Transverse plasmon in a two-dimensional electron gas at finite temperature

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The temperature dependence of the dispersion relation of the transverse plasmon in a two-dimensional electron gas is calculated using the Sommerfeld expansion for the retarded current response function and the self-consistent linear response approximation for the electromagnetic field. The index of refraction and the penetration depth for the electromagnetic wave propagating in the two-dimensional electron gas are obtained.

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I. INTRODUCTION

Recent experimental progress in semiconductor superlattices and quantum wells[1,2] have stimulated renewed theoretical interests in the electromagnetic properties of a stack of layers of two-dimensional electron gas (2DEG) systems.[3–5] The propagating electromagnetic wave in such a system, whose dispersion relation is equivalent to that of the transverse plasmon excitation in the many-electron system, seems to be of particular importance to understand the microscopic mechanism of the semiconductor laser such as AlGaAs/GaAs double-heterostructure diode laser.[6]

In 1977 Dahl and Sham[7] calculated the nonlocal dielectric tensor of quasi-two-dimensional electrons and investigated the electromagnetic properties, particularly in the limiting case where the retardation effects are negligible. In 1984 Toyoda, Gudmundsson, and Takahashi[8] calculated the retarded current response function of a 2DEG with finite thickness and obtained the dispersion relation of the transverse plasmon by fully taking account of the retardation effects. In the same year Tselis and Quinn[9] calculated the current response function of a 2DEG under a static magnetic field and also obtained the corresponding dispersion relation.

The response of the electron current to an external electromagnetic field can be calculated in terms of the retarded current response function. Then the dispersion relation of the transverse plasmon can be derived by using the linear response formula for the electron current expectation value with the Maxwell equation in a self-consistent way, i.e., the self-consistent linear response approximation (SCLRA).[8–10] The effects of electron-electron interaction on the current response can also be included by solving the Bethe-Salpeter equation for the two-particle Green's function.[11,12]

Although there have been further developments in the theoretical investigation of the transverse plasmon in 2DEG,[13,14] the thermal effects on the transverse plasmon have not been fully understood. The aim of this paper is to investigate the effects of finite temperature on the dispersion relation of the transverse plasmon in a 2DEG. There have been no calculations of the temperature dependence of the retarded current response function of a 2DEG nor that of the dispersion relation of the transverse plasmon, to the best of the knowledge of the present authors.

We first calculate the retarded current response function of a 2DEG at finite temperatures. Starting with the same formula for the retarded current response function given in Ref. 8, we use the Sommerfeld expansion to evaluate the temperature dependence up to T^2 order. Then, using the SCLRA, we obtain the dispersion relation of the transverse plasmon with the temperature dependence up to the same order. Using the dispersion relation we also calculate the index of refraction and the penetration length.

In Sec. II we define a 2DEG with a finite thickness in terms of the second quantized field operators describing the electrons. In Sec. III we define the retarded current response function following Ref. 8. In Sec. IV we apply the Sommerfeld expansion to the formula for the retarded current response function and calculate the temperature correction up to T^2 order in the small wave number expansion. In Sec. V we use the SCLRA to derive the dispersion relation of the transverse plasmon with the finite temperature effects. In Sec. VI we calculate the index of refraction and the penetration length.

II. CURRENT RESPONSE FUNCTION

We assume the electrons are confined in the x_1-x_2 plane by a potential $V(x_3)$, which yields the ground-state wave function $\chi(x_3)$ satisfying

$$\left[\frac{-\hbar^2}{2m}\frac{d^2}{dx_3^2} + V(x_3)\right]\chi(x_3) = E_3\chi(x_3), \tag{1}$$

where m is the electron effective mass and E_3 is the lowest energy eigenvalue with respect to the confining potential. The second quantized field operator Ψ_s that describes the dynamics of the electrons in the x_1-x_2 plane has the form

$$\Psi_s(\mathbf{r},x_3,t) = \chi(x_3)\Phi_s(\mathbf{r},t), \tag{2}$$

where s is the spin variable and $\mathbf{r}\equiv(x_1,x_2)$. The field operator $\Phi_s(\mathbf{r},t)$ and $\Phi_s^\dagger(\mathbf{r},t)$ are assumed to satisfy the equal-time canonical anticommutation relation

$$\Phi_s(\mathbf{r},t)\Phi_{s'}^\dagger(\mathbf{r}',t) + \Phi_{s'}^\dagger(\mathbf{r}',t)\Phi_s(\mathbf{r},t) = \delta_{ss'}\delta(\mathbf{r}-\mathbf{r}'). \tag{3}$$

