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Nonequilibrium relaxation analysis of gauge glass models in two dimensions

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The Kosterlitz-Thouless (KT) transition is investigated for gauge glass models in two dimensions by means of the nonequilibrium relaxation (NER) method. Two kinds of models, which have the same symmetry, are analyzed. Using the scaling analysis of the NER function on a large lattice with $L=1000$, we confirm the KT transition numerically for both models. This indicates the stability of the KT phase against a small disorder, which was previously claimed by perturbation expansion and renormalization group arguments.

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The gauge glass (GG) model is a classical spin system with quenched disorder and has attracted much attention. It describes the thermodynamics of various systems such as disordered magnets with random Dzyaloshinskii-Moriya interaction,[1] Josephson-junction arrays with positional disorder in a magnetic field,[2] vortex glasses,[3] and crystal systems on disordered substrates.[4] In three dimensions, the spin-glass (SG) transition for a strongly disordered regime has been confirmed in the GG systems theoretically,[5,6] as well as experimentally.[7] In two dimensions, there is a controversy about the existence of the SG-like phase in the strongly disordered regime. Some numerical simulations suggested the quasi-long-range order,[8,9] while experimental observation[10] and other numerical simulations[6,11–14] deny it. Although the long-range SG order is denied in two dimensions following the Marmine-type argument,[15] there is a possibility of a phase in which the SG correlation decays in a power law.[16]

In weakly disordered regime, there has been another controversy about the existence of the reentrant transition from the Kosterlitz-Thouless (KT) phase[17] to the non-KT one. Earlier works with real-space renormalization group (RG) analysis suggest reentrance.[1,2,18,19] The analysis has been modified and provides the absence of it.[20–22] The same results are obtained by Monte Carlo simulations.[4,6,9,13,23,24] and the RG analyses.[20,25] With all these studies, the instability of the KT phase against a small disorder is pointed out by the perturbation expansion and the RG analysis.[26,27] However, it is denied by numerical simulations [4,6,9,13] and other RG analyses.[9,20–22] Analytically, the gauge theory, which has provided several exact relations in Ising SG models, shows the absence of reentrance if the KT phase appears in a finite disordered regime.[16] The same result is also derived from a dynamical point of view obtained by the dynamical gauge theory.[28] While the results of gauge theory are plausible, it is necessary to assume the stability of the KT phase. Investigation of the phase diagram of these random *XY* models is still at a primitive stage as compared to the case of the Ising model.

In the present study, we apply the nonequilibrium relaxation (NER) analysis to the GG models in two dimensions in order to clarify the stability of the KT phase against a small disorder. The NER method has been an efficient numerical technique for analyzing equilibrium phase transitions. It provides the critical temperature and critical exponents accurately for second-order transition systems,[29–31] and has been used successfully to study various problems, including frustrated and/or random systems.[32] It has also been extended beyond second-order transitions; e.g., the KT transition[31,33,34] and the first-order transition systems.[35] In the NER analysis, the equilibration step is not necessary. Simulation is made only up to steps when the asymptotic behavior indicates the equilibrium state. Thus, one can analyze large systems as compared with equilibrium simulations. This advantage becomes more effective for slow-relaxation systems.

We consider two kinds of GG models, which have the same symmetry, in two dimensions. The one that we call the "cosine-type" is based on the plane rotator model with random gauge variables

$$\mathcal{H} = -J\sum_{\langle ij\rangle} \cos(\theta_i - \theta_j + A_{ij}), \tag{1}$$

where $J>0$ and $0\leqslant\theta_i\leqslant 2\pi$. The summation $\langle ij\rangle$ is taken over all nearest-neighboring sites on the square lattice. An independent quenched random variable, A_{ij}, obeys the distribution function

$$P(A_{ij}) = \frac{\exp(D\cos A_{ij})}{2\pi I_0(D)}, \tag{2}$$

with $0\leqslant A_{ij}\leqslant 2\pi$, where $I_0(D)$ is the modified Bessel function. The function (2) is chosen so that the model satisfies the gauge symmetry that derives various analytic properties.[16,28] We consider that it behaves similarly to the Gaussian distribution and the difference between them is irrelevant (see Fig. 1, for example), since the same properties are derived by the gauge theory if one considers the other model, the "Villain-type" GG model. It is defined by the local Boltzmann factor based on the periodic Gaussian potential[36]

