Resolving the order parameter of high-T_c superconductors through quantum pumping spectroscopy

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The order parameter of high-T_c superconductors through a series of experiments has been quite conclusively demonstrated to not be of the normal s-wave type. It is either a pure $d_{x^2-y^2}$-wave type or a mixture of a $d_{x^2-y^2}$-wave with a small imaginary s-wave or d_{xy}-wave component. In this work a distinction is brought out among the four types (i.e., s-wave, $d_{x^2-y^2}$-wave, $d_{x^2-y^2}+is$-wave, and $d_{x^2-y^2}+id_{xy}$-wave types) with the help of quantum pumping spectroscopy. This involves a normal-metal double-barrier structure in contact with a high-T_c superconductor. The pumped current, heat, and noise show different characteristics with change in order parameter revealing quite easily the differences among these.

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I. INTRODUCTION

One of the outstanding issues of high-T_c superconductor research involves the identification of the order-parameter symmetry and the underlying mechanism.[1,2] Although a host of experiments have indicated the order-parameter symmetry to be of a $d_{x^2-y^2}$-wave type,[3,4] there are theoretical works[5–8] that indicate that an imaginary s-wave or d_{xy}-wave component is necessary to explain some of the experimental results. These experimental results[9] being notably the splitting of the zero energy peak in conductance spectra, which indicates the presence of an imaginary s-wave or d_{xy}-wave component, that would break the time-reversal symmetry. Many theoretical attempts have been made to bring out the differences among the different order parameters. Early theoretical attempts were made by Hu[10] where the existence of a sizable areal density of midgap states on the {110} surface of a $d_{x^2-y^2}$-wave superconductor was brought out. Furthermore, using tunneling spectroscopy, Tanaka and Kashiwaya[11] brought out the fact that zero bias conductance peaks (which were seen earlier in many experiments[12]) are formed when a normal metal is in contact with a $d_{x^2-y^2}$-wave superconductor enabling a distinction between s-wave and $d_{x^2-y^2}$-wave superconductors. A shot-noise analysis by Zhu and Ting[13] also revealed differences between s-wave and $d_{x^2-y^2}$-wave superconductors. Further inclusion of phase breaking effects[14] in double barriers formed by normal and superconducting electrodes revealed a double-peaked structure in case of s-wave whereas a dramatic reduction of zero bias maximum for $d_{x^2-y^2}$-wave superconductors. These are in addition to many other works that involve spin-polarized transport in ferromagnet-superconductor junctions,[15–17] which reveal differences between different possible high-T_c order parameters. In a recent review, Deutscher[18] has used the Andreev–Saint James reflections to indicate the presence of an additional imaginary component in the order parameter. Also in another review,[19] Lofwander *et al.* arrived at some conclusive arrivals for $d_{x^2-y^2}$-wave superconductivity in the cuprates. Recently, Ng and Varma[20] studied some of the proposed order parameters and also suggested experiments to bring out the subtle differences among these. In this work we apply the principles of quantum adiabatic pumping to bring out the differences between the different types of order parameters. Quantum adiabatic pumping involves the transport of particles without the application of any bias voltage. This is done by varying in time at least two independent parameters of the mesoscopic system out of phase. The physics of the adiabatic quantum pump is based on two independent works by Brouwer[21] and by Zhou *et al.*[22] which built on earlier works by Büttiker *et al.*[23] The first experimental realization of an adiabatic quantum pump was made in Ref. 24. The phenomenon of quantum adiabatic pumping has been extended to pump a spin current,[25] it has also been used in different mesoscopic systems, such as quantum-hall systems,[26] luttinger liquid-based mesoscopic conductor,[27] in the context of quantized charge pumping because of surface acoustic waves,[28] a quantum dot in the Kondo regime,[29] and, of course in the context of enhanced pumped currents in hybrid mesoscopic systems involving a superconductor.[30,31] In Ref. 30, Wang *et al.* showed that Andreev reflection at the junction between a normal metal and a superconductor (of s-wave type) can enhance the pumped current as much as four times that in a purely normal-metal structure. Blaauboer[31] showed that for slightly asymmetric coupling to the leads, this enhancement can be slightly increased. Recently, Taddei *et al.*[32] generalized the adiabatic quantum pumping mechanism wherein several superconducting leads are present.

