Chiral symmetry breaking in three-dimensional quantum electrodynamics in the presence of irrelevant interactions: A renormalization group study

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Motivated by recent theoretical approaches to high temperature superconductivity, we study dynamical mass generation in three-dimensional quantum electrodynamics (QED_3) in the presence of irrelevant four-fermion quartic terms. The problem is reformulated in terms of the renormalization group flows of certain four-fermion couplings and charge, and then studied in the limit of a large number of fermion flavors N. We find that the critical number of fermions N_c below which the mass becomes dynamically generated depends continuously on a weak chiral symmetry breaking interaction. One-loop calculation in our gauge-invariant approach yields $N_{c0}=6$ in pure QED_3. We also find that chiral symmetry preserving mass cannot become dynamically generated in pure QED_3.

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I. INTRODUCTION

It has been proposed recently that the low-energy theory of gapless quasiparticles in a two-dimensional *d*-wave superconductor (dSC) with strong phase fluctuations can be represented by the two-flavor massless quantum electrodynamics in three dimensions (QED_3).[1,2] The coupling constant (or the "charge") in such an effective theory is the vortex condensate, i.e., the order parameter dual to the usual superconducting order parameter. It is well known that QED_3 is inherently unstable toward the dynamical mass generation,[3,4] which in the context of *d*-wave superconductivity implies the transition into one of several possible insulating ground states. Each of the insulators corresponds to a broken generator of the U(4) *chiral* symmetry of QED_3, which emerges at low energies in the standard dSC.[2,5] Most important among the insulating ground states is the spin-density wave, which turns out to be favored by the repulsive interactions.[2,6] This approach then provides a viable unified description of the known low-temperature phases of underdoped high-temperature superconductors.

Dynamical mass generation, however, occurs only if the number of Dirac fermions N in QED_3 does not exceed the critical number N_{c0}. If the value of N_{c0} turns out to be less than the number of Dirac fermions, which for a single-layer dSC is $N=2$, then quantum disordering of the phase of the dSC will yield a spin liquid, instead of a spin-density wave insulator. It is thus of importance to establish whether QED_3 with $N=2$ lies below or above the critical value for spontaneous chiral symmetry breaking in the theory.

The estimates of N_{c0} at the moment strongly disagree, however. Early studies of Schwinger-Dyson equations in large-N approximation gave $N_{c0}=32/\pi^2\approx 3.24$.[4] Vertex corrections,[7] or the next-to-leading-order terms in the $1/N$ expansion[8] did not change N_{c0} significantly, and if anything, only increased its value. On the other hand, Appelquist *et al.* have argued that $N_{c0}<3/2$.[9] Adding to the controversy, recent lattice calculations have found no decisive signal for chiral symmetry breaking for $N=2$, but did detect a significant fermion mass for $N=1$.[10] It has been argued, however, that although greatly increased compared to early studies, the sizes of the systems considered in the lattice calculations may still not be close enough to the thermodynamic limit.[11] In fact, due to the essential singularity at $N=N_c$, the value of the mass at $N=2$, if finite, should be rather small and the results of numerical simulations are not necessarily in conflict with the values obtained from the Schwinger-Dyson equations.[10,12]

In the context of high-temperature superconductivity, however, an additional issue arises. Chirally symmetric, Lorentz invariant QED_3 emerges only asymptotically at low energies, when all the irrelevant perturbations may be ignored. For example, large anisotropy between the two characteristic velocities of the dSC, although marginally irrelevant,[13–15] reduces the U(4) symmetry of the two-flavor theory to the $U(2)\otimes U(2)$ over a wide crossover region.[2] The (irrelevant) repulsive interaction between electrons breaks each U(2) factor per flavor further down to $U(1)\otimes U(1)$. It is presently unclear how, and if at all, the presence of these irrelevant perturbations affects the value of N_c in the more complete theory. This is the issue we wish to address in the present paper.

We apply the momentum-shell renormalization group (RG) to QED_3 theory with N fermion flavors, and with four-fermion interactions which break the U(2) symmetry per flavor. The gauge-invariant β functions for the charge and the four-fermion couplings are computed to the leading order in $1/N$. The value of N_c may be obtained from the RG flow simply by inverting the dependence of the critical coupling(s) g on N. In case of symmetry breaking interaction we show that N_c obtained this way is necessarily a monotonic function of the interaction coupling, i.e., that an infinitesimal interaction, although irrelevant, alters the value of N_c. In particular, this suggests that even if $N_{c0}<2$ in pure QED_3, the low-energy theory of underdoped cuprates with repulsive interactions included[2] is likely to lie below the (shifted) critical point for dynamical mass generation. The flow of the chirally symmetric interactions, on the other hand, suggests that the chirally symmetric mass cannot get spontaneously generated in pure QED_3.

