Generation of spin current and polarization under dynamic gate control of spin-orbit interaction in low-dimensional semiconductor systems

C. S. Tang,[1] A. G. Mal'shukov,[2] and K. A. Chao[3]
[1]*Physics Division, National Center for Theoretical Sciences, P.O. Box 2-131, Hsinchu 30013, Taiwan*
[2]*Institute of Spectroscopy, Russian Academy of Science, 142190, Troitsk, Moscow oblast, Russia*
[3]*Solid State Theory Division, Department of Physics, Lund University, S-22362 Lund, Sweden*
(Received 7 December 2004; published 13 May 2005)

Based on the Keldysh formalism, the Boltzmann kinetic equation and the drift-diffusion equation have been derived for studying spin-polarization flow and spin accumulation under effect of the time-dependent Rashba spin-orbit interaction in a semiconductor quantum well. The time-dependent Rashba interaction is provided by time-dependent electric gates of appropriate shapes. Several examples of spin manipulation by gates have been considered. Mechanisms and conditions for obtaining the stationary spin density and the induced rectified dc spin current are studied.

DOI: 10.1103/PhysRevB.71.195314 PACS number(s): 72.25.Dc, 73.40.Lq, 72.30.+q, 71.70.Ej

I. INTRODUCTION

Spin transport in semiconductor heterostructures has recently attracted much attention owing to a perspective of its practical application for quantum computing and communications.[1–3] In the way of fundamental studies and applications of the spin transport the key problem is how to generate, detect, and manipulate the electron spin polarization. Many ideas have been proposed to achieve control of the spin using magnetic materials, external magnetic fields, and optical excitation (for a review see Ref. 3). At the same time, a challenging goal remains to employ only the electric control integrated by means of gates into a high-mobility semiconductor heterostructure.

A promising opportunity in this way is opened due to the strong Rashba spin-orbit interaction (SOI) in narrow-gap two-dimensional (2D) electron systems.[4] An important feature of this interaction is its tunability which can be achieved by varying the gate voltage.[5,6] The field-effect transistor was the first proposal utilizing this phenomenon.[7] The gate control of the spin current employing the Aronov-Casher effect[8] was considered in Ref. 9. The electric-dipole spin resonance controlled by the time-dependent gate was studied in Ref. 10.

The SO interaction effect alone or in combination with the external electric field allows one to approach an important goal to create spin currents and spin polarization by entirely electrical means, not involving any of the magnetic materials or optical excitation. One of the examples is the recently predicted[11] (see also Ref. 12) and observed[13,14] spin-Hall effect in 2D and 3D electron and hole gases, where the spin current is driven by the electric field. A closely related phenomenon is the spin polarization of 2D electron gas (2DEG) in response to the parallel electric field.[15] Another method utilizes the time-dependent gate to modulate the shape of a quantum dot[16] or the strength of the SO coupling constant in 1D[17,18] and 2D systems.[18] In the latter case an efficient spin-current generation can be attained in the presence of high-frequency (hundreds of MHz or higher) gate-voltage variations. The physics of this phenomenon is simple. The Rashba spin-orbit interaction resembles an interaction with a spin-dependent gauge field. Therefore, its time derivative produces a motive force on electrons, similar to the electromotive force from the time-dependent electric vector potential. The important difference is, however, that the force created by the SOI acts in opposite directions for oppositely polarized spins. Such a method of spin-current generation is convenient for implementations in conventional semiconductor heterostructures. By a proper choice of gate shape it allows one to create and rectify the ac spin current or accumulate the spin polarization at a given location. However, the theory in Ref. 18 is restricted to the spatially homogeneous case. Therefore, within this theory one cannot properly consider boundary effects, as well as spin-current generation due to a time-dependent gate of small area. Also, it is impossible to study any effects of spin-current generation accompanied by spin accumulation.

In the present work we develop a theory which is based on the Boltzmann transport equations. This approach is quite universal and it allows us to study the spin transport under an arbitrary space-time-dependent SO interaction, providing that a characteristic scale of this dependence is within the range of applicability of the semiclassical approximation. This means that the spatial variations of the SO interaction are assumed to have the scale Δr larger than the electron Fermi wavelength and the scale of its time variations $\Delta t \gg \hbar/E_F$, where E_F is the Fermi energy. For even larger time-space scales, such as $\Delta r \gg l$ and $\Delta t \gg \tau$, where l and τ are, respectively, the mean free path and the mean free scattering time, we use the Boltzmann equation to derive the drift-diffusion equation. Within this theory we consider the following problems.

(i) Spin-current generation by a finite-size time-dependent gate. In this case the spin polarization is pumped by the gate into 2D regions adjacent to it. This polarization further diffuses away from the gate, as well as in the backward direction.

(ii) Spin-current generation in a gas confined within an infinite 2D strip. In such a geometry the generated spin current with the polarization perpendicular to boundaries flows freely along the strip. At the same time, spins polarized parallel to the boundaries are accumulated near them, opposite spins near opposite banks.

(iii) Rectification of the ac spin current. It will be shown that the spin polarization generated by a finite-size ac gate in some cases contains the dc component due to periodic variations of the electron density under the gate. The dc component appears as a result of the interplay of two periodic processes: oscillations of the SO coupling constant and oscillations of the 2D electron density.

This paper is organized as follows. In Sec. II, the Boltzmann equation is derived for calculation of the spin transport in the presence of the time-dependent SO interaction. In Sec. III, we apply the Boltzmann equation to some of the spin-transport problems. The derivation of the drift-diffusion equation and its applications are presented in Sec. IV. The results of this work are summarized in Sec. V.

II. BOLTZMANN TRANSPORT THEORY

We consider spin transport in a noninteracting 2DEG confined within a narrow-gap semiconductor quantum well (QW). The 2DEG is applied atop by a gate under time-dependent bias. The inversion asymmetry of the quantum-well-confining potential is controllable by an external gate, and hence electrons experience a tunable Rashba-type SO interaction.[5,6] The system is therefore described by the effective mass Hamiltonian

$$H(t) = \frac{\mathbf{p}^2}{2m^*} + \mathbf{M}\cdot\frac{1}{2}[\alpha(\mathbf{r},t)\mathbf{p} + \mathbf{p}\alpha(\mathbf{r},t)] + U(\mathbf{r},t), \tag{1}$$

where m^* represents the effective mass, $\mathbf{p}=(p_x,p_y)$ is the electron momentum in the 2DEG plane, and $\alpha(\mathbf{r},t)$ denotes the time-dependent Rashba coupling constant. $\alpha(\mathbf{r},t)$ contains both a static part α_0 and a dynamic part $\alpha_1(\mathbf{r},t)$ due to the time-dependent gate. In addition to time dependence, α_1 can also vary in space depending on the gate configuration and applied bias. We denote $\mathbf{M}=\boldsymbol{z}\times\boldsymbol{\sigma}$ where $\boldsymbol{z}$ is the unit vector along the growth direction and $\boldsymbol{\sigma}=\{\sigma_x,\sigma_y,\sigma_z\}$ is the vector of the Pauli matrices. Further, $U(\mathbf{r},t)$ indicates the external potential energy of an electron in the electric field produced by the gate. The static part of the Rashba coupling constant lifts the spin degeneracy, resulting in a momentum-dependent spin splitting of the conduction band: namely,

$$\varepsilon_{\pm}(\mathbf{k}) = \frac{\mathbf{k}^2}{2m^*} \pm \alpha_0 k. \tag{2}$$

For the 2DEG density of interest the Rashba spin-splitting energy $\Delta_0=2\alpha_0 k$ will be assumed to be small compared to the Fermi energy, $\Delta_0 \ll E_F$. In the following, we set $\hbar=1$ for convenience.

