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Switching probabilities for single-domain magnetic particles

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Solving the stochastic Landau-Lifshitz equation numerically, we compute as a function of time t the probability per unit time, $P_s(t)$, that a classical, single-domain magnetic particle with an easy uniaxial anisotropy and a collinear applied magnetic field will reverse its magnetization ("switch") via thermal activation over the energy barrier. The $P_s(t)$ curves increase with t for small t, achieving a maximum at some time τ_P before decaying exponentially with time constant τ_D at long time, as per the standard Neel-Brown picture. Both τ_P and τ_D increase (the latter exponentially) with increasing barrier height; τ_P grows logarithmically with τ_D, consistent with a recent phenomenological "energy-ladder" model, and experiments on submicron-sized magnetic thin films.

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I. INTRODUCTION

Both the relentless increase in the areal density of magnetic storage devices[1] and the attempt to fabricate magnetic dynamic random access computer memories whose performance exceeds that of standard semiconductor technology[2,3] have fueled the need for progressively more refined understanding of the thermal magnetization reversal, or switching, of small magnetic systems. For example, the probability that a given element or bit of a magnetic device will spontaneously switch due to thermal fluctuations increases rapidly with decreasing volume of the bit. Since even a minute probability of such spontaneous switching is unacceptable for memory or magnetic storage elements,[4] this phenomenon potentially limits the size to which individual bits can be decreased, at least at room temperature. The intricacies of thermal switching on very short—say nanosecond—time scales also have important implications for technology as areal densities and read and write speeds increase. For example, one needs to be able to switch or "write" a given bit very rapidly by applying a short, localized magnetic field pulse to it, without switching nearby bits as well. To do this requires detailed understanding of the short-time switching probabilities of small magnetic systems at finite temperature (T).

In a recent paper,[3] we studied experimentally the switching probabilities $P_s(t)$ of submicron magnetic thin films on microsecond time scales, as a function of applied field. At sufficiently long times t, the probabilities fell off exponentially with time, consistent with the Neel-Brown[5,6] picture of a single characteristic time τ_D governing the long-time decay of probability as $t \to \infty$. At short times, however, the probabilities *increased* with t, achieving a peak value at another characteristic time τ_P before commencing their ultimate exponential decay. Both τ_P and τ_D were found to increase as the energy barrier to switching was increased by reducing the applied magnetic field in the experiments, the rate of increase of τ_P being far slower than that of τ_D.

We rationalized these observations in Ref. 3 by constructing and analyzing an elementary one-variable "energy-ladder" model wherein the switching process is idealized as the stochastic, thermally activated progression of the system through a series ("ladder") of states of steadily increasing energy. In the model, switching can occur only after the system has reached the highest state, the energy difference between the lowest and highest states defining the height of the energy barrier, ΔE, which must be surmounted in the switching process. The ladder model produces switching probabilities with the same qualitative peaked structures observed experimentally, as well as the exponential decays at large times. Both τ_P and τ_D in the model diverge as $\Delta E/k_BT$ diverges, with τ_P growing like $\log(\tau_D)$ asymptotically.

In Ref. 3, we hypothesized that the phenomenological ladder model and the resulting peaked switching curves ought to apply quite generally to switching phenomena involving thermal activation over barriers. A natural starting place to investigate this hypothesis is the magnetization reversal of a single classical, monodomain magnetic particle, the theory of which has been studied for more than half a century. Calculations of switching probabilities have been carried out,[5,6] at least in the long-time limit that has typically been of greatest experimental relevance, and for the simplest models involving only uniaxial anisotropy and magnetic fields. The familiar result, $P_s(t) \sim e^{-t/\tau_D}$, valid for $t \gg \tau_D$, has emerged, together with the Arrhenius formula $\tau_D = \tau_0 e^{\Delta E/k_BT}$, and approximations for the microscopic time τ_0 (which is typically of order 10^{-9} s), in certain limits.[6,7]

While verifying these results experimentally has not proven easy,[8] recent work by Wernsdorfer *et al.*[9] has suggested the correctness of the Neel-Brown picture of magnetization reversal for ideal monodomain magnets on the nanoscale. However, little attention has been paid to the short-time switching behavior of a single-domain magnetic particle.[10] In this paper, we compute switching probabilities for such a particle numerically from the stochastic Landau-Lifshitz dynamical equation in the simplest case of easy uniaxial anisotropy and a collinear applied field. We find that the $P_s(t)$ curves indeed have the peaked structure predicted by the ladder model. Moreover, the data are consistent with a logarithmic dependence of τ_P upon τ_D, at least when τ_D becomes large compared to microscopic times.