Our method relies on identification of the RG runaway flow of the chiral symmetry breaking interaction coupling constant with the dynamical mass generation. This conjecture is supported by the exact solution in the limit $N=\infty$ and of zero charge. The idea is rather general, however, and similar to the standard way of determining a spontaneously broken symmetry in statistical physics: first allow a weak explicit symmetry breaking perturbation, take the thermodynamic limit, and only then take the perturbation to zero. Thermodynamic limit would, in the RG language, correspond to letting the momentum cutoff go to zero.

The article is organized as follows: In Sec. II we introduce the symmetry breaking and the symmetry preserving four-fermion interactions. In Sec. III we formulate the problem of dynamical mass generation in the RG language. In Secs. IV and V the RG flows are derived in the full theory with all the important quartic interactions taken into account. Concluding remarks are given in Sec. VI, and some technical details are presented in the Appendix.

II. QED$_3$ AND QUARTIC INTERACTIONS

We begin by reviewing briefly the spin sector of the low-energy theory of the phase-disordered d-wave superconductor,[2] described by the action $S=\int d^3xL$, with the Lagrangian

$$L = L_{\mathrm{QED}_3} + L_{\mathrm{int}} + L_{\mathrm{hd}},$$
$$L_{\mathrm{QED}_3} = \bar{\Psi}_i \gamma_\mu (\partial_\mu + i a_\mu)\Psi_i + \frac{1}{2e^2}(\nabla \times \mathbf{a})^2. \tag{1}$$

Ψ_i, $i=1,2$ represent the electrically neutral spin-1/2 fermions (spinons), $\bar{\Psi}=\Psi^\dagger\gamma_0$, γ_μ's are the usual Dirac gamma matrices ($\mu=0,1,2$) and we define $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$, and $\gamma_{35}=i\gamma_3\gamma_5$ for later use.[2] The charge $e^2 \sim |\langle\Phi\rangle|^2$, where Φ is the vortex loop condensate.[2,16] The complementary charge sector of the theory may be shown to be describing an insulator.[17]

The short-range repulsive interaction may be written in terms of Dirac fermions as

$$L_{\mathrm{int}} = U(i\bar{\Psi}_i\gamma_5\gamma_1\Psi_i)^2. \tag{2}$$

Higher derivatives in the kinetic energy, similarly, take the form

$$L_{\mathrm{hd}} \sim \bar{\Psi}_i\gamma_5[\gamma_1 f(\partial^2) - \gamma_2 g(\partial^2)]\Psi_i, \tag{3}$$

where the functions $f(z)$ and $g(z)$ come from the expansion of the quasiparticle dispersion near the nodes.[2]

The velocity anisotropy neglected in Eq. (1) in principle reduces the full U(4) symmetry of QED$_3$ to $\mathrm{U}(2)\otimes\mathrm{U}(2)$. Each $\mathrm{U}(2)=\mathrm{U}(1)\otimes\mathrm{SU}_c(2)$ factor is generated by the algebra $\{\mathbf{1},\gamma_3,\gamma_5,\gamma_{35}\}$ where $\mathrm{SU}_c(2)$ is the chiral symmetry subgroup generated by the last three generators. Inclusion of L_{int} and L_{hd} reduces the $\mathrm{SU}_c(2)$ symmetry further down to the $\mathrm{U}_c(1)$ generated by γ_5, which is simply the generator of translations in the nodal direction in this language. Since the mass that turns out to be dynamically generated in S is $m\sim\langle\bar{\Psi}\Psi\rangle$, which preserves γ_{35},[6] it will prove more convenient to consider interactions that directly preserve that particular generator.

Let us first consider the case of single fermion species, and then generalize to $N>1$. To construct the quartic interaction that breaks the $\mathrm{SU}_c(2)$ symmetry down to $\mathrm{U}_c(1)$ we notice that the three-component objects,

$$\mathbf{A} = (\bar{\Psi}\Psi,\ \bar{\Psi}i\gamma_3\Psi,\ \bar{\Psi}i\gamma_5\Psi),$$
$$\mathbf{B}_\mu = (\bar{\Psi}\gamma_\mu\gamma_{35}\Psi,\ \bar{\Psi}i\gamma_\mu\gamma_3\Psi,\ \bar{\Psi}i\gamma_\mu\gamma_5\Psi), \tag{4}$$

are the only triplets under the chiral group. Upon breaking the symmetry to $\mathrm{U}_c(1)$, we look at the projection of $\mathbf{A}$ and $\mathbf{B}_\mu$ along the direction corresponding to the remaining generator of the $\mathrm{SU}_c(2)$. In this case, these are $\bar{\Psi}\Psi$ and $\bar{\Psi}\gamma_\mu\gamma_{35}\Psi$ which remain invariant under the action of γ_{35}. Thus, the required quartic chiral symmetry breaking (CSB) interaction will have the form

$$L_{\mathrm{CSB}} = \frac{g}{N}(\bar{\Psi}\Psi)^2 + \frac{g'}{N}(\bar{\Psi}\gamma_\mu\gamma_{35}\Psi)^2. \tag{5}$$