Let us consider the time-modulated nonequilibrium electron system in the temperature regime $T\ll E_F$, assuming the semiclassical conditions to be satisfied. Hence, the time intervals Δt and distances Δr over which all quantities vary significantly satisfy the inequalities $E_F\Delta t \gg 1$ and $p_F\Delta r\gg 1$. These conditions are necessary for derivation of the kinetic equation. We start this derivation by introducing the matrix of the nonequilibrium Green's function in Keldysh space:[19]

$$\hat{\mathcal{G}}_{\alpha\beta}(1,2) = \begin{bmatrix} G^{--}_{\alpha\beta}(1,2) & G^{-+}_{\alpha\beta}(1,2) \\ G^{+-}_{\alpha\beta}(1,2) & G^{++}_{\alpha\beta}(1,2)\end{bmatrix}, \tag{3}$$

where 1 and 2 denote two points $\mathbf{r}_1,t_1$ and $\mathbf{r}_2,t_2$, in the time-space, and α,β are spinor indices. We will assume that besides the time-dependent forces in the Hamiltonian (1), the electrons are also subject to scattering from randomly distributed static impurities. This scattering can be taken into account by introducing the self-energy

$$\hat{\Sigma}_{\alpha\beta} = \begin{bmatrix} \Sigma^{--}_{\alpha\beta} & \Sigma^{-+}_{\alpha\beta} \\ \Sigma^{+-}_{\alpha\beta} & \Sigma^{++}_{\alpha\beta}\end{bmatrix}. \tag{4}$$

The corresponding matrix Dyson equation is of the form

$$\hat{\mathcal{G}}_{\alpha\beta}(1,2) = \hat{\mathcal{G}}^0_{\alpha\beta}(1,2) + \int d4d3[\hat{\mathcal{G}}^0_{\alpha\gamma}(1,4)\hat{\Sigma}_{\gamma\delta}(4,3)\hat{\mathcal{G}}_{\delta\beta}(3,2)] \tag{5}$$

or the conjugate form

$$\hat{\mathcal{G}}_{\alpha\beta}(1,2) = \hat{\mathcal{G}}^0_{\alpha\beta}(1,2) + \int d3d4[\hat{\mathcal{G}}_{\alpha\gamma}(1,3)\hat{\Sigma}_{\gamma\delta}(3,4)\hat{\mathcal{G}}^0_{\delta\beta}(4,2)], \tag{6}$$

where the functions in the integrand are matrices both in real space and in spin space being combined by the rule of matrix multiplication. Acting on the Dyson equation from the left (or from the right on its conjugate) by the operator

$$[G^0_{\alpha\beta}(j)]^{-1} \equiv i\frac{\partial}{\partial t_j}\delta_{\alpha\beta} - H_{\alpha\beta}(t_j), \tag{7}$$

where the suffix $j=1$ or 2 indicates that the differentiation is with respect to the variables t_j and $\mathbf{r}_j$. After some algebra, we obtain the equation[20]

$$\begin{aligned}(\hat{\mathcal{I}}_{\rm sc})_{\alpha\beta} = &- i\left(\frac{\partial}{\partial t_1} + \frac{\partial}{\partial t_2}\right)\hat{\mathcal{G}}_{\alpha\beta}(1,2) + \frac{1}{2m}(\Delta_2 - \Delta_1)\hat{\mathcal{G}}_{\alpha\beta}(1,2) \\ &+ [U(1) - U(2)]\hat{\mathcal{G}}_{\alpha\beta}(1,2) \\ &- i[\alpha(2)\boldsymbol{\nabla}_2\cdot(\hat{\mathcal{G}}_{\alpha\gamma}(1,2)\mathbf{M}_{\gamma\beta}) \\ &+ \alpha(1)\boldsymbol{\nabla}_1\cdot(\mathbf{M}_{\alpha\gamma}\hat{\mathcal{G}}_{\gamma\beta}(1,2))] - \frac{i}{2}[\boldsymbol{\nabla}_2\alpha(2)\cdot(\hat{\mathcal{G}}_{\alpha\gamma}\mathbf{M}_{\gamma\beta}) \\ &+ \boldsymbol{\nabla}_1\alpha(1)\cdot(\mathbf{M}_{\alpha\gamma}\hat{\mathcal{G}}_{\gamma\beta}(1,2))],\end{aligned} \tag{8}$$

which is equivalent to the set of integro-differential equations for the component Green functions. Here the scattering integral is defined by[20]

$$(\hat{\mathcal{I}}_{\rm sc})_{\alpha\beta} = \int d3[\hat{\tau}\hat{\Sigma}_{\alpha\gamma}(1,3)\hat{\mathcal{G}}_{\gamma\beta}(3,2) - \hat{\mathcal{G}}_{\alpha\gamma}(1,3)\hat{\Sigma}_{\gamma\beta}(3,2)\hat{\tau}], \tag{9}$$

where $\hat{\tau}$ is the matrix, such that $\tau^{++}=-\tau^{--}=1$ and $\tau^{+-}=\tau^{-+}=0$.

In the semiclassical regime, it is convenient to transform the variables to the Wigner coordinates: $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$; $t=t_1-t_2$, $T=\frac{1}{2}(t_1+t_2)$. The difference variables $\mathbf{r}$ and t

vary on a microscopic scale, while the center variables **R** and T are macroscopic variables. In the quasiclassic approximation the Green functions and self-energies vary slowly with respect to the center variables. Accordingly, only linear gradient expansions will be taken into account in Eq. (8).[21] It is appropriate to define the space-time Fourier transform to the fast variables,

$$G^{ij}_{\alpha\beta}(\mathbf{R},T;\mathbf{r},t)=\sum_{\mathbf{k}}\int\frac{d\omega}{2\pi}G^{ij}_{\alpha\beta}(\mathbf{R},T;\mathbf{k},\omega)\times\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)],\qquad(10)$$

where the indices i,j take the values "+" or "−." Denoting as $G^{ij}(\mathbf{R},T;\mathbf{k},\omega)$ a matrix with elements $G^{ij}_{\alpha\beta}(\mathbf{R},T;\mathbf{k},\omega)$ and making the gradient expansion of Eq. (8) one can write the quantum kinetic equation in the form

