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Strongly anisotropic waveguide as a nonmagnetic left-handed system

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We develop an approach to build a material with negative refraction index that can be implemented for optical and infrared frequencies. In contrast to conventional designs that require simultaneously negative dielectric permittivity and magnetic permeability and rely on a resonance to achieve a nonzero magnetic response, our material is intrinsically nonmagnetic and makes use of an anisotropic dielectric constant to provide a lefthanded behavior in waveguide geometry. We demonstrate that the proposed material can support surface (polariton) waves, and show the connection between the polaritons and the enhancement of evanescent fields, also known as superlensing.

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A large number of potential applications in optics, materials science, biology, and biophysics has instigated an extensive research in the area of materials with negative phase velocity, also known as left-handed media (LHM).[1–6] The LHMs, based on simultaneously negative dielectric permittivity and magnetic permeability have been successfully demonstrated in microwave (GHz) frequencies.[7–10] However, while the direct scale down of the experimentally verified systems is possible only to the THz region,[11] most of practical applications of these unique materials are in the "faster" (*optical* and *infrared*) part of the spectrum. Furthermore, the requirement to have negative magnetic response at high frequencies implies the presence of a resonance,[12] which, usually accompanied by strong losses, makes the current LHM designs almost impractical for real-life applications. In this paper we propose a *nonmagnetic*, *nonresonant* approach to build LHM, show the connection between the existence of surface (polariton) waves and enhancement of exponentially decaying (evanescent) fields, and present several implementations of our LHM design in optical and infrared frequencies.

Unlike most of the present LHM composites,[7–9,13–15] our system is based on a planar waveguide with anisotropic dielectric core, and does not have any magnetic response. Moreover, in contrast to current *composite, resonance-based* LHMs, the proposed material may be homogeneous, and does not require a resonance to achieve a negative phase velocity, the fundamental property of LHM. We describe the electromagnetic properties of our system, and derive the conditions for its right-, and left-handed response, and for excitation of surface waves (polaritons).

We consider a planar waveguide, parallel to the (y,z) plane of coordinate system, with the boundaries at $x=\pm d/2$. We assume that the material inside the waveguide is nonmagnetic ($\mu=1$), and has an anisotropic *uniaxial* dielectric constant ϵ, with $\epsilon_x=\epsilon_\perp$ and $\epsilon_y=\epsilon_z=\epsilon_\|$ (see Fig. 1).

Similarly to uniaxial crystals, our system may support two different kinds of electromagnetic waves.[16] The waves of the first kind have their electric field vector in the (y,z) plane. The propagation of such waves depends only on $\epsilon_\|$, and is not affected by anisotropy. These waves are also known as ordinary waves. The waves of the second kind (known as extraordinary waves) have their magnetic field in the (y,z) plane. Correspondingly their electromagnetic properties are affected by both $\epsilon_\|$ and $\epsilon_\perp$.

As we show below, the ordinary and extraordinary waves are fundamentally distinct as they have different dispersion relations and refraction properties.

A wave propagating in the proposed system can be represented as a series of the waves with their electric (magnetic) field perpendicular to the direction of propagation, known as TE (TM) waves, correspondingly.[16,17] In our case of the planar waveguide with an anisotropic core, the extraordinary wave has TM polarization, while the ordinary wave has the TE form (see Fig. 1). As it can be explicitly verified, the $\{x,y,z\}$ components of ordinary $(E^{(o)},H^{(o)})$, and extraordinary $(E^{(e)},H^{(e)})$ waves propagating in the (y,z) direction can be represented by the following expressions:

$$E^{(e)}=\left\{i\frac{k_z^{(e)^2}+k_y^{(e)^2}}{k_z^{(e)}\varkappa^{(e)^2}}\frac{\epsilon_\|}{\epsilon_\perp}E_0^{(e)\prime};\quad \frac{k_y^{(e)}}{k_z^{(e)}}E_0^{(e)};\quad E_0^{(e)}\right\},$$

$$H^{(e)}=\left\{0;\quad i\frac{k\epsilon_\|}{\varkappa^{(e)^2}}E_0^{(e)\prime};\quad -i\frac{kk_y^{(e)}\epsilon_\|}{k_z^{(e)}\varkappa^{(e)^2}}E_0^{(e)\prime}\right\},$$