This work is organized as follows. After generalizing the formula for the adiabatically pumped current through a normal-metal lead in presence of a high-T_c superconductor, we derive the amount of pumped charge current into the normal metal in the vicinity of a high-T_c superconductor with different types of order-parameter symmetry. Next we focus on the heat transported and noise generated in the pumping process in the case of each of the specific order-parameter symmetries. Finally, we juxtapose all the obtained results in case of different order-parameter symmetry in the amount of pumped current, heat, and noise to have some conclusive arrivals and to propose experiments that would fulfill this theoretical proposal.

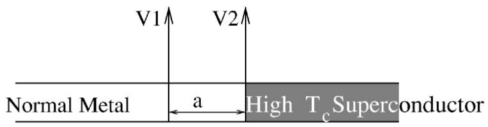

FIG. 1. The model system. A normal-metal double-barrier structure in proximity with a high-T_c superconductor. The double-barrier structure is modeled by two δ barriers a distance a apart.

II. THEORY OF THE PUMPED CHARGE CURRENT

The model system is shown in Fig. 1. It consists of a normal-metal double-barrier structure in junction with a high-T_c superconductor. The double-barrier structure is modeled by two δ barriers of strengths V_1 and V_2, a distance a apart. Quantum pumping is enabled by adiabatic modulations in the strength of the δ barriers, i.e., $V_1=V_0+V_p\sin(wt)$ and $V_2=V_0+V_p\sin(wt+\phi)$, where V_p is the strength of the pumping amplitude. Andreev reflection mechanism[33,34] is what takes place when a normal metal is brought in contact with a superconductor. The scattering matrix for the entire system is given by

$$S_{NS}(\epsilon)=\begin{pmatrix} S_{ee}(\epsilon) & S_{eh}(\epsilon) \\ S_{he}(\epsilon) & S_{hh}(\epsilon) \end{pmatrix}, \tag{1}$$

wherein $S_{ee}(\epsilon)$, $S_{eh}(\epsilon)$, $S_{he}(\epsilon)$, $S_{hh}(\epsilon)$ are 1×1 matrices, since we are considering single-channel leads. The explicit analytical form of the expressions are given by[35]

$$S_{ee}(\epsilon)=S_{11}(\epsilon)+\frac{S_{12}(\epsilon)\alpha^h S_{22}^*(-\epsilon)\alpha^e S_{21}(\epsilon)}{1-\alpha^h\alpha^e S_{22}(\epsilon)S_{22}^*(-\epsilon)},$$

$$S_{he}(\epsilon)=\frac{S_{12}^*(-\epsilon)\alpha^e S_{21}^*(\epsilon)}{1-\alpha^h\alpha^e S_{22}(\epsilon)S_{22}^*(-\epsilon)},$$

$$S_{eh}(\epsilon)=\frac{S_{12}(\epsilon)\alpha^h S_{21}^*(-\epsilon)}{1-\alpha^h\alpha^e S_{22}(\epsilon)S_{22}^*(-\epsilon)},$$

$$S_{hh}(\epsilon)=S_{11}^*(-\epsilon)+\frac{S_{12}^*(-\epsilon)\alpha^e S_{22}(\epsilon)\alpha^h S_{21}^*(-\epsilon)}{1-\alpha^h\alpha^e S_{22}(\epsilon)S_{22}^*(-\epsilon)}. \tag{2}$$

$$\text{with, } \alpha^h=e^{-i\arccos[\epsilon/\Delta(k_h)]+i\phi(k_h)},\ \alpha^e=e^{-i\arccos[\epsilon/\Delta(k_e)]-i\phi(k_e)},$$

$$e^{i\phi(k_e)}=\frac{\Delta(k_e)}{|\Delta(k_e)|},\ \text{and}\ e^{i\phi(k_h)}=\frac{\Delta(k_h)}{|\Delta(k_h)|}, \tag{3}$$

where $\phi(k_e)$ and $\phi(k_h)$ are the phase of the order parameter for electroniclike quasiparticles and holelike quasiparticles, respectively, with k_e and k_h being the respective wave vectors for the electroniclike quasiparticles and holelike quasiparticles.[14]

