# Tunable negative refraction in a two-dimensional active magneto-optical photonic crystal

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A two-dimensional square photonic crystal (PC) with magnetically active semiconductor constituents, for example GaAs, is proposed to realize tunable negative refractions. By the Voigt effect, the magnetic-dependent permittivity of the semiconductor, then PC's band structure and equifrequency surface could be tuned by an external magnetic field. Taking a real model into consideration with absorption, the calculation reveals that the refraction could be manipulated from positive to negative, and then to positive by the applied fields. With the tunable negative refraction, tunable superlenses with all angle negative refraction could be established by a relatively small magnetic field (0.262–0.315 T), which might lead to great potential applications in photoelectronic devices.

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## I. INTRODUCTION

Negative refraction of electromagnetic (EM) waves was theoretically predicted in the left-handed material (LHM) by Veselago in 1968.[1] Due to its potential applications, the study of negative refraction has been attracting a great deal of attention recently. In contrast to the positive refraction resulted conventionally used curve lens to form an image, through double focusing effect based on negative refraction, a flat LHM slab with $\varepsilon=-1$ and $\mu=-1$ was proposed by Pendry as a perfect lens in which both propagating and evanescent waves contribute to the image.[2] Some relevant phenomena have been experimentally demonstrated by constructing two-dimensional arrays consisted of split-ring resonators and wires.[3–6]

On the other hand, negative refraction also exists in photonic crystals (PCs), resulted by intense scatterings near the Brillouin zone boundaries instead of simultaneously negative permittivity and permeability in LHM.[7–11] The behaviors of negative refraction in PCs have been also discussed both theoretically and experimentally. Based on the negative refraction in PC, another kind of flat lens was also proposed by Luo *et al.* as a superlens to make use of both evanescent and propagating waves to produce a real image beyond the diffraction limit.[9–11] The relevant properties have been observed experimentally in the microwave range,[12,13] showing great potential in the optical regime.

For ordinary PCs, photonic band structures (PBS) and related equifrequency surfaces (EFS) will remain immutable once they have been constructed. For many applications, however, it is highly desired to obtain some degrees of tunability in PBS (Refs. 14–18) and EFS.[19,20] Recently, several ideas to tune PBS have been proposed that a nematic-liquid PC was responsible to an applied electric field, that PCs constructed with ferroelectric or ferromagnetic materials could be tuned by an external electric or magnetic field,[15] that tunabilities of the PCs incorporating superconductor constituents were discussed,[16] and that the possibility to tune PCs by the Faraday effect was also described.[17] Can negative refractions be tunable like PBS? And how large can the tuning range of refractions reach? In this paper, we will attempt to address these questions by tunable PCs with semiconductor constituents based on another magneto-optical effect, Voigt effect.[18] The theoretical study shows the refraction of the EM wave could be manipulated from positive to negative, thus realizing a tunable superlens. To the best of our knowledge, the tunable negative refraction is discussed for the first time, whose very large tuning range will allow great potential applications.

In Sec. II, the Voigt effect is described briefly. Then based on the Voigt effect, we propose tunable negative refractions in PCs in Sec. III and tunable superlensing effect in Sec. IV. Section V is the discussion of the feasibility of tunable negative refraction and Sec. VI is the conclusion.

## II. THE DESCRIPTION OF THE VOIGT EFFECT

The Voigt effect is a well-known magneto-optical effect where the permittivity is changed by an applied magnetic field,[21] whose direction is perpendicular to the propagating direction of the EM wave. In the effect, the permittivity for the EM wave with its electric field perpendicular to the external magnetic field is different from that parallel to the applied field. When the electric field of the EM wave is parallel to the external magnetic field, the permittivity keeps unchanged with various external magnetic fields, by neglecting the absorption, which is[22]

$$\varepsilon_{\parallel}=\varepsilon_0\left(1-\frac{\omega_P^2}{\omega^2}\right), \tag{1}$$

where $\varepsilon_0$ is the static dielectric constant and $\omega_P$ is the screened plasma frequency, which depends on the densities $n$ and effective masses $m^*$ of free carriers and could be described as