The electromagnetic electron current density along the x_1-x_2 plane is

$$J_\mu^{(3D)}(\mathbf{r},x_3,t) = -ej_\mu^{(3D)}(\mathbf{r},x_3,t) - \frac{e^2}{mc}|\chi(x_3)|^2 A_\mu(\mathbf{r},x_3,t)\sum_s \Phi_s^\dagger(\mathbf{r},t)\Phi_s(\mathbf{r},t), \tag{4}$$

where the electron charge is $-e$ and $\mu=1,2$. The three-dimensional current density operator $j_\mu^{(3D)}(\mathbf{r},x_3,t)$ is defined by

$$j_\mu^{(3D)}(\mathbf{r},x_3,t) = \frac{-i\hbar}{2m}\sum_s \Psi_s^\dagger(\mathbf{r},x_3,t)(\partial_\mu - \overleftarrow{\partial_\mu})\Psi_s(\mathbf{r},x_3,t). \tag{5}$$

Using the two-dimensional current density operator

$$j_\mu(\mathbf{r},t) = \frac{-i\hbar}{2m}\sum_s \Phi_s^\dagger(\mathbf{r},t)(\partial_\mu - \overleftarrow{\partial_\mu})\Phi_s(\mathbf{r},t), \tag{6}$$

the three-dimensional current density operator can be written as

$$j_\mu^{(3D)}(\mathbf{r},x_3,t) = |\chi(x_3)|^2 j_\mu(\mathbf{r},t). \tag{7}$$

Because the wave function χ is real, the component $j_3^{(3D)}$ simply vanishes.

The retarded current response function with respect to the current density $j_\mu^{(3D)}$ is defined by

$$\Lambda_{\mu\nu}^{(3D)}(\mathbf{r},x_3,t;\mathbf{r}',x_3',t') = -i\theta(t-t')\langle[j_\mu^{(3D)}(\mathbf{r},x_3,t),j_\nu^{(3D)}(\mathbf{r}',x_3',t')]\rangle. \tag{8}$$

Here the notation $\langle\cdots\rangle$ stands for the grand canonical ensembel expectation value with the Hamiltonian of the form

$$H = \sum_s \int d^2\mathbf{r}\Phi_s^\dagger(\mathbf{r},t)\{-\hbar^2(2m)^{-1}\nabla^2 - \mu\}\Phi_s(\mathbf{r},t), \tag{9}$$

where μ is the chemical potential for the two-dimensional electrons including the effects of E_3. We define the two-dimensional transverse current density

$$j_\mu^t(\mathbf{r},t) = \sum_{\nu=1}^{2}\mathbf{T}_{\mu\nu}^{2D}(\nabla)j_\nu(\mathbf{r},t) \quad (\mu=1,2), \tag{10}$$

with the transverse projection operator

$$\mathbf{T}_{\mu\nu}^{2D}(\nabla) = \delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\nabla^2}, \quad \nabla^2 = \partial_1^2 + \partial_2^2. \tag{11}$$

Then, the transverse part of the response function can be written as

$$\Lambda_{\mu\nu}^{(3D)t}(\mathbf{r},x_3,t;\mathbf{r}',x_3',t') = |\chi(x_3)\chi(x_3')|^2\Lambda_{\mu\nu}^t(\mathbf{r},t;\mathbf{r}',t'), \tag{12}$$

where $\Lambda_{\mu\nu}^t$ is the two-dimensional transverse current response function defined by

$$\Lambda_{\mu\nu}^t(\mathbf{r},t;\mathbf{r}',t') = -i\theta(t-t')\langle[j_\mu^t(\mathbf{r},t),j_\nu^t(\mathbf{r}',t')]\rangle \tag{13}$$