From Refs. 30 and 31, the adiabatically pumped electronic current into the normal lead in presence of the high-T_c superconducting lead is given by

$$I_e=\frac{wq_e}{2\pi}\int_0^\tau d\tau\left(\frac{dN_L^e}{dV_1}\frac{dV_1}{dt}+\frac{dN_L^e}{dV_2}\frac{dV_2}{dt}\right), \tag{4}$$

The quantity dN_L^e/dV (wherein, the subscript L denotes left lead or the normal lead) is the electronic injectivity given at zero temperature by

$$\frac{dN_L^e}{dV_j}=\frac{1}{2\pi}\mathcal{I}(S_{ee}^*\partial_{V_j}S_{ee}+S_{eh}^*\partial_{V_j}S_{eh}). \tag{5}$$

In the above equation and below, $\mathcal{I}$ represents the imaginary part of the quantity in parentheses. Similarly, the adiabatically pumped hole current into the normal lead in presence of the high-T_c superconducting lead is given by

$$I_h=\frac{wq_h}{2\pi}\int_0^\tau d\tau\left(\frac{dN_L^h}{dV_1}\frac{dV_1}{dt}+\frac{dN_L^h}{dV_2}\frac{dV_2}{dt}\right). \tag{6}$$

The quantity dN_L^h/dV (wherein, the subscript L denotes left lead or the normal lead) is the hole injectivity given at zero temperature by

$$\frac{dN_L^h}{dV_j}=\frac{1}{2\pi}\mathcal{I}(S_{hh}^*\partial_{V_j}S_{hh}+S_{he}^*\partial_{V_j}S_{he}) \tag{7}$$

with $q_e=-q_h$ as per the usual convention, and in the weak pumping regime the adiabatically pumped electronic current similar to the analysis in Refs. 21 and 30, is given by[36]

$$I_e=\frac{wq_e\sin(\phi)V_p^2}{\pi}\mathcal{I}(\partial_{V_1}S_{ee}^*\partial_{V_2}S_{ee}+\partial_{V_1}S_{eh}^*\partial_{V_2}S_{eh}) \tag{8}$$

and the adiabatically pumped hole current in the weak pumping regime is

$$I_h=\frac{wq_h\sin(\phi)V_p^2}{\pi}\mathcal{I}[\partial_{V_1}S_{hh}^*\partial_{V_2}S_{hh}+\partial_{V_1}S_{he}^*\partial_{V_2}S_{he}], \tag{9}$$

whereas for a normal-metal structure, the expression for the pumped electronic current in the weak pumping regime is given by

$$I(N)=\frac{wq_e\sin(\phi)V_p^2}{\pi}\mathcal{I}(\partial_{V_1}S_{11}^*\partial_{V_2}S_{11}+\partial_{V_1}S_{21}^*\partial_{V_2}S_{21}). \tag{10}$$

III. PUMPED CURRENT FOR DIFFERENT ORDER PARAMETERS

In Ref. 30, the pumped current for a normal metal-superconductor (NS) system (where the superconductor is of s-wave type) has been shown to be four times of that in a purely normal-metal junction. The system considered in Ref. 30 is also a double-δ barrier structure. We rederive the results for the pumped current in a normal-metal–s-wave superconductor junction and subsequently derive the results for the pumped current in a normal-metal–$d_{x^2-y^2}$-wave superconductor junction, for the pumped current in a normal metal–$d_{x^2-y^2}+is$-wave superconducting junction, and finally for the pumped current in a normal-metal–$d_{x^2-y^2}+id_{xy}$-wave superconducting junction. We consider in the examples below as well as in the succeding sections the Fermi energy to match the chemical potential of the superconducting lead so that