$$e^{V(\theta_i-\theta_j+A_{ij})} = \sum_{n=-\infty}^{\infty} e^{-K(\theta_i - \theta_j + A_{ij} - 2\pi n)^2/2}, \tag{3}$$

instead of the Hamiltonian (1), where $K=J/k_BT$. The random variable $-\infty<A_{ij}<\infty$ obeys the Gaussian distribution with mean 0 and variance $1/D$. Note that the thermodynamic be-

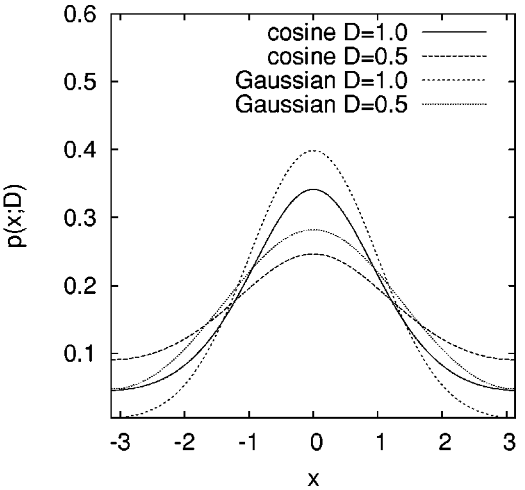

FIG. 1. Typical behaviors of distribution functions: the cosine-type, Eq. (2), with $D=1.0$ and 0.5, and the periodic Gaussian one, Eq. (3), with $K=1.0$ and 0.5, are shown.

havior for the Gaussian distribution is identical with that for the periodic Gaussian one, Eq. (3), with $0 \leqslant A_{ij} \leqslant 2\pi$. For both types of models, the parameter D controls the randomness, and the KT phase is observed in the pure case ($D=\infty$); the transition temperature for the pure case has been estimated numerically as $T_{\mathrm{KT}}^{0} \sim 0.894$ for the cosine-type[34] and $T_{\mathrm{KT}}^{0} \sim 1.33$ for the Villain-type.[37] The temperature is measured in a unit of J/k_{B} in the following.

The gauge theory[16,28] suggests that if the KT transition appears in the regime of finite disorder ($D<+\infty$), the line $K=D$, which is called the Nishimori line, intersects the KT transition line at the smallest D value in the KT phase, and the phase boundary below this temperature is likely parallel to the K axis (see Fig. 2). This point can be the multicritical point if the quasi-long-range SG order appears (indicated as the dashed line). Therefore, we may consider the transition only along the line $K=D$ in order to confirm the stability of the KT phase against a small disorder.

In the NER analysis of the KT phase, we choose a complete FM state ($\theta_i=0$ for all i) as the initial state, and calculate the relaxation of the FM order parameter $m(t) \equiv \Sigma_i[\langle \cos \theta_i(t)\rangle]/N$, where $\langle \cdots \rangle$ represents the dynamical average and $[\cdots]$ is the average for disorder. We use the skew

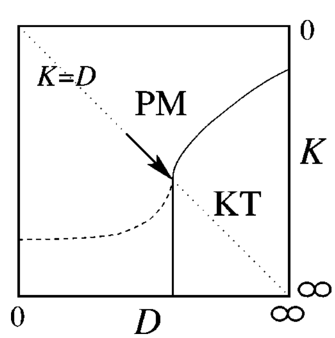

FIG. 2. Schematic phase diagram in two dimensions suggested by the gauge theory. The present analysis will be made along the Nishimori line $K=D$ (indicated by the arrow).