and j_μ^t is the transverse part of the two-dimensional current density defined by Eq. (6). Because $\Lambda_{3\nu}^t=\Lambda_{\mu 3}^t=0$, it is straightforward to show $\Sigma_{\mu=1}^3\partial_\mu\Lambda_{\mu\nu}^{(3D)t}=0$. Using the Fourier transforms of the transverse current response function

$$\Lambda_{\mu\nu}^t(\mathbf{r},t;\mathbf{r}',t') = \frac{1}{(2\pi)^3}\int d^2\mathbf{k}\int d\omega e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')-i\omega(t-t')}\Lambda_{\mu\nu}^t(\mathbf{k},\omega) \tag{14}$$

and the transverse projection operator

$$\mathbf{T}_{\mu\nu}(\mathbf{k}) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \tag{15}$$

the retarded current response function reduces to the form

$$\Lambda_{\mu\nu}^t(\mathbf{k},\omega) = \mathbf{T}_{\mu\nu}(\mathbf{k})\Lambda(\mathbf{k},\omega). \tag{16}$$

With the explicit time dependence of the electron field operators, which is given by the Hamiltonian (9), the real part of the retarded current response function $\Lambda(\mathbf{k},\omega)$ becomes

$$\mathrm{Re}\,\Lambda(\mathbf{k},\omega) = \frac{\hbar}{\pi mk}\int_0^\infty dp f(p)p^2\{-u_+ + \mathrm{sgn}(u_+)\theta(u_+^2-1)\times\sqrt{u_+^2-1} + u_- - \mathrm{sgn}(u_-)\theta(u_-^2-1)\sqrt{u_-^2-1}\}, \tag{17}$$

where $u_\pm$, $\mathrm{sgn}(x)$, and $\theta(x)$ are defined by

$$u_\pm = \frac{m\omega}{\hbar kp} \pm \frac{k}{2p}, \tag{18}$$

$$\mathrm{sgn}(x) \equiv \frac{x}{|x|}, \quad \theta(x) \equiv \frac{1}{2}[1+\mathrm{sgn}(x)], \tag{19}$$

and $f(p)$ is the Fermi distribution function

$$f(p) = \frac{1}{1+e^{\beta[\varepsilon(p)-\mu]}}, \quad \varepsilon(p) = \frac{\hbar^2p^2}{2m}. \tag{20}$$

Similarly, the imaginary part of $\Lambda(\mathbf{k},\omega)$ can be written as

$$\mathrm{Im}\,\Lambda(\mathbf{k},\omega) = \frac{\hbar}{\pi mk}\int_0^\infty dp f(p)p\left\{\theta\left(p-\left|\frac{m\omega_+}{\hbar k}\right|\right)\sqrt{p^2-\left|\frac{m\omega_+}{\hbar k}\right|^2} - \theta\left(p-\left|\frac{m\omega_-}{\hbar k}\right|\right)\sqrt{p^2-\left|\frac{m\omega_-}{\hbar k}\right|^2}\right\}, \tag{21}$$

where $\omega_\pm$ is defined by

$$\omega_\pm = \omega \pm \frac{\hbar k^2}{2m}. \tag{22}$$

In the zero-temperature limit the integrations in Eqs. (17) and (21) can be carried out.[8] For finite temperatures, the integrals cannot be calculated analytically.

III. SOMMERFELD EXPANSION

The Fourier transforms of the retarded current response function given by Eqs. (17) and (21) contain the Fermi distribution functions in the integrals and the Sommerfeld expansion can be applied to calculate the temperature dependence. The result of the Sommerfeld expansion of the real part of the retarded current response function $\mathrm{Re}\,\Lambda$ can be cast in the form

$$\mathrm{Re}\,\Lambda(\mathbf{k},\omega) = R_0(\mathbf{k},\omega) + R_2(\mathbf{k},\omega) + O(T^4), \tag{23}$$

where R_0 is the zero-temperature term and R_2 is the T^2 correction term. Since we are interested in the dispersion relation of the transverse plasmon, we consider the small k region. We have calculated R_0 and R_2 by expanding them in powers of k. The zero-temperature term has been calculated in Ref. 8:

