Magneto-optical rotation of a one-dimensional all-garnet photonic crystal in transmission and reflection

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We present spectra of transmittance, reflectance, and Faraday rotation of transmitted and reflected light for a periodic garnet multilayer structure with a central defect layer. The multilayer consists of alternating layers of bismuth and yttrium iron garnet, is 1.5 μm thick, and was prepared by pulsed laser deposition. For the reflection measurements, a silver mirror was evaporated on top of the multilayer. Faraday rotation is strongly enhanced at resonances in transmission and reflection. The peak value obtained at 748 nm in transmission is 5.3° and at 733 nm in reflection is 18°. A single layer BIG film of equivalent thickness shows 2.2° Faraday rotation at 748 nm. We find rather good agreement between measured and calculated spectra. Using calculations of the distributions of light intensities at different wavelengths inside the multilayer, we are able to give consistent qualitative explanations for the enhancement of Faraday rotation. We also find numerically that—at moderate strengths of the optical resonances—a linear relation exists between Faraday rotation and the intensity integrated over all magneto-optically active layers, if absorption is neglected.

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I. INTRODUCTION

Transparent multilayers with periodic regions where single layers have thicknesses of a quarter wavelength in the respective material can be denoted one-dimensional photonic crystals.[1] The term "magnetophotonic crystal" (MPC) was coined by the researchers who presented the first concepts and experimental realizations of one-dimensional photonic crystals that contained magneto-optical (MO) layers.[2] Since then, more advanced theoretical concepts have been proposed, aiming especially at possible applications in optical isolators.[3–5] In MPCs, it is possible to enhance the MO rotation of the polarization plane of incident linearly polarized light by means of optical resonances.

Iron garnets exhibit low absorption and strong Faraday rotation in visible and/or infrared light. The MO effect is enhanced by different substitutions, e.g., when the material contains bismuth. Iron garnets are therefore the materials of choice for MPCs. The first MPC contained iron garnet in a defect layer (i.e., in a layer of a thickness that breaks the periodicity of the photonic crystal) of the MPC and nonmagnetic dielectric materials in the periodic regions,[2] while we have recently reported an all-garnet MPC.[6] The operation of MPCs in reflection has also been considered[7] and has a few advantages as compared to the operation in transmission: The multilayer structure (which is challenging to prepare) can be much shorter, the problem of limited bandwidth due to narrow transmission resonances vanishes, and the reduction of signal intensity due to splitting of the transmission resonances for left- and right-circularly polarized waves is not any issue either. In this paper, we present the MO response in transmission and in reflection from an all-garnet MPC.

The most common and simplest physical explanation for the enhancement of Faraday rotation in MPCs and Fabry-Perot interferometers is that the light reflects back and forth many times inside the defect layer or cavity before it passes out. Faraday rotation, which is proportional to the optical path length and nonreciprocal, is therefore enhanced. However, it is not so straightforward to find the number of multiple reflections and this number is not necessarily an integer. Inoue *et al.*[2,8] have suggested that the enhanced Faraday rotation is probably related to the localization of light in MPCs. We will show in Sec. IV that intensity distributions of the light inside the MPC are useful for qualitative explanations. In Sec. V, we search for a quantitative relation between the angle of FR and the intensities integrated over the MO layers in the MPC. We find numerically that the enhanced angle of FR is to a good approximation a single-valued and linear function of the integrated intensity for our case of weak optical resonances.

II. EXPERIMENTAL AND CALCULATIONAL PROCEDURE

The MPC discussed here was prepared on a (111)-oriented gadolinium gallium garnet (GGG) substrate and consists of 17 layers of bismuth iron garnet $Bi_3Fe_5O_{12}$ (BIG) and yttrium iron garnet $Y_3Fe_5O_{12}$ (YIG), see Fig. 1. This sample has already been described in more detail in Ref. 6. The central BIG layer has thickness of λ_0/n_{BIG}, $\lambda_0 = 748$ nm, and all other layers have quarter-wavelength thickness in the respective material with regard to λ_0. This means that the central BIG layer is 279 nm thick whereas the other BIG and YIG layers have thicknesses of 70 and 81 nm, respectively. The sum of the thicknesses of all BIG layers is 836 nm and of all YIG layers 652 nm. At 748 nm, BIG and YIG have refractive indices $n_{BIG}=2.7$ and $n_{YIG}=2.3$. For measurements in reflection, a silver layer was deposited on top of the MPC by thermal evaporation, after the MPC had been characterized by all other measurements.

