hat{G}(\varepsilon) = (\varepsilon - H)^{-1}$ is the operator Green's function of electrons described by the Hamiltonian H, which includes the disorder, $\hat{v}_i$ is the corresponding velocity operator, the trace goes over any eigenstates in the space of operator H (e.g., eigenstates of the Hamiltonian H itself), and $\langle...\rangle$ means the disorder average.

It was shown by Středa[22] and later in context of the AHE by Crépieux and Bruno[7] that a general formula for the static conductivity $\sigma_{ij}(\omega=0)$ can be presented in a form of two different terms $\sigma_{xy} = \sigma_{xy}^{I} + \sigma_{xy}^{II}$, one of which σ_{xy}^{I}, stems from the contribution of electrons at the Fermi surface

$$\sigma_{ij}^{I} = \frac{e^2}{2}\mathrm{Tr}\int\frac{d\varepsilon}{2\pi}\left(-\frac{\partial f(\varepsilon)}{\partial\varepsilon}\right)\langle\hat{v}_i[\hat{G}^R(\varepsilon) - \hat{G}^A(\varepsilon)]$$

$$\times\hat{v}_j\hat{G}^A(\varepsilon) - \hat{v}_i\hat{G}^R(\epsilon)\hat{v}_j[\hat{G}^R(\varepsilon) - \hat{G}^A(\varepsilon)]\rangle, \tag{4}$$

and the second one σ_{xy}^{II}, formally contains the contribution of all filled states below the Fermi energy

$$\sigma_{ij}^{II} = \frac{e^2}{2}\mathrm{Tr}\int\frac{d\varepsilon}{2\pi}f(\varepsilon)\left\langle\hat{v}_i\frac{\partial\hat{G}^A(\varepsilon)}{\partial\varepsilon}\hat{v}_j\hat{G}^A(\varepsilon) - \hat{v}_i\hat{G}^A(\varepsilon)\hat{v}_j\frac{\partial\hat{G}^A(\varepsilon)}{\partial\varepsilon}\right.$$

$$\left. + \hat{v}_i\hat{G}^R(\varepsilon)\hat{v}_j\frac{\partial\hat{G}^R(\varepsilon)}{\partial\varepsilon} - \hat{v}_i\frac{\partial\hat{G}^R(\varepsilon)}{\partial\varepsilon}\hat{v}_j\hat{G}^R(\varepsilon)\right\rangle. \tag{5}$$

In this paper, for the model of Eq. (1) with the impurity disorder, using a different way of calculation, we come to a similar result in accordance with the Středa formula. We calculate the off-diagonal conductivity σ_{xy} in the loop approximation, changing the disorder average in Eq. (3) as $\langle v_x G\, v_y G\rangle \rightarrow \mathcal{T}_x\langle G\rangle v_y\langle G\rangle$, where $\mathcal{T}_x$ is the renormalized velocity related to the vertex corrections.[25] In this approximation we do not take into account the localization corrections,[24] which are vanishingly small in the limit of small concentration of impurities. The vertex corrections are known to vanish in the case of short-range impurity potential and simple parabolic energy spectrum. As we show in Sec. III, in the case of the Rashba model, the vertex corrections do not vanish even for the short-range impurity potential.

To simplify the notations, in the following we omit the angular brackets for $\langle G\rangle$, and we denote the disorder-averaged Green's function as $G_{\mathbf{k}}(\varepsilon)$. Here we use the momentum representation for $G_{\mathbf{k}}(\varepsilon)$ since the disorder-averaged Green's function is diagonal in this representation.

Hence, we will calculate the off-diagonal conductivity from

$$\sigma_{xy}(\omega) = \frac{e^2}{\omega}\mathrm{Tr}\int\frac{d\varepsilon}{2\pi}\frac{d^2\mathbf{k}}{(2\pi)^2}\mathcal{T}_x(\varepsilon,\omega)G_{\mathbf{k}}(\varepsilon+\omega)v_yG_{\mathbf{k}}(\varepsilon),$$

$$\tag{6}$$

where the trace runs over the spin states.

In the following calculation of the static off-diagonal conductivity $\sigma_{xy}(\omega=0)$, we will be interested in the case when the density of impurities N_i is small but finite. It also includes the "clean limit" of $N_i \rightarrow 0$, which can be physically realized in samples with vanishingly small concentration of impurities and defects. Correspondingly, when we calculate the static conductivity tensor of a clean sample using Eq. (6), we take the limit of $\omega \rightarrow 0$ before the limit of $N_i \rightarrow 0$.