$$\begin{aligned} R_0 &= \frac{2\mu}{\pi\hbar}\left\{\frac{-1}{2} + \frac{u^2}{12} + \left(\frac{\nu}{u}\right)^2 - g_+(u,\nu) + g_-(u,\nu)\right\} \\ &= \frac{\mu}{4\pi\hbar}\left\{\left(\frac{u}{\nu}\right)^2 + \frac{1}{2}\left(\frac{u}{\nu}\right)^4 + O(u^6)\right\}, \end{aligned} \tag{24}$$

where u, ν, $v_\pm$, and $g_\pm$ are defined as

$$u \equiv \frac{k}{k_F}, \quad \nu \equiv \frac{m\omega}{\hbar k_F^2}, \quad v_\pm \equiv \frac{\nu}{u} \pm \frac{u}{2} \tag{25}$$

and

$$g_\pm(u,\nu) \equiv \frac{1}{3u}\mathrm{sgn}(v_\pm)\,\theta(v_\pm^2 - 1)\sqrt{(v_\pm^2 - 1)^3}. \tag{26}$$

Calculating the T^2 correction term in the Sommerfeld expansion, we have obtained

$$\begin{aligned} R_2 &= \frac{\pi\mu}{12\hbar(\beta\mu)^2 u}\left[\mathrm{sgn}(v_-)\frac{1}{\sqrt{v_-^2 - 1}}\theta(v_-^2 - 1)\right. \\ &\qquad \left. - \mathrm{sgn}(v_+)\frac{1}{\sqrt{v_+^2 - 1}}\theta(v_+^2 - 1)\right] \\ &= \frac{\pi\mu}{12\hbar(\beta\mu)^2}\left\{\left(\frac{u}{\nu}\right)^2 + \frac{3}{2}\left(\frac{u}{\nu}\right)^4 + O(u^6)\right\}, \end{aligned} \tag{27}$$

where $\beta = 1/k_B T$. Combining Eqs. (24) and (27), the explicit temperature dependence of the real part of Λ becomes

$$\begin{aligned} \mathrm{Re}\,\Lambda(\mathbf{k},\omega) &= \frac{\mu}{4\pi\hbar}\left[\left\{1 + \frac{\pi^2}{3\beta^2\mu^2}\right\}\left(\frac{u}{\nu}\right)^2 + \frac{1}{2}\left\{1 + \frac{\pi^2}{3\beta^2\mu^2}\right\}\right. \\ &\qquad \left. \times\left(\frac{u}{\nu}\right)^4 + O(u^6;T^4)\right]. \end{aligned} \tag{28}$$

In order to express the result of the Sommerfeld expansion applied to the imaginary part of the retarded current response function $\mathrm{Im}\,\Lambda$, we introduce the function $f_\pm$ such that

$$f_\pm(u,\nu) = \frac{1}{3u}\theta(1 - v_\pm^2)\sqrt{(1 - v_\pm^2)^3}. \tag{29}$$

Because of the step funcion θ in this function, we have to consider the following four domains separately in the first quadrant of the k-ω plane:

$$\text{I} \quad \frac{\hbar}{2m}(k + k_F)^2 - \frac{\hbar k_F^2}{2m} < \omega, \tag{30}$$

$$\text{II} \quad \left|\frac{\hbar}{2m}(k - k_F)^2 - \frac{\hbar k_F^2}{2m}\right| < \omega < \frac{\hbar}{2m}(k + k_F)^2 - \frac{\hbar k_F^2}{2m}, \tag{31}$$

$$\text{III} \quad \omega < \frac{-\hbar}{2m}(k - k_F)^2 - \frac{\hbar k_F^2}{2m}, \tag{32}$$

$$\text{IV} \quad \omega < \frac{\hbar}{2m}(k - k_F)^2 - \frac{\hbar k_F^2}{2m}. \tag{33}$$