$$\omega_P^2=\sum_i \frac{4\pi n_i e^2}{m_i^*\varepsilon_0}, \tag{2}$$

where the footnote $i$ is the relevant conducting or valence band of free electrons and holes and $e$ is the charge. While the electric field is perpendicular to the external magnetic field $B$, the permittivity will be dependent on the magnetic field by[21]

$$\varepsilon_{\perp} = \varepsilon_0\left(1 - \frac{\omega_P^2}{(\omega^2 - \omega_c^2)} - \frac{\omega_P^4\omega_c^2}{\omega^2(\omega^2 - \omega_c^2)(\omega^2 - \omega_c^2 - \omega_P^2)}\right), \tag{3}$$

where the cyclotron frequency $\omega_c = eB/m^*c$ is linearly proportional to the external magnetic field and $c$ is the light speed in a vacuum. There exists a resonant frequency $\sqrt{\omega_c^2+\omega_p^2}$. It is well known that the absorption is important and expected near the plasma frequency and resonant frequency due to the finite lifetimes of electrons, holes, and phonons. Therefore, the dielectric constant has a damping term, so $\omega^2$ should be replaced by $\omega^2 + i(\omega/\tau)$, namely,

$$\varepsilon_{\parallel}(\omega) = \varepsilon_0\left(1 - \frac{\omega_P^2}{\omega^2 + i\omega/\tau}\right), \tag{4}$$

$$\varepsilon_{\perp}(\omega) = \varepsilon_0\left(1 - \frac{\left(\omega^2 + i\dfrac{\omega}{\tau}\right)\omega_P^2 - \omega_P^4}{\left(\omega^2 + i\dfrac{\omega}{\tau}\right)\left(\omega^2 - \omega_c^2 - \omega_P^2 + i\dfrac{\omega}{\tau}\right)}\right), \tag{5}$$

where $\tau$ is the scattering time.

### III. TUNABLE NEGATIVE REFRACTION IN PCs

We consider a 2D square magneto-optical-active photonic crystal (MOAPC) made up of $n$-doped GaAs with "hole-in-dielectric" structure. Note that the polarization of the electric field perpendicular (parallel) to the external magnetic field corresponds to the TE (TM) mode of wave propagation in MOAPC. The static dielectric constant and plasma frequency of GaAs at 300 K could be taken to be 12.9 and 0.785 THz, respectively[18,22] and the proper scattering time is chosen as $\tau = 20$ ps, which is easy to reach in doping GaAs.[23] As an example, a MOAPC with the lattice constant of 0.360 mm and the radii of cylindrical air holes of 0.108 mm was calculated in detail by using the plane-wave expansion (PWE) method.[24]

To investigate the characteristics of MOAPC, the change of TE permittivity $\varepsilon_{\perp}$ must be discussed. For the frequency lower than the plasma frequency $\omega_p$, the permittivity increases monotonously to $\varepsilon_0$ at the infinite field. For the frequency higher than $\omega_p$, however, the permittivity value could reach much larger [Fig. 1(a)]. And when the frequency is a little bit higher than $\omega_p$, TE permittivity $\varepsilon_{\perp}$ is very sensitive to applied fields so that a relatively small field could alter $\varepsilon_{\perp}$ dramatically. Figure 1(a) shows the dependence of TE permittivity of GaAs on applied magnetic fields, in which the operating frequency is $\omega = 1.1\omega_p = 0.864$ THz. The permittivity experiences a field-dependent resonance which corresponds to the resonant frequency $\sqrt{\omega_c^2+\omega_p^2}$. When the external field is smaller than the resonant field, the real part of the permittivity varies from 2.29 with $B=0$ to $-8.37$ with $B=0.12$ T. Due to the existence of the absorption near the resonance, the permittivity could not reach infinitude so that the real part of the permittivity varies from $-8.37$ to 25.4 near the resonance. While the external field is stronger than the resonant field, the permittivity varies from 25.4 with $B=0.17$ T to 12.9 with the infinite field, in which the ratio of the imaginary part to the real part of $\varepsilon_{\perp}(\omega)$ is very small, indicating a weak absorption.