On the other hand, the two $\mathrm{SU}_c(2)$ singlets

$$C_\mu = \bar{\Psi}\gamma_\mu\Psi, \quad C_{35} = \bar{\Psi}\gamma_{35}\Psi, \tag{6}$$

may be used to construct the chiral symmetry preserving (CSP) quartic interactions, as

$$L_{\mathrm{CSP}} = \frac{\lambda}{N}(\bar{\Psi}\gamma_{35}\Psi)^2 + \frac{\lambda'}{N}(\bar{\Psi}\gamma_\mu\Psi)^2. \tag{7}$$

For a general N we will therefore define the following $\mathrm{U}(N)\otimes\mathrm{U}(N)$ symmetric theory

$$L = L_{\mathrm{QED}_3} + L_{\mathrm{CSB}} + L_{\mathrm{CSP}}$$
$$= \Bigg\{ \bar{\Psi}_i\gamma_\mu(\partial_\mu + ia_\mu)\Psi_i + \frac{1}{2e^2}(\nabla\times\mathbf{a})^2 + \frac{g}{N}(\bar{\Psi}_i\Psi_i)^2 + \frac{g'}{N}(\bar{\Psi}_i\gamma_\mu\gamma_{35}\Psi_i)^2 + \frac{\lambda}{N}(\bar{\Psi}_i\gamma_{35}\Psi_i)^2 + \frac{\lambda'}{N}(\bar{\Psi}_i\gamma_\mu\Psi_i)^2 \Bigg\},$$
$$i = 1,\ \ldots,\ N. \tag{8}$$

In principle, one could imagine other interaction terms satisfying the required symmetry. However, it can be shown that these would have to be a linear combination of the already introduced quartic terms. For example, the ("Nambu-Jona-Lasinio") interaction $g_1|\mathbf{A}|^2+g_2|\mathbf{B}_\mu|^2$ can be written as a linear combination of C_μ^2 and C_{35}^2. This follows from Fierz identities which imply that there are only two linearly independent quartic terms invariant under the $\mathrm{U}(2N)$. For $\mathrm{U}(N)\otimes\mathrm{U}(N)$ theory, the number of independent couplings doubles to four, which are precisely the introduced g, g', λ, and λ'. For a more detailed discussion we refer the reader to the Appendix.

In the next section we focus on a single four-fermion interaction and try to understand the spontaneous chiral symmetry breaking within the renormalization group approach.

III. DYNAMICAL MASS GENERATION IN THE RG LANGUAGE

An exactly solvable case of the theory in Eq. (8) is in the limit of infinite number of fermion flavors and of zero charge ($e=0$). Let us first consider a single CSB interaction term, $(g/N)(\bar{\Psi}\Psi)^2$, and set $g'=\lambda=\lambda'=0$ (i.e., the Gross-Neveu model). For $N\to\infty$, such interaction gives rise to a dynamically generated mass, $m\sim\langle\bar{\Psi}\Psi\rangle$, determined by the gap equation

$$-\frac{1}{g}=8\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2+m^2}, \tag{9}$$

which after the integration gives

$$1=\frac{4g\Lambda}{\pi^2}\left(\frac{m}{\Lambda}\tan^{-1}\frac{\Lambda}{m}-1\right), \tag{10}$$

with $\Lambda\gg m$ being the assumed ultraviolet (UV) cutoff. Demanding m to be invariant under the change of cutoff $\Lambda\to\Lambda/b$, the β function at $N=\infty$ is readily obtained to be exactly

$$\beta_g=\frac{dg}{d\ln b}=-g-g^2, \tag{11}$$

where g has been rescaled as $4g\Lambda/\pi^2\to g$. We see that a weak coupling g is irrelevant, but that the flow for $g<g_*=-1$, which represents the infrared (IR) unstable fixed point, is toward negative infinity. Since the same values of g yield a finite mass from the gap equation, it is natural to identify the runaway flow of g with the dynamical mass generation. Note that the same Eq. (11) can alternatively be obtained in the standard Wilson's momentum-shell one-loop RG.