The result of the Sommerfeld expansion of the imaginary part $\mathrm{Im}\,\Lambda$ can be written as

$$\mathrm{Im}\,\Lambda = I_0 + I_2 + O(T^4), \tag{34}$$

where I_0 is the zero-temperature term and I_2 is the T^2 correction term. The zero-temperature term I_0 has been calculated in Ref. 8,

$$I_0 = \frac{2\mu}{\pi\hbar}\{f_+(u,\nu) - f_-(u,\nu)\}. \tag{35}$$

We have calculated the temperature dependent term I_2. The results depend on the domains

$$\text{I} \quad I_2 = 0, \tag{36}$$

$$\text{II} \quad I_2 = \frac{-\pi m}{6\hbar^3\beta^2 k_F k}\frac{1}{\sqrt{1 - v_-^2}}, \tag{37}$$

$$\text{III} \quad I_2 = \frac{\pi m}{6\hbar^3\beta^2 k_F k}\left\{\frac{1}{\sqrt{1 - v_+^2}} - \frac{1}{\sqrt{1 - v_-^2}}\right\}, \tag{38}$$

$$\text{IV} \quad I_2 = 0. \tag{39}$$

Note that in the domains I and IV, the imaginary part vanishes. This means even at finite temperatures there are the same undamped domains as the zero-temperature case. That the imaginary part vanishes in the domain I even at finite temperatures indicates that the possibility of the undamped transverse plasmon mode as in the zero-temperature case.

To conclude this section it should be remarked that mathematically the Sommerfeld expansion fails in the vicinity of these domain boundaries defined by Eqs. (30)–(33). These singularities are responsible for the Friedel oscillations and further investigation of the temperature effects in the boundary areas seems to be of great interest.

IV. TRANSVERSE PLASMON

In the linear response theory, the response of the electrons to a transverse electromagnetic vector potential $\vec{A}^t$ is expressed as the expectation value of the current density

$$\langle J_\mu^{(3D)t}(\mathbf{r},x_3,t)\rangle = \frac{-ne^2}{mc}|\chi(x_3)|^2 A_\mu^t(\mathbf{r},x_3,t) - \frac{e^2}{\hbar c}|\chi(x_3)|^2\int_{-\infty}^{\infty}dt'\int d^2\mathbf{r}'\int dx_3'|\chi(x_3')|^2 \times\sum_{\nu=1}^{3}\Lambda_{\mu\nu}^t(\mathbf{r},t;\mathbf{r}',t')A_\nu^t(\mathbf{r}',x_3',t'), \tag{40}$$

where the retarded current response function $\Lambda_{\mu\nu}^t$ is calculated for an ideal electron gas and n is the two-dimensional electron number density. The basic idea of the self-consistent linear response approximation is to assume the electromagnetic field $\vec{A}^t$ in the linear response formula is the induced field due to the transverse current of the electrons and satisfies the Maxwell equation. Theoretically this idea can be realized by replacing the current term in the Maxwell equation by the linear response expectation value

$$\left(\sum_{\eta=1}^{3}\partial_\eta^2 - c^{-2}\partial_t^2\right)A_\mu^t(\mathbf{r},x_3,t) = -4\pi c^{-1}\langle J_\mu^{(3D)t}(\mathbf{r},x_3,t)\rangle. \tag{41}$$

As the right-hand side contains the electromagnetic vector potential $\vec{A}^t$ linearly, this equation yields a linear wave equation. If certain conditions are met, it may produce a propagating electromagnetic wave. We can also eliminate $\vec{A}^t$ from Eqs. (40) and (41) to obtain a wave equation for the expectation value of the electron transverse current $\langle J_\mu^{(3D)t}\rangle$, whose propagating mode corresponds to the transverse plasmon of the electrons. In the following we shall pursue this approach. In order to eliminate the electromagnetic field $\vec{A}^t$ from the Maxwell equation and the linear response expectation value for the electron transverse current, it is convenient to introduce the Fourier transform of Eq. (40)

$$\langle J_\mu^{(3D)t}(\mathbf{k},k_3,\omega)\rangle = \frac{-ne^2}{2\pi mc}\int dq_3\rho(k_3-q_3)A_\mu^t(\mathbf{k},q_3,\omega) - \frac{e^2}{2\pi\hbar c}\rho(k_3)\Lambda_{\mu\nu}^t(\mathbf{k},\omega)\int dq_3\rho(-q_3) \times A_\nu^t(\mathbf{k},q_3,\omega), \tag{42}$$