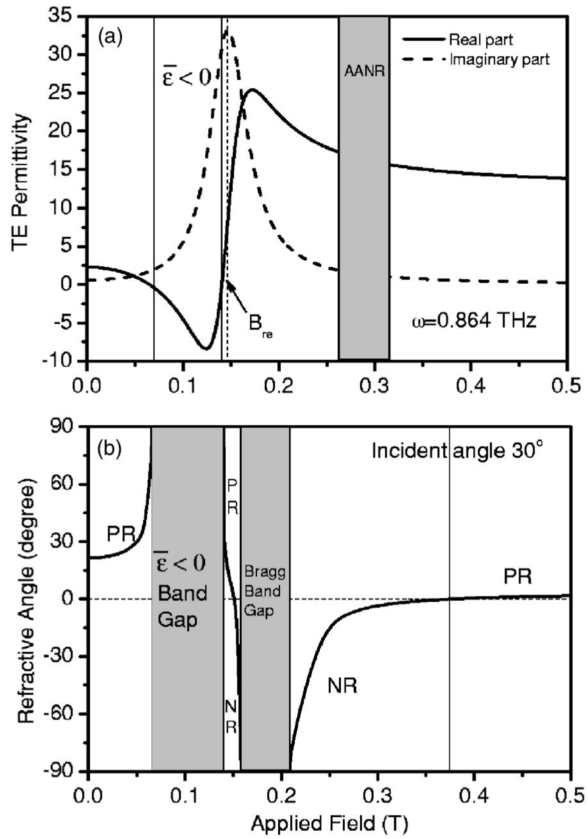


FIG. 1. (a) The permittivity of TE mode varies with applying external fields in the Voigt effect, whose real and imaginary parts are solid and dashed curves, respectively. The dashed line indicates the resonant field of the permittivity, where $B_{re} = (\sqrt{\omega^2 - \omega_p^2}mc)/e$ and $\omega = \sqrt{\omega_c^2+\omega_p^2}$. The shaded region is the field that enables MOAPC to be AANR at the frequency of $\omega = 0.864$ THz. (b) The relationship between refractive angles of EM waves of the TE mode and applied magnetic fields with the incident angle of 30° at the frequency of $\omega = 0.864$ THz. The positive refraction (PR) is manipulated to the negative refraction (NR), and then to PR. The shaded regions indicate the band gap resulting from $\overline{\varepsilon} < 0$ and the first Bragg band gap, respectively.

The propagating direction of EM wave in PC could be investigated by analyzing the direction of group velocity via EFS, which was proposed to analyze the superprism effect.[25] In our structure, the relation between absorption and EFS must be considered. It is well known that EFS is the dispersion of the frequency on wave vectors like band structures. In Ref. 26, compared to the case without absorption, the PC's band gap is well preserved though the transmittance of EM waves decrease due to the absorption in materials when absorption is taken into account, indicating the validity that PC's band structure could be calculated without absorption. So EFS could also keep stable regardless of the existence of absorption so that the direction of wave propagation will not

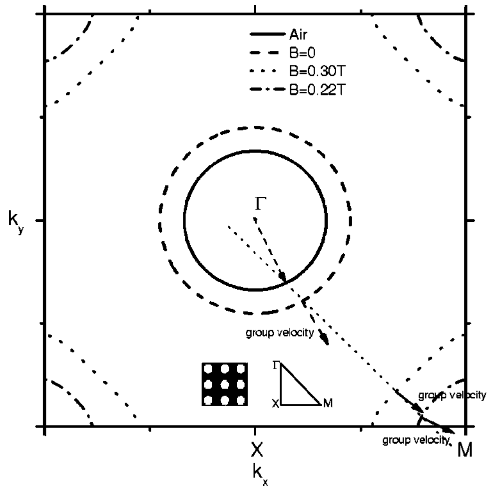


FIG. 2. EFSs of air (solid line) and MOAPC at the frequency of $\omega=0.864$ THz. The dashed, dotted, and dashed–dotted lines correspond to the EFS with $B=0$, $B=0.30$ T, and $B=0.22$ T, respectively. With the normal direction along Γ-M, the directions of incident wave (dashed arrow) and three different refractive group velocities in MOAPC (solid arrow) are shown.

change. In addition, MOAPC mainly operates in the field more than 0.20 T, in which the ratio of imaginary part of permittivity to the real part is less than 0.2, indicating a weak absorption. Hence, in our PWE calculation, we will only consider the real part of permittivity.