$$iI^{ij}_{\rm sc}=\frac{\partial G^{ij}}{\partial T}+\mathbf{v}\cdot\nabla_{\mathbf{R}}G^{ij}+i\alpha[\mathbf{k}\cdot\mathbf{M},G^{ij}]+\frac{1}{2}\alpha\{\mathbf{M},\nabla_{\mathbf{R}}G^{ij}\}-\frac{1}{2}\nabla_{\mathbf{R}}\alpha\cdot\{\mathbf{k}\cdot\mathbf{M},\nabla_{\mathbf{k}}G^{ij}\}+\frac{1}{2}\frac{\partial\alpha}{\partial T}\left\{\mathbf{k}\cdot\mathbf{M},\frac{\partial G^{ij}}{\partial\omega}\right\}-\nabla U\cdot\nabla_{\mathbf{k}}G^{ij}+\frac{\partial U}{\partial T}\frac{\partial G^{ij}}{\partial\omega},\qquad(11)$$

where $\mathbf{v}=\mathbf{k}/m^*$ and $\{A,B\}$ denotes the anticommutator. The scattering part of this equation is obtained from Eq. (9) in the leading order of the gradient expansion with respect to the "slow" $\mathbf{R},T$ variables.[20] In calculations below we will employ the three Green functions. There are G^{-+}, retarded G^r, and advanced G^a functions. G^r and G^a are defined by the equations

$$G^r=G^{--}-G^{-+}=G^{+-}-G^{++},$$

$$G^a=G^{-+}-G^{++}=G^{--}-G^{+-}.\qquad(12)$$

The same equations are valid also for the retarded and advanced self-energies with the only difference that Σ^{+-} and Σ^{-+} enter with the signs opposite to signs of G^{+-} and G^{-+} in Eqs. (12). In terms of these functions the corresponding scattering parts of Eq. (11) are written as

$$I^{-+}_{\rm sc}=-\Sigma^rG^{-+}+G^{-+}\Sigma^a-G^r\Sigma^{-+}+\Sigma^{-+}G^a,\qquad(13)$$

$$I^r_{\rm sc}=-[\Sigma^r,G^r];\quad I^a_{\rm sc}=-[\Sigma^a,G^a].\qquad(14)$$

In order to determine the self-energy we assume, for simplicity, that the impurity scattering potential is isotropic, spin independent, and short range. Hence, it can be simply written as $V(\mathbf{r})=V\delta(\mathbf{r})$ so that the corresponding Born-scattering amplitude does not depend on the electron wave vector. In the quasiclassic approximation ignoring weak localization effects the averaged over random impurity positions self-energy is thus given by[22]

$$\Sigma^{ij}=(-1)^{i+j}|V|^2\sum_{\mathbf{k}}G^{ij}(\mathbf{R},T;\mathbf{k},\omega).\qquad(15)$$

One can see that the so-defined self-energy does not depend on the wave vector. In the thermodynamic equilibrium the unperturbed retarded and advanced Green functions are easily found from Eqs. (15) and (1):

$$G^r_0=G^{a\dagger}_0=(\omega-H_0+i\Gamma)^{-1},\qquad(16)$$

where $H_0=(k^2/2m^*)+\alpha_0\mathbf{M}\cdot\mathbf{k}$. The elastic scattering rate Γ is obtained from Eq. (15). Substituting Eq. (16) into Eq. (15) one easily finds $\Gamma=\pi|V|^2N_F$, where $N_F=m^*/(2\pi)$ is the one-particle 2D density of states.[22] The "−+" unperturbed Green function can be found from the equation[20]

$$G^{-+}_0=-n_F(\omega)(G^r_0-G^a_0),\qquad(17)$$

where $n_F(\omega)$ is the equilibrium Fermi distribution.

The 2×2 matrix Wigner distribution function is defined as $-iG^{-+}(\mathbf{R},T;\mathbf{k},\omega)$. At the same time, the density matrix which is used for calculations of observable physical quantities—for example, spin density and spin current—can be obtained from the Wigner function by integration over ω. Usually, the Boltzmann equation for this function is obtained by integration over ω of the quantum kinetic equation, such as the "−+" component of Eq. (11). In our case, however, this method cannot be used because of the term proportional to $\partial\alpha/\partial T$, which vanishes after integration. Moreover, it becomes a not simple task to write the scattering part in terms of the density function. A similar problem arises when one derives the Boltzmann equation for the system driven out of equilibrium by the electric field represented by a time-dependent vector potential. This difficulty is resolved by shifting variables from wave vectors to kinematic momenta or to velocities.[22] In the problem considered here such a shift is not very helpful, because the velocity $\mathbf{k}/m^*+\alpha(\mathbf{R},T)\mathbf{M}$ is a matrix in spin space, not a number. We will employ a different method. Let us represent the Wigner function in the form

$$-iG^{-+}=in_F(\omega)(G^r-G^a)+F.\qquad(18)$$

At $U(\mathbf{R},T)=0$ and $\alpha(\mathbf{R},T)=\alpha_0$ the first term on the right-hand side turns into the unperturbed Wigner function. Hence, the F function is not zero only due to deviation of the system from the original homogeneous thermodynamically equilibrium state. Assuming this deviation to be small, we will linearize Eq. (11), omitting all products of F with terms containing the time and space derivatives of α and U. After substitution of Eq. (18) to the "−+" component of Eq. (11), taking into account Eq. (13) and the corresponding equations for G^r and G^a with the scattering terms given by Eq. (14), we arrive at the following equation for F:

$${\rm St}[F]=\frac{\partial F}{\partial T}+\mathbf{v}\cdot\nabla_{\mathbf{R}}F+i\alpha[\mathbf{k}\cdot\mathbf{M},F]+\frac{1}{2}\alpha\{\mathbf{M},\nabla_{\mathbf{R}}F\}+\frac{i}{2}\frac{\partial\alpha}{\partial T}\{\mathbf{k}\cdot\mathbf{M},G^r-G^a\}\frac{\partial n_f(\omega)}{\partial\omega}+i\frac{\partial U}{\partial T}(G^r-G^a)\frac{\partial n_f(\omega)}{\partial\omega},\qquad(19)$$

where ${\rm St}[F]$ is given by

$$\mathrm{St}[F] = \Sigma^r F - F\Sigma^a - G^r\Sigma(F) + \Sigma(F)G^a, \qquad (20)$$

with $\Sigma(F)$ defined as $\Sigma(F)=|V|^2\Sigma_{\mathbf{k}}F$.