It should be noted that in the previous consideration of the intrinsic mechanism of AHE, the σ_{xy}^{I} contribution related to impurities, was neglected. The impurities were involved only through the finite lifetime of carriers in the intrinsic σ_{xy}^{II} term,[16] and it was assumed that the effect of impurities is
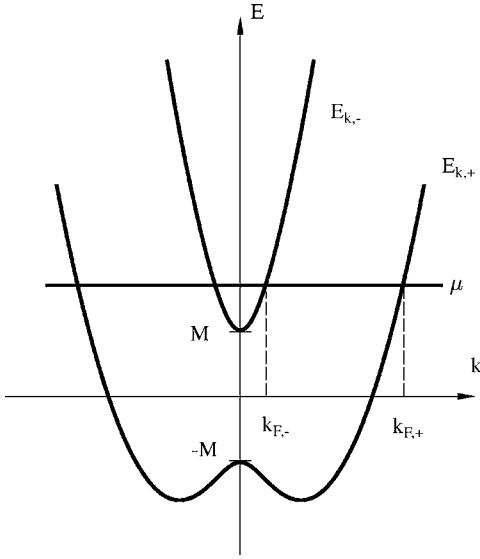
FIG. 1. Energy spectrum of electrons in a two-dimensional ferromagnet with Rashba SO interaction (schematically).

essentially "extrinsic." Here we emphasize that this was essentially wrong. Performing the calculations, which include the scattering from impurities, here we find an additional contribution, which is also intrinsic, i.e., it does not vanish in the limit of $N_i \to 0$. This result is quite similar to the known side-jump mechanism of AHE.[7] As we will see, the additional contribution to AHE comes from the states at the Fermi surface, for which the finite relaxation time due to impurity scattering is important. If we totally neglect the scattering from impurities, we miss this term. The real structures always have some impurities or defects. Therefore, we can justify taking the limit of $\omega \to 0$ before $N_i \to 0$ as a physical realization of the clean system as a system with very small but still nonzero density of impurities.

In the absence of impurity scattering, the electron Green's function can be found using Hamiltonian (1)

$$G_{\mathbf{k}}^0(\varepsilon) = \frac{\varepsilon - \varepsilon_{\mathbf{k}} + \mu + \alpha(k_y \sigma_x - k_x \sigma_y) - M\sigma_z}{\varepsilon - E_{\mathbf{k},+} + \mu + i\delta \operatorname{sgn} \varepsilon}$$

$$\times \frac{1}{\varepsilon - E_{\mathbf{k},-} + \mu + i\delta \operatorname{sgn} \varepsilon}, \qquad (7)$$

where μ is the chemical potential and the electron energy spectrum $E_{\mathbf{k}}$ consists of two branches, which we label as "+" and "−" corresponding mostly to the spin-up and -down electrons, respectively (even though they contain an admixture of the opposite spin due to the SO interaction),

$$E_{\mathbf{k},\pm} = \varepsilon_{\mathbf{k}} \mp \lambda(k), \qquad (8)$$

and we denote $\lambda(k) = (M^2 + \alpha^2 k^2)^{1/2}$.

We consider a general case when the chemical potential μ can be situated in both spin-up and -down subbands, corresponding to $\mu > M$, as shown schematically in Fig. 1. When only the spin-up subband is filled with electrons $-M < \mu < M$, only the contribution of the filled subband, $E_{\mathbf{k},+}$, should be kept.

Due to the scattering from impurities and defects, the Green's function (7) is modified. It is important to include the effect of scattering for the correct evaluation of the contribution to the off-diagonal conductivity from the Fermi surface. For simplicity, we consider the model of disorder created by weak short-range scatterers, which can be treated in the Born approximation. The corresponding self energy of electrons is calculated as

$$\Sigma_i(\varepsilon) = N_i V_0^2 \int \frac{d^2 \mathbf{k}}{(2\pi)^2} G_{\mathbf{k}}^0(\varepsilon), \qquad (9)$$

where V_0 is the Fourier transform of the impurity potential, and N_i is the impurity density. The result of calculation using Eqs. (7) and (9) can be presented as

$$\Sigma_i(\varepsilon) = -\frac{i \operatorname{sgn} \varepsilon}{2} \left(\frac{1}{\tau} + \frac{1}{\tilde{\tau}} \sigma_z \right), \qquad (10)$$

where

$$\frac{1}{\tau} = \pi N_i V_0^2 (\nu_+ + \nu_-), \qquad (11)$$

$$\frac{1}{\tilde{\tau}} = \pi N_i V_0^2 M \left(\frac{\nu_+}{\lambda_+} - \frac{\nu_-}{\lambda_-} \right), \qquad (12)$$

$\lambda_\pm \equiv \lambda(k_{F,\pm})$, and

$$\nu_+ = \frac{m}{2\pi} \left| 1 - \frac{m\alpha^2}{\lambda_+} \right|^{-1},$$

$$\nu_- = \theta(\mu - M) \frac{m}{2\pi} \left| 1 - \frac{m\alpha^2}{\lambda_-} \right|^{-1} \qquad (13)$$

are the densities of states at the Fermi surfaces of two different subbands for $\mu > -M$. Here $\theta(x)$ is the Heaviside step function, and $k_{F,\pm}$ are the Fermi momenta of the majority and minority electrons, respectively.

Taking into account the self-energy correction (10), we find the Green's function of the electron system with impurities

$$G_{\mathbf{k}}(\varepsilon) = \frac{\varepsilon + i\Gamma - \varepsilon_{\mathbf{k}} + \mu + \alpha(k_y \sigma_x - k_x \sigma_y) - \sigma_z(M + i\tilde{\Gamma})}{\varepsilon - E_{\mathbf{k},+} + \mu + i \operatorname{sgn} \varepsilon / 2\tau_+}$$

$$\times \frac{1}{\varepsilon - E_{\mathbf{k},-} + \mu + i \operatorname{sgn} \varepsilon / 2\tau_-}, \qquad (14)$$

where

$$\Gamma = \frac{\operatorname{sgn} \varepsilon}{2\tau}, \quad \tilde{\Gamma} = \frac{\operatorname{sgn} \varepsilon}{2\tilde{\tau}}, \quad \frac{1}{\tau_\pm} = \frac{1}{\tau} \pm \frac{1}{\tilde{\tau}} \frac{M}{\lambda_\pm}. \qquad (15)$$

The values of τ_+ and τ_- determined by Eq. (15) are the relaxation times of electrons in different subbands.