With 289 plane waves, EFSs in the first Brillouin zone for TE mode in MOAPC have been calculated. Figure 2 shows the calculated EFSs at the frequency of $\omega=0.864$ THz. The anisotropy of wave propagation is determined by the curvature of EFS. The more dramatic modification of the EFS, the greater the anisotropy. In comparison with no external magnetic field case, the ratio of GaAs permittivity to air's will increase from 2.29 to 20 when the external magnetic field is 0.22 T, which results in dramatic modifications of MOAPC's EFS and anisotropy of wave propagation. With the external magnetic field increasing to 0.30 T, though the deformation of EFS is still obvious comparing to the EFS without the applied field, the anisotropy is not as great as that at 0.22 T. With the supposed parameters of MOAPC, applying a quite smaller field more than resonant field, the anisotropy of EFS could be effectively enhanced.

The anisotropy of EFS determines the propagation and refraction of EM waves in PC. Due to sensitivity of anisotropy to external fields, the wave refraction varies with the applied fields. From Fig. 2, with the surface normal of MOAPC along Γ-M (1, 1) direction, it is obvious that the great anisotropy at the field of 0.22 T or 0.30 T produces negative refraction, while the refraction is positive without the applied field. To illustrate the effect of fields on wave refraction in MOAPC clearly, the refraction of a single beam is shown in Fig. 1(b), in which the incident angle is 30° and the frequency is $\omega=0.864$ THz. The refraction varies dramatically with the applied fields, which attributes to the resonance of the permittivity. When the applied field is much smaller than the resonant field, the refractive angle in MOAPC is positive. If the applied field can satisfy the condition that the average permittivity of MOAPC ($\overline{\varepsilon}=f_{\mathrm{GaAs}}\varepsilon_{\perp}+f_{\mathrm{air}}$, where $f$ is the filling fraction) is $0<\overline{\varepsilon}<1$, the refractive angle is positive and bigger than the incident angle, and even reaches 90°, where $k_{\Gamma\text{-}M}=\sin 30°\, k_{\mathrm{air}}=\frac{1}{2}k_{\mathrm{air}}$ corresponding to the directional band gap of 30° incidence. When the applied field is between 0.066 T and 0.140 T, the average permittivity is $\overline{\varepsilon}<0$ which corresponds to band gap where the propagation of the EM wave no matter its direction will be forbidden in MOAPC. Near the resonance from 0.140 T to 0.158 T, the refractive angle varies from −90° to 90°. Of course, due to the existence of large absorption, the transmittance will be low in this range. So the operating field in MOAPC should be far away from the resonance. On the other hand, when applying field greater than the resonant field, the increase of the applied field results in the decrease of permittivity, which may lead to PBS move upwards,[18] meaning that the defined frequency of $\omega=0.864$ THz shifts from higher band to lower band so that the first Bragg band gap occupies from 0.158 T to 0.208 T and the negative refraction in the lowest band only exists when the field is above 0.208 T which corresponds to the field with the refractive angle of −90°. When $B>0.208$ T, the absorption is weak $(\mathrm{Im}(\varepsilon)/\mathrm{Re}(\varepsilon)\leqslant 0.2)$, so it is feasible to operate MOAPC in this range. And in this range, the negative refractive angle will be smaller due to the decrease of the anisotropy. So there exists a critical field of $B=0.374$ T to the refraction varying from negative to positive. Notice that the critical field will vary with different frequencies and incident directions. The higher frequency and bigger incident angle may make the critical field greater, and vice versa. Hence, with strengthening the external field, the refraction of a single beam EM wave in MOAPC varies from positive refraction to totally internal reflection in bandgaps, then to negative refraction and finally back to positive refraction in the lowest band. The angle of refraction could deflect from 90° to −90° with varying applied magnetic fields, which is more effective than the deflection resulted by the tunable PC with electro-optical effect[19,20] because of the greater magneto-optical response of Voigt effect. Therefore, MOAPC is possible to construct an optical scanner in which the refractive route of a single beam could be manipulated, from positive to negative, or to some particular refractive angle with different applied fields.