The second solvable limit of the theory is pure QED_3 without any four-fermion interactions, again in the limit $N\to\infty$. The flow of the charge is then

$$\beta_e=\frac{de^2}{d\ln b}=e^2-Ne^4, \tag{12}$$

where the dimensionless charge is defined as $(4/3)[e^2/(2\pi^2\Lambda)]\to e^2$. Although the theory is free in the UV region, there is a nontrivial IR stable fixed point at $e_*^2=1/N$. (Notice that the quartic interactions, even when present, cannot appear in β_e to the leading order in large N as a consequence of the Ward-Takahashi identity.)

Next, we want to consider the interplay of the charge e and the quartic coupling g, and in particular to examine the influence of a weak charge on the value of g_*. One expects the effect of the gauge field on β_g to be

$$\frac{dg}{d\ln b}=-g-g^2+(\text{const.})e^2g, \tag{13}$$

to the leading order in e^2 and $1/N$. In particular, $\beta_g(g=0)=0$ even when $e\neq 0$, since otherwise it would be possible to *generate* the CSB interaction in the chirally symmetric theory. So $g=0$ is *always* a fixed point. Since at the fixed point $e_*^2=1/N$, decreasing N is the same as increasing the charge in Eq. (13). Since the factor in front of the last term is expected to be positive (as it indeed turns out to be the case), decreasing N will reduce the absolute value of the nontrivial critical coupling g_*, until it eventually merges with the trivial fixed point, permanently located at $g=0$. There will therefore exist a critical charge $e_c^2=1/N_{c0}$, above which an *infinitesimal* symmetry breaking interaction suffices to cause the runaway flow of g. We identify this point with the *spontaneous* chiral symmetry breaking in pure QED_3. The phase diagram with this structure has been depicted in Fig 1.

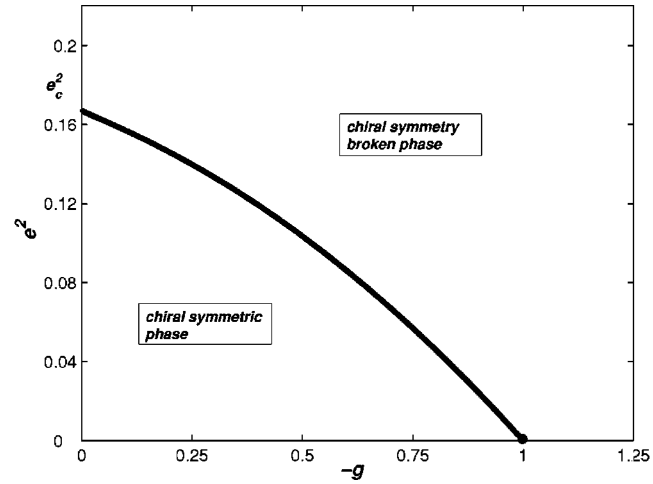

FIG. 1. The phase diagram in the interaction-charge plane, for the chiral symmetry breaking interaction. The value of the charge is $e^2=1/N$. N_c is a continuous function of the symmetry breaking interaction, as a consequence of the existence of the fixed point at $g=0$ at any charge.

When $N<\infty$ the terms with an explicit N dependence in β_g, such as g^3/N, should also be included. These terms may be understood as contributing to the $1/N$ corrections to N_{c0} in the following way. One can expand the critical charge (corresponding to the double root of β_g at $g=0$) in powers of $1/N$ as

$$e_c^2=a_0+\frac{a_1}{N_{c0}}+\frac{a_2}{N_{c0}^2}+\cdots . \tag{14}$$

Since $e_c^2=1/N_{c0}+O(1/N_{c0}^2)$ from β_e, this effectively generates then the $1/N$ expansion for N_{c0}.

One may analogously consider the CSP interaction $(\lambda/N)(\bar{\Psi}\gamma_{35}\Psi)^2$, which when alone leads to the dynamically generated mass $m\sim\langle\bar{\Psi}\gamma_{35}\Psi\rangle$ for $\lambda<-1$, in the $N\to\infty$ limit. In presence of the charge, however, there is a crucial difference between the β_λ and β_g. Since the CSP interaction term has the same full chiral symmetry as the pure QED_3, finite charge may, and in fact does, *generate* the coupling λ. This manifests itself as the e^4 contribution in β_λ, which will now take the form

$$\frac{d\lambda}{d\ln b}=-\lambda-\lambda^2+(\text{const.})e^2\lambda+(\text{const.})e^4. \tag{15}$$