$\epsilon=0$. In which case, from Eq. (2) we have $S_{ee}=S^*_{hh}$. The system we consider is a normal-metal double-barrier structure at resonance in junction with a high-T_c superconductor. The resonance condition in the normal-metal quantum dot structure is exemplified by the fact that the reflection coefficients are zero, while the transmission coefficients are unity. Thus, $|S_{11}|^2=|S_{22}|^2=0$, while $|S_{12}|^2=|S_{21}|^2=1$, with $S_{12}=S_{21}=e^{-2ika}$ for the double-barrier quantum dot at resonance. Furthermore, from Eq. (2), we have, $S_{eh}=\alpha^h$ and $S_{he}=\alpha^e$. With this we get $\partial_{V_j}S_{eh}=\partial_{V_j}S_{he}=0$, for $j=1,2$. Thus, by the arguments above, the pumped electron and hole currents are exactly one and the same in both magnitude as well as direction and reduce to

$$I_h = I_e = \frac{wq_e \sin(\phi)V_p^2}{\pi}\mathcal{I}[\partial_{V_1}S^*_{ee}\partial_{V_2}S_{ee}]. \tag{11}$$

Furthermore, for the double-barrier structure at resonance from Eq. (2), one has the normal scattering amplitude, $S_{ee}=S_{11}+\alpha^h\alpha^e(S_{12})^2S^*_{22}$, and for the partial derivatives appearing in Eq. (11), we have $\partial_{V_1}S_{ee}=\partial_{V_1}S_{11}+\alpha^h\alpha^e(S_{12})^2\partial_{V_1}S^*_{22}$, with the help of the Dyson equation,[37] $\partial_{V_j}G^r_{\alpha\beta}=G^r_{\alpha j}G^r_{j\beta}$, and the Fisher-Lee relation,[38] $S_{\alpha\beta}=-\delta_{\alpha\beta}+i2kG^r_{\alpha\beta}$, one can easily derive $\partial_{V_1}S_{11}=-i/2k$, and $\partial_{V_1}S_{22}=-i/2k(S_{12})$.[2] Thus for a double barrier quantum dot at resonance, we have for the partial derivatives appearing in Eq. (11),

$$\partial_{V_1}S_{ee} = \frac{-i}{2k}(1-\alpha^h\alpha^e), \text{ and } \partial_{V_2}S_{ee} = \frac{-i}{2k}e^{-4ika}(1-\alpha^h\alpha^e). \tag{12}$$

With these formulas in mind we herein below derive the results for the pumped charge current for a normal-metal double-barrier structure in junction with a high-T_c superconductor, which we assume to have $d_{x^2-y^2}$-wave, $d_{x^2-y^2}+is$-wave, and $d_{x^2-y^2}+id_{xy}$-wave order parameters. For the sake of completeness and comparison we rederive the already known results for a pure normal-metal structure and that of a normal-metal double-barrier structure in junction with an isotropic s-wave superconductor.

A pure normal-metal double-barrier structure: From the discussion above the pumped current in case of a normal-metal double-barrier structure at resonance reduces to [from Eq. (10)]

$$I(N) = \frac{-wq_e \sin(\phi)V_p^2}{4\pi k^2}\sin(4ka). \tag{13}$$

An isotropic s-wave superconductor: For a normal s-wave superconductor, which is isotropic $\Delta(k_h)=\Delta(k_e)=\Delta$ and $\alpha^h=\alpha^e=-i$. Thus, $\partial_{V_1}S_{ee}=-i/k$ and $\partial_{V_2}S_{ee}=(-i/k)e^{-4ika}$, and therefore in the weak pumping regime for an isotropic s-wave superconductor in junction with a normal-metal double-barrier heterostructure the pumped current denoted by $I(NS)$ is four times that in a pure normal-metal structure,[30]

$$I(NS) = 4I(N), \tag{14}$$

with $I(N)$ as given in Eq. (13).