III. VERTEX CORRECTION

The equation for the renormalized vertex $\mathcal{T}(\varepsilon, \omega)$ can be presented using the diagrams with the impurity ladder in-

cluded into the vertex part.[25] For a short-range impurity potential, this equation has the following form:

$$\mathcal{T}_i(\varepsilon,\omega) = v_i + N_i V_0^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2} G_\mathbf{k}(\varepsilon)\mathcal{T}_i(\varepsilon,\omega)G_\mathbf{k}(\varepsilon+\omega). \quad (16)$$

In the limit of $\omega \to 0$, the integral is not zero only at the Fermi surface, i.e., for energies $\varepsilon \ll \tau_{\uparrow,\downarrow}^{-1}$. We assume that the density of impurities is low, which corresponds to the large relaxation times $\tau_{\uparrow,\downarrow}$. Thus, we calculate the vertex renormalization for $\varepsilon=0$, otherwise we take $\mathcal{T}_x = v_x$.

Denoting $\mathcal{T}_x = \mathcal{T}_x(\varepsilon=0, \omega \to 0)$, we look for a solution of Eq. (16) in the form

$$\mathcal{T}_x = ak_x + b\sigma_x + c\sigma_y \quad (17)$$

with some coefficients a, b, and c. After substituting this expression into Eq. (16) and using Eq. (14), we find that in the limit of low impurity density $a=1/m$ and $b=0$ and the constant c is defined by

$$c\left[1 - N_i V_0^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{(\mu-\varepsilon_\mathbf{k})^2 - M^2}{D_+ D_-}\right]$$
$$= -\alpha\left[1 + 2N_i V_0^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{\varepsilon_\mathbf{k}(\mu-\varepsilon_\mathbf{k})}{D_+ D_-}\right], \quad (18)$$

where $D_\pm = (\mu - E_{\mathbf{k},+} \pm i/2\tau_+)(\mu - E_{\mathbf{k},-} \pm i/2\tau_-)$.

Using Eqs. (2), (6), (17), and (18), we conclude that the vertex renormalization in Eq. (6) for σ_{xy} results in a substitution of the velocity v_x by $\mathcal{T}_x = k_x/m - \alpha^* \sigma_y$ where

$$\alpha^* = \alpha\left[1 + 2N_i V_0^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{\varepsilon_\mathbf{k}(\mu-\varepsilon_\mathbf{k})}{D_+ D_-}\right]$$
$$\times\left[1 - N_i V_0^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{(\mu-\varepsilon_\mathbf{k})^2 - M^2}{D_+ D_-}\right]^{-1}. \quad (19)$$

We should emphasize that the vertex renormalization refers only to the states near the Fermi surface.

The integrals in Eq. (19) can be calculated by transforming them to integrals over $\varepsilon_\mathbf{k}$, i.e., $\int d^2\mathbf{k}(2\pi)^{-2}\cdots = \int \nu_0(\varepsilon_\mathbf{k})d\varepsilon_\mathbf{k}\cdots$, where $\nu_0(\varepsilon)=(m/2\pi)\theta(\varepsilon)$. Finally, we find

$$\alpha^* = \alpha\left\{1 + \frac{2\pi N_i V_0^2}{(\mathcal{E}_1 - \mathcal{E}_2)^2}\left[\nu_0(\mathcal{E}_1)\frac{\mathcal{E}_1(\mu-\mathcal{E}_1)}{\Gamma_1}\right.\right.$$
$$\left.\left.+ \nu_0(\mathcal{E}_2)\frac{\mathcal{E}_2(\mu-\mathcal{E}_2)}{\Gamma_2}\right]\right\}\left\{1 - \frac{\pi N_i V_0^2}{(\mathcal{E}_1 - \mathcal{E}_2)^2}\right.$$
$$\times\left[\nu_0(\mathcal{E}_1)\frac{[(\mu-\mathcal{E}_1)^2 - M^2]}{\Gamma_1}\right.$$
$$\left.\left.+ \nu_0(\mathcal{E}_2)\frac{[(\mu-\mathcal{E}_2)^2 - M^2]}{\Gamma_2}\right]\right\}^{-1}, \quad (20)$$

where

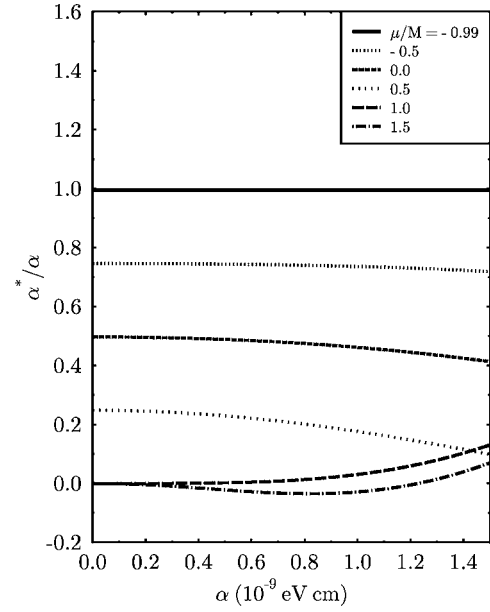$$\mathcal{E}_{1,2} = \mu + m\alpha^2 \pm (m^2\alpha^4 + 2\mu m\alpha^2 + M^2)^{1/2} \quad (21)$$

and

FIG. 2. Renormalized constant of SO interaction α^* in the vertex $\mathcal{T}_x$ at the Fermi surface for different locations of the Fermi level.