The setup used for optical and MO measurements in transmission and reflection is shown in Fig. 1. In order to eliminate the influence of reflections from components of the

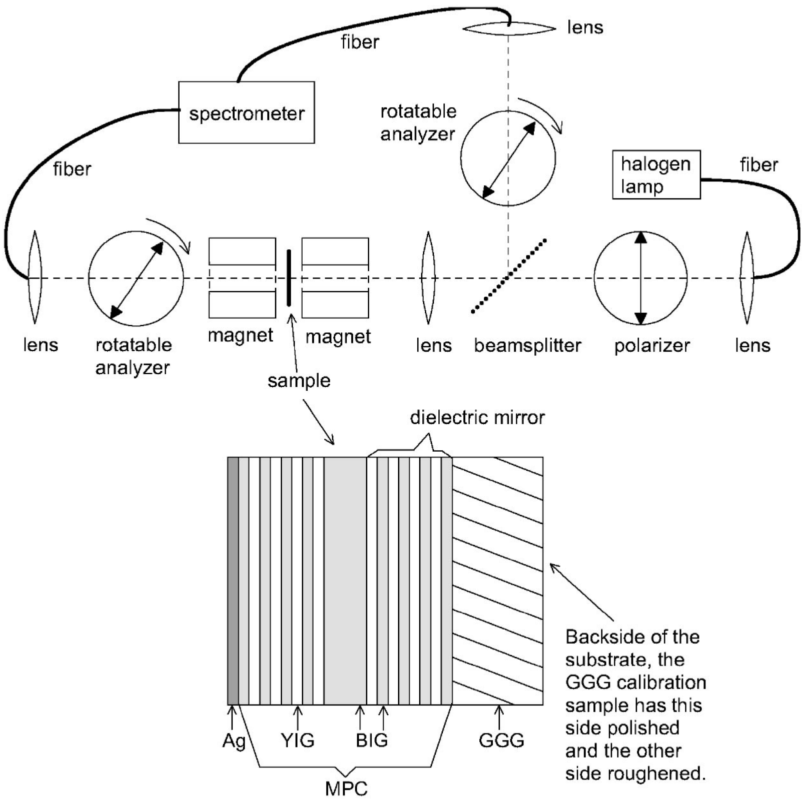

FIG. 1. Setup for measurement of transmittance, reflectance, and Faraday rotation of transmitted and reflected light. The position of the rotatable analyzer depends on whether the measurement is performed in transmission or reflection.

setup and from the backside of the substrate, the spectra from a single-side polished GGG substrate were substracted from the MPC/GGG spectra at each analyzer angle. Reflectance was measured with respect to an iridium mirror whose reflectance was assumed to be 72%.[9] Due to our rather imprecise calibration procedures, we estimate the measurement errors to be 5%–10% in measurements in reflection.

Calculations of transmittance, reflectance, Faraday rotation in transmission (FR), and Faraday rotation of reflected light (FRR) were carried out with parameters from Ref. 10 as described in Ref. 6, using the full 4×4 transfer matrix method.[11,12] Faraday ellipticity was neglected. We can neglect the Faraday rotation in the YIG layers because Faraday rotation is almost two orders of magnitude lower in YIG than in BIG. The intensity profiles inside the multilayer were calculated by dividing every single layer into several slices. The complex refractive index of the silver layer, $0.27+5i$, was taken from Ref. 9. Interface and surface roughnesses have not been included into the calculations.