$d_{x^2-y^2}$-wave superconductor: Now we consider the case of a $d_{x^2-y^2}$-wave superconductor, in junction with a normal-metal double-barrier structure at resonance. The effective order parameter of the $d_{x^2-y^2}$-wave superconductor for electronlike quasiparticles is $\Delta(k_e)=\Delta_d\cos(2\theta_s-2\alpha)$ and for holelike quasiparticles it is $\Delta(k_h)=\Delta_d\cos(2\theta_s+2\alpha)$, with θ_s being the injection angle between the electron wave vector (k_e) and the x axis, whereas α is the misorientation angle between the a axis of the crystal and the interface normal. Now for a $d_{x^2-y^2}$-wave superconductor with a (110) orientation, $\alpha=\pi/4$. Thus, $\Delta(k_e)=\Delta_d\sin(2\theta_s)$ and $\Delta(k_h)=-\Delta_d\sin(2\theta_s)$. In light of this we have, $e^{i\phi(k_e)}=1$, and $e^{i\phi(k_h)}=-1$, and thus $\alpha_e=-i$ and $\alpha_h=i$, therefore we have $\partial_{V_1}S_{ee}=\partial_{V_2}S_{ee}=0$, and hence in the weak pumping regime for a $d_{x^2-y^2}$-wave superconductor in junction with a normal-metal double-barrier heterostructure the pumped current denoted by $I(ND)$ regardless of the injection angle is zero.

$$I(ND) = 0. \tag{15}$$

$d_{x^2-y^2}+is$-wave superconductor: Now, we consider the order parameter of the high-T_c superconductor to be a mixture of the $d_{x^2-y^2}+is$ type. The $d_{x^2-y^2}$ component has a (110) oriented surface, with $\alpha=\pi/4$. The effective order parameter for electron- and holelike quasiparticles becomes

$$\Delta(k_e) = \Delta_d\sin(2\theta_s) + i\Delta_s, \text{ and } \Delta(k_h) = -\Delta_d\sin(2\theta_s) + i\Delta_s.$$

For the phases of the pairing symmetries for electron- and holelike quasiparticles, we have

$$e^{i\phi(k_e)} = \frac{\Delta_d\sin(2\theta_s) + i\Delta_s}{\sqrt{\Delta_d^2\sin^2(2\theta_s)+\Delta_s^2}},$$

and

$$e^{i\phi(k_h)} = \frac{-\Delta_d\sin(2\theta_s) + i\Delta_s}{\sqrt{\Delta_d^2\sin^2(2\theta_s)+\Delta_s^2}},$$

and hence, the product $\alpha^h\alpha^e$ reduces to

$$\alpha^h\alpha^e = \frac{\Delta_d\sin(2\theta_s) - i\Delta_s}{\Delta_d\sin(2\theta_s) + i\Delta_s},$$

and finally for the partial derivatives appearing in Eq. (11) one has

$$\partial_{V_1}S_{ee} = \frac{\Delta_s}{k[\Delta_d\sin(2\theta_s)+i\Delta_s]}$$

and

$$\partial_{V_2}S_{ee} = \frac{\Delta_s e^{-4ika}}{k[\Delta_d\sin(2\theta_s)+i\Delta_s]}. \tag{16}$$

Thus, the pumped charge current reduces to

$$I(NDs) = \frac{-wq_e\sin(\phi)V_p^2}{\pi k^2}\frac{\Delta_s^2}{\Delta_d^2\sin^2(2\theta_s)+\Delta_s^2}\sin(4ka). \tag{17}$$

From Eqs. (13) and (17), the ratio of the pumped current in presence of the high-T_c superconductor to that in a pure normal-metal double-barrier structure becomes