With the last term, however, $\lambda=0$ *is not* a fixed point any longer. Further, the sign of the e^4 term turns out to be *positive*, so that the critical coupling actually *increases* with

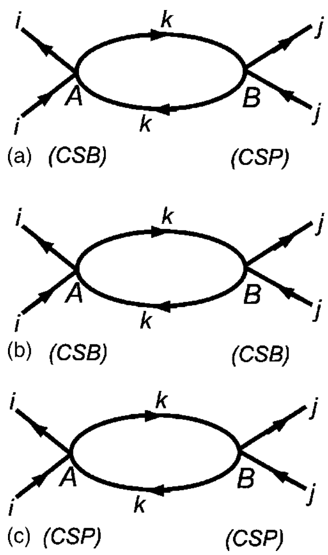

FIG. 2. Particle-hole diagrams to the leading order in $1/N$.

charge. We interpret the latter feature as that the spontaneous dynamical generation of the chiral symmetry preserving mass in pure QED_3 is not possible. This would be in agreement with conclusions of the earlier studies.[18,19]

IV. RG FOR CSB INTERACTIONS AND THE VALUE OF N_{c0}

In general, the β functions for all four quartic interactions will be coupled and the flow is nontrivial. To the leading order in $1/N$, however, the calculation simplifies considerably. In the following two theorems we show that the β functions for the CSB and CSP interactions are completely decoupled in this limit.

Theorem I: To the leading order in $1/N$ and for $e=0$, different β functions decouple.

Proof: Only particle-hole diagrams, as in Fig. 2 contribute to leading order in large N. Such diagrams are proportional to

$$g_A g_B \cdot \int d^3q \, \mathrm{Tr}(\Gamma_A G(q) \Gamma_B G(q)) \propto \mathrm{Tr}(\Gamma_A \Gamma_B). \quad (16)$$

Here, Γ_A and Γ_B's are the matrices in the kernel of the quadratic form accompanying either g_A or g_B,

$$\Gamma_A, \Gamma_B \in \{1, \gamma_\mu, \gamma_{35}\gamma_\mu, \gamma_{35}\}. \quad (17)$$

It is easy to see that these diagrams are zero unless $g_A = g_B$. For diagrams that mix CSB and CSP interactions in Fig. 2(a), Eq. (16) contains the trace of an odd number of γ matrices and thus yields zero. For the CSB−CSB or CSP−CSP diagrams in Figs. 2(b) and 2(c), the identities $\mathrm{Tr}(\gamma_\mu \gamma_\nu) = 4\delta_{\mu\nu}$ and $\mathrm{Tr}(\gamma_\mu \gamma_{35}) = 0$, imply that all the mixing terms are zero unless $\mu = \nu$, i.e., $g_A = g_B$.

So, to the leading order, the coupling between different quartic interactions in the β functions can only be mediated through charge. One may easily see, however, that the symmetry requires that the β functions for the CSB and CSP couplings still remain decoupled. We will state it in the form of the following theorem:

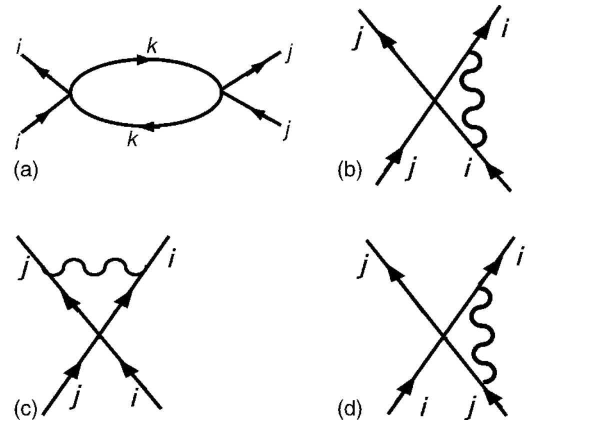

FIG. 3. Diagrams contributing to the renormalized couplings to the leading order in $1/N$.

Theorem II: There are no $\sim \lambda e^2$ or $\sim \lambda' e^2$ terms in β_g or $\beta_{g'}$, nor $\sim g e^2$ and $\sim g' e^2$ terms in β_λ and $\beta_{\lambda'}$, to the leading order in $1/N$.

Proof: λe^2 and $\lambda' e^2$ terms obey the full chiral symmetry, and thus cannot generate a CSB interaction. To prove the equivalent statement for the CSB couplings, we notice that to the leading order in $1/N$ the ge^2 and $g'e^2$ terms differ from the λe^2 and $\lambda' e^2$ terms by a single γ_{35} matrix, and thus necessarily break the chiral symmetry. They therefore cannot generate a CSP coupling.