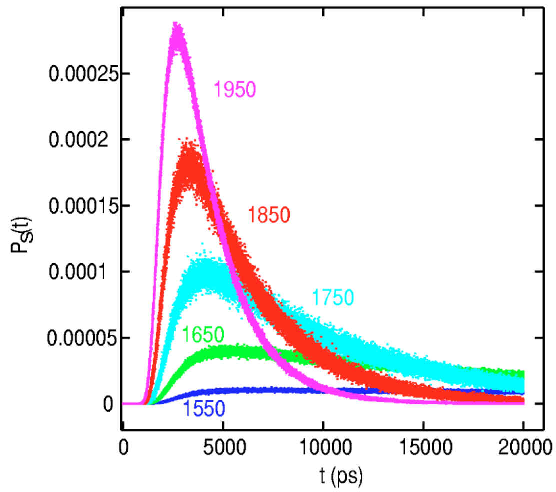

FIG. 1. (Color online) Switching probability $P_s(t)$ as a function of time t (in units of 10^{-12} s), for a magnetic particle with parameters given in the text, and h values (respectively describing the curves in decreasing order of peak heights) of 1950, 1850, 1750, 1650, and 1550 Oe.

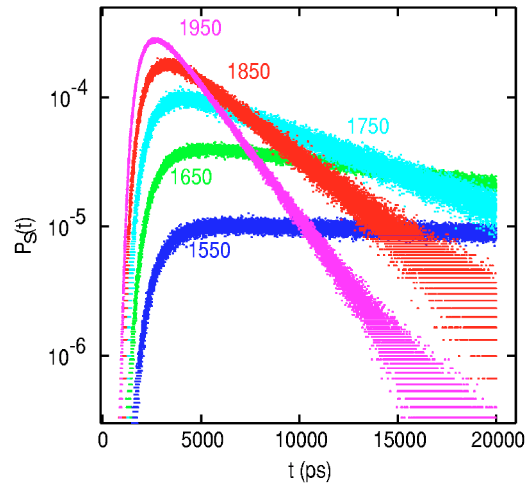

FIG. 2. (Color online) Same as Fig. 1, except plotted on a semilog scale.

II. THE MODEL

The system studied here is an idealized single-domain classical magnetic particle with easy uniaxial anisotropy and a magnetic field applied along the easy (z) axis. It is modeled by a single Heisenberg spin vector, $\hat{S}$, of unit length, described by the classical energy function

$$E = -K_z V S_z^2 - h M_S V S_z, \tag{1}$$

where V is the particle volume, K_z (>0) the anisotropy energy density, h the applied field, and M_s the saturation magnetization. For h less than the value $h_c \equiv 2K_z/M_s$, this model has a local, metastable energy minimum at $S_z=-1$, a global minimum at $S_z=1$, and a maximum at $S_z=-h/h_c$. The energy barrier separating the metastable minimum and the maximum has height $\Delta E = K_z V(1-h/h_c)^2$. At $h=h_c$, the metastable minimum and maximum merge, whereupon ΔE vanishes.

The dynamics of the spin at finite temperature T is described by the noisy Landau-Lifshitz equation,[11,6,12,13]

$$d\hat{S}/dt = -D\hat{S} + \gamma\hat{S}\times\partial E/\partial\hat{S} + \alpha\gamma[\hat{S}\times(\hat{S}\times\partial E/\partial\hat{S})] + \hat{S}\times\vec{\eta}. \tag{2}$$

Here $\vec{\eta}$ is a Gaussian random noise variable of strength D, with correlations $\langle\eta_i(t)\eta_j(t')\rangle = D\delta_{ij}\delta(t-t')$, $\gamma\equiv\gamma_0/M_sV(1+\alpha^2)$, where γ_0 is the gyromagnetic ratio,[14] and α is the phenomenological damping constant. The choice $D=2k_BT\alpha\gamma$ ensures[15,6] that the stationary Boltzmann distribution corresponding to the energy function E and temperature T is achieved in the long-time limit. Here k_B is Boltzmann's constant. [We use the Itô interpretation of the multiplicative noise term[16–18] in Eq. (2), whereupon the extra term $-D\hat{S}$ must be included[12,13,19] on the right side of Eq. (2) in order both that the magnitude of $\hat{S}$ be preserved by the equation, and that the Boltzmann distribution be reached as $t\to\infty$.]