A good approximation to the functions G^r and G^a is the local equilibrium functions G_l^r and G_l^a defined by

$$G_l^r = G_l^{a\dagger} = (\omega - H_l + i\Gamma)^{-1}, \qquad (21)$$

where $H_l=(k^2/2m^*)+\alpha(\mathbf{R},T)\mathbf{M}\cdot\mathbf{k}+U(\mathbf{R},T)-E_F$. The local function has the same form as the equilibrium function (16) with the electron energy and spin splitting parametrically dependent on time and space. It can be seen from Eq. (11), using Eqs. (12), (14), and (15), that the corrections to the local functions are proportional to the Green function gradients times the small parameter α/v_F, where v_F is the Fermi velocity. Such a small parameter can be important for the spin-Hall effect,[11] but not for the case considered here of the spin current driven by the time-dependent SO coupling constant. One can further simplify Eq. (19), omitting proportional to this parameter terms in Eq. (19), such as $\alpha\{\mathbf{M},\nabla_{\mathbf{R}}F\}$, which enters together with the much bigger $\mathbf{v}\cdot\nabla_{\mathbf{R}}F$. After this simplification the linear operator acting upon F on the right-hand side of Eq. (19) decouples into scalar- and spin-dependent parts. Further, since we are interested in the spin transport driven by the term in Eq. (19) proportional to the time derivative of α, one can ignore the term containing $\partial U/\partial T$, because it contributes additively into the linear response and gives rise to the spin-Hall effect which will not be considered in the present work. It should be noted that the linearization is undertaken only with respect to the time derivatives, while local values of α and U entering into $G_l^{r(a)}$ can vary noticeably within macroscopic time-space scales.

Now let us take a look at the scattering part (20). One can easily see that since the SOI is an odd function with respect to $\mathbf{k}$, the self-energies $\Sigma^{r,a}$ calculated from Eq. (15) with the local equilibrium functions (21) are contributed only by the scalar parts of these functions. Therefore, omitting the small corrections of the order of Δ/E_F, where E_F is the Fermi energy, from Eqs. (21), (15), and (12) we get

$$\Sigma^r F - F\Sigma^a = -2i\Gamma F. \qquad (22)$$

The other terms of Eq. (20), the ones which contain $\Sigma(F)$, can be also simplified. We notice that the spin-dependent parts of $G_l^{r(a)}$ contribute effectively with the small parameter Δ/E_F. Although small, these terms provide a coupling of the particle density represented by the scalar $\mathrm{Tr}[\Sigma(F)]$ to the spin-distribution function associated with $\mathrm{Tr}[\sigma F]$ and vice versa. They are important in the spin-Hall effect.[23,24] In our case we can ignore them. Taking into account all these simplifications we are ready to derive the Boltzmann equation for the particle distribution function in the space of particle coordinates and momenta. This function is obtained by integration over ω of Eq. (18),[20] with $G^{r,a}$ given by Eq. (21). In the leading approximation we thus get

$$-i\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}G^{-+} = n_{Fl} + \alpha\mathbf{M}\cdot\mathbf{k}\frac{\partial n_{Fl}}{\partial E} + f_{\mathbf{k}}(\mathbf{R},T), \qquad (23)$$

where $n_{Fl}=n_F[E+U(\mathbf{R},T)]$ is the local Fermi distribution function, where the coordinate- and time-dependent potential energy is added to the electron kinetic energy E. The function $f_{\mathbf{k}}(\mathbf{R},T)$ is defined by

$$f_{\mathbf{k}}(\mathbf{R},T) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}F(\mathbf{R},T;\mathbf{k},\omega). \qquad (24)$$

After integrating Eq. (19) over the frequency we get the Boltzmann equation in the form

$$\mathrm{St}[f] = \frac{\partial f_{\mathbf{k}}}{\partial T} + \mathbf{v}\cdot\nabla_{\mathbf{R}}f_{\mathbf{k}} + i\alpha(\mathbf{R},T)[\mathbf{k}\cdot\mathbf{M},f_{\mathbf{k}}] + \mathbf{k}\cdot\mathbf{M}\frac{\partial\alpha(\mathbf{R},T)}{\partial T}\frac{\partial n_{Fl}}{\partial E}. \qquad (25)$$

The scattering term is obtained as

$$\mathrm{St}[f] = \frac{2\pi}{\tau m^*}\sum_{\mathbf{k}'}\delta(E'-E)f_{\mathbf{k}'} - \frac{f_{\mathbf{k}}}{\tau}, \qquad (26)$$

where $\tau=1/2\Gamma$ is the elastic scattering time.

III. SIMPLE EXAMPLES OF SPIN TRANSPORT UNDER THE TIME-DEPENDENT GATE

In this section, we will employ Eq. (25) to investigate the electron transport properties of two-dimensional electron systems in the presence of a time-dependent SOI. Two simple examples of application of the Boltzmann equation will be considered: spin transport driven by a homogeneous infinite gate and the ballistic transport due to a narrow time-dependent gate.

Before proceeding with these examples, let us define the spin current and spin density in terms of $f_{\mathbf{k}}$. According to definition of G^{-+},[20] the spin distribution in the space of particle coordinates and momenta is given by the spin-dependent part of Eq. (23). Therefore, the spin density is obtained by integration of nonscalar terms in Eq. (23) over $\mathbf{k}$. Taking into account that the second term in Eq. (23) turns to zero after averaging over $\mathbf{k}$ directions, we get the spin density

$$P^i(\mathbf{R},T) = \frac{1}{2}\sum_{\mathbf{k}}\mathrm{Tr}[\sigma^i f_{\mathbf{k}}(\mathbf{R},T)]. \qquad (27)$$

The spin current definition is based on the one-particle spin-flux operator $\frac{1}{4}\{\sigma^i,v\}$, where the velocity operator $v=\mathbf{v}+\alpha\mathbf{M}$. Hence, using Eq. (23), the spin-current density can be written in the form

$$\mathcal{J}^i(\mathbf{R},T) = \frac{1}{4}\sum_{\mathbf{k}}\mathrm{Tr}\left[\{v,\sigma^i\}\left(n_{Fl}+\alpha\mathbf{M}\cdot\mathbf{k}\frac{\partial n_{Fl}}{\partial E}\right)\right] + \frac{1}{4}\sum_{\mathbf{k}}\mathrm{Tr}[\{v,\sigma^i\}f_{\mathbf{k}}(\mathbf{R},T)]. \qquad (28)$$

It is easy to see by a direct calculation that the first sum is zero. Therefore, only the nonequilibrium part of the spin current associated with $f_{\mathbf{k}}(\mathbf{R},T)$ contributes to Eq. (28). In this connection, it should be noted that the equilibrium spin current was found to be nonzero in a homogeneous 2D gas.[25] This is due to quantum effects which are beyond the semiclassical approach used in the present work.

A. Homogeneous case

As a simple, yet nontrivial application, we consider the case of a large ac-biased gate such that the time-dependent region can be treated homogeneously. In this case one can omit $\boldsymbol{\nabla}_{\mathbf{R}} f_{\mathbf{k}}$ in Eq. (25) and look for a solution of Eq. (25) in the form $f_{\mathbf{k}}=A(k)\mathbf{M}\cdot\mathbf{k}$. Since $f_{\mathbf{k}}=-f_{-\mathbf{k}}$, the first term in Eq. (26) turns to zero and the solution of Eq. (25) is easily obtained as

$$f_{\mathbf{k}} = \mathbf{M}\cdot\mathbf{k}\frac{i\Omega\tau}{1-i\Omega\tau}\alpha(\Omega)\frac{\partial n_{Fl}}{\partial E}, \tag{29}$$

where $\alpha(\Omega)$ is the Fourier transform of $\alpha(T)$ at the frequency Ω. For simplicity we have assumed that the electron density, as well as $\partial n_{Fl}/\partial E$, does not change in time. On the other hand, this dependence can be important for obtaining the rectified dc spin current. This effect will be discussed in the next section. Substituting Eq. (29) into Eq. (28) we find the spin-current expression in accordance with Ref. 18:

$$\mathcal{J}^i_j(\Omega) = \frac{1}{2}\frac{\rho\Omega}{\Omega+2i\Gamma}\varepsilon^{ij3}\alpha(\Omega), \tag{30}$$

where $\rho=2\Sigma_{\mathbf{k}} n_F(k)$ is the electron density and the indices i and j denote the spin polarization and direction of the current flow, respectively. With the spin-distribution function (29) the spin polarization obtained from Eq. (27) is $\boldsymbol{P}=0$. Hence, in the homogeneous case no spin polarization is induced.