and the Fourier transform of the Maxwell equation (41)

$$A_\mu^t(\mathbf{k},k_3,\omega) = \frac{4\pi}{c}D^{-1}(\mathbf{k},k_3,\omega)\langle J_\mu^{(3D)t}(\mathbf{k},k_3,\omega)\rangle, \tag{43}$$

where we have defined $D(\mathbf{k},k_3,\omega)\equiv k^2+k_3^2-c^{-2}(\omega+i0^+)^2$. Using Eq. (43) it is straightforward to eliminate the vector potential $\vec{A}^t$ from Eq. (42). After eliminating the vector potential field, we find

$$\langle J_\mu^{(3D)t}(\mathbf{k},k_3,\omega)\rangle = \frac{-ne^2}{2\pi mc}\int dq_3\rho(k_3-q_3)\Omega_{\mu\nu}(\mathbf{k},q_3,\omega) \times\langle J_\nu^{(3D)}(\mathbf{k},q_3,\omega)\rangle - \frac{e^2}{2\pi\hbar c}\rho(k_3)\Lambda_{\mu\lambda}^t(\mathbf{k},\omega) \times\int dq_3\rho(-q_3)\Omega_{\lambda\nu}(\mathbf{k},q_3,\omega) \times\langle J_\nu^{(3D)}(\mathbf{k},q_3,\omega)\rangle, \tag{44}$$

where we have defined

$$\Omega_{\mu\nu}(\mathbf{k},k_3,\omega) \equiv \frac{4\pi}{c}D^{-1}(\mathbf{k},k_3,\omega)\mathbf{T}_{\mu\nu}^{3D}(\mathbf{k},k_3) \tag{45}$$

with the transverse projection operator

$$\mathbf{T}_{\mu\nu}^{3D}(\mathbf{k},k_3) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2+k_3^2}. \tag{46}$$

Equation (44) gives the collective excitation of the transverse current of the electrons and contains the dispersion relation of the transverse plasmon. Since we are interested in the transverse plasmon propagating in the x_1-x_2 plane, we consider the case $k_3=0$. Then, the first term on the right-hand side of Eq. (44) can be written as

$$\frac{-ne^2}{2\pi mc}\int dq_3\rho(-q_3)\Omega_{\mu\nu}(\mathbf{k},q_3,\omega)\langle J_\nu^{(3D)}(\mathbf{k},q_3,\omega)\rangle = \frac{-ne^2}{2\pi mc}\Gamma(\mathbf{k},\omega)\langle J_\mu^t(\mathbf{k},\omega)\rangle, \tag{47}$$

where we have defined

$$\Gamma(\mathbf{k},\omega) \equiv \frac{4\pi}{c}\int dq_3\rho(-q_3)\rho(q_3)D^{-1}(\mathbf{k},q_3,\omega). \tag{48}$$

The second term on the right-hand side of Eq. (44) can be written as

$$-\frac{e^2}{2\pi\hbar c}\rho(k_3)\Lambda_{\mu\lambda}^t(\mathbf{k},\omega)\int dq_3\rho(-q_3)\Omega_{\lambda\nu}(\mathbf{k},q_3,\omega) \times\langle J_\nu^{(3D)}(\mathbf{k},q_3,\omega)\rangle = \frac{-e^2}{2\pi\hbar c}\Lambda_{\mu\lambda}^t(\mathbf{k},\omega)\Gamma(\mathbf{k},\omega)\langle J_\lambda^t(\mathbf{k},\omega)\rangle. \tag{49}$$

