Rotation-induced three-dimensional vorticity in ^{4}He superfluid films adsorbed on a porous glass

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A detailed study of torsional oscillator experiments under steady rotation up to 6.28 rad/sec is reported for a ^{4}He superfluid monolayer film formed in 1-μm-pore-diameter porous glass. We discovered a new dissipation peak, whose height changes in proportion to the angular speed. The new peak appears at a temperature lower than the vortex pair unbinding peak observed in the static state. A model of three-dimensional (3D) coreless vortices ("pore vortices") is proposed to explain this additional peak in terms of 2D vortex pair dynamics in the presence of a flow field caused by 3D vortices. The new peak originates from dissipation close to the pore vortex lines, where a large superfluid velocity shifts the vortex pair unbinding dissipation to lower temperature. This explanation is consistent with the observation of nonlinear effects at high oscillation amplitudes.

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The system of superfluid ^{4}He films adsorbed in porous media provides a unique opportunity to study the interplay between two-dimensional (2D) and 3D physics.[1] The system shows similar behavior as 2D films (the Kosterlitz-Thouless transition[2]) such as the superfluid density being proportional to the transition temperature T_c, with an energy dissipation peak around T_c.[1,3,6] Simultaneously, the system behaves like 3D bulk ^{4}He with the critical index of superfluid density ρ_s close to 2/3 (Refs. 1, 4, and 5) and a sharp cusp of the specific heat[7] appearing at T_c. The thermally excited vortex-antivortex pairs (VAP's) play a crucial role in the Kosterlitz-Thouless 2D transition. For a λ transition of the 3D ^{4}He system, a similar mechanism was proposed where the vortex rings play an important role.[8] Multiple connectivity of superfluid film in porous media allows a variety of vortex configurations other than VAP's—e.g., vortex rings and 3D coreless pore vortices.[8–11] Therefore, the response of films on porous substrates is fundamentally different from that on a plane substrate and a 3D phase transition is expected.[10]

Experimentally the importance of 2D coherence to determine the transition temperature T_c has been reported.[3] Then what would be a manifestation of the 3D superfluidity of the system other than the sharp specific heat peak and critical behavior? One obvious thing is the appearence of 3D vortex lines through the system. Observations of these films under rotation are of great interest for understanding in this respect and further to see the role of these 3D vortices on the superfluid transition.

An effective experimental method to study the superfluidity of films, both on a plane and on a porous substrate, is the torsional oscillator (TO) technique.[12] The theory of the vortex dynamics of 2D films, probed in TO experiments, was developed by Ambegaokar, Halperin, Nelson, and Siggia (AHNS).[13] Applying the theory to experiments revealed that close to the critical temperature, the VAP's gave rise to a dissipation peak, where VAP's started to dissociate and free vortices appeared. The number of vortices in a 2D superfluid is dependent on the temperature and other parameters, which are difficult to measure or estimate independently. On the other hand, rotation with angular velocity Ω produces free vortices with temperature-independent areal density $2\Omega/\kappa$ ($\kappa=h/m$ is the ^{4}He circulation quantum), and this simplifies analysis of the dynamical properties. Adams and Glaberson[14] have shown that rotation-induced vortices in a plane superfluid film are responsible for major dissipation at the low-temperature side of the dissipation peak, where the density of thermally created vortices is negligible and dissipation is not detectable without rotation. They obtained valuable information on 2D vortex diffusivity from their experiment. TO with a resonance frequency of 0.5–1 kHz is appropriate for studying 2D vortex dynamics.