B. Current generation by a narrow gate

We now consider the case of a narrow ac-biased gate, which is infinitely long in the $\boldsymbol{y}$ direction while the width w in the $\boldsymbol{x}$ direction is much smaller than the electron mean free path l, so that, except for a small number of particles moving in the $\boldsymbol{y}$ direction, the motion of electrons under the gate is ballistic. We are interested in spin current flowing in the $\boldsymbol{x}$ direction. Since $M_x=-\sigma_y$, it follows from Eq. (25) that this current is polarized along the $\boldsymbol{y}$ axis. Neglecting the scattering term we obtain, from Eqs. (25) and (28),

$$\mathcal{J}^y_x(q_x,\Omega) = \frac{m^{*2}}{4\pi}\int dE\left[\frac{\partial\alpha(x,T)}{\partial T}\frac{\partial n_{Fl}}{\partial E}\right]_{\Omega,q_x} \times\int\frac{d\phi}{2\pi}\frac{v^2\cos^2\phi}{-i\Omega+iq_x v\cos\phi+0^+}, \tag{31}$$

where q_x is the wave number in the Fourier transform of the current with respect to the $\boldsymbol{x}$ coordinate and $[\cdots]_{\Omega,q_x}$ denotes the Ω,q_x Fourier component of the product in the square brackets. We assume that n_{Fl} is coordinate and time dependent due to the gate electric potential. At $\Omega w \ll v_F$ the frequency in the denominator of Eq. (31) can be neglected. Representing $(iq_x v\cos\phi+0^+)^{-1}$ as $\pi\delta(q_x)/(v|\cos\phi|)$ and returning to the coordinate and time representation we obtain

$$\mathcal{J}^y_x(x,T) = -\frac{m^*}{2\pi}\int dx'\frac{\partial\alpha(x',T)}{\partial T}k_F(x',T), \tag{32}$$

where $k_F(x',T)=\sqrt{2m^*[E_F-U(x',T)]}$. The above expression is valid in the near vicinity of the gate, within the length of the electron mean free path. This ballistic result gives only a part of the spin current, the one associated with a direct generation action of the time dependent gate. It does not take into consideration the backflow of diffusion current due to the spin polarization accumulated on both sides of the gate. Such a diffusion current and the accumulated polarization will be calculated within the drift-diffusion theory in the next section. It is interesting to note that besides the ac component, the spin current also contains the dc component due to time dependence of k_F in the integrand of Eq. (32). It can be easily seen that the dc current is obtained from Eq. (32) if harmonic oscillations of $\alpha(T)$ and $k_F(T)$ are phase shifted with respect to each other. Such a rectification effect will be studied in more detail in the next section.

IV. DRIFT-DIFFUSION EQUATION

In this section, we are interested in the time-dependent spin dynamics in a disordered system, such that the characteristic frequency of the gate time dependence is much smaller then the elastic scattering rate $1/\tau$, while the characteristic length of the spatial variation of $f_{\mathbf{k}}(\mathbf{R},T)$ is larger than the mean free path l. To this end, we start from Eq. (25) and represent $f_{\mathbf{k}}$ in the form

$$f_{\mathbf{k}}(\mathbf{R},T) = \boldsymbol{\sigma}\cdot\boldsymbol{g}_{\mathbf{k}}(\mathbf{R},T). \tag{33}$$

Substituting Eq. (33) into Eq. (25) we obtain

$$\frac{\partial\boldsymbol{g}_{\mathbf{k}}}{\partial T} + (\mathbf{v}\cdot\boldsymbol{\nabla}_{\mathbf{R}})\boldsymbol{g}_{\mathbf{k}} + 2\alpha_0 k(\boldsymbol{g}_{\mathbf{k}}\times\boldsymbol{h}) + \boldsymbol{h}k\frac{\partial\alpha}{\partial T}\frac{\partial n_{Fl}}{\partial E} = \frac{1}{\tau}(\boldsymbol{P}_E-\boldsymbol{g}_{\mathbf{k}}), \tag{34}$$

where the unit vector $\boldsymbol{h}=(\mathbf{k}\times\mathbf{z})/k$ and $\boldsymbol{P}_E=(2\pi/m^*)\Sigma_{\mathbf{k}'}\delta(E'-E)\boldsymbol{g}_{\mathbf{k}'}$. With this definition of $\boldsymbol{P}_E$, from Eqs. (33) and (27), the spin polarization can be expressed as

$$\boldsymbol{P} = \frac{m^*}{2\pi}\int dE\boldsymbol{P}_E. \tag{35}$$

For simplicity we have assumed small variations of α and put $\alpha=\alpha_0$ in the third term of the left-hand side of Eq. (34).

In order to derive the drift-diffusion equation the function $\boldsymbol{g}_{\mathbf{k}}$ is expressed from Eq. (34) in the form

$$\boldsymbol{g}_{\mathbf{k}} = \hat{\Lambda}^{-1}\left[\frac{\boldsymbol{P}_E}{\tau} - \Delta_0(\boldsymbol{g}_{\mathbf{k}}\times\boldsymbol{h}) - hk\frac{\partial\alpha}{\partial T}\frac{\partial n_{Fl}}{\partial E}\right], \tag{36}$$

where $\hat{\Lambda}$ is the operator inverse to $-i\Omega+\mathbf{v}\cdot\boldsymbol{\nabla}_{\mathbf{R}}+1/\tau$. As we mentioned in the beginning of this section, in the diffusion approximation one can expand $\hat{\Lambda}^{-1}$ with respect to the small Ω and $\mathbf{v}\cdot\boldsymbol{\nabla}_{\mathbf{R}}$ compared to $1/\tau$, so that

$$\hat{\Lambda}^{-1} \approx \tau[1 + i\Omega\tau - \tau(\mathbf{v}\cdot\nabla_{\mathbf{R}}) + \tau^2(\mathbf{v}\cdot\nabla_{\mathbf{R}})^2]. \qquad (37)$$

The next step is to express $\boldsymbol{g}\times\boldsymbol{h}$ on the right-hand side of Eq. (36) via $\boldsymbol{P}_E$. It can be done by decomposing $\boldsymbol{g}$ into parallel and perpendicular to $\boldsymbol{h}$ parts, according to

$$\boldsymbol{g} = (\boldsymbol{g}\cdot\boldsymbol{h})\boldsymbol{h} + \boldsymbol{g}_\perp. \qquad (38)$$