$$\Gamma_{1,2} = \mp\frac{\mu - \mathcal{E}_{1,2} - (M^2 + 2m\alpha^2\mathcal{E}_{1,2})^{1/2}}{2\tau_+(\mathcal{E}_1 - \mathcal{E}_2)}$$
$$\mp\frac{\mu - \mathcal{E}_{1,2} + (M^2 + 2m\alpha^2\mathcal{E}_{1,2})^{1/2}}{2\tau_-(\mathcal{E}_1 - \mathcal{E}_2)}. \quad (22)$$

If $\mu > M$ (Fermi level is located within two subbands like that shown in Fig. 1) and in the limit of $\alpha \to 0$, we can find from Eq. (22) that $\alpha^*/\alpha \to 0$.

In the case of $-M < \mu < m$ (only the lowest energy subband is partly filled with electrons) and $\alpha \to 0$, we obtain

$$\frac{\alpha^*}{\alpha} = 1 - \frac{\mu + M}{2M}, \quad (23)$$

which gives us $\alpha^*/\alpha \to 1$ for $\mu \to -M$, and $\alpha^*/\alpha \to 0$ for $\mu \to M$. The dependence of α^*/α on α for different values of μ is shown in Fig. 2.

IV. ANOMALOUS HALL EFFECT AND TOPOLOGY OF THE ENERGY BANDS

The integration over energy ε in Eq. (6) with Green's function (14) leads to the separation of σ_{xy} into two parts $\sigma_{xy} = \sigma_{xy}^I + \sigma_{xy}^{II}$ corresponding to the above-mentioned contribution of energy states at the Fermi surface and the states below the Fermi energy, respectively.[7,22]

First we calculate the contribution from the states below the Fermi energy σ_{xy}^{II}. Using Eqs. (2), (6), and (14), we find σ_{xy}^{II} as an integral over momentum with the Fermi-Dirac functions $f(E_{\mathbf{k},\pm})$. Thus, it accounts for the contribution of all states with $E_{\mathbf{k},+} < \mu$ and $E_{\mathbf{k},-} < \mu$
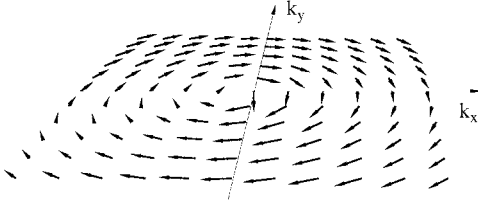
FIG. 3. The field $\mathbf{n}(\mathbf{k})$ in the momentum space.

$$\sigma_{xy}^{II}(\omega) = -\frac{2ie^2}{\omega}\mathrm{Tr}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{f(E_{\mathbf{k},+})-f(E_{\mathbf{k},-})}{(E_{\mathbf{k},+}-E_{\mathbf{k},-})^3}\left(\frac{k_x}{m}-\alpha\,\sigma_y\right)$$

$$\times(E_{\mathbf{k},+}-\varepsilon_{\mathbf{k}}+\alpha k_y\sigma_x-\alpha k_x\sigma_y-M\sigma_z)\left(\frac{k_y}{m}+\alpha\,\sigma_x\right)$$

$$\times(-\omega+E_{\mathbf{k},+}-\varepsilon_{\mathbf{k}}+\alpha k_y\sigma_x-\alpha k_x\sigma_y-M\sigma_z). \quad (24)$$

In the static limit of $\omega\to 0$, and after calculating the trace, we obtain from Eq. (24)

$$\sigma_{xy}^{II} = -4e^2 M\alpha^2\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{f(E_{\mathbf{k},+})-f(E_{\mathbf{k},-})}{(E_{\mathbf{k},+}-E_{\mathbf{k},-})^3}. \quad (25)$$

In the low-temperature limit, Eq. (25) coincides with the expression previously found by Culcer et al.[16] Using Eq. (8) we can calculate the integral over momentum and obtain finally

$$\sigma_{xy}^{II} = \frac{e^2}{4\pi}\left[1-\frac{M}{\lambda_+}-\theta(\mu-M)\left(1-\frac{M}{\lambda_-}\right)\right]. \quad (26)$$

In the limit of weak SO interaction, $\alpha k_{F,\pm}\ll M$, we get from Eq. (26)

$$\sigma_{xy}^{II} \simeq \frac{e^2}{2\pi}\frac{m\alpha^2}{M}\left[\theta(M-\mu)\frac{\mu+M}{2M}+\theta(\mu-M)\right]. \quad (27)$$