III. MEASURED AND CALCULATED SPECTRA

Measured and calculated spectra of transmittance, reflectance, FR, and FRR as well as the measured degree of polarization are shown in Fig. 2. The transmittance spectrum exhibits a peak at λ_0, in the center of the photonic band gap, which extends approximately from 670 to 830 nm. The high reflectance of the multilayer for wavelengths in the photonic band gap is caused by the alternating BIG and YIG layers of quarter-wavelength thickness that act as dielectric mirror. The specifically chosen thickness of the central layer, which breaks the periodicity, generates the transmission resonance. FR is enhanced at λ_0 and reaches the value of 5.3°, which is higher than both the value of 2.2° that would be obtained if all BIG layers were assembled into a single layer film and the value of 3.9° that would be obtained from the MPC if all YIG layers (which are assumed not to be magneto-optically active) were replaced by BIG layers of the same thickness.[10]

The spectrum of FRR possesses two resonances, at 733 and 809 nm, where FRR reaches values of 18° and 14°, respectively. Both the central layer and the interface between the top BIG and the silver layer break the periodicity of the photonic crystal and create one resonance each. Reflectance is in general high in the photonic bandgap, except close to the resonances where the absorption in the material is enhanced by the resonant multilayer structure. The depolarization of approximately 5% at the resonance wavelengths of the measurements in reflection is probably caused by the combined effect of interface and surface roughness and multiple reflections.

The agreement between calculated and measured spectra is better for transmission than for reflection. One of the reasons for this could be that we did not know the real optical properties of the interface between the top BIG film and the silver layer. The discrepancy between the measured and calculated peak heights is also stronger for reflection because the role of interface roughness increases when the resonances become stronger. The resonances of reflected light are stronger than those of transmitted light because the reflected light can be considered to pass twice through the structure.

IV. QUALITATIVE EXPLANATIONS IN TERMS OF INTENSITY PROFILES

Intensity profiles can give a qualitative physical understanding of the influence of the multilayer structure on the

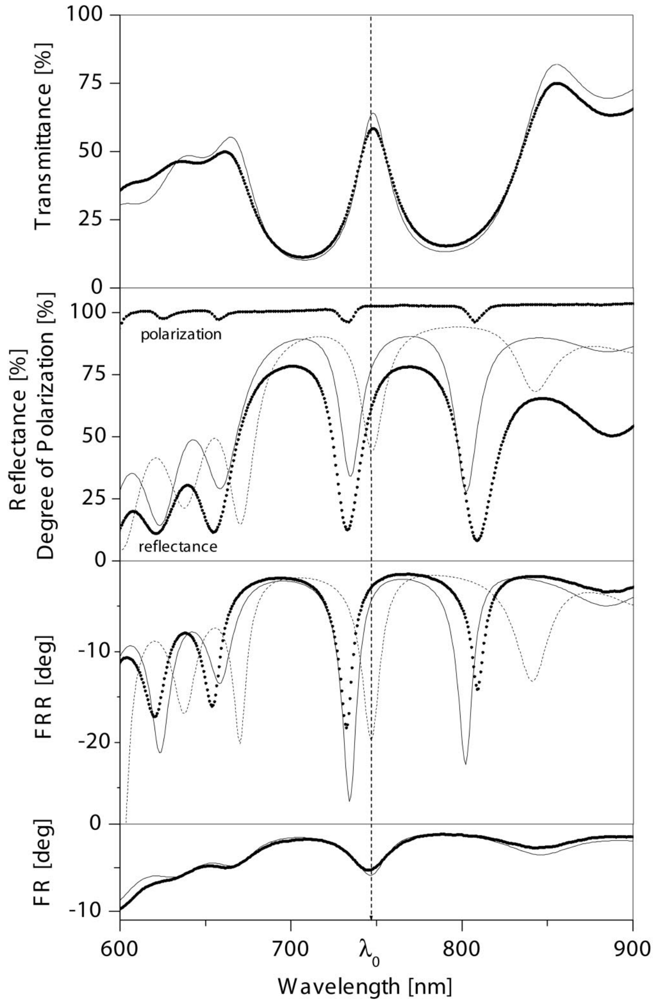

FIG. 2. Spectra of transmittance, reflectance, Faraday rotation in reflection (FRR), Faraday rotation in transmission (FR), and the degree of polarization of the BIG-YIG MPC. Symbols represent measured data, solid lines represent calculated spectra of the MPC, and dashed lines represent spectra calculated for a MPC where the thickness of the top BIG layer has been increased by 64%. Measurements and calculations of reflectance and FRR were performed on the MPC covered by a silver layer.

optical and magneto-optical properties, as is explained in the following. In this section, we plot intensities just as squares of the electric field amplitudes, $(E_+^i(\lambda)+E_+^r(\lambda))^2+(E_-^i(\lambda)+E_-^r(\lambda))^2$, where the electric field amplitudes E have indices $i,r,+$, and $-$ for incident (or right-going), reflected (or left-going), left- and right-circularly polarized waves. It should be noted, though, that the refractive indices of BIG and YIG decrease with increasing wavelength,[10] and that light intensity (or irradiance) is proportional to the refractive index.[13]

The intensities in Fig. 3 were calculated for the resonance wavelengths obtained in the calculations of Fig. 2 (734.5, 748, and 802 nm) assuming that light of intensity 1 is incident from the GGG substrate on the MPC.