Taking the perpendicular projection of Eq. (34) we find

$$\boldsymbol{g}_{\mathbf{k}} \times \boldsymbol{h} = \frac{1}{\tau}(\hat{\Lambda}^2 + \Delta_0^2)^{-1}[\Delta_0 \boldsymbol{P}_{E\perp} + \hat{\Lambda}(\boldsymbol{P}_E \times \boldsymbol{h})]. \qquad (39)$$

After inserting this expression into Eq. (36) we use the expansion (37) and a similar expansion for $(\hat{\Lambda}^2+\Delta_0^2)^{-1}$. It will be further assumed that $\Delta_0 \ll 1/\tau$ and only the terms not smaller than $(\Delta_0\tau)^2$ will be retained on the right-hand side of Eq. (36). Since the current source in Eq. (34) is proportional to $\partial n_{Fl}/\partial E$, for a degenerate electron gas the function $\boldsymbol{P}_E$ has a peak at the local Fermi energy $E_F - U(\mathbf{R},T)$. Hence, taking into account the definition (35), this function can be represented as $\boldsymbol{P}_E = (2\pi/m^*)\delta(E+U(\mathbf{R},T)-E_F)\boldsymbol{P}$. Integrating Eq. (36) over energies and averaging over the angles of $\mathbf{k}$ we arrive at the diffusion equation for the spin transport:

$$\frac{\partial \boldsymbol{P}}{\partial T} = (\nabla\cdot D\,\nabla)\boldsymbol{P} - 4Dm^*\alpha_0[(\mathbf{z}\times\nabla)\times\boldsymbol{P}] - \Gamma_s\boldsymbol{P} - \Gamma_s P^z\mathbf{z} - \frac{1}{2}(\nabla\times\mathbf{z})\rho\tau\frac{\partial\alpha}{\partial T}, \qquad (40)$$

where $D = v_{Fl}^2\tau/2$ is the diffusion constant and $\Gamma_s = 4D(m^*\alpha_0)^2$ is the spin-relaxation rate. It should be noted that the diffusion constant can be coordinate and time dependent via the local Fermi velocity v_{Fl}. Except for the last term on the right-hand side of Eq. (40) the above spin-diffusion equation coincides with that derived earlier from the Green function formalism.[26] In this equation the first three terms represent, respectively, spin diffusion, spin precession due to the SOI, and the D'yakonov-Perel'[27] spin relaxation. The last term is the spin-current source provided by the time-dependent SOI. The spin relaxation gives the natural time scale $T_D = \Gamma_D^{-1}$ for the most of spin-diffusion processes. One can also define the characteristic length of the spin-density spatial variations as the spin-diffusion length $L_D = \sqrt{DT_D}$. The drift-diffusion approach is valid when $T_D \gg \tau$. This condition is provided by the small Δ_0 in comparison with $1/\tau$.

For the following calculations we need an expression for the spin current. It can be found from its initial definition (28). Inserting there $f_{\mathbf{k}}$ written in the form (33), the spin current is found as

$$\mathcal{J}_j^i = \sum_{\mathbf{k}} v_j g_{\mathbf{k}}^i(\mathbf{R},T). \qquad (41)$$

Expressing $g_{\mathbf{k}}^i$ according Eq. (36), from Eqs. (37)–(39) we obtain

$$\mathcal{J}_j^i = -D\nabla_j P^i - 2Dm^*\alpha(\delta^{ij}P^z - P^j\delta^{iz}) + \frac{1}{2}\varepsilon^{ij3}\rho\tau\frac{\partial\alpha}{\partial T}. \qquad (42)$$

This equation shows that the spin current contains three components. The first and second terms represent the usual diffusion current and the current associated with spin precession. These two contributions have been found in earlier works.[26] The third term is the new one. It represents the spin-current generation due to the time-dependent SOI.

It is interesting to note that in the homogeneous case $\boldsymbol{P} = 0$ and the spin current is simply of the form $\mathcal{J}_j^i = \frac{1}{2}\varepsilon^{ij3}\rho\tau(\partial\alpha/\partial T)$. This result coincides with Eq. (30) by taking the limit of $\Omega\to 0$ in the denominator of Eq. (30).

A. Finite-size ac-biased gate

We consider an ac-biased gate which is supposed to be infinite along $\boldsymbol{y}$ and with a width w wider then the mean free path l along $\boldsymbol{x}$, with a Rashba coupling constant $\alpha(x)$. The finiteness of the gate in the $\boldsymbol{x}$ direction results in an $\boldsymbol{x}$ dependence of $\boldsymbol{P}$. At the same time $\boldsymbol{P}$ does not depend on $\boldsymbol{y}$. Retaining in Eq. (40) only derivatives with respect to $\boldsymbol{x}$, one can easily see that a pair of equations for P^x and P^z polarization components is decoupled from the equation for P^y and that the current source term enters only into the latter equation. Therefore, the solution of the diffusion equation is such that $P^x = P^z = 0$ and the equation for the retained component is

$$\frac{\partial P^y}{\partial T} = \frac{\partial}{\partial x}D\frac{\partial}{\partial x}P^y - \Gamma_s P^y + \frac{\partial}{\partial x}S(x,T), \qquad (43)$$

where the time-dependent spin-current source is given by $S(x,T) = \rho\tau\partial\alpha/\partial T$.

We will apply Eq. (43) to two problems. In the first problem the width of the gate will be assumed large, so that $w \gg L_D$. The opposite case of $w \ll L_D$ will be considered in the second example.

1. $w \gg L_D$

Let us assume that in the interval $x_1 < x < x_2$ the diffusion constant is $D(T)$ and $\alpha = \alpha_0 + \alpha(T)$, while outside this interval $D = D_0$ and $\alpha = \alpha_0$. We will consider relatively slow variations in time of $\alpha(T)$, such that their frequency is much less than Γ_s. Consequently, the time derivative on the left-hand side of Eq. (43) will be neglected. The spin current induced in the range $x_1 < x < x_2$ will be injected near edges at the points x_1 and x_2, and the injected polarization will diffuse outward, as well as backward to the modulation gate region, as shown in Fig. 1. For analysis of this injection process it is enough to consider the vicinity of either edge. Let it be the right edge. Then, the solution of Eq. (43) has the form

$$P^y = A\exp\left(-\frac{|x-x_2|}{L_D}\right), \qquad (44)$$

where $L_D = (\alpha_0 m^*)^{-1}$. The coefficient A, in its turn, can be found from the spin-current conservation at the boundary x_2. On the right of the boundary the current is represented by the