We report[15] on our studies of monolayer superfluid films covering a 1-μm-pore porous-glass substrate[3,16] under rotation by the TO technique.[17] We have chosen porous glass with this pore diameter because of the clear thin-film 2D feature of ^{4}He and apparent 3D connectivity. That is, the de Broglie wavelength of ^{4}He is much smaller than the pore diameter under the conditions of the present study. Our situation is closely connected to the one considered by theoretical works for thin films.[9–11] The 2D superfluid areal density ρ_{s2} is derived from the period change of the oscillation, while information about the vortex dynamics is obtained from the dissipation, characterized by the change ΔQ^{-1} of the quality factor Q. Traditionally, the response of the superfluid film in TO experiments is described by the "dielectric permeability" ε, which is the ratio of the effective mass of the superfluid component participating in the oscillation to the total or "bare" superfluid mass. Then $\Delta Q^{-1}=(M_s/M_{TO})\mathrm{Im}(\varepsilon^{-1})$, where $M_s=A\rho_{s2}$ is the total superfluid mass of the film of area A and M_{TO} is the oscillator's effective mass.[13,18]

The design and performance of the rotating dilution refrigerator used have been reported previously.[19] It can be rotated with an angular velocity up to 6.28 rad/sec, while the temperature of the mixing chamber is controlled in the range from 50 mK to 2 K. Our TO (Ref. 17) is made of Be—Cu alloy and filled with a stack of porous glass disks (diameter $\Phi=15$ mm, total height 9 mm) glued by a tiny amount of epoxy to the sidewall of the bob. It is attached to the mixing

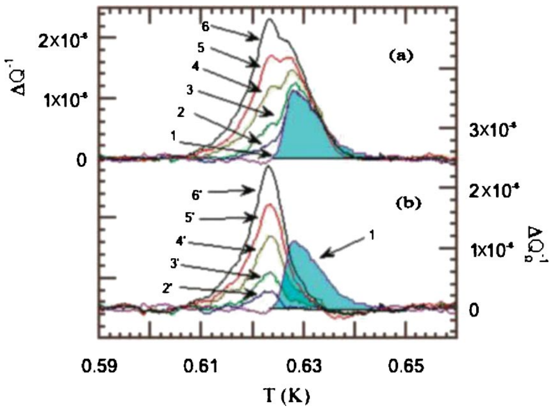

FIG. 1. (Color online) Energy dissipation peak under rotation (a), the number from 1 to 6 each corresponding to rotation speed 0(1), 0.79, 1.57, 3.14, 4.71, 6.28(6) ~ rad/sec, respectively. It is realized that the data in (a) are actually the summation of the two peaks—namely, the static peak 1 and a rotationally induced peak—by subtracting the former from the data in (a). The rotationally induced peak ΔQ_Ω^{-1} is displayed in (b) together with the static peak 1 with a dashed number, each corresponding to the number in (a). Data are all for the film with a superfluid transition temperature T_c=628 mK, which is determined as in Ref. 3. See also Fig. 2 caption.

chamber through a vibration isolation scheme as described in Ref. 3 and has a resonance at f_0=477 Hz and $Q=10^6$ at 100 mK.

The experimental procedure is first to measure without rotation the superfluid density (period shift) and dissipation (Q value) from ~50 mK to above T_c. The TO is then cooled down to ~$0.94T_c$, where T_c is the superfluid onset temperature determined in the same manner as in Ref. 3. Then rotation of the cryostat is started at a fixed angular velocity $\Omega \leqslant 6.28$ rad/sec, and the period and amplitude of the TO are recorded while the temperature is slowly raised up to $T = 1.05T_c$ during 20 h. The process is repeated for each Ω. This procedure provides reproducibility of the period and amplitude data of the TO. The observed temperature dependence of ΔQ^{-1} in the vicinity of the superfluid transition is shown for different angular velocities in frame (a) of Fig. 1. In order to see the effect of rotation, we have plotted $\Delta Q_\Omega^{-1} \equiv \Delta Q^{-1} - \Delta Q_s^{-1}$ in frame (b). It can be seen that ΔQ^{-1} nicely decomposes into two contributions from static and rotational states.

The observed relative location of the static peak to the superfluid density in our porous glass is different from the flat film case.[3] The peak in ΔQ_s^{-1} is located at a higher temperature than where the rapid change in superfluid density occurs. There was no detectable change in ρ_s for $0 \leqslant \Omega \leqslant 6.28$ rad/sec (see Fig. 2). Two pronounced features of dissipation are observed in our experiment. These are dramatically different from the response of flat films to rotation: (1) There is no low-temperature tail of dissipation, which characterizes the response of flat films, to rotation.[14] Instead, a sharp cutoff, in comparison to the flat film of dissipation at the low-temperature side of the peak is observed. (2) The double-peak structure of the dissipation was revealed with sharp peaks at constant temperatures independent of Ω.