$$E^{(o)}=\left\{0;\quad E_0^{(o)};\quad -\frac{k_y^{(o)}}{k_z^{(o)}}E_0^{(o)}\right\},$$

$$H^{(o)}=\left\{-\frac{k_z^{(o)^2}+k_y^{(o)^2}}{kk_z^{(o)}}E_0^{(o)};\quad -\frac{ik_y^{(o)}}{kk_z^{(o)}}E_0^{(o)\prime};\quad -\frac{i}{k}E_0^{(o)\prime}\right\}, \tag{1}$$

where $k=\omega/c$, and prime $(\prime)$ denotes the differentiation with respect to x. Similar to Ref. 16, the field $E_0^{(e|o)}(x,y,z;t)=E_0^{(e|o)}(x)e^{-i\omega t+ik_y^{(e|o)}y+ik_z^{(e|o)}z}$ is defined from the equation

$$E_0^{(e|o)\prime\prime}+\varkappa^{(e|o)^2}E_0^{(e|o)}=0, \tag{2}$$

with the conventional boundary conditions for the tangential (y,z) components of the electric field at the waveguide walls. For simplicity, here we consider the case of perfectly conducting waveguide boundaries; the straightforward extension

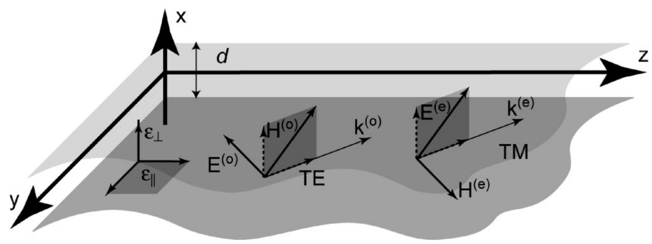

FIG. 1. The extraordinary (TM) and ordinary (TE) waves propagating in a planar waveguide with an anisotropic core.

of the presented theory to the case of dielectric walls, where similar effects are anticipated, will be presented elsewhere (see also Fig. 2).

Equation (2) yields a series of solutions (modes) $E_0^{(e|o)}(x)=A_m^{(e|o)}\cos(\varkappa x)$ with $\varkappa=(2m+1)\pi/d$, and $E_0^{(e|o)}(x)=A_m^{(e|o)}\sin(\varkappa x)$, $\varkappa=2m\pi/d$ (where m is an integer number). Note that the structure of the mode in the x direction is fully described by the parameter $\varkappa$, which for the case of perfectly conducting walls, considered here is fully determined by waveguide thickness d and does not depend on the dielectric properties of the core. Each waveguide mode has its own dispersion relation

$$k_z^{(e|o)^2}+k_y^{(e|o)^2}=\epsilon^{(e|o)}\nu^{(e|o)}k^2, \tag{3}$$

where

$$\epsilon^{(e)}=\epsilon_\perp;\epsilon^{(o)}=\epsilon_\parallel;\nu^{(e|o)}=1-\varkappa^{(e|o)^2}/(\epsilon_\parallel k^2). \tag{4}$$

Note that due to different geometry the TM and TE modes defined here are somewhat different from the conventional waveguide solutions presented in common textbooks.[16] Here we focus on the planar waveguide unbounded in the (y,z) plane with an anisotropic core in contrast to bounded in the (x,y) directions "tubular" one-dimensional (1D) structure with isotropic filling, where waves can propagate in z direction alone. It is straightforward to obtain the well-known TE ($E_z=0$) and TM ($H_z=0$) isotropic tubular solutions as the linear combination of the waves from Eq. (2). Also, as an alternative to the formalism presented in this paper, our system may be described in the terms of introduced in Ref. 6 generalized dielectric tensor with spatial dispersion.

An arbitrary wave inside a planar waveguide can be represented as a linear combination of waveguide modes (corresponding to different values of $\varkappa$). For simplicity for the rest of the paper we limit ourselves to the case when only a single mode is excited. This assumption does not restrict the generality of our approach since (i) it does not limit the (y,z) structure of the solutions nor their polarization and (ii) different modes of the waveguide do not couple to each other. The generalization of expressions presented here to a multiplemode case is straightforward.