We write the left-hand side of Eq. (44) as $\langle J_\mu^t(\mathbf{k},\omega)\rangle$. Then we combine Eqs. (44), (47), and (49) to obtain

$$\langle J_\mu^t(\mathbf{k},\omega)\rangle = \frac{-ne^2}{2\pi mc}\Gamma(\mathbf{k},\omega)\langle J_\mu^t(\mathbf{k},\omega)\rangle - \frac{e^2}{2\pi\hbar c}\Lambda_{\mu\lambda}^t(\mathbf{k},\omega)\Gamma(\mathbf{k},\omega)\langle J_\lambda^t(\mathbf{k},\omega)\rangle, \tag{50}$$

which can be written as

$$\left\{1 + \frac{e^2 n}{2\pi mc}\Gamma(\mathbf{k},\omega) + \frac{e^2}{2\pi\hbar c}\Lambda(\mathbf{k},\omega)\Gamma(\mathbf{k},\omega)\right\}\langle J^t_\mu(\mathbf{k},\omega)\rangle = 0. \tag{51}$$

This is the SCLRA equation for the transverse current of a 2DEG.[8]

V. DISPERSION RELATION

If the SCLRA equation for the two-dimensional transverse current density (51) has a nontrivial solution for the expectation value of the electron current, then it can be concluded that there is a collective excitation mode of the electron transverse current, i.e., the transverse plasmon. Therefore, the dispersion relation for the transverse plasmon can be obtained by solving the equation

$$\frac{1}{\Gamma(\mathbf{k},\omega)} + \frac{e^2 n}{2\pi mc} + \frac{e^2}{2\pi\hbar c}\Lambda(\mathbf{k},\omega) = 0. \tag{52}$$

To proceed further it is necessary to specify the functional form of the wave function $\chi(x_3)$. In Ref. 8 three different models for the wave function were calculated and it was found that there are no essential differences. Therefore, here we consider the Gaussian model

$$|\chi(x_3)|^2 = \frac{a}{\sqrt{2\pi}}\exp\left(\frac{-a^2x^2}{2}\right). \tag{53}$$

The Fourier transform of this Gaussian model is

$$\rho(q_3) = \exp\left(\frac{-q_3^2}{2a^2}\right). \tag{54}$$

By making use of the fact that $\rho(q)$ has a sharp peak at $q=0$, we approximate the integral in Γ by[8]

$$\Gamma(\mathbf{k},\omega) = \frac{-4\pi^{3/2}ac}{\omega^2 - c^2k^2 + i\epsilon}. \tag{55}$$

Substituting this result into Eq. (52), we obtain

$$\omega^2 - c^2k^2 - \frac{2\sqrt{\pi}ae^2n}{m} - \frac{2\sqrt{\pi}ae^2}{\hbar}\Lambda(\mathbf{k},\omega) = 0. \tag{56}$$

From this equation one can derive the dispersion relation for the transverse plasmon. In order to calculate the temperature dependence of the dispersion relation of the transverse plasmon under the condition of a fixed electron number, the chemical potential appeared in the Sommerfeld expansion of $\Lambda(\mathbf{k},\omega)$ given by Eq. (27) must be expressed as a function of the electron number density and temperature. The relation between the chemical potential and the other thermodynamical variables in general depends on the temperature. However, in the two-dimensional case, the result

$$k_F^2 = 2\pi n \equiv \frac{2m\mu}{\hbar^2} \tag{57}$$

holds for the T^2 order. Using the result of the Sommerfeld expansion for $\Lambda(\mathbf{k},\omega)$ given by Eq. (27) and taking into account the chemical potential relation (57), we have obtained

$$\omega^2 = \omega_p^2 + c^2k^2 + \frac{\omega_p^2\mu}{2m}\left[1 + \frac{\pi^2}{3}\left(\frac{k_BT}{\mu}\right)^2\right]\left(\frac{k}{\omega}\right)^2 + \frac{\omega_p^2\mu^2}{2m^2}\left[1 + \pi^2\left(\frac{k_BT}{\mu}\right)^2\right]\left(\frac{k}{\omega}\right)^4 + O(k^6;T^4), \tag{58}$$

where the plasma frequency ω_p is defined as

$$\omega_p^2 = \frac{2\sqrt{\pi}ane^2}{m}. \tag{59}$$

This dispersion relation (58) has a strong resemblance to the three-dimensional result.[6] The main reason for this resemblance between the present two-dimensional case and the three-dimensional case is that the Maxwell equation for the present calculation is written in the three-dimensional space. Only the current term in the Maxwell equation (41) has been assumed to be confined in the x_1-x_2 plane. The two-dimensional characteristics appears through the current response function.