The expression for σ_{xy}^{II} can be also presented in a different form demonstrating the topological character of this contribution.[20] Let us introduce a 3D unit vector $\mathbf{n}(\mathbf{k})$ at each point of two-dimensional momentum plane

$$\mathbf{n}(\mathbf{k})=\left(\frac{\alpha k_y}{\lambda(k)},\ -\frac{\alpha k_x}{\lambda(k)},\ -\frac{M}{\lambda(k)}\right). \quad (28)$$

By using the $\mathbf{n}$-field (28), we parametrize the manifold of 2×2 Hermitian matrices corresponding to Hamiltonian (1), since $H_0=\varepsilon_{\mathbf{k}}+\lambda(k)\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{k})$. At $\mathbf{k}=0$, the vector $\mathbf{n}$ is perpendicular to the $\mathbf{k}$ plane, whereas for large $|\mathbf{k}|\gg M/\alpha$, it lies in the $\mathbf{k}$ plane and is oriented perpendicular to the momentum $\mathbf{k}$, like that schematically presented in Fig. 3. The dependence $\mathbf{n}(\mathbf{k})$ is a mapping of the $\mathbf{k}$ plane to the unit sphere S_2. As we can see from Fig. 3, the total $\mathbf{k}$ plane maps onto the lower hemisphere of S_2.

Using Eqs. (24) and (28) we find that in terms of the $\mathbf{n}$ field, the contribution σ_{xy}^{II} can be written as

$$\sigma_{xy}^{II} = -\frac{e^2}{2}\int\frac{d^2\mathbf{k}}{(2\pi)^2}[f(E_{\mathbf{k},+})-f(E_{\mathbf{k},-})]\epsilon_{\alpha\beta\gamma}n_\alpha\frac{\partial n_\beta}{\partial k_x}\frac{\partial n_\gamma}{\partial k_y}, \quad (29)$$

where $\epsilon_{\alpha\beta\gamma}$ is the unit antisymmetric tensor. The integral

$$\Omega = \frac{1}{2}\int\frac{d^2\mathbf{k}}{(2\pi)^2}[f(E_{\mathbf{k},+})-f(E_{\mathbf{k},-})]\epsilon_{\alpha\beta\gamma}n_\alpha\frac{\partial n_\beta}{\partial k_x}\frac{\partial n_\gamma}{\partial k_y} \quad (30)$$

is the spherical angle on S_2 enclosed by two contours L_+ and L_-, where $L_\pm$ are the mappings of the Fermi surfaces (circles) $E_{\mathbf{k}_F,+}=\mu$ and $E_{\mathbf{k}_F,-}=\mu$ to the sphere S_2, respectively.

Notice that for $-M<\mu<M$, and in the case of $M/\alpha\ll k_{F,+}<2m\alpha(1+M/m\alpha^2)^{1/2}$, we obtain $\Omega\simeq 2\pi$, corresponding to the mapping of the $\mathbf{k}$ plane to the whole unit hemisphere. This is possible only in the case of large SO interaction or very small magnetization, when $M\ll m\alpha^2$.

There is also a contribution to the Hall conductivity from the vicinity of the Fermi surface. The calculation of the corresponding term using Eqs. (6), (14), and (15) gives us

$$\sigma_{xy}^{I} = -e^2 M\alpha\alpha^*\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{1}{(E_{\mathbf{k},+}-E_{\mathbf{k},-})^2}\left[\left(1-\frac{\tau_+}{\tau_-}\right)\right.$$

$$\left.\times\left(-\frac{\partial f(E_{\mathbf{k},+})}{\partial\varepsilon}\right)+\left(1-\frac{\tau_-}{\tau_+}\right)\left(-\frac{\partial f(E_{\mathbf{k},-})}{\partial\varepsilon}\right)\right]. \quad (31)$$

Here the presence of factor $(-\partial f/\partial\varepsilon)$ restricts Eq. (31) to the integral over Fermi surface (we consider only the low-temperature limit of $T/(\mu+M)\ll 1$). Calculating the integral over momentum (31), we find

$$\sigma_{xy}^{I} = -\frac{e^2\alpha\alpha^* M}{4}\left[\left(1-\frac{\tau_+}{\tau_-}\right)\frac{\nu_+}{\lambda^2(k_{F,+})}+\left(1-\frac{\tau_-}{\tau_+}\right)\frac{\nu_-}{\lambda^2(k_{F,-})}\right]. \quad (32)$$

Using Eqs. (11), (12), and (15), we can present σ_{xy}^{I} depending only on the parameters of the energy spectrum and density of states at the Fermi level

$$\sigma_{xy}^{I} = -\frac{e^2\alpha\alpha^* M^2(\xi_+-\xi_-)}{2}\left\{\frac{\xi_+}{\lambda_+[\nu_++\nu_-+M(\xi_+-\xi_-)]}\right.$$

$$\left.+\frac{\xi_-}{\lambda_-[\nu_++\nu_--M(\xi_+-\xi_-)]}\right\}, \quad (33)$$

where $\xi_\pm=\nu_\pm/\lambda_\pm$. As we see from Eq. (33), the contribution of σ_{xy}^{I} does not depend on impurities at all but is an intrinsic property of the crystal.