It is clearly seen from Eq. (3) that a propagating solution (described by real k_z and k_y) is only possible in the case when the corresponding parameters ϵ and ν are of the same sign. The case $\epsilon>0;\ \nu>0$ is usually realized for an isotropic material inside the planar (transmitting) waveguide;[16] the case $\epsilon>0;\ \nu<0$ corresponds to the so-called subcritical waveguide, which does not support propagating modes and reflects all "incident" radiation. The third case that can be realized in a waveguide with an isotropic core, $\epsilon<0;\ \nu>0$, describes a perfectly conducting interior, which again does not support propagating waves.

Finally the case $\epsilon<0;\ \nu<0$, which is a primary focus of this paper, can only be realized only for the extraordinary wave in the anisotropic material. The corresponding structure is transparent for a TM wave; the TE solution exponentially decays into such a waveguide.

While Eq. (3) defines the magnitude of the phase velocity of the mode, the *sign* of the phase velocity cannot be determined by Eq. (3) alone. To define the sign of the phase velocity, and consequently the "handedness" of a media, we consider the refraction of a wave at the interface between the transparent isotropic (right-handed) media and a media with $\epsilon<0;\ \nu<0$ inside the same waveguide. We assume that the interface coincides with the coordinate plane $z=0$ (see Fig. 2).

We first consider the special case of the normal (z) propagation of a TM-polarized wave. Since in such a wave $H_z=H_x=0$, neither refracted nor reflected ordinary waves are excited. Since for $k_y=0$ the components H_y and E_x are related to each other: $H_y=(k\epsilon_\perp/k_z)E_x$ [see Eq. (1)], the requirement for continuity of tangential fields across the boundary $z=0$ immediately shows that the sign of k_z should coincide with the one of $\epsilon_\perp$. This is a clear indication that the media with $\epsilon<0,\nu<0$ is lefthanded.

The analysis of a general case of an obliquely incident wave (shown in Fig. 2) is more complicated, as in the general ordinary reflected wave is also excited, and the direction of the refracted (extraordinary) wave should be determined by the causality principle.[16] We perform such an analysis via exact three-dimensional (3D) numerical calculations. We represent the fields as a series of waveguide modes as described above and use a full set of boundary conditions for the E and H fields to find the necessary coefficients. We assert that the propagating in the real (absorbing) media wave decays in the direction of its propagation. Our results are shown in Fig. 2. It is clearly seen that Snell's law is reversed, meaning that phase velocity in the medium with $\epsilon<0;\ \nu<0$ is negative and the resulting wave is lefthanded for a general case of oblique incidence. As it is shown in Ref. 1, all optical effects directly related to a phase velocity (Snell's law, Doppler effect, Cherenkov radiation, etc.) are reversed in such a medium.

Another class of phenomena commonly associated with LHMs (e.g., enhancement of the evanescent fields,[2] nonlinear surface waves[5]), however requires the propagation of surface waves, also known as polaritons, at the left- and right-handed media interface. In the following calculations we note that the surface wave on nonmagnetic interface always has a TM structure.[16] We represent the fields and electromagnetic constant of the righthanded media (which fills the region $z<0$) with superscript $(-)$ and the ones in the LHM region $z>0$ with $(+)$. We search for a polariton solution $(E,H)^{(-)}\propto\exp[ik_y y+\xi^{(-)}z]$; $(E,H)^{(+)}\propto\exp[ik_y y-\xi^{(+)}z]$, with *real* k_y, and positive $\xi^{(-|+)}$ (the exponentially growing away from the interface "antipolariton" solution corresponding to negative $\xi^{(-|+)}$ can exist only in finite region of space).

While the LHM region is bound to have $\epsilon_{\perp} < 0, \epsilon_{\parallel} > 0$, the "right-handed" medium can be constructed by either $\epsilon_{\perp} > 0, \epsilon_{\parallel} > 0$ or by $\epsilon_{\perp} > 0, \epsilon_{\parallel} < 0$. These two combinations of the dielectric constants lead to different conditions for polariton propagation.