VI. CONCLUDING REMARKS

We have calculated the temperature dependence of the real part and the imaginary part of the retarded current response function of an ideal two-dimensional electron gas. Then, using the SCLRA, we have derived the dispersion relation of the transverse plasmon at finite temperatures up to the T^2 order in the Sommerfeld expansion. We have shown that in the domain I the transverse plasmon is undamped even at finite temperatures and obtained the dispersion relation (58).

Starting with the Maxwell equation (41) and the linear response equation (42), we have eliminated the transverse vetor potential and obtained the SCLRA equation for the transverse current expectation value of the electrons (51), from which we have derived the dispersion relation (58). It is also possible to eliminate the transverse current expectation value of the electrons and to obtain a SCLRA equation for the transverse vector potential. Then we can derive the same dispersion relation. That is, the dispersion relation for the transverse electromagnetic wave propagating in the two-

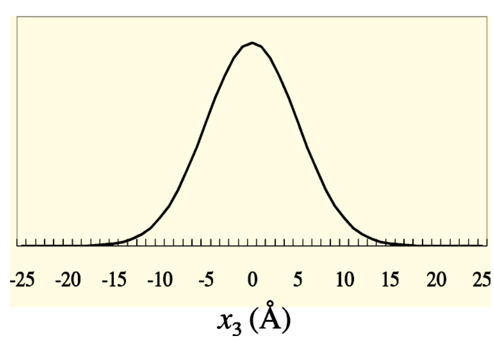

FIG. 1. Plot of $|\chi(x_3)|^2$ given by Eq. (53) with $a=2\times10^7$ (cm^{-1}).

dimensional electrons is identical to that of the transverse plasmon.

In view of a possible application of the present theoretical formulation to the physics of laser diodes, it seems to be of great physical interest to compute the index of refraction and the penetration depth. By keeping up to the k^2 terms in the dispersion relation (58), we have calculated the index of refraction

$$n_{2D} = \sqrt{\frac{\omega^2 - \omega_p^2}{\omega^2 + \Delta}} \tag{60}$$

and the penetration depth

$$\delta_{2D} = \frac{c}{\sqrt{\omega_p^2 - \omega^2}} \sqrt{1 + \frac{\Delta}{\omega^2}}, \tag{61}$$

where we have defined

$$\Delta = \frac{\omega_p^2 \mu}{2mc^2}\left[1 + \frac{\pi^2}{3}\left(\frac{k_B T}{\mu}\right)^2\right]. \tag{62}$$

As it can be expected from the form of the dispersion relation (58), the results (60) and (61) have the same forms as the three-dimensional case.[15] The two-dimensional characteristics appears in Δ which is the result of the retarded current response function of the 2DEG.

It is interesting to estimate the plasma frequency ω_p for the parameters of the inversion layer of a typical GaAs semiconductor. Although the electron number density n and the effective mass m can be specified without ambiguity,[1] the parameter a, which is directly connected to the thickness of the two-dimensional electron system, is a subtle quantity due mainly to the Gaussian assumption of $|\chi(x_3)|^2$ given by Eq. (53). For the thickness 50 Å[1,6] we assume $a = 2 \times 10^7$ (cm^{-1}), which corresponds to the Gaussian $|\chi(x_3)|^2$ shown in Fig. 1. With the values of the parameters $n = 10^{12}$ (cm^{-2}) and $m = 0.074 m_e$ (Ref. 2) the plasma frequency is $\omega_p = 4.9 \times 10^{14}$ (s^{-1}), which corresponds to the vacuum electromagnetic wave length $\lambda_{\text{vac}} = 3.8 \times 10^3$ (nm). This value is much larger than the typical wave length of GaAs laser 840 (nm). These values of the parameters also give $\Delta = 0.10 \times (n^2 + 2.3 \times 10^{19} T^2)$. Although this Δ is much smaller than ω_p^2, the temperature term can make significant contribution comparable to the n^2 term at room temperature and the effects may be observed in the measurement of the penetration depth given by Eq. (61).

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