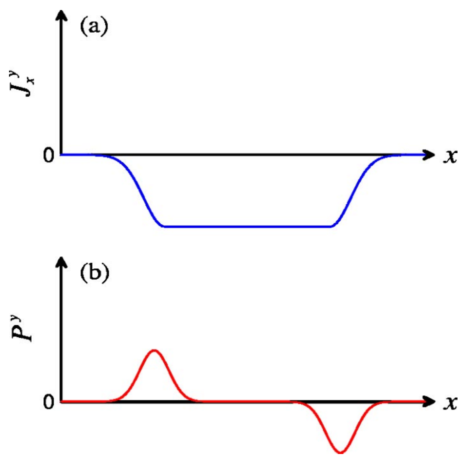

FIG. 1. Illustration of the spin-current flow induced by a wide ac gate: (a) spin current $\mathcal{J}_x^y$ as a function of x, which is uniform under the gate and exponentially decays outside the gate; (b) spin polarization P^y as a function of x, which is accumulated at the gate edges.

diffusion current AD_0/L_D. It must be equal to the current on the left, which is the sum of the diffusion and the source terms $-AD(T)/L_D - S(T)$. The factor A is thus

$$A = -\frac{\rho(T)\tau}{2m^*\alpha_0[D_0 + D(T)]}\frac{\partial\alpha}{\partial T}. \qquad (45)$$

Using the expression $D = v_F^2\tau/2$ and $\rho = k_F^2/2\pi$ we obtain the polarization at the right edge:

$$P^y = -\frac{m^*\rho(T)}{\pi\hbar\alpha_0[\rho(T) + \rho_0]}\frac{\partial\alpha}{\partial T}. \qquad (46)$$

We restored conventional dimensionality by writing $\hbar$ in the proper place. An important feature of this expression is that P^y does not depend on the absolute value of the SOI coupling constant, but rather from the relative amplitude of its variations in time. Hence, the same spin-injection effect is expected for both QW's with a large Rashba SOI and QW's with a not large SOI, providing that the relative variations of α in time are the same in both cases. Another interesting phenomenon, which follows from Eq. (46), is that the ac-modulated SOI can result in stationary spin accumulation near the gate edges. This is due to the time dependence of the electron density in Eq. (46). The dc spin density is obtained by time-averaging this expression. A phase shift between $\rho(T)$ and $\alpha(T)$ is required to have this average nonzero. Such a shift can be achieved, for example, by manipulating the front and back gates, as shown in Fig. 2. Both the back and front gates are equally efficient to induce electron density oscillations. At the same time, as shown by Grundler,[6] the front gate close to the 2DEG is necessary to control efficiently α. Therefore, the required phase shift can be obtained by choosing appropriate phases of V_1 and V_2.

2. $w \ll L_D$

Another interesting regime is the case of a narrow gate with size w (in x direction) much shorter than the characteristic diffusion length L_D. Outside the gate region the solution of Eq. (43) has the same form as in the previous example: namely,

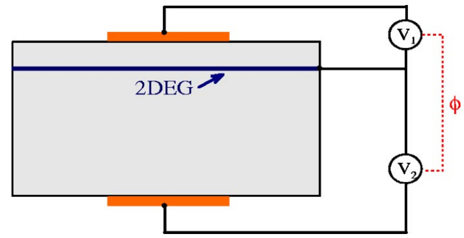

FIG. 2. Illustration of the setup for spin-current generation. The front gate is close to 2DEG. This gate controls the spin-orbit coupling constant via periodic oscillations of V_1. ϕ denotes the phase shift between electric potentials of the front and back gates.

$$P^y = \pm A\exp\left(-\frac{|x|}{L_D}\right), \qquad (47)$$

where the $-$ and $+$ signs refer to the $x > 0$ and $x < 0$ regions, respectively. Inside the gated region, as seen from Fig. 2 near $x=0$, the polarization varies very fast. Therefore, within this range one can retain in Eq. (43) only terms containing derivatives of P^y and S. This gives

$$-D(x)\frac{\partial P^y}{\partial x} - S(x) = C. \qquad (48)$$

This equation has the form of the spin-current conservation law. The constant C is, obviously, equal to the current just outside the gated region, where $S(x)=0$, but still $x \ll L_D$. Hence, from Eq. (47) one obtains $C = D_0A/L_D$. Further, integrating Eq. (48) we arrive at the expression for P^y in the near vicinity of $x=0$:

$$P^y = -\frac{A}{L_D}x - \int_0^x dx'\frac{S(x')}{D(x')} - \frac{A}{L_D}\int_0^x dx'\left(\frac{D_0}{D(x')} - 1\right). \qquad (49)$$

This expression must coincide with Eq. (47) at $w < x \ll L_D$: namely, with $A\exp(-x/L_D) \simeq A - Ax/L_D$. Substituting the latter instead of P^y into Eq. (49) and setting $x \to \infty$, one obtains

$$A = -\left[1 + \frac{1}{L_D}\int_0^\infty dx\left(\frac{D_0}{D(x)} - 1\right)\right]^{-1}\int_0^\infty dx\frac{S(x)}{D(x)}. \qquad (50)$$

One can neglect the integral in the square brackets because it is small by the parameter w/L_D, except for special cases when $D_0/D(x)$ becomes very large at some points. Expressing $D(x)$ via $\rho(x)$, as has been done in the previous problem, we arrive at the simple formula

$$A = -\frac{m^{*2}}{\pi\hbar^3}\int dx\frac{\partial\alpha}{\partial T}. \qquad (51)$$

Since the integral in Eq. (51) is of the order of $wd\alpha/dT$, the polarization injected into the case of a narrow gate is smaller by a parameter w/L_D than in the previous case of the wide gate. That is because of a strong counterflow of the diffusion current reducing the effect of the current $-S$ pumped by the

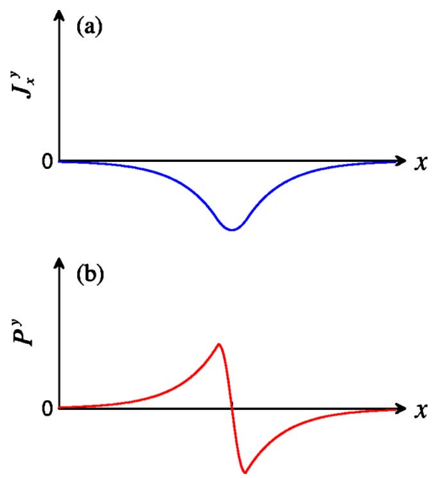

FIG. 3. Illustration of the spin-current flow induced by a narrow ac gate: (a) spin current $\mathcal{J}_x^y$ as a function of x, which is exponentially decays outside the gate; (b) spin polarization P^y as a function of x, which is accumulated around the ac-biased narrow gate. The polarization has opposite signs on two sides of the gate.

gate. Also, unlike the previous example, the narrow gate cannot inject a stationary spin polarization. The rectification effect is absent in this case because in Eq. (51) there are no time-dependent parameters beside $\partial\alpha/\partial T$.

Figure 3 illustrates the spin current [Fig. 3(a)] and spin polarization [Fig. 3(b)] induced by the narrow ac gate. The diffusion current exponentially decays far from the gate region with spin accumulation shown in Fig. 3(b). The spin polarization has opposite signs on two sides of the gate.