The wave propagation and refraction are dramatically related to the frequency, thus the deflection of refraction is different with frequencies. Figure 3(a) shows the dependence of the refractive angle of a singe beam on frequencies with the incident angle of 30°. At a fixed field, the enhancement of the anisotropy with increasing frequency may produce a bigger refractive angle and larger deflection. The refractive angle varies monotonically toward negative to −90° with the increase of frequencies.

Due to the anisotropy of EFS, the wave propagation and refraction are also strongly dependent on the incident angle of the EM wave. For the MOAPC, the anisotropy is greatly amplified by the Voigt effect. With the surface normal along the Γ-M (1, 1) direction, three dispersion relations of TE waves incident and refractive angle have been calculated as shown in Fig. 3(b) with magnetic fields of $B=0$, $B=0.30$ T, and $B=0.22$ T, respectively, at the frequency of $\omega$

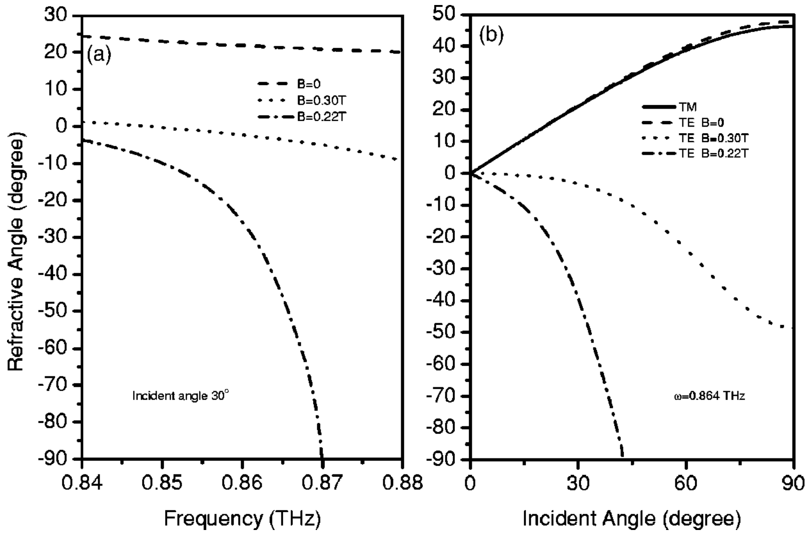


FIG. 3. (a) The relationship between refractive angles of the EM waves of the TE mode and frequencies with the incident angle of 30°. (b) The dependence of refractive angles on the incident angles of the EM wave at the frequency of $\omega = 0.864$ THz. The solid, dashed, dotted, and dashed–dotted lines correspond to the relation for TM, TE with $B=0$, $B=0.30$ T, and $B=0.22$ T, respectively.

$=0.864$ THz. Meanwhile, insusceptible permittivity of the TM mode assures the steadiness of the TM refraction with the fields. By using this kind of different response to the external magnetic field, for TE and TM modes mixed EM waves, MOAPC could also be regarded as a polarizer. Notice that a single beam could evolve into two separate parallel beams after propagating through the slab of MOAPC, one as a TE wave and the other as a TM wave, whose distance could be tunable by adjusting the TE wave refraction with applied fields. When the incident angle is large enough with a relatively small field, such as 50° at 0.22 T, only the TM waves could transmit and all the TE waves reflect, then MOAPC could also be used as a filter to remove TE waves.