It follows from Eq. (33) that in the limit of large magnetization (weak SO interaction) $M\gg\alpha k_{F,+}$, the contribution of this term is

$$\sigma_{xy}^{I} \simeq \frac{e^2}{4\pi}\frac{m\alpha\alpha^*}{M}\left[\theta(M-\mu)\frac{m\alpha^2}{M}-4\ \theta(\mu-M)\left(\frac{m\alpha^2}{M}\right)^2\right]. \quad (34)$$

The dependence of two contributions to the Hall conductivity on the magnitude of coupling constant α for different positions of the Fermi level are presented in Figs. 4 and 5. We calculated them using the parameters $M=0.01$ eV and $m=m_0$. Note that the contribution of σ_{xy}^{I} does not depend on the impurity density but only on the ratio between the relaxation times for majority and minority carriers. The figures demonstrate that σ_{xy}^{I} and σ_{xy}^{II} can be of the same order of magnitude, and for $\mu<M$, the effect of σ_{xy}^{I} can dominate. The total Hall conductivity is presented in Fig. 6. It shows
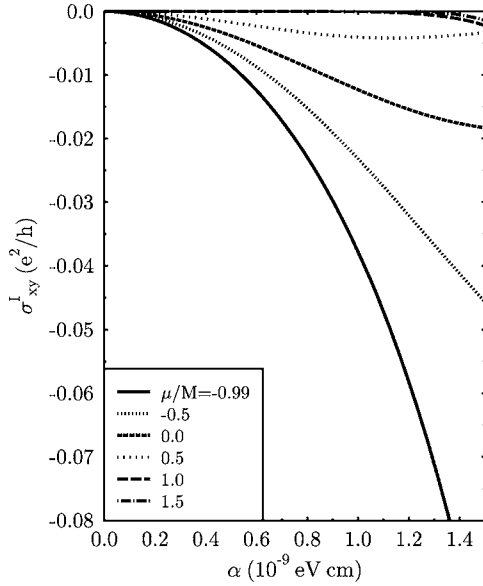
FIG. 4. σ^I_{xy} in units of e^2/h as a function of SO coupling constant α for different filling of the energy bands.

that the AHE can change the sign with the density of carriers.

The magnitude of SO coupling constant α can be roughly estimated if we take the SO splitting in a semiconductor like GaAs, $\Delta E_{SO} \simeq \alpha k = 0.5$ meV corresponding to the Fermi momentum $k = (2\pi N_s)^{1/2}$ for the carrier density $N_s = 10^{11}$ cm^{-2} as a characteristic value for GaAs/GaAlAs heterostructures.[26] It gives us $\alpha \simeq 6.3 \times 10^{-10}$ eV cm. The experimentally obtained magnitude of α in InGaAs/InAlAs heterostructures is about 10^{-9} eV cm.[27]

V. SEPARATION OF THE CONTRIBUTIONS TO HALL CONDUCTIVITY: GENERAL CASE

Let us consider now a general case of the energy spectrum described by a Hamiltonian H_0, which is a matrix in a certain
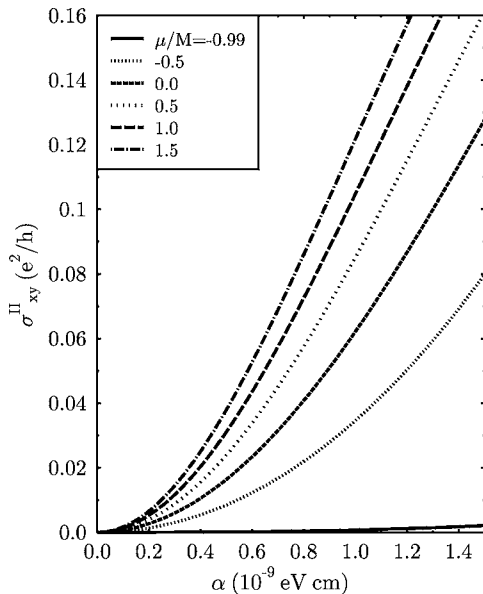

FIG. 5. σ^{II}_{xy} in units of e^2/h as a function of α for different μ.
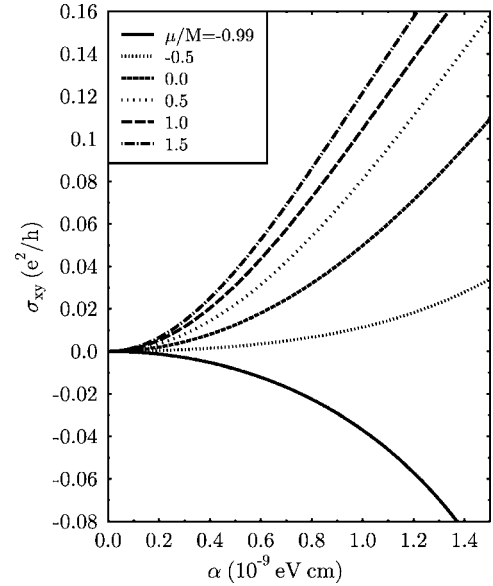

FIG. 6. Total Hall conductance $\sigma_{xy} = \sigma^I_{xy} + \sigma^{II}_{xy}$ in units of e^2/h as a function of α for different μ.

basis of wave functions. The off-diagonal conductivity can be calculated using the Kubo formula

$$\sigma_{xy}(\omega) = \frac{e^2}{\omega} \int \frac{d\varepsilon}{2\pi} \sum_{\mathbf{k}} \sum_{nn'mm'} (v_x)_{nm}$$

$$\times G_{\mathbf{k}\,mm'}(\varepsilon + \omega)(v_y)_{m'n'}G_{\mathbf{k}\,n'n}(\varepsilon), \qquad (35)$$

where we can take the basis of eigenfunctions of the Hamiltonian H_0 and assume a proper renormalization of velocities due to the impurity corrections. In this representation and in the absence of impurities, the Green's function $G^0_{\mathbf{k}}(\varepsilon)$ is diagonal. However, the scattering from impurities induces nonzero off-diagonal elements, as we can see from the above-considered model (1) leading to the self energy with nonvanishing off-diagonal matrix elements after diagonalization of the Hamiltonian H_0.