The previous theorems allow us to significantly reduce the number of relevant Feynman diagrams. The straightforward calculation of the diagrams in Figs. 3 and 4 leads to the following β functions for the symmetry breaking interactions:

$$\frac{de^2}{d \ln b} = e^2 - Ne^4,$$

$$\frac{dg}{d \ln b} = -g - g^2 + 4e^2 g + 18 e^2 g', \quad (18)$$

$$\frac{dg'}{d \ln b} = -g' + g'^2 + \frac{2}{3} e^2 g,$$

with conveniently rescaled parameters

$$4g\Lambda/\pi^2 \to g, \; 4g'\Lambda/(3\pi^2) \to g', \; 2e^2/(3\pi^2 \Lambda) \to e^2. \quad (19)$$

Note that the coupling g' becomes generated by g and e even if absent initially, so in principle it must be included into the analysis. A notable feature of the previous β functions is also

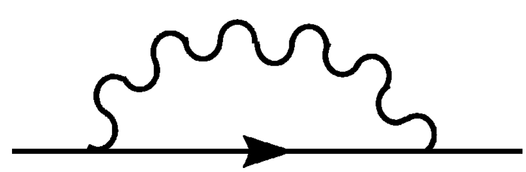

FIG. 4. Diagram contributing to the wave function renormalization.

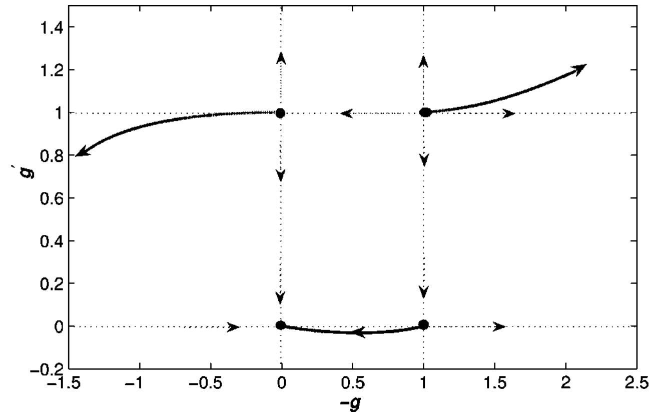

FIG. 5. The RG flow (dashed lines) in the plane of the two symmetry breaking interactions g and g' for $N=\infty$. Full lines mark the evolution of the four fixed points with decrease of N.

their independence on the gauge-fixing parameter ξ. This derives from the exact cancellation between the gauge-dependent part of the diagrams in Fig. 3 and the wave function renormalization factor Z (Fig. 4):

$$Z = 1 + \left(\xi - \frac{2}{3}\right)e^2 \ln b. \tag{20}$$

The flow diagram on the $g-g'$ plane for $N=\infty$ ($e^2=0$) is given in Fig. 5. For $N<\infty$, the fixed point value of the charge becomes $e^2=1/N$, and the locations of all the fixed points, except the trivial one at the origin, shift in the directions as indicated. The point at which the RG trajectory that starts at the purely repulsive fixed point [initially at $(-1,1)$] and terminates at the "Gross-Neveu" fixed point [initially at $(-1,0)$] intersects the g axis determines the location of the phase boundary in the $g-e^2$ ($g'=0$) plane. At small charge we obtain such a phase boundary at

$$g = -1 + 4e^2 + O(e^4), \tag{21}$$

whereas at low g

$$g = -\frac{144}{13}\left(\frac{1}{6} - e^2\right) + O\left[\left(\frac{1}{6} - e^2\right)^2\right]. \tag{22}$$

Numerical solution at a general coupling is given at Fig. 1. The critical point in pure QED_3, N_{c0}, is determined by the value of N for which Gross-Neveu fixed point reaches the origin. For $N>N_{c0}$, the flow beginning at an infinitesimal negative g and $g'=0$ then runs away to infinity. To the leading order in $1/N$, this criterion yields $N_{c0}=6$. At $N=N_{c0}$ the other two nontrivial fixed points are still at finite values. The role of g' is therefore only to modify the phase boundary in the $g-e^2$ plane and the value of N_{c0} quantitatively, but not qualitatively. Neglecting the flow of g' entirely would lead, for example, to $N_{c0}=4$. This would correspond to the value at which the dimension of the coupling g at the charged fixed point changes sign.