## IV. TUNABLE SUPERLENSING EFFECT

For the case of $B=0.30$ T, there exists an all angle negative refraction (AANR), which is necessary to get the best resolution of the image by the superlensing effect.[9] By calculating the −90° negative-refraction field of the EM wave with the incident angle of 90° and the critical field of the wave with incident angle of 0°, for the frequency of $\omega = 0.864$ THz, AANR superlens could be expected to establish within the range from 0.262 T to 0.315 T [shown in Fig. 1(a)]. The frequency range of AANR could also be tuned by applied fields. Corresponding to higher frequency, the range of AANR is greater and wider, and vice versa. In order to simulate the superlensing effect, we have performed finite difference time domain (FDTD) simulations with perfectly matched layer boundary conditions (FullWAVE v3.0.1, Rsoft Design Group, Inc.) on a (1, 1)-oriented slab of MOAPC as shown in Fig. 4. And the spatial resolution of the simulated structure is given as $20\times 20$ mesh points to discrete space within one unit cell. What are depicted here are figures of the magnetic (electric) field of TE (TM) mode for a continuous wave point source placed at a distance of 0.108 mm from the left surface of MOAPC at the frequency of $\omega = 0.864$ THz. Based on Figs. 1–3, for the TM mode and the TE mode without applied field, EFS is not concave to the M point and all group refractive angles are positive, while for the TE mode with the magnetic field of 0.22 T, only waves with the incident angle smaller than 42° could refract negatively and waves with bigger incident angles undergo totally internal reflection because PC's EFS is smaller than air's. For the TE mode with the magnetic field of $B=0.30$ T, however, all waves no matter their incident directions could converge at the right side of the PC to contribute to the formation of a real "point" image. Hence, such tunable superlensing effect could be expected in the MOAPC by different applied magnetic fields.

To validate the feasibility of PWE calculations, we have considered two cases, one with absorption (Fig. 4 a1–d1), and the other without absorption (Fig. 4 a2–d2). When $B=0.30$ T, the image quality with absorption are almost same as that without absorption because the absorption is very weak ($\mathrm{Im}(\varepsilon)/\mathrm{Re}(\varepsilon)=0.07$). When $B=0.22$ T, the difference between two cases are more obvious due to its larger absorption ($\mathrm{Im}(\varepsilon)/\mathrm{Re}(\varepsilon)=0.19$), but the tunable effect by MOAPC is still preserved except the decrease of the transmittance. From these comparisons, it is reasonable to expect the tunable effect based on MOAPC regardless of the existence of absorption, especially in the range of $B>0.208$ T in which the absorption is weak. On the other hand, FDTD simulations with absorption agree very well with our PWE calculations that ignore the absorption, showing the correctness of our predictions. Meanwhile, the results of Ref. 26 and our analysis about the effect of absorption on EFS and wave propagation are also proved.

## V. THE FEASIBILITY OF TUNABLE NEGATIVE REFRACTION

Besides applied fields, tunabilities of MOAPC are also dependent on the plasma frequency of magneto-optical-active materials. From Eq. (2), the plasma frequency $\omega_p$ could be modulated by changing the densities of the free electrons and holes.

A higher plasma frequency $\omega_p$ could reach resonance $\omega = \sqrt{\omega_c^2+\omega_p^2}$ with a smaller cyclotron frequency $\omega_c$, i.e., a smaller external field. As all the properties of MOAPC are