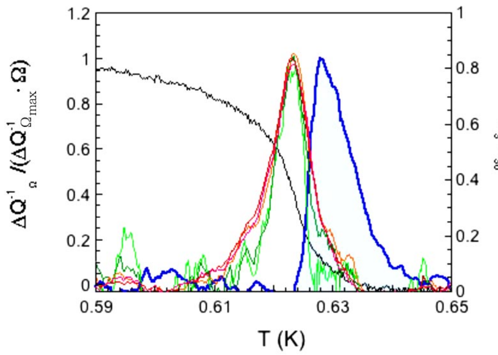

FIG. 2. (Color online) The blue peak is the static one specified as 1 in Fig. 1. The group of the left peaks is composed of rotation-induced ones, 2′–6′, for various Ω divided by Ω. The fact that all curves collapse more or less on the same curve proves linearity on Ω. In addition, the superfluid density scaled by its zero temperature value is displayed. The lagest slope is extrapolated to zero density to T_c=628 mK where the static peak is located (Ref. 3).

One peak, which is seen on the high-T side of the rapid change of the superfluid density in Fig. 2, is present at all Ω including the static state. The other peak appears only under rotation at a lower temperature. The peak appears at a temperature ~1% below T_c, where no excess dissipation was detected when Ω=0. The rotation-induced peak ΔQ_Ω^{-1} is normalized by the product of Ω and $\Delta Q^{-1}_{\Omega\ max=6.28\ rad/sec}$ as shown in Fig. 2. They all collapse onto the same curve except for small deviations on the low-temperature side. The ΔQ_Ω^{-1} peak height is proportional to Ω. The half width at half maximum of the rotational peak is about $0.01T_c$.

The amplitude of the torsional oscillation, used for recording of the data in Fig. 1, corresponds to the ac velocity amplitude of the TO, V_{ac}= ~0.03 cm/sec, dimensions evaluated at half its radius. This velocity amplitude is in the linear response region.[20] The peak height and shape are independent of excitation under the linear condition.[20] Figure 3 presents ΔQ_s^{-1} under static condition in the range of drive ac velocity, $\sim 0.09 < V_{ac} < \sim 0.9$ cm/sec. The nonlinear effect becomes pronounced above $V_{ac} \sim 0.25$ cm/sec.

Now, we shall utilize these nonlinear response data in

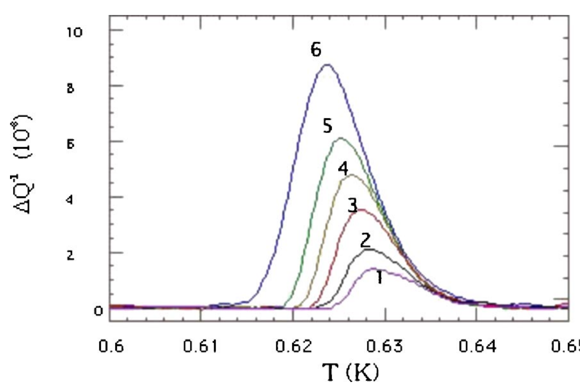

FIG. 3. (Color online) Energy dissipation curves in the static condition for the nonlinear regime. The ac drive velocity amplitude for each curve corresponds to V_{ac}=0.095(1), 0.19, 0.36, 0.52, 0.66, 0.94(6) cm/sec, respectively.