Specifically, for the case $\epsilon_{\parallel}^{(-)} > 0, \epsilon_{\perp}^{(-)} > 0$, usually realizable in an isotropic right-handed medium, the polaritons are only possible for $k_y = 0$ and have the dispersion relation [see Eqs. (1) and (2)]

$$\nu^{(-)}/\epsilon_{\perp}^{(-)} = \nu^{(+)}/\epsilon_{\perp}^{(+)} \tag{5}$$

Such waves, however, assume propagation along the x direction. The existence of these waves in the waveguide geometry considered here is limited to a number of "modes," each forming a standing wave between the waveguide plates and fulfilling the corresponding boundary conditions [see also Eq. (2) and the discussion afterwards].

However if the right-hand medium has $\epsilon_{\parallel} < 0$, and $\epsilon_{\perp} > 0$, the propagation of polaritons with nonzero k_y is also possible when

$$\epsilon_{\parallel}^{(-)} \nu^{(-)} = \epsilon_{\parallel}^{(+)} \nu^{(+)}. \tag{6}$$

This equation again relates $\varkappa$ to k. When Eq. (6) is satisfied, the surface wave exists for any given $|k_y|^2 > \epsilon \nu k^2$, and the relation between k_y and ξ is given by Eq. (3), where we substitute $k_z^2 = -\xi^2$. Note that a similar situation takes place in 3D geometry on the boundary between the right-handed medium ($\epsilon^{(-)} > 0, \mu^{(-)} > 0$) and "conventional" LHM ($\epsilon^{(+)} < 0, \mu^{(+)} < 0$), where for the same frequency the polaritons exist for any wave vector provided that $\epsilon^{(-)} = -\epsilon^{(+)}$, $\mu^{(-)} = -\mu^{(+)}$.

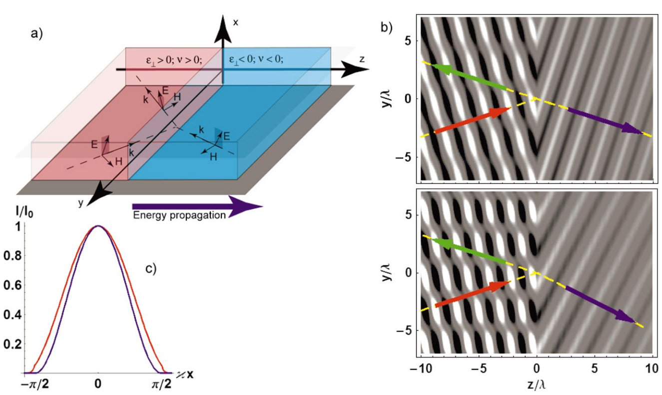

FIG. 2. (Color) Reflection and refraction at the boundary between right- (RHM) and left-handed media (LHM). (a) Schematic illustration of refraction of a TM wave at the RHM- LHM interface (ordinary wave not shown). (b) (top) The results of the exact numerical calculations of refraction of the mode in planar waveguide with perfectly conducting walls; $\varkappa = k/2$; RHM parameters ($z < 0$): $\epsilon = \nu = 1/2 + 0.002i$; LHM ($z > 0$): $\epsilon = \nu = -1/2 + 0.003i$, angle of incidence: $\pi/10$, normalized real part of E_x shown. (bottom): the same as (top), but with a finite-conductive waveguide (silver; $\lambda = 0.75\ \mu$m; $\epsilon_w = -25 + 0.3i$). The red, green, and blue arrows show the direction of incident, reflected, and refracted waves correspondingly. (c) The intensity profile of E_z for the systems in (b); (blue) and (red) correspond to perfect metal (top) and silver (bottom) waveguide walls.

We stress that it is the existence of surface waves for a wide range of wave vectors that makes the proposed in Ref. 2 phenomenon of superlensing possible. The evanescent components, which carry the information about the subwavelength features of the source, exponentially decay away from the object plane. Their resonant enhancement by a slab of either planar (described here) or 3D (described in Refs. 1 and 2) LHM can be represented as a resonant coupling of the original evanescent wave to the surface modes on both interfaces of the LHM lens. In such a process, the original evanescent wave excites antipolariton (surface mode growing away from the interface) on the front interface (see Fig. 3), which, in turn, excites the true-polariton mode on the back interface of the slab. The exponentially decaying away from the lens part of this surface mode represents the LHM-enhanced evanescent wave. This concept is illustrated in Fig. 3 where we calculate the transmission of an evanescent component through the slab of planar LHM proposed here. It is clearly seen that decaying through the right-handed media evanescent wave, is resonantly enhanced inside LHM slab only in the presence of polaritons.[18]