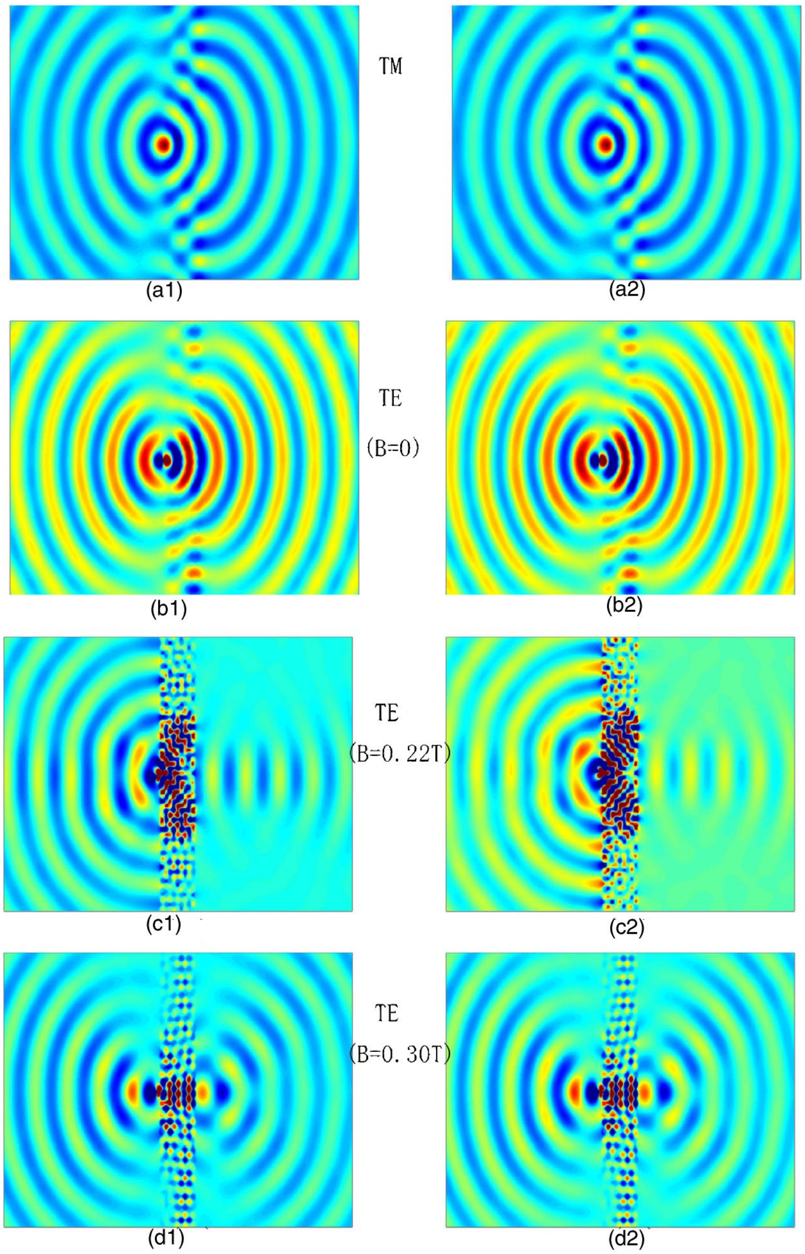


FIG. 4. (Color online only) The FDTD simulations of the tunable superlensing effect at the frequency of $\omega = 0.864$ THz with absorption (a1–d1) and without absorption (a2–d2). (a) TM mode, (b) TE mode with $B=0$, (c) TE mode with $B=0.22$ T, and (d) TE mode with $B=0.30$ T.

scalable, such tunable negative refractions may also be realized in higher frequency region by selecting proper magneto-optical constituents with higher plasma frequency. But for lower frequency, such as microwave region, a small external field could always reach the resonance. So in lower frequency, it is only needed to change the electrons and holes' densities to make the plasma frequency less than operating frequency.

On the other hand, the absorption must be considered in MOAPC. Due to the existence of the absorption near the resonant frequency, the permittivity cannot reach the infinite value. But with a large scattering time, the change of the permittivity is still obvious (for example, from −200 to 500 with the scattering time of 300 ps). So to eliminate the effect of the absorption, a large scattering time is preferable. Meanwhile when the applied field is far away from the resonance, the effect of absorption can be neglected, corresponding to the region of tunable negative refraction in the lowest band [more than 0.208 T in Fig. 1(b)]. So tunable negative refraction by external magnetic fields is feasible although there exists strong absorption near the resonant field.

Hence the MOAPC is quite flexible in materials selection and operating frequency regions. By choosing a proper plasma frequency and a large scattering time, which can be easily realized by changing the densities of the free electrons and holes, tunable negative refraction is sure to be established in MOAPC, leading to great potential applications in photoelectronic devices.