The off-diagonal matrix elements of the Green's function can be formally presented in the following form resulting from an expansion over off-diagonal self-energy matrix $\Sigma^{(\text{off})}$

$$G_{\mathbf{k}\,nn'}(\varepsilon) \simeq G^{(0)}_{\mathbf{k}\,nn}(\varepsilon)\Sigma^{(\text{off})}_{nn'}(\varepsilon)G^{(0)}_{\mathbf{k}\,n'n'}(\varepsilon)$$

$$= \frac{i\,\text{sgn}\,\varepsilon}{\tilde{\tau}_{nn'}}\frac{P_{nn'}(\mathbf{k})}{D_{\mathbf{k}}(\varepsilon)}, \quad n \neq n', \qquad (36)$$

where $\tilde{\tau}_{nn'}$ is the relaxation time associated with impurity-induced interband transitions, $P_{nn'}(\mathbf{k})$ is a coefficient determined by the specific form of Hamiltonian, $D_{\mathbf{k}}(\varepsilon) = \Pi_n(\varepsilon - E_{\mathbf{k}n} + i\,\text{sgn}\,\varepsilon/2\tau_n)$, $E_{\mathbf{k}n}$ is the dispersion of electrons in the nth subband, and τ_n is the corresponding relaxation time, which includes all intra- and interband transitions.

In the above-studied two-band model of Sec. II, the relaxation time $\tilde{\tau}_{12}$ equals to $\tilde{\tau}$ of Eq. (12), and $P_{12}(\mathbf{k}) = \alpha k/\lambda$. It should be emphasized that both $\tilde{\tau}_{nn'}$ and τ_n are inversely proportional to the density of impurities and to the matrix elements of scattering from impurities.

We consider first the terms in Eq. (35) with diagonal elements of the Green's functions (i.e., $m=m'$ and $n=n'$). In the static limit of $\omega \to 0$, the nonvanishing contribution comes from the states close to the Fermi surface, belonging to the same energy branch $E_{\mathbf{k}n}$ (in other words, we should also take $n=m$). Then we find

$$\sigma_{xy}^{IA} = e^2 \sum_n \sum_{\mathbf{k}} \left(-\frac{\partial f(E_{\mathbf{k}n})}{\partial \varepsilon} \right) (v_x)_{nn} (v_y)_{nn} \tau_n. \qquad (37)$$

This contribution to the Hall conductivity is proportional to the relaxation time of electrons. Correspondingly, it looks like the skew scattering.[7,8] However, in the case when the energy spectrum $E_{\mathbf{k}n}$ is symmetric with respect to $k_x \to -k_x$ or $k_y \to -k_y$, integral (37) vanishes, and the nonzero skew scattering can occur only through a more sophisticated mechanism involving SO interaction with impurity. In particular, there is no such term within the simple Rashba model.

Let us consider now the contribution from terms with $n=m$ and $n'=m'$ in Eq. (35). For $\omega \to 0$ and using Eq. (36) we find

$$\sigma_{xy}^{IB} = e^2 \sum_{n \neq n'} \sum_{\mathbf{k}} \left(-\frac{\partial f(E_{\mathbf{k}n})}{\partial \varepsilon} \right) (v_x)_{nn} (v_y)_{n'n'}$$

$$\times \frac{\tau_n}{\tilde{\tau}_{nn'}} \frac{P_{nn'}(\mathbf{k})}{\prod\limits_{m(\neq n)} (E_{\mathbf{k}n} - E_{\mathbf{k}m})}. \qquad (38)$$

As follows from Eq. (38), this contribution does not depend on the impurity density and on the scattering amplitude because both inverse relaxation times $1/\tau_n$ and $1/\tilde{\tau}_{nn'}$ are proportional to the same impurity density and scattering amplitude. Then the ratio $\tau_n/\tilde{\tau}_{nn'}$ can be presented using only the band parameters like in the case of the model in Sec. II, see Eq. (33). Hence, the contribution (38) should be also identified as an intrinsic mechanism even though it was calculated using the impurity scattering. In this result we recognize the well-known property of the side-jump mechanism.[7] The only difference from the earlier considered models is that the SO interaction is included here into the Hamiltonian of free electrons instead of the impurity potential.