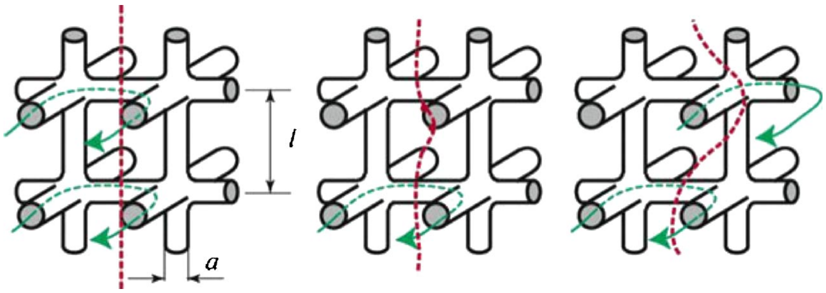

FIG. 4. (Color online) Vortex creep in the jungle gym structure: the vortex (dashed) line crosses a cylinder between two cells, creating a VAP in the cylinder. The arrowed curved lines show circulation around the cells where the vortex line is located.

explaining the rotational dissipation peak. We model the porous medium with a "jungle gym" structure,[9,10] which is a 3D cubic lattice of intersecting cylinders of diameter a with period l (Fig. 4). If this structure is the substrate for a superfluid film, multiple connectivity allows a new type of vortices, which is impossible on a flat single connected film: a 3D vortex without a 2D core—i.e., without any phase singularity in the film. The position of vortex line is confined within the pores around which there is a superfluid velocity circulation. Vortex motion is possible as a creep process, when the vortex line shifts from one pore to another by crossing cylinders. Crossing of a cylinder means creation of a VAP in the film, which covers this cylinder, as shown in Fig. 4.

If the vortex line moves over distances much larger than the size distance l, one may describe the vortex motion by a continuous displacement of the vortex line. But for small-amplitude oscillations the position vector can be defined only within the accuracy of l. In practice this means that the vortex is locked along some chain of pores by "intrinsic pinning" (a concept suggested for vortices in layered superconductors). However, if the vortex line cannot move, the question arises as to how it is possible to detect its presence at all. We argue that though the 3D vortex lines cannot move easily themselves once 3D coherence is established, they can influence the gas of VAP's, and the rotation experiment reveals this influence. This immediately explains the first feature of the rotation response: no low-temperature dissipation. Indeed, a significant number of VAP's is possible only close to the transition temperature. Since one can notice the rotation-induced vortex lines only via their effect on VAP's, one can reveal the presence of locked 3D coreless vortices also only at high temperatures where 2D VAP's are present.

In order to explain the second feature of the rotation response (the double-peak structure), we shall consider the effect of circular currents around the 3D vortices on the VAP dissipation. At the highest rotation speed $\Omega=6.28$ rad/sec, the average inter-3D vortex distance is $L_v=\sqrt{\kappa/2\Omega}\simeq 90\ \mu$m. This means that the 3D vortices are well separated: L_v is at least $\simeq 40$ times larger than ℓ under rotations in question. As a result, the 3D-vorticity-generated superfluid velocity relative to the substrate is effectively zero except for regions close to the 3D vortex centers. For each 3D vortex, the length of the squares of cylinders surrounding the center is $4(2n-1)\ell$. Here $n=1$ is of the closest square to the center, $n=2$ is of the second closest one, etc. The superfluid velocity for the circulation in the nth closest square is then $v_s^{(n)}=\kappa/\{4(2n-1)\ell\}$. (As n becomes larger, the current stream line changes to circular from square. Here, we neglect this change because it changes $v_s^{(n)}$ only by a numerical factor.) We assume $\ell=2.5a$, which agrees with the aspect ratio seen from scanning tunneling microscope (SEM) observations.[16] For $a=1\ \mu$m, we obtain $v_s^{(1)}=1.0$ cm/s, $v_s^{(2)}=0.33$ cm/s, $v_s^{(3)}=0.2$ cm/s, $v_s^{(4)}=0.14$ cm/s, $v_s^{(5)}=0.11$ cm/s, and so on.