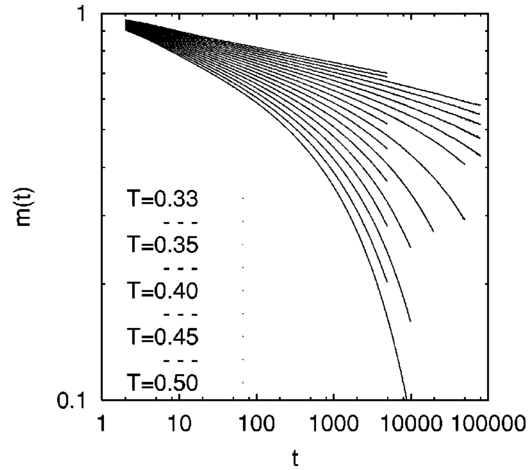

FIG. 3. The relaxation of FM order parameter $m(t)$ in double-log plot for the cosine-type model in $0.33 \leqslant T \leqslant 0.50$. With an interval of $\Delta T=0.01$, the corresponding temperature is increasing from top to bottom.

boundary condition for the purpose of efficient calculations. Furthermore, the spin space is discretized instead of the continuous one. That is the 1024-state clock model. For both models, calculations are carried out on the 1001×1000 lattice up to the observation time 10^5 Monte Carlo steps (MCS). About 320 independent runs are performed for averaging. The result for the cosine-type is plotted in Fig. 3. The size dependence is checked to be negligible, when we compare the data with those for 1501×1500. This reveals that the effect of the boundary condition on thermodynamic behaviors is negligible. The discretization of spin space is found to be irrelevant by comparing the data with the 1536-state model.

In Fig. 3, we observe quite slow relaxations of $m(t)$ in the low-temperature regime indicating the phase transition. For all data points in Fig. 3 (as well as Fig. 6), the statistical

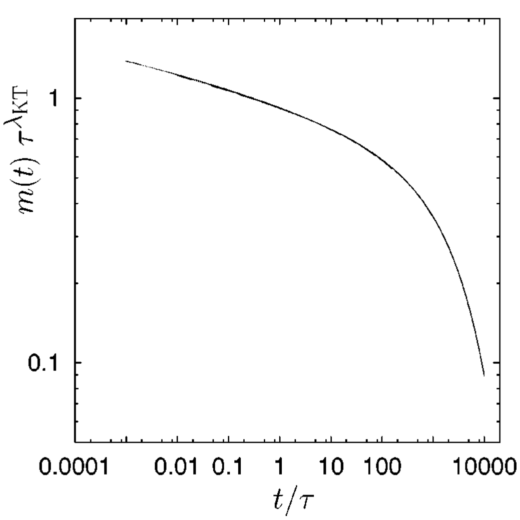

FIG. 4. Scaling plot of $m(t)$ for the cosine-type model fitted to Eq. (4) with appropriately chosen $\tau(T)$ (see Fig. 5) and $\lambda_{\mathrm{KT}}=0.050$. Curves for all the temperatures in $0.37 \leqslant T \leqslant 0.50$ are plotted.

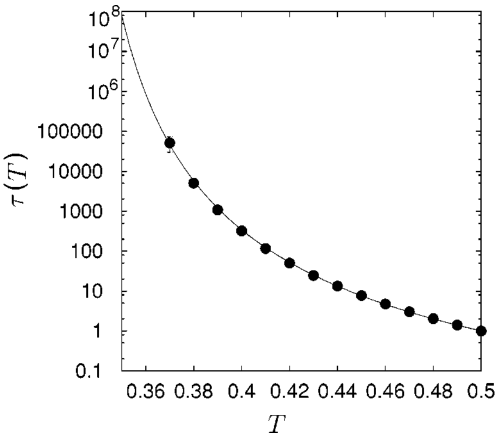

FIG. 5. Relaxation time for the cosine-type model in a unit of τ at $T=0.50$. The curve fitted to Eq. (5) with $T_{KT}=0.325$ is shown.