B. Spin flow in a channel

We consider a 2D channel (strip) of width d, so that particles are confined in the y direction and free to move in the x direction. The time-dependent gate atop of this channel causes time variations of the Rashba SOI. We assume $d \gg l$ and apply the drift-diffusion equation to describe the spin transport in the channel. Since the gate is supposed to be infinite in the x direction, the spin polarization does not depend on x and only derivatives with respect to y and T have to be retained in Eq. (40).

In this case one can look for a solution of Eq. (46) such that $P^y = P^z = 0$ and $P^x = P^x(y)$. It is easy to check that with such a choice of $\boldsymbol{P}$ the spin current, Eq. (42), in the y direction does not contain y and z polarization components. The homogeneous y-polarized current given by the last term in Eq. (40) flows only in the x direction. Hence, the only current flowing along the y axis is x polarized. For this current the boundary condition is imposed that $(\mathcal{J}_y^x)_{x=\pm d/2} = 0$. Since the time-dependent α in the last term of Eq. (42) generates the spin current $\mathcal{J}_y^x = (1/2)\rho\tau\partial\alpha/\partial T$, to satisfy the boundary condition this current must can be compensated by counterflow of the spin-diffusion current given by the first term in Eq. (42). Hence, the boundary condition is

$$\left[D\nabla_y P^x - \frac{1}{2}\rho\tau\frac{\partial\alpha}{\partial T}\right]_{x=\pm d/2} = 0. \quad (52)$$

In its turn, the equation for P^x has the form

$$\frac{\partial P^x}{\partial T} = D(T)\frac{\partial^2 P^x}{\partial y^2} - \Gamma_s P^x(y). \quad (53)$$

Assuming that the frequency of α variations is small compared to the spin-relaxation rate Γ_s, we obtain a general solution of Eq. (53) in the form

$$P^x(y,\Omega) = C_1 e^{\kappa y} + C_2 e^{-\kappa y}, \quad (54)$$

where $\kappa = 1/L_D$. The solution satisfying the boundary condition is easily obtained as

$$P^x(y,T) = \frac{m^*}{2\hbar\pi\alpha}\frac{\sinh(\kappa y)}{\cosh\left(\kappa\frac{d}{2}\right)}\frac{\partial\alpha}{\partial T}. \quad (55)$$

For an extended 2D electron system with $d \to \infty$, it is easy to see that $P^x(y,T) \to 0$ indicating the absence of the bulk spin density, in agreement with the result of Sec. III A. At the same time, as expected, the polarization is accumulated near $y = \pm d/2$, decreasing exponentially when the distance from the boundary increases.

V. SUMMARY AND DISCUSSION

We have employed the electric gate effect on the Rashba spin-orbit interaction in narrow-gap semiconductor QW's and considered spin transport in a 2DEG with the space-time-dependent Rashba spin-orbit interaction. The variations of the SOI in time and space were assumed to be provided by gates of various shapes. Spin transport was considered in the framework of the Keldysh formalism which has been applied to derive the Boltzmann equation for the spin-distribution function in phase space. This equation was further employed in the derivation of the drift-diffusion equation for the spin density. We found that besides the usual terms, both the Boltzmann and drift-diffusion equations contain a spin-current motive force proportional to the time derivative of the SO coupling parameter.

We have considered several examples with various gate geometries. Although these examples do not embrace many other interesting possibilities, nevertheless, they demonstrate how spin-current flow and spin-density accumulation can be electrically controlled by means of gates. It has been shown that in some geometries the ac bias applied to the gate can result in dc spin current and stationary spin accumulation. This is a simplest self-rectification effect. Probably, more efficient could be a special rectification setup consisting of several gates in series and combinations of back and forward gates to control separately the electron density and SO parameter.

ACKNOWLEDGMENTS

This work was partly funded by the Taiwan National Science Council (NSC) under Grant No. NSC93-2119-M-007-002 (NCTS), Russian Academy of Sciences and RFBR Grant No. 03-02-17452, and the Swedish Royal Academy of Science. A.G.M. acknowledges the hospitality of NCTS where this work was initiated.

[1] *Semiconductor Spintronics and Quantum Computation*, edited by D. D. Awschalom, N. Samarth, and D. Loss (Springer-Verlag, Berlin, 2002).
[2] S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnar, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, Science **294**, 1488 (2001).
[3] I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
[4] E. I. Rashba, Fiz. Tverd. Tela (S.-Peterburg) **2**, 1224 (1960) [Sov. Phys. Solid State **2**, 1109 (1960)]; Y. A. Bychkov and E. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 66 (1984) [JETP Lett. **39**, 78 (1984)].
[5] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997).
[6] D. Grundler, Phys. Rev. Lett. **84**, 6074 (2000).
[7] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).
[8] H. Mathur and A. D. Stone, Phys. Rev. Lett. **68**, 2964 (1992).
[9] A. G. Mal'shukov, V. Shlyapin, and K. A. Chao, Phys. Rev. B **66**, 081311(R) (2002); C. H. Chang, A. G. Mal'shukov, and K. A. Chao, Phys. Lett. A **326**, 436 (2004).
[10] L. S. Levitov and E. I. Rashba, Phys. Rev. B **67**, 115324 (2003).
[11] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. **92**, 126603 (2004); S. Murakami, N. Nagaosa, and S. Zhang, Science **301**, 1348 (2003).
[12] M. I. Dyakonov and V. I. Perel, Phys. Lett. **35A**, 459 (1971); J. E. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999).
[13] J. Wunderlich, B. Kästner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).
[14] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science **306**, 1910 (2004).
[15] V. M. Edelstein, Solid State Commun. **73**, 233 (1990).
[16] P. Sharma and P. W. Brouwer, Phys. Rev. Lett. **91**, 166801 (2003).
[17] M. Governale, F. Taddei, and R. Fazio, Phys. Rev. B **68**, 155324 (2003); L. Y. Wang, C. S. Tang, and C. S. Chu, cond-mat/0409291 (unpublished).
[18] A. G. Mal'shukov, C. S. Tang, C. S. Chu, and K. A. Chao, Phys. Rev. B **68**, 233307 (2003).
[19] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
[20] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, New York, 1981).
[21] H. Haug and A. P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors*, Springer Series in Solid-State Sciences, Vol. 123 (Springer-Verlag, Berlin, 1996).
[22] J. Rammer and H. Smith, Rev. Mod. Phys. **58**, 323 (1986).
[23] E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. **93**, 226602 (2004).
[24] A. A. Burkov, Alvaro S. Nunez, and A. H. MacDonald, Phys. Rev. B **70**, 155308 (2004).
[25] E. I. Rashba, Phys. Rev. B **68**, 241315(R) (2003).
[26] A. G. Mal'shukov and K. A. Chao, Phys. Rev. B **61**, R2413 (2000).
[27] M. I. D'yakonov and V. I. Perel', Fiz. Tverd. Tela (S.-Peterburg) **13**, 3581 (1971) [Sov. Phys. Solid State **13**, 3023 (1972)]; Zh. Eksp. Teor. Fiz. **60**, 1954 (1971) [Sov. Phys. JETP **33**, 1053 (1971).