Figure 3(a) shows the intensities at the wavelength λ_0 of the transmission resonance. The intensity is significantly enhanced in the central MO BIG layer, which explains the enhancement of FR.

In Fig. 3(b), we show the intensity distribution calculated at 734.5 nm for our MPC with silver cover. Light intensity is enhanced both in the central BIG layer and in the BIG layers between the central layer and the silver layer. As BIG is both MO and absorbent and silver is strongly absorbent, this explains the strong enhancement in FRR and the decrease in reflectance.

At 802 nm [see Fig. 3(c)], the highest intensity occurs in the BIG layer in front of the silver layer. Especially in this case, strong losses should be expected in the silver layer, which explains why absorption is higher (reflectance is lower) than at 734.5 nm, although the extinction coefficients of BIG and YIG decrease towards longer wavelengths.[10]

The phase shift at the silver mirror can in principle be compensated easily by increasing the thickness of the topmost BIG layer by 64%. The resulting calculated spectra and the intensity profile are shown in Figs. 2 and 3(d). The second resonance has disappeared and the resonance now occurs at λ_0.

At wavelengths within the photonic bandgap and away from resonances, intensity decreases quickly with distance from the interface between GGG substrate and MPC.

We would like to emphasize that our explanations have so far been based on a pure plausibility argument: If the light intensity is high at a certain position in the sample, the (magneto-) optical properties at this position should contribute more strongly to the overall (magneto-) optical response of the multilayer.

V. QUANTATIVE RELATION BETWEEN FARADAY ROTATION AND INTEGRATED INTENSITIES

As we have discussed in the previous section, it is a plausible assumption for us, that the MO rotations generated in the MPC are functionals of the intensity distributions $I(\lambda)$

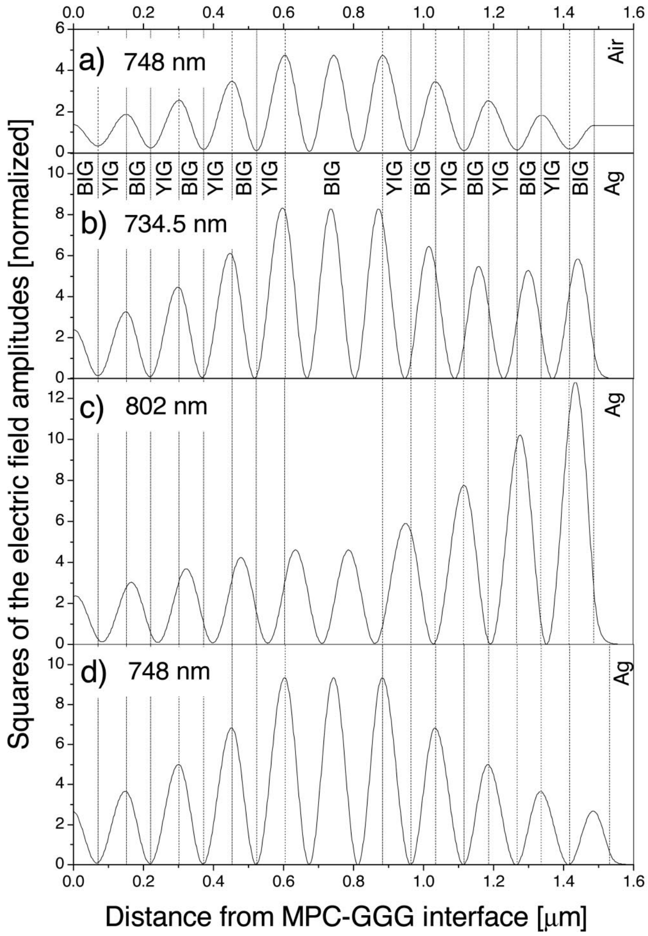

FIG. 3. Calculated intensity distributions inside BIG-YIG MPCs. (a) MPC without silver cover at resonance. (b) MPC covered with silver layer at the first resonance. (c) MPC with silver cover at the second resonance. (d) MPC with silver cover and compensation for phase shift at resonance.