errors are less than the line thickness. Since it is always a power law below the KT transition point T_{KT}, one cannot evaluate the lower bound of T_{KT} numerically from $m(t)$. To estimate T_{KT} definitely, we have proposed a scaling relation[31,34] for $T>T_{KT}$,

$$m(t) = \tau^{-\lambda_{KT}} g(t/\tau), \tag{4}$$

where τ is the relaxation time depending on the temperature and λ_{KT} is the dynamic exponent (constant of temperature). Fitting the calculated curves to this relation, one obtains $\tau(T)$ for each temperature, and the exponent λ_{KT}. The resulting fit for the data in $0.37 \leqslant T \leqslant 0.50$ is shown in Fig. 4 with $\tau(T)$ plotted in Fig. 5 and $\lambda_{KT}=0.050(3)$. Note that we cannot use the data for temperatures closer to the transition point, since it shows almost a straight line in the observed time regime, which is useless for the fitting. The error bar of λ_{KT} is estimated by the range where the value of χ^2 is less than twice that of the minimum one. Then, we estimate T_{KT} from the estimated $\tau(T)$. As T approaches T_{KT}, the correlation length diverges exponentially[17] as $\xi = \tilde{a}\exp(\tilde{b}/\sqrt{T-T_{KT}})$. We expect that the relaxation time diverges in the same way,

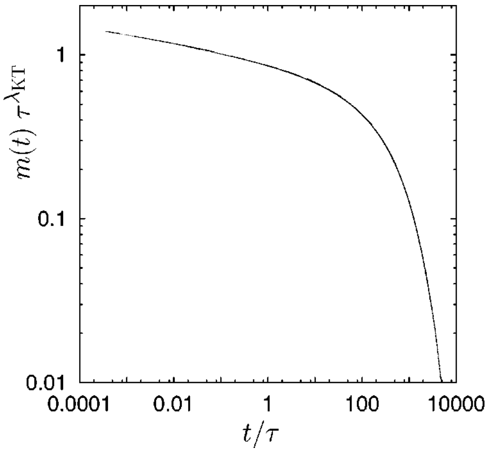

FIG. 7. Scaling plot of $m(t)$ for the Villain-type model fitted to Eq. (4) with appropriately chosen $\tau(T)$ (see Fig. 8) and $\lambda_{KT}=0.046(5)$. The curves for all the temperatures in $0.50 \leqslant T \leqslant 0.90$ are plotted.

$$\tau(T) = a\exp(b/\sqrt{T-T_{KT}}), \tag{5}$$

instead of a power-law divergence in standard second-order transitions. It is reasonable if one assumes the relation $\tau \sim \xi^z$ with a definite value of z. Using the χ^2 fitting with parameters a, b, and T_{KT}, we obtain the best fitting as shown in Fig. 5 with $T_{KT}=0.325(6)$.

The same analysis is applied to the Villain-type model. The result of $m(t)$ is plotted in Fig. 6. The scaling plot of the data in $0.50 \leqslant T \leqslant 0.90$ fitted to Eq. (4) with $\lambda_{KT}=0.046(5)$ is shown in Fig. 7. The estimated relaxation times are plotted in Fig. 8. The fitting to Eq. (5) is also shown with $T_{KT}=0.382(12)$.

For both models, we have clearly observed the KT transition with $T_{KT}>0$ on the Nishimori line indicating the stability of the phase against a small disorder. It is noted that, since the transition temperature T_{KT} is estimated by the scaling analysis, the reliability of it is less than those obtained by the NER analysis for second-order transitions. Therefore, the

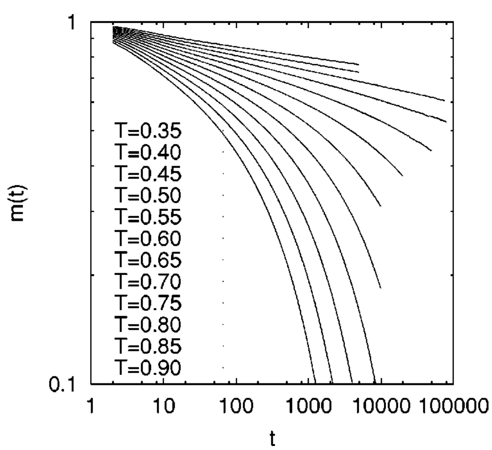

FIG. 6. The relaxation of the FM order parameter $m(t)$ in double-log plot for the Villain-type model.