V. RG FOR CSP INTERACTIONS

We now turn to the analysis of the theory in Eq. (8) with $g=g'=0$, i.e., when the quartic terms respect the full chiral symmetry. Although somewhat artificial from the point of view of the effective theory for underdoped cuprates, this exercise underlines the important role of symmetry in the phase diagram. The diagrams are still the same as in the CSB case, with the addition of the two diagrams in Fig. 6. These new terms *generate* the coupling λ, and thus change the evolution of the flow diagram with N in an important way, as mentioned in the introduction and depicted in Fig. 7. We obtain the following β functions for the couplings λ, λ', and e^2:

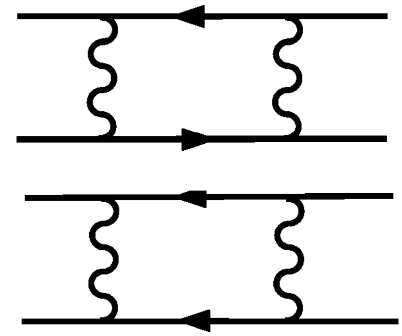

FIG. 6. The diagrams that give e^4 term in β_λ.

$$\frac{de^2}{d\ln b} = e^2 - Ne^4,$$

$$\frac{d\lambda}{d\ln b} = -\lambda - \lambda^2 + 4e^2\lambda + 18e^2\lambda' + 9Ne^4, \tag{23}$$

$$\frac{d\lambda'}{d\ln b} = -\lambda' + \lambda'^2 + \frac{2}{3}e^2\lambda.$$

Notice that the flow equations for the CSB and CSP cases are identical, apart from the positive e^4 term. This term, however, prevents the fixed point that was located at $(-1,0)$ for $N=\infty$ to ever merge with the Gaussian fixed point, and consequently, no spontaneous generation of the chiral symmetry preserving mass should be allowed in pure QED_3.

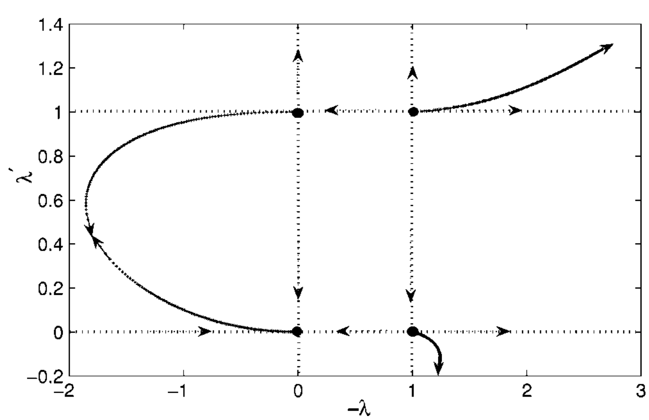

FIG. 7. The evolution of the chiral symmetry preserving fixed points in the $\lambda-\lambda'$ plane with the increase of charge.

VI. CONCLUSION

In conclusion, by reformulating the problem of dynamical mass generation in QED_3 with four-fermion interactions in terms of the renormalization group flows, we found that the critical number of fermions N_c is a continuous function of the chiral symmetry breaking interaction. By taking the limit of vanishing interactions, we estimated that $N_{c0}=6$ in pure QED_3. Our analysis of the chiral symmetry preserving interactions suggests that the chiral symmetry preserving mass cannot become dynamically generated in pure QED_3.

The result that the N_c may depend on an infinitesimal symmetry breaking interaction should be contrasted with the previous studies of Schwinger-Dyson equations in QED_3 with symmetry preserving interactions (the "gauged Nambu-Jona-Lasinio model"). There, the N_c was found to depend on the quartic interaction only if the latter is larger than a certain value.[20] In the RG language this would correspond to the merger of the two fixed points, like the Gaussian and the "Gross-Neveu" fixed points in our case, at a *finite* value of the coupling. In fact, we find that occurring in Eqs. (23) for CSP interactions: the "Gaussian" fixed point [initially at (0,0)] and the "Thirring" fixed point [initially at (0,1)] for $N=4.83$ meet at (1.78,0.43). For $N>4.83$, both couplings become complex, and the flow that begins at the line $\lambda=0$ is always toward infinite λ'. It is tempting to identify this runaway flow with the phase with broken chiral symmetry and the dynamically generated mass, as proposed in Ref. 21. We refrain from doing so, however, since the runaway flow for $\lambda'>1$ at $e=\lambda=0$ (the "Thirring model") actually *does not* correspond to the broken symmetry phase, as one can easily check by directly solving the gap equation in this case at $N=\infty$. The transition in the Thirring model occurs only at the order of $1/N$,[22] and so we suspect that the above runaway flow of λ' may be an artifact of the $N=\infty$ limit. This issue is left for future studies.