$=I(\lambda,z)$, $z\,\epsilon[0,d]$ inside the MPC. Light propagates along the z axis, and d is the MPC thickness. As we only consider the MO effect in BIG, it is also plausible that the MO rotations only depend on the intensities within the BIG layers. The assumption that rotation increases monotonously when the intensities in the BIG layers increase is true for our structure, if we neglect absorption. For strong dielectric mirrors, however, this assumption must be wrong because splitting of the resonances for left and right circular waves leads to a *reduction* of the intensities at the wavelengths of maximum rotation. Therefore we propose to relate the intensities calculated *for zero intrinsic Faraday rotation* to the calculated MO rotations. Absorption complicates the picture because it is also enhanced at the resonances and reduces intensity. We will therefore not consider absorption here. Dispersions of the refractive indices and Faraday rotation are retained, and intensities in the BIG layers are calculated as $I(\lambda,z)=n_{\mathrm{BIG}}(\lambda)[(E_+^i(\lambda)+E_+^r(\lambda))^2+(E_-^i(\lambda)+E_-^r(\lambda))^2]$.

Since our structure without silver cover both transmits and reflects part of the incident light, we have to take into account Faraday rotation from both transmitted and reflected light, $\mathrm{FR}(\Theta_t(\lambda))$ and $\mathrm{FRR}(\Theta_r(\lambda))$. This is done by an "averaging"

$$\Theta_{\mathrm{ave}}(\lambda) = T(\lambda)\Theta_t(\lambda) + R(\lambda)\Theta_r(\lambda), \tag{1}$$

where $T(\lambda)$ and $R(\lambda)$ are transmittance and reflectance. If we make the further (naive) assumption that every small slice of BIG material contributes to $\Theta_{\mathrm{ave}}(\lambda)$ an amount that is proportional to the intensity in that slice and the intrinsic Faraday rotation $\theta_F(\lambda)$ generated per path length of the light in the BIG material, we obtain the simple relation where we again assume that light of intensity 1 is incident from the GGG substrate on the MPC.

$$\frac{\Theta_{\mathrm{ave}}(\lambda)}{\theta_F(\lambda)} = C \times \int_{\mathrm{BIG\ layers}} I(\lambda,z)\mathrm{d}z, \quad C=\mathrm{const}. \tag{2}$$

We calculated $\Theta_{\mathrm{ave}}(\lambda)/\theta_F(\lambda)$, $\Theta_t(\lambda)/\theta_F(\lambda)$, and $\Theta_r(\lambda)/\theta_F(\lambda)$, as well as the integral on the right-hand side of Eq. (2) as functions of λ for the range from 600 to 900 nm and plotted the MO rotations against the integrated intensities in Fig. 4. Certain values of MO rotation and integrated intensity occur several times in the considered wavelength range, compare Fig. 2 and the inset of Fig. 4. While the graphs for $\Theta_t(\lambda)/\theta_F(\lambda)$ and $\Theta_r(\lambda)/\theta_F(\lambda)$ are neither linear nor single-valued, we see that $\Theta_{\mathrm{ave}}(\lambda)/\theta_F(\lambda)$ indeed exhibits a linear dependence on the right-hand side of Eq. (2), with $C=0.53$.