Finally, we consider the fabrication perspectives of the proposed LHM materials. In GHz frequencies the required strongly anisotropic response may be realized in a composite of metallic wires aligned along the x axis proposed in Ref. 19. This idea to achieve $\epsilon_{\perp} < 0$ is realized in Refs. 7 and 8 (note that $\epsilon_{\parallel} > 0$). Also, we note that the "left-handed" transmission line[10] (while it is not a homogeneous 3D material) may be described in terms of the proposed formalism since the inductive elements formally corresponds to $\epsilon < 0$, while capacitors corresponds to $\epsilon > 0$.

In optical and near-infrared frequencies negative dielectric constant is achieved due to plasmon resonance of a free electron gas inside the metal (Ag, Au, etc.) or doped semiconductor (Si) (plasmonic) structures. There, electron concentration and effective mass define the region of $\epsilon < 0$.[20] In the midinfrared spectrum range negative dielectric constant naturally occurs in polar crystals (e.g., SiC).[21] Also, both plasmonic and polar materials generally have relatively small

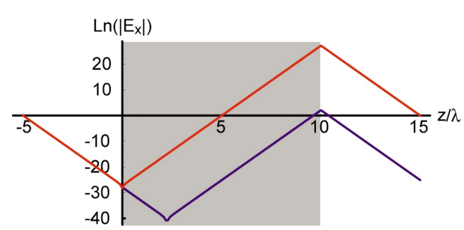

FIG. 3. (Color) Amplification of an evanescent field by a parallel slab of planar LHM ($\varkappa = k/2$). The blue line corresponds to nonplasmonic case. The right-hand media (RHM) parameters: $\epsilon^{(RHM)} = \nu^{(RHM)} = 1/2$, LHM parameters: $\epsilon^{(LHM)} = \nu^{(LHM)} = -1/2$; $k_y = 2k$. The red line shows the case of resonant excitation of polariton waves $\epsilon^{(RHM)} = 3/2$, $\nu^{(RHM)} = 4$; $\epsilon^{(LHM)} = -6/5$, $\nu^{(LHM)} = -5$; $k_y = \sqrt{9/8}k$; the LHM is positioned between $z = 0$ and $z = 10\lambda$. The resonant enhancement of evanescent components with surface waves (often attributed to superlens, originally proposed in Ref. 2) is clearly seen.

absorption, which in conjunction with negative ϵ makes them excellent candidates for proposed LHM. The anisotropic dielectric response described here, may be achieved in the following composites:

(i) A composite of subwavelength (nanostructured) inclusions with anisotropic (e.g., spheroidal) shape in the isotropic dielectric host. In this approach all the inclusions have to be aligned, and homogeneously distributed in the dielectric host. The shape of the inclusion defines the frequency range of the LHM response. We stress that no special arrangement of the inclusions (except for their aligning) is necessary to achieve desired dielectric properties. To give just one example, for the composite of 10% of SiC nanospheroids with an aspect ratio of $1/2$, aligned with their shorter axis along the x axis and embedded in quartz, for a wavelength of CO_2 laser of 12 μm we obtain $\epsilon_\perp \approx -2.7+6\times 10^{-4}i$; $\epsilon_\parallel \approx 1.6+1\times 10^{-5}i$.[22]

(ii) A composite based on isotropic (spherical) inclusions in a dielectric host. The anisotropy may be achieved by anisotropic distribution of inclusions. For example, one may deposit a dielectric spacer followed by a (random) deposition of inclusions or deform the composite with isotropic inclusion distribution. Our numerical calculations show that the composite of 15% of Ag nanospheres in TiO_2 with an average separation between inclusions in the x direction half of that in y and z directions, has $\epsilon_\parallel \approx 90+10i$; $\epsilon_\perp \approx -25+2i$ at $\lambda = 0.75$ μm. The detailed theory of such composites is not presented here due to space limitations and is deferred to our future work.

(iii) A layered structure based either on multiple semiconductor quantum wells where the mobility of the electrons is different the x direction and the y-z plane[23] or on layered plasmonic (polar) materials as described in Ref. 15.

We also anticipate the desired response from intrinsically anisotropic semimetal crystals (Bi and its alloys).[24]

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