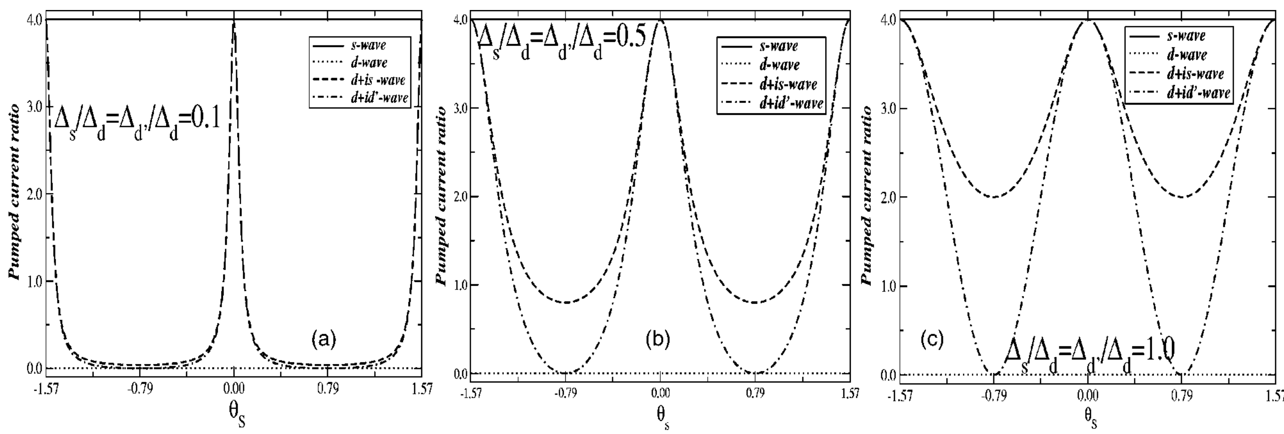

FIG. 2. The ratio of the pumped current for double-barrier quantum dot at resonance in junction with a high-T_c superconductor to that in a pure normal-metal double-barrier structure. (a) Magnitude of the subdominant component in the mixed-order parameter cases is 10% of the dominant component, (b) magnitude of the subdominant component in the mixed order parameter cases is *half* of the dominant component, and (c) magnitude of the subdominant component in the mixed order-parameter cases is *equal* to the dominant component.

$$\frac{I(NDs)}{I(N)} = 4\frac{\Delta_s^2}{\Delta_d^2 \sin^2(2\theta_s) + \Delta_s^2}. \quad (18)$$

From the expression it is evident that the maximum enhancement of the pumped current is four times of that in a pure normal-metal structure. Depending on the relative magnitudes of Δ_s and Δ_d and the injection angle θ_s, the ratio $I(NDs)/I(N)$ can be as low as zero as in the pure $d_{x^2-y^2}$ case or as large as 4 as in the pure s-wave case.

$d_{x^2-y^2}+id_{xy}$-wave superconductor: Finally, we consider the order parameter of the high-T_c superconductor to be a mixture of the $d_{x^2-y^2}+id_{xy}$ type. The order parameter for electronlike quasiparticles is $\Delta(k_e)=\Delta_d\cos(2\theta_s-2\alpha)+i\Delta_d'\sin(2\theta_s-2\alpha)$, whereas for holelike quasiparticles it becomes $\Delta(k_h)=\Delta_d\cos(2\theta_s+2\alpha)+i\Delta_d'\sin(2\theta_s-2\alpha)$. The $d_{x^2-y^2}$ and d_{xy} component have a (110) oriented surface, with $\alpha=\pi/4$. Thus, $\Delta(k_e)=\Delta_d\sin(2\theta_s)-i\Delta_d'\cos(2\theta_s)$, and $\Delta(k_h)=-\Delta_d\sin(2\theta_s)-i\Delta_d'\cos(2\theta_s)$.

For the phases of the order parameter for electron- and holelike quasiparticles, we have

$$e^{i\phi(k_e)} = \frac{\Delta_d\sin(2\theta_s) - i\Delta_d'\cos(2\theta_s)}{\sqrt{\Delta_d^2\sin^2(2\theta_s) + \Delta_d'^2\cos^2(2\theta_s)}},$$

and

$$e^{i\phi(k_h)} = \frac{-\Delta_d\sin(2\theta_s) - i\Delta_d'\cos(2\theta_s)}{\sqrt{\Delta_d^2\sin^2(2\theta_s) + \Delta_d'^2\cos^2(2\theta_s)}}$$

and hence the product $\alpha^h\alpha^e$ reduces to

$$\alpha^h\alpha^e = \frac{\Delta_d\sin(2\theta_s) + i\Delta_d'\cos(2\theta_s)}{\Delta_d\sin(2\theta_s) - i\Delta_d'\cos(2\theta_s)}.$$