Mimicking the measurements performed in Ref. 3, we study switching in the model according to the following protocol: We start the system off in the metastable energy minimum at $S_x = S_y = 0$, $S_z=-1$, for fixed, chosen values of all parameters except h. Choosing a value of h somewhat below h_c, we then solve Eq. (2) numerically, using the elementary Euler method for a chosen discrete time step δt. We record the elapsed time at which S_z first becomes positive, defining that as the time taken for the system to switch. Performing many such measurements for a given h value, we accumulate statistics for the switching time, thereby computing a discrete approximant to the probability $P_s(t)$ that the system switches at time t. We repeat these calculations for a series of values of h to study the dependence of the $P_s(t)$ curve on the energy barrier height.

III. RESULTS

Figure 1 shows representative data for $P_s(t)$ versus t for parameter values $K_z=4\times10^5$ erg/cm^3, $V=2.5\times10^{-19}$ cm^3, $M_s=400$ emu/cm^3, $\alpha=0.01$, $\gamma_0=2.0\times10^7$ Oe^{-1} s^{-1}, $T=7.25$ K, $\delta t=10^{-12}$ s, and a series of h values between 1550 and 1950 Oe.[20]

Each curve represents the results of between 2 000 000 and 15 000 000 runs, and each exhibits the same qualitative peaked structure seen in Ref. 3. This supports the argument in that reference that any switching process involving thermal activation over an energy barrier should give rise to $P_s(t)$ curves with this characteristic shape. $P_s(t)$ increases with t for $t<\tau_P$ because it takes the particle some time to activate thermally up to the top of the barrier where it can switch with high probability.

Figure 2 shows a semilog plot of the data displayed in Fig. 1. For times t exceeding its peak time τ_P, each curve appears linear, indicating the exponential decay, e^{-t/τ_D}, of $P_s(t)$ predicted at long times by the Neel-Brown theory.[5,6] The inverse slopes of these curves are the decay times τ_D. As expected, τ_D increases with decreasing h, that is, with increasing energy barrier. The peak times, τ_P, also increase as

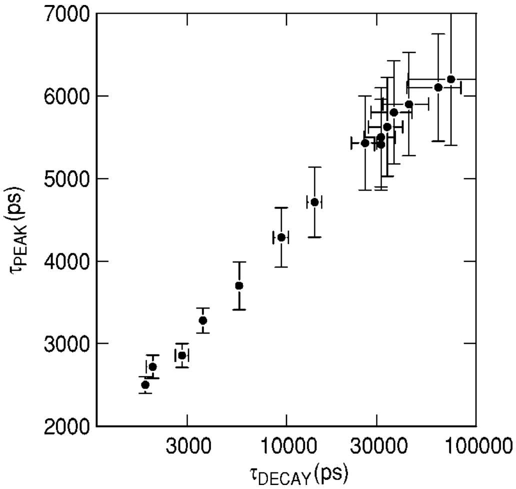

FIG. 3. (Color online) Semilog plot of peak time, τ_P, vs decay time, τ_D, for switching curves computed with parameters given in the text. Data are consistent with τ_P growing logarithmically with τ_D.

h is lowered, though far more slowly than does τ_D.

This phenomenology again agrees qualitatively with that emerging from the ladder model of Ref. 3. To make the comparison more quantitative, note that when the ratio $\Delta E/k_BT$ becomes large, τ_P in that model increases as the logarithm of τ_D (which in turn grows like $e^{\Delta E/k_BT}$, consistent with the Neel-Brown picture). Figure 3 shows a plot of τ_P versus $\log(\tau_D)$ for the set of curves whose representatives are displayed in Figs. 1 and 2. Over the roughly two-decade range of τ_D's studied, τ_P does indeed seem to grow logarithmically with τ_D. In fact, given that the logarithmic dependence has only been derived in the ladder model for asymptotically large values of $\Delta E/k_BT$, the agreement is surprisingly good.

Of course it is always difficult to distinguish numerically between a logarithmic dependence and a power law with a sufficiently small exponent. In Fig. 4 we show the τ_P versus τ_D data on a log-log plot. The data exhibit clear, systematic

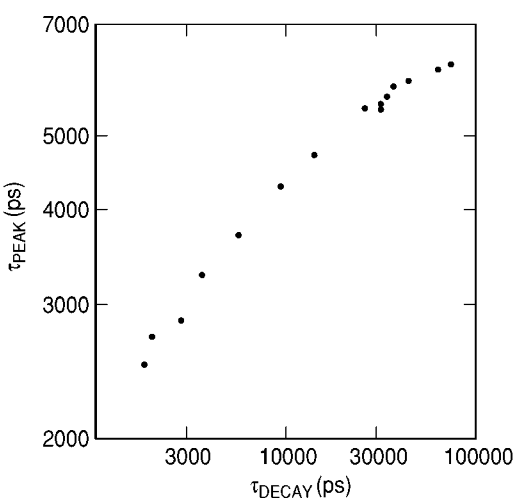

FIG. 4. (Color online) Same as Fig. 3, except on a log-log plot, which shows noticeable downward curvature.