If we deposit a Ag layer on top of our MPC, no light is transmitted and $\Theta_{\mathrm{ave}}(\lambda)=\Theta_r(\lambda)$. In order to consider this case for zero absorption as well, we set $n_{\mathrm{Ag}} \ll 1$ and real, which means that we replace the silver layer by a perfectly reflecting lossless interface. Since the phase shift upon reflection has now disappeared, there will be just a single reso-

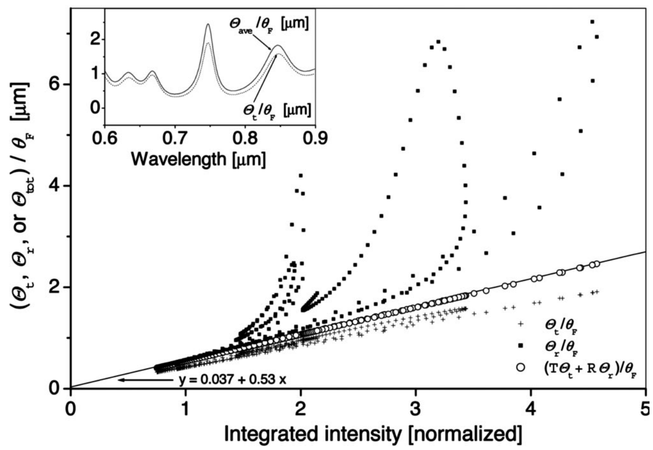

FIG. 4. Calculated FR $[\Theta_t(\lambda)]$, FRR $[\Theta_r(\lambda)]$, and averaged MO rotation $[\Theta_{\rm ave}(\lambda)]$ normalized to the bulk rotation per μm $[\theta_F(\lambda)]$as a function of the integrated intensity [see RHS of Eq. (2)]. The inset shows $\Theta_t(\lambda)/\theta_F(\lambda)$ and $\Theta_{\rm ave}(\lambda)/\theta_F(\lambda)$ vs wavelength.

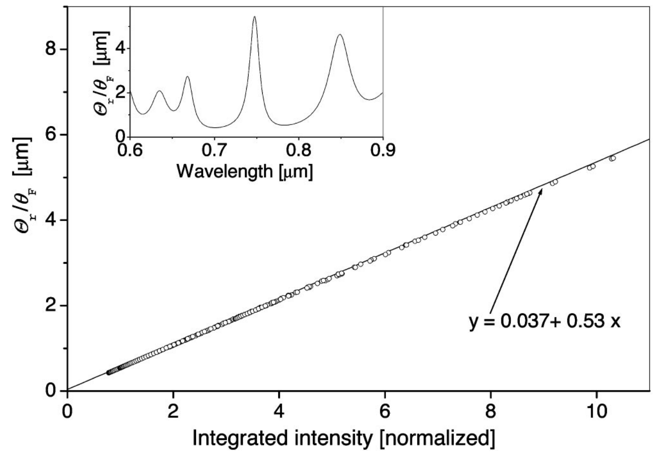

FIG. 5. FRR $[\Theta_r(\lambda)]$ normalized to the bulk rotation per μm $[\theta_F(\lambda)]$ as a function of the integrated intensity [see RHS of Eq. (2)]. Note that the straight line is identical to the straight line in Fig. 4. The inset shows $\Theta_r(\lambda)$ vs wavelength.

nance. Figure 5 demonstrates that the same linear relation with $C=0.53$ between MO rotation and integrated intensity is true for this case as well.

We made analogous calculations under the assumption that $n_{\rm YIG}$ is smaller by 0.2 and 0.5 than it actually is. This increases the strengths of the dielectric mirrors and the enhancement of the MO rotations. For $n_{\rm YIG}$ reduced by 0.2, the maximum FR and FRR are 11° and 31° and Eq. (2) still holds with $C=0.54$. If $n_{\rm YIG}$ is reduced by 0.5, however, Eq. (2) is not a good approximation because the relation between averaged Faraday rotation and integrated intensity is neither linear nor single-valued any longer. In that case, FR and FRR reach maximum values of 28° and 77°. Clearly, the linear relation is but an approximation to a more general theory.

VI. SUMMARY

All-garnet MPCs have been grown by pulsed laser deposition. Their Faraday rotations in transmission and reflection are significantly enhanced at wavelengths of optical resonances. The agreement of the measured with calculated spectra is rather good and demonstrates good quality of the prepared MPC. Calculated distributions of the light intensity within the MPC help to gain a qualitative understanding of the enhancement of Faraday rotation and absorption at different wavelengths. If none of the materials of the MPC is absorbent, the average Faraday rotation of the transmitted and reflected light is proportional to the intensity integrated over all BIG layers. However, if we assume a much stronger contrast in refractive index between BIG and YIG, the relation between Faraday rotation and integrated intensity is neither linear nor single-valued any longer.

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