Furthermore, for the partial derivatives of the scattering amplitudes appearing in Eq. (11), we have

$$\partial_{V_1}S_{ee} = -\frac{\Delta_d'\cos(2\theta_s)}{k[\Delta_d\sin(2\theta_s) - i\Delta_d'\cos(2\theta_s)]} \quad (19)$$

and

$$\partial_{V_2}S_{ee} = -\frac{\Delta_d'\cos(2\theta_s)e^{-4ika}}{k[\Delta_d\sin(2\theta_s) + i\Delta_d'\cos(2\theta_s)]},$$

and thus the pumped current into the normal-metal lead for this order parameter becomes

$$I(NDd') = \frac{-wq_e\sin(\phi)V_p^2}{\pi k^2}\frac{\Delta_d'^2\cos^2(2\theta_s)}{\Delta_d^2\sin^2(2\theta_s) + \Delta_d'^2\cos^2(2\theta_s)} \times\sin(4ka). \quad (20)$$

Furthermore, the ratio of the pumped current in presence of the high-T_c superconductor with pairing symmetry of the type $d_{x^2-y^2}+id_{xy}$ to that in a pure normal-metal double-barrier structure [see Eq. (13)] is

$$\frac{I(NDd')}{I(N)} = 4\frac{\Delta_d'^2\cos^2(2\theta_s)}{\Delta_d^2\sin^2(2\theta_s) + \Delta_d'^2\cos^2(2\theta_s)}. \quad (21)$$

From the above expression it is evident that the maximum enhancement of the pumped current is four times of that in a pure normal-metal structure. Depending on the relative magnitudes of Δ_d' and Δ_d and, of course, also depending on the injection angle, the ratio $I(NDd')/I(N)$ can be as low as zero as in the pure $d_{x^2-y^2}$ case or as large as 4 as in the pure s-wave case.

To conclude this section we have seen contrasting results in all the four cases. Although as seen before for the s-wave case there is fourfold enhancement as compared to the normal-metal case, in case of a $d_{x^2-y^2}$-wave superconductor there is no pumped current at all, and for the case of $d_{x^2-y^2}+is$-wave and $d_{x^2-y^2}+id_{xy}$-wave superconductor the enhancement depends on the relative magnitude of the components as well as the injection angle. To probe the dependence of the injection angle and relative magnitudes of the different components in the cases where we have considered mixed pairing symmetry, in Fig. 2 we plot the the ratio of the pumped current in presence of the high-T_c superconductor as function of the injection angle for different ratios of the relative magnitudes where mixed pairing symmetry is considered along with the pure s-wave and $d_{x^2-y^2}$-wave cases.

From Fig. 2, it is quite evident that the s-wave and $d_{x^2-y^2}$-wave cases are completely independent of injection angle. Furthermore, one can clearly see that whatever the strength of the subdominant component in the mixture (i.e., s or d_{xy} for the injection angle $\theta_s=0,\pm\pi/2$), one has for the pumped current in $d_{x^2-y^2}+is$-wave and $d_{x^2-y^2}+id_{xy}$-wave cases four times that in the pure normal-metal structure. Also it is evident especially for the relative magnitudes of the subdominant component in the mixture being around half or more, that there is a marked difference between the $d_{x^2-y^2}+is$-wave and $d_{x^2-y^2}+id_{xy}$-wave cases at injection angles $\theta_s=\pm\pi/4$, where as for the $d_{x^2-y^2}+is$-wave case the pumped current is almost same as that in a normal-metal structure in Fig. 2(b). In Fig. 2(c) it is almost twice of that in a normal-metal structure, but in both Figs. 2(b) and 2(c) the pumped current in the $d_{x^2-y^2}+id_{xy}$-wave case is zero at the same injection angle values. These differences can be easily exploited in distinguishing the different pairing symmetries considered here.

IV. PUMPED HEAT AND NOISE

A time-dependent scatterer always generates heat flows and can be considered as a mesoscopic (phase coherent) heat source, which can be useful for studying various thermoelectric phenomena in mesoscopic structures. The adiabatic quantum pump thus not only generates an electric current, but also heat current which can be expressed as the sum of noise power and the joule heat dissipated.[39–41] In this section we look into the heat pumped and the noise generated for the various order parameters of the high-T_c superconductors considered above to further unravel the differences among them.