Assuming that these circular flows around vortex lines produce the same effect on dissipation as large-amplitude ac flows, we shall utilize the corresponding large-amplitude ac dissipation data in Fig. 3, respectively at, $v_s^{(1)}\leqq 0.94$ cm/s, $v_s^{(2)}=0.36$ cm/s, $v_s^{(3)}=0.19$ cm/s, and $v_s^{(n)}=0.095$ cm/s for $n\geqslant 4$: The rotational dissipation ΔQ_Ω^{-1} mainly comes from the nonlinear dissipation of VAP's in $v_s^{(n)}$ with $n\leqslant 3$ of the 3D vortices:

$$\Delta Q_\Omega^{-1}=\frac{1}{S_{tot}}\sum_{n\geqslant 1} S_n(\Delta Q_{(n)}^{-1}-\Delta Q_s^{-1})$$

$$=C\sum_{n=1}^{3}(2n-1)(\Delta Q_{(n)}^{-1}-\Delta Q_s^{-1}), \quad (1)$$

where $\Delta Q_{(n)}^{-1}$ are data from Fig. 3 for $v_s^{(n)}$, $S_n\equiv 4(2n-1)\ell\pi a$ is the substrate surface area in the nth closest square, and $S_{tot}\equiv 2\pi a\ell\times L_v^2/\ell^2$ is the total substrate area per single vortex. The prefactor is $C=4\Omega\ell^2/\kappa\simeq 1.6\times 10^{-3}\Omega$ where Ω is given in radians per second. Then, the height of ΔQ_Ω^{-1} has turned to be

$$[\Delta Q_\Omega^{-1}]_{max}=3.5\times 10^{-10}\Omega. \quad (2)$$

This value is about 10 times smaller than the experimental one. The peak position, however, is in good agreement with

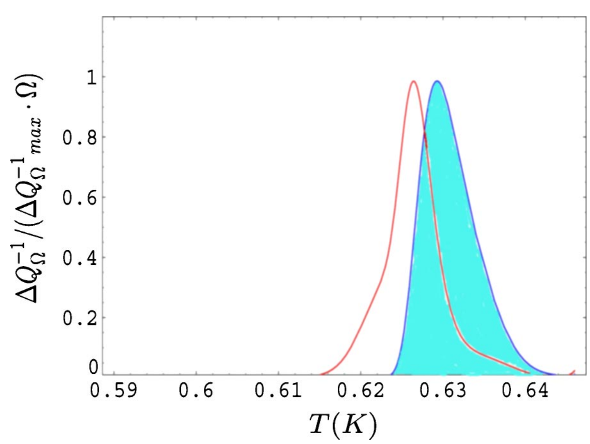

FIG. 5. (Color online) Calculated rotation-induced energy dissipation peak ΔQ_Ω^{-1} divided by Ω (thin line) along with the static state ΔQ_s^{-1} (blue color peak). These two peaks are scaled so that their peak height becomes unity to compare with Fig. 2.

the experiment. In Fig. 5, the calculated rotation-induced dissipation is plotted along with the experimental dissipation peak 1, of the static state shown in Fig. 2. The peak width is somewhat narrower than the experimental one. This disagreement does not appear to be serious bearing in mind the complicated geometry for ac and dc external flows and the fact that dissipation peak of He films in porous glass is very much dependent on the pore size. Indeed, the 1-μm system has $1/10$ of the dissipation peak height of flat film.[3] Another reason why our model predicts a weaker dissipation is that the large-amplitude ac flows oscillate and the oscillation should weaken their effect on dissipation compared with static circular flows generated by 3D vorticity.

In conclusion, we demonstrated the existence of an additional dissipation peak whose height is proportional to the rotation speed in a torsional-oscillator experiment with submonolayer superfluid ^{4}He films in porous glass. We relate the rotation-induced peak to 3D coreless pore vortices threading through the jungle gym structure cells. We argue that the rotation peak is due to high superfluid velocity around centers of 3D vortices. This effect is similar to the nonlinear effect of high-amplitude oscillating velocity, which was revealed both for flat and multiply connected porous substrates. Using our own measurements of this nonlinear effect for our system, we were able to reproduce the structure of the observed rotation dissipation peak within a reasonable numerical agreement. One can expect larger separation of the peaks for films condensed on smaller-pore-size porous glass. For larger-pore systems we would expect a merging of the two peaks into one, but it would still be different from the flat film response. Our preliminary data on a 10-μm porous medium is consistent with the present analysis.

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