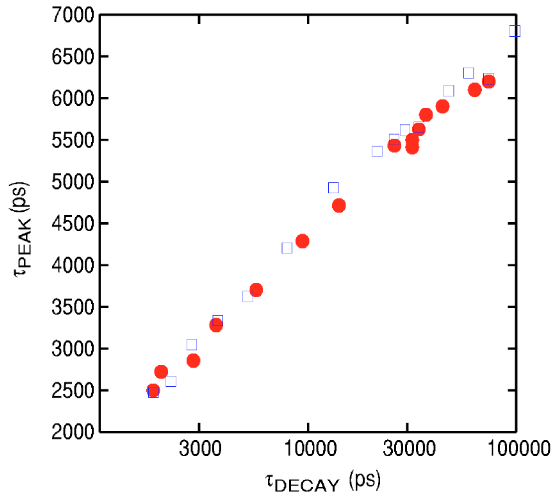

FIG. 5. (Color online) Semilog plots of τ_P vs τ_D for parameters identical to those of Fig. 3 (solid circles); and for parameters identical to those of Fig. 3, except that the discrete time step, δt, is 3.33×10^{-13} s, rather than 10^{-12} s (open squares).

curvature in this plot, but if one insists upon fitting them to a straight line, one arrives at an exponent of roughly 0.2, i.e., $\tau_P \sim \tau_D^{0.2}$.

To check that these results are characteristic of the model (1) and (2), and not artifacts of the time discretization, we performed computations with the parameters listed above, but with $\delta t = 3.33\times10^{-13}$ s rather than 10^{-12} s. The results are illustrated in Fig. 5, which shows the τ_P versus τ_D data on a semilog plot, together with the data from Fig. 3, for comparison. The two approximately linear curves are indistinguishable within the accuracy of our numerical measurements. Thus Fig. 5 strongly suggests that the relationship $\tau_P \sim \log(\tau_D)$ continues to hold even with the finer time mesh.

We conclude that the switching of a single classical spin is in accord with the phenomenology of the ladder model. This furthers one's confidence that the simple arguments advanced in Ref. 3 indeed capture the essential physics of thermal switching: At short times, the process is governed by the stochastic climb of the system to the top of the energy barrier, producing a switching probability $P_s(t)$ that is zero at $t=0$, and increases with t up to τ_P;[21] at long times, the familiar exponential decay of the switching probability, characterized by a single decay time, τ_D, is obtained.

While switching probabilities exhibiting peaks have been observed in Monte Carlo simulations of spatially extended systems such as 1D micromagnetic models[22] and 2D Ising models,[23,24] the peaks have been ascribed—very reasonably—to the fact that it takes time for the boundary of a switched region or droplet to propagate spatially through the entire system. The ladder model, which consists of a single variable, and the single-domain particle studied here, make clear that peaked switching probabilites should be expected to occur very generally, even in the absence of any spatial extension: Surmounting any energy barrier via thermal fluctuations will necessarily involve some "climbing" time which will produce a peak in $P_s(t)$.

It is obviously of some importance to see how widely this prediction can be verified experimentally.[25] To give one example of clear significance: At short enough times[26] t the switching probabilities $P_s(t)$ measured in Ref. 9 should increase with t. On a more quantitative level, it also remains to be seen how universally the relationship $\tau_P \sim \log(\tau_D)$, predicted by the ladder model, holds. Recent important advances[27] in the numerical identification of the most probable switching paths for experimentally realistic and technologically relevant magnetic systems can doubtless be brought fruitfully to bear upon this question.

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[20] These values for K_z, V, and M_s are roughly appropriate for a Co sphere with an 8 nm diameter.

[21] Though one cannot infer from our numerical data the functional form of the initial rise of $P_s(t)$ from zero for very small t, we speculate that this initial increase is very gradual, bounded above by the expression familiar from the random diffusion of probability density, namely $e^{-C/t}/t^x$, for constants C and x.

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