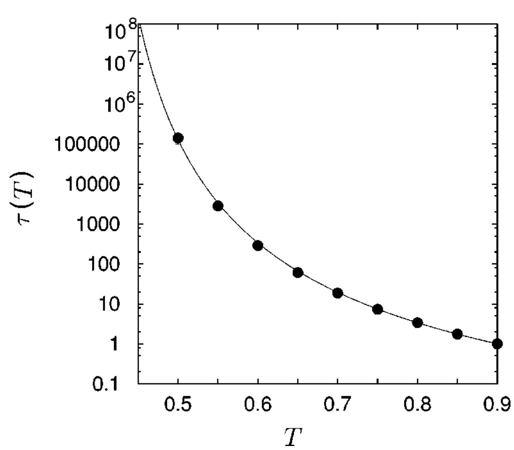

FIG. 8. Relaxation time for the Villain-type model in a unit of τ at $T=0.9$. The curve fitted to Eq. (5) with $T_{KT}=0.382$ is shown.

accuracy of the transition temperature is not so high. However, the increasing behavior of the relaxation time estimated from a standard dynamic scaling (4), which is observed independently for both models, strongly supports the divergence toward a definite temperature. Consequently, we have numerically confirmed the stability of the KT phase against a small disorder for the GG models in two dimensions, which is consistent with other analyses.[4,6,9,13,20–22] With the result of gauge theory,[16,28] this provides the strong evidence for the absence of reentrance.

Since the equilibration is not necessary in the NER analysis, the results do not suffer the difficulty due to the slow relaxation in frustrated systems. It is remarkable that the present simulations are performed on a large lattice with $L=1000$, which remove the finite-size effect and make it possible to observe thermodynamic behaviors. While the simulated size is large enough to eliminate the finite-size effect, the finite time observation cannot avoid the possibility of crossover phenomena that could modify the physics. It is necessary to proceed with further investigations for a longer time behavior to confirm the physics more reliably.