The expressions for pumped heat and noise in the presence of a superconducting (s-wave) lead have been derived earlier in Ref. 40. Below we extend the description to include the $d_{x^2-y^2}$-wave, $d_{x^2-y^2}+is$-wave, and $d_{x^2-y^2}+id_{xy}$-wave superconductors. The pumped current in Eq. (4), can be reexpressed as follows:

$$I=\frac{wq}{2\pi}\int dE(-\partial_E f)\int_0^\tau dt\sum_{j=1,2}[\mathcal{I}(S^*_{ee}\partial_{V_j}S_{ee}+S^*_{eh}\partial_{V_j}S_{eh})]\frac{dV_j}{dt}. \tag{22}$$

In Eq. (22), $\mathcal{I}$ represents the imaginary part of the quantity in parenthesis. Furthermore, as in the adiabatic regime, $\partial_t S_{\alpha\beta}=\Sigma_i(\partial_{V_i}S_{\alpha\beta}\partial_t X_i+\dots)$, and from complex algebra $\mathcal{I}(S^*_{ee}\partial_t S_{ee})=-i(S^*_{ee}\partial_t S_{ee})$, the pumped current becomes

$$I=\frac{wq}{2\pi}\int dE\int_0^\tau dt\left\{S_{NS}\left[f\left(E+i\frac{\partial_t}{2}\right)-f(E)\right]S^\dagger_{NS}\right\}_{ee} \tag{23}$$

with S_{NS} being the 2×2 matrix as defined in Eq. (1). In Eq. (23), the Fermi-Dirac distribution is expanded to first order in ∂_t only and $\{\dots\}_{ee}$ represents the eeth element of the quantity in brackets.

The heat current pumped is defined as the magnitude of the electric current multiplied by energy measured from the Fermi level

$$H=\frac{1}{\pi\tau}\int_0^\tau\int dE(E-E_F)\times\left\{S_{NS}(E,t)\left[f\left(E+i\frac{\partial_t}{2}\right)-f(E)\right]S^\dagger_{NS}(E,t)\right\}_{ee}. \tag{24}$$

Expanding $f(E+i(\partial_t/2))$ up to second order one gets a nonvanishing contribution to the heat current in the zero temperature limit as

$$H=\frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{ee} \tag{25}$$

and since two parameters are being varied, we have

$$H=\frac{1}{8\pi\tau}\int_0^\tau dt\sum_{i,j=1,2}[\partial_{V_i}S_{ee}\partial_{V_j}S^*_{ee}+\partial_{V_i}S_{eh}\partial_{V_j}S^*_{eh}]\frac{\partial V_i}{\partial t}\frac{\partial V_j}{\partial t}. \tag{26}$$

By integrating Eq. (26) up to $\tau=2\pi$ we get the pumped current in the weak pumping regime as

$$H=\frac{w^2V_p^2}{16\pi}\left[\sum_{\beta=e,h}|\partial_{V_1}S_{e\beta}|^2+\sum_{\beta=e,h}|\partial_{V_2}S_{e\beta}|^2+2\cos(\phi)\sum_{\beta=e,h}\mathcal{R}(\partial_{V_1}S_{e\beta}\partial_{V_2}S^*_{e\beta})\right] \tag{27}$$

$\mathcal{R}$ refers to the real part of the quantity in parentheses. Similar to the above one can derive expressions for the noise and joule heat dissipated. The expression for the heat current can be reexpressed as

$$H=\frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{ee}+\frac{1}{8\pi\tau}\int_0^\tau dt\sum_{\beta=e,h}[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)]_{e\beta}\times[S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{\beta e}. \tag{28}$$

The diagonal term is identified as the joule heat, whereas the off-diagonal element is the noise power.[40]

$$H=J+N,$$
$$=\frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)]_{ee}[S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{ee}+\frac{1}{8\pi\tau}\int_0^\tau dt[\partial_t S_{NS}(E,t)S^\dagger_{NS}(E,t)]_{eh}[S_{NS}(E,t)\partial_t S^\dagger_