Coming back to Eq. (35) with $n \neq m$ and $n' \neq m'$, we find one more contribution to the AHE, which has been discovered by Karplus and Luttinger[4] and recently interpreted in terms of the Berry phase in momentum space[16–18] (also called *intrinsic* mechanism of the AHE)

$$\sigma_{xy}^{II}(\omega) = \frac{e^2}{\omega} \int \frac{d\varepsilon}{2\pi} \sum_{\mathbf{k}} \sum_{n \neq m} (v_x)_{nm} G_{\mathbf{k}\,mm}(\varepsilon + \omega) (v_y)_{mn} G_{\mathbf{k}\,nn}(\varepsilon), \qquad (39)$$

where we omit some small corrections related to the off-diagonal elements of the Green's functions. After integrating over energy, this result can be presented in the following form:[16–18]

$$\sigma_{xy}^{II} = e^2 \sum_n \sum_{\mathbf{k}} f(E_{\mathbf{k}n}) \left(\frac{\partial A_y(\mathbf{k}n)}{\partial k_x} - \frac{\partial A_x(\mathbf{k}n)}{\partial k_y} \right), \qquad (40)$$

where

$$A_\alpha(\mathbf{k}n) = -i \left\langle \mathbf{k}n \left| \frac{\partial}{\partial k_\alpha} \right| \mathbf{k}n \right\rangle \qquad (41)$$

is the gauge potential in the momentum space related to the transformation of the Hamiltonian H_0 to the diagonal form. In a general case, this transformation is local in the $\mathbf{k}$ space, leading to the nonvanishing gauge potential $A_\alpha(\mathbf{k}n)$.

In the model of 2DEG with the Rashba Hamiltonian we can calculate explicitly the eigenfunctions

$$\left\langle \mathbf{k}, \pm \right| = \sqrt{\frac{\lambda(k) \pm M}{2\lambda(k)}} \left(1, -\frac{i\alpha(k_x - ik_y)}{M \pm \lambda(k)} \right). \qquad (42)$$

Then using Eq. (41) we find the gauge potential

$$\mathbf{A}(\mathbf{k}, \pm) = \left(-\frac{\alpha^2 k_y}{2\lambda(k)[M \pm \lambda(k)]}, \frac{\alpha^2 k_x}{2\lambda(k)[M \pm \lambda(k)]} \right), \qquad (43)$$

and from Eq. (39) we come again to the same result of Eq. (25). Note that Eq. (43) can be also found as a gauge potential corresponding to the local transformation of vector field (28) to the homogenous field oriented along axis z in the momentum space (like in the case of local transformations in the real space[15]).

In the 2D case, the flux of curl of the gauge potential $\mathbf{A}(\mathbf{k}, \pm)$ in Eq. (40) through the surface $E_{\mathbf{k},\pm} = \mu$ can be presented as the circulation of vector $\mathbf{A}(\mathbf{k}, \pm)$ along the circle (Fermi surface in 2D). In other words, the contribution of the filled states below the Fermi surface can be also presented by the integral of the gauge field over the Fermi surface. Recently, it was shown by Haldane[28] that such a reduction of the integral in momentum space takes place in any dimensionality, in accordance with the Landau concept of the Fermi liquid stating that the transport properties are fully determined by the properties of electrons near the Fermi surface.

VI. CONCLUSIONS

We calculated the off-diagonal conductivity using a model of 2DEG with Rashba-type SO interaction. Starting from the Kubo formalism, we found two contributions to the AHE, one of which, σ_{xy}^{II} presents the "intrinsic" mechanism and is related to the topology of electron energy bands in the momentum space. The other contribution σ_{xy}^{I} is related to impurities, and it is not vanishing in the limit of small impurity density. We demonstrate that both contributions can be of the same order of magnitude.

As we see from Eqs. (25) and (31), the Hall conductivity σ_{xy} is proportional to the second order of SO coupling α. The even order in α is related to the form of SO interaction in Eq. (1), for which the space inversion, $\mathbf{k} \to -\mathbf{k}$ is equivalent to the change of sign $\alpha \to -\alpha$. Indeed, the nonvanishing integral over momentum in Eq. (6) requires the integrand to be in-

variant with respect to $\mathbf{k} \rightarrow -\mathbf{k}$, which excludes any odd in α factors in σ_{xy}.

It is worth noting that the impurity-related mechanisms of AHE (both side-jump and skew scattering) lead to the non-vanishing off-diagonal conductivity in the first order of α.[7] The SO interaction included in the impurity scattering potential has the form of $i g V_{\mathbf{k}\mathbf{k}'} \boldsymbol{\sigma}(\mathbf{k} \times \mathbf{k}')$, where g is a constant and $V_{\mathbf{k}\mathbf{k}'}$ is the matrix element of impurity potential. This type of SO interaction is invariant with respect to spatial inversion. Thus, in the case of small SO interaction, the side-jump or skew scattering can be dominating as they both are of the first order of SO interaction.

The essential point of our consideration is that we do not neglect the impurity scattering leading to a finite relaxation time of electrons at the Fermi surface. Hence, the transition to the "clean limit" in our approach means that we should take the limit of small density of impurities only in the final formula for conductivity after the limit of $\omega \rightarrow 0$. The opposite way of calculation would lead us to break of continuous transition between the "clean limit" and any small impurity density. This point was completely neglected in the previous consideration of the intrinsic mechanism of AHE. Fortunately, it does not affect the "geometric ingredient" of AHE σ_{xy}^{II} but, as we demonstrated in this paper, it is crucial for the competing contribution σ_{xy}^{I}.

Note added: Recently, we were informed that similar results have been obtained by Sinitsyn *et al.*,[29] using a different method. We thank Jairo Sinova for this information.

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