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[1] M. Rubinstein, B. Shraiman, and D. R. Nelson, Phys. Rev. B **27**, 1800 (1983).
[2] E. Granato and J. M. Kosterlitz, Phys. Rev. B **33**, R6533 (1986).
[3] M. P. A. Fisher, T. A. Tokuyasu, and A. P. Young, Phys. Rev. Lett. **66**, 2931 (1991).
[4] M.-C. Cha and H. A. Fertig, Phys. Rev. Lett. **74**, 4867 (1995).
[5] D. A. Huse and H. S. Seung, Phys. Rev. B **42**, R1059 (1990); J. D. Reger, T. A. Tokuyasu, A. P. Young, and M. P. A. Fisher, *ibid.* **44**, R7147 (1991); M. Cieplak, J. R. Banavar, and A. Khurana, J. Phys. A **24**, L145 (1991); M. S. Li, T. Nattermann, H. Rieger, and M. Schwartz, Phys. Rev. B **54**, 16024 (1996) J. Maucourt and D. R. Grempel, *ibid.* **58**, 2654 (1998); T. Olson and A. P. Young, *ibid.* **61**, 12467 (2000); M. J. P. Gingras, *ibid.* **44**, R7139 (1991); M. Cieplak, J. R. Banavar, M. S. Li, and A. Khurana, *ibid.* **45**, 786 (1991).
[6] J. M. Kosterlitz and M. V. Simkin, Phys. Rev. Lett. **79**, 1098 (1997); J. M. Kosterlitz and N. Akino, *ibid.* **81**, 4672 (1998); N. Akino and J. M. Kosterlitz, Phys. Rev. B **66**, 054536 (2002).
[7] R. H. Koch, V. Foglietti, W. J. Gallagher, G. Koren, A. Gupta, and M. P. A. Fisher, Phys. Rev. Lett. **63**, 1511 (1989); P. L. Gammel, L. F. Schneemeyer, and D. J. Bishop, *ibid.* **66**, 953 (1991); C. Dekker, W. Eidelloth, and R. H. Koch, *ibid.* **68**, 3347 (1992).
[8] Y.-H. Li, Phys. Rev. Lett. **69**, 1819 (1992).
[9] G. S. Jeon, S. Kim, and M. Y. Choi, Phys. Rev. B **51**, 16211 (1995); B. J. Kim, M. Y. Choi, S. Ryu, and D. Stroud, *ibid.* **56**, 6007 (1997); M. Y. Choi and S. Y. Park, *ibid.* **60**, 4070 (1999).
[10] C. Dekker, P. J. M. Wöltgens, R. H. Koch, B. W. Hussey, and A. Gupta, Phys. Rev. Lett. **69**, 2717 (1992).
[11] J. D. Reger and A. P. Young, J. Phys. A **26**, L1067 (1993).
[12] R. A. Hyman, M. Wallin, M. P. A. Fisher, S. M. Girvin, and A. P. Young, Phys. Rev. B **51**, 15304 (1995).
[13] J. Maucourt and D. R. Grempel, Phys. Rev. B **56**, 2572 (1997).
[14] H. G. Katzgraber, Phys. Rev. B **67**, 180402(R) (2003).
[15] H. Nishimori and H. Kawamura, J. Phys. Soc. Jpn. **62**, 3266 (1993).
[16] Y. Ozeki and H. Nishimori, J. Phys. A **26**, 3399 (1993); Y. Ozeki, *ibid.* **29**, 5805 (1996).
[17] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973); J. M. Kosterlitz, *ibid.* **7**, 1046 (1974).
[18] M. G. Forrester, H. J. Lee, M. Tinkham, and C. J. Lobb, Phys. Rev. B **37**, R5966 (1988).
[19] M. Paczuski and M. Kardar, Phys. Rev. B **43**, 8331 (1991).
[20] T. Nattermann, S. Scheidl, S. E. Korshunov, and M. S. Li, J. Phys. (Paris) I **5**, 565 (1995); S. E. Korshunov and T. Nattermann, Phys. Rev. B **53**, 2746 (1996).
[21] L.-H. Tang, Phys. Rev. B **54**, 3350 (1996).
[22] S. Scheidl, Phys. Rev. B **55**, 457 (1997).
[23] A. Chakrabarti and C. Dasgupta, Phys. Rev. B **37**, 7557 (1988).
[24] M. G. Forrester, S. P. Benz, and C. J. Lobb, Phys. Rev. B **41**, 8749 (1990).
[25] M. J. P. Gingras and E. S. Sørensen, Phys. Rev. B **46**, 3441 (1992).
[26] S. E. Korshunov, Phys. Rev. B **48**, 1124 (1993).
[27] C. Mudry and X.-G. Wen, Nucl. Phys. B **549**, 613 (1999).
[28] Y. Ozeki, J. Phys. A **36**, 2673 (2003).
[29] N. Ito, Physica A **192**, 604 (1993); **196**, 591 (1993).
[30] N. Ito, K. Hukushima, K. Ogawa, and Y. Ozeki, J. Phys. Soc. Jpn. **69**, 1931 (2000).
[31] Y. Ozeki, N. Ito, and K. Ogawa, in *Activity Report 1999* (Supercomputer Center, ISSP University of Tokyo, 2000), pp. 37–51.
[32] Y. Ozeki and N. Ito, J. Phys. A **31**, 5451 (1998); N. Ito, Y. Ozeki, and H. Kitatani, J. Phys. Soc. Jpn. **68**, 803 (1999); K. Ogawa and Y. Ozeki, *ibid.* **69**, 2808 (2000); Y. Ozeki and N. Ito, Phys. Rev. B **64**, 024416 (2001); Y. Ozeki, N. Ito, and K. Ogawa, J. Phys. Soc. Jpn. **70**, 3471 (2001).
[33] H. J. Luo, L. Schulke, and B. Zheng, Phys. Rev. Lett. **81**, 180 (1998); Phys. Rev. E **57**, 1327 (1998).
[34] Y. Ozeki, K. Ogawa, and N. Ito, Phys. Rev. E **67**, 026702 (2003); Y. Ozeki and N. Ito, Phys. Rev. B **68**, 054414 (2003);
[35] Y. Ozeki, K. Kasono, N. Ito, and S. Miyashita, Physica A **321**, 271 (2003).
[36] J. Villain, J. Phys. (Paris) **36**, 581 (1975).
[37] W. Janke and K. Nather, Phys. Lett. A **157**, 11 (1991); Phys. Rev. B **48**, 7419 (1993); P. Archambault, S. T. Bramwell, and P. C. W. Holdsworth, J. Phys. A **30**, 8363 (1997).