Finally, although our scheme provides a systematic way of computing N_{c0}, for example, it becomes rapidly complicated. To the next order in $1/N$, CSP and CSB coupling constants mix in the β functions. Since the couplings λ and λ' get generated by the charge, and then mix into β_g, one necessarily has to track the flow of all four couplings.

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APPENDIX: FIERZ IDENTITIES AND GENERALITY OF THE INTERACTION LAGRANGIAN

In this appendix, we will construct the linear relationship between the quartic terms invariant under a unitary $U(N)$ group, known as Fierz identities. These are the direct consequences of the completeness relation for the generators of the symmetry group.

Defining $\mathrm{Tr}(A\cdot B)$ as the inner product between matrices A and B, we write down the completeness relation for the basis constructed out of generators of a $U(N)$ group, $\{\lambda^\alpha,1\}$, as

$$\frac{1}{N}\delta_{ab}\delta_{cd}+\frac{1}{2}\sum_{\alpha}^{N^2-1}\lambda_{ab}^{\alpha}\lambda_{cd}^{\alpha}=\delta_{ad}\delta_{cb}. \tag{A1}$$

As the special case of U(2), using Pauli matrices, Eq. (A1) simplifies to

$$\delta_{ab}\delta_{cd}+\sum_{\alpha}\sigma_{ab}^{\alpha}\sigma_{cd}^{\alpha}=2\delta_{ad}\delta_{cb}. \tag{A2}$$

Using the previous relations, one can derive the requisite linear relationship between different quartic terms. First, it is convenient to represent the $4N$-component spinor Ψ in terms of $2N$-component ones as

$$\Psi=\begin{pmatrix}\chi^i\\ \phi^i\end{pmatrix}. \tag{A3}$$

It is then possible to apply the previous completeness relations to the quartic terms of the form

$$\sum_{\alpha}(\bar{\chi}_a^{i}\lambda_{ij}^{\alpha}\chi_a^{j})(\bar{\chi}_b^{k}\lambda_{kl}^{\alpha}\chi_b^{l}), \tag{A4}$$

where χ^i stands for both χ^i and ϕ^i. (The spinor index is indicated by subscripts.) Applying Eq. (A2) for spinor degrees of freedom and Eq. (A1) for flavor degrees of freedom, one ends up with the following identity

$$\left(1+\frac{1}{N}\right)(\bar{\chi}\chi)^2+\sum_{\mu}(\bar{\chi}\sigma_{\mu}\chi)^2+\sum_{\alpha}(\bar{\chi}\lambda^{\alpha}\chi)^2=0, \tag{A5}$$

where we have suppressed both the spinor and flavor (large N) indices for convenience, and replaced N with $2N$, since QED_3 is $U(2N)$ symmetric. Similarly, beginning with the quartic term

$$\sum_{\alpha,\mu}(\bar{\chi}_a^{i}\lambda_{ij}^{\alpha}\sigma_{ab}^{\mu}\chi_b^{j})(\bar{\chi}_c^{k}\lambda_{kl}^{\alpha}\sigma_{cd}^{\mu}\chi_d^{l}), \tag{A6}$$

it is easy to derive the other identity,[23]

$$\sum_{\alpha,\mu}(\bar{\chi}\lambda^{\alpha}\sigma_{\mu}\chi)^2+\sum_{\alpha}(\bar{\chi}\lambda^{\alpha}\chi)^2+\frac{1}{N}\sum_{\mu}(\bar{\chi}\sigma_{\mu}\chi)^2+\left(4+\frac{1}{N}\right)\times(\bar{\chi}\chi)^2=0. \tag{A7}$$

The previous identities applied to a $U(2N)$-symmetric theory with the S_{int} of the form

$$\tilde{g}_1(\bar{\chi}\chi)^2+\tilde{g}_2(\bar{\chi}\lambda^{\alpha}\chi)^2+\tilde{g}_3(\bar{\chi}\sigma_{\mu}\chi)^2+\tilde{g}_4(\bar{\chi}\sigma_{\mu}\lambda^{\alpha}\chi)^2, \tag{A8}$$

leave only two of the terms as independent. Noticing that the Eq. (A8) is equivalent to the interaction term written in $4N$-component representation: $g_1|\mathbf{A}|^2+g_2|\mathbf{B}_\mu|^2+g_3|C_\mu|^2+g_4|C_{35}|^2$, we can see that our choice of C_μ and C_{35} as the most general CSP quartic terms is justified.

The CSB case is not very different. One can consider the interaction of the form in Eq. (A8) for each $U(N)$ sector separately (i.e., χ and ϕ). Repeating the same argument would reduce the number of independent interaction couplings in each sector to two, so that the overall number of independent couplings will be four, as assumed in Eq. (8).

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