## VI. CONCLUSIONS

In summary, we have theoretically analyzed a 2D square MOAPC by the Voigt effect, which results in effective modifications of PBS, then EFS and anisotropy of wave propagation with applied magnetic fields. In this effect, a relatively small field could manipulate the refraction of EM waves from positive to negative, or to some desired angle. Based on this tunability, a tunable AANR superlens could be established by MOAPC with external magnetic fields. Hence, MOAPC, combined external magnetic fields with microstructure lattice and materials flexibilities, could be used as the tunable optical controller and polarizer for a single beam and tunable superlens for a point source and will lead to great potential applications in photoelectronics.

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[1] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[2] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[3] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[4] R. A. Shelby, D. R. Smith, S. C. Nemat-Nasser, and S. Schultz, Appl. Phys. Lett. **78**, 4i89 (2001).
[5] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[6] A. A. Houck, J. B. Brock, and I. L. Chuang, Phys. Rev. Lett. **90**, 137401 (2003).
[7] M. Notomi, Phys. Rev. B **62**, 10696 (2000).
[8] E. Cubukcu, K. Aydin, E. Ozbay, S. Foteinopoulou, and C. M. Soukoulis, Nature (London) **423**, 604 (2003).
[9] C. Luo, S. G. Johnson, J. D. Joannopoulos, and J. B. Pendry, Phys. Rev. B **65**, 201104 (R) (2002).
[10] C. Luo, S. G. Johnson, and J. D. Joannopoulos, Appl. Phys. Lett. **81**, 2352 (2002).
[11] C. Luo, S. G. Johnson, J. D. Joannopoulos, and J. B. Pendry, Phys. Rev. B **68**, 045115 (2003).
[12] P. V. Parimi, W. T. Lu, P. Vodo, and S. Sridhar, Nature (London) **426**, 404 (2003).
[13] E. Cubukcu, K. Aydin, E. Ozbay, S. Foteinopoulou, and C. M. Soukoulis, Phys. Rev. Lett. **91**, 207401 (2003).
[14] K. Busch and S. John, Phys. Rev. Lett. **83**, 967 (1999).
[15] N. Yamamoto, S. Noda, and A. Sasaki, Jpn. J. Appl. Phys., Part 1 **36**, 1907 (1997); J. E. G. Wijnhoven and W. L. Vos, Science **281**, 802 (1998).
[16] Y. B. Chen, C. Zhang, Y. Y. Zhu, S. N. Zhu, and N. B. Ming, Mater. Lett. **55**, 12 (2002).
[17] W. L. Jia, Y. Z. Li, Y. G. Xi, P. Jiang, X. H. Xu, X. H. Liu, R. T. Fu, and J. Zi, J. Phys.: Condens. Matter **15**, 6731 (2003).
[18] C. Xu, X. H. Hu, Y. Li, X. Liu, R. Fu, and J. Zi, Phys. Rev. B **68**, 193201 (2003).
[19] D. Scrymgeour, N. Malkova, S. Kim, and V. Gopalan, Appl. Phys. Lett. **82**, 3176 (2003).
[20] S. Xiong and H. Fukshima, J. Appl. Phys. **94**, 1286 (2003).
[21] C. R. Pidgeon, in *Handbook on Semiconductors*, edited by M. Balkanski (North-Holland, Amsterdam, 1980), Vol. 2, p. 223.
[22] C. Kittel, *Introduction to Solid State Physics*, 5th ed. (Wiley, New York, 1976).
[23] W. M. Zheng, M. P. Halsall, P. Harrison, J. P. R. Wells, I. V. Bradley, and M. J. Steer, Phys. Status Solidi B **235**, 54 (2003).
[24] M. Plihal and A. A. Maradudin, Phys. Rev. B **44**, 8565 (1991).
[25] H. Kosaka, T. Kawashima, A. Tomita, M. Notomi, T. Tamamura, T. Sato, and S. Kawakami, Appl. Phys. Lett. **74**, 1212 (1999).
[26] Z. L. Wang, C. T. Chan, W. Y. Zhang, N. B. Ming, and P. Sheng, Phys. Rev. B **64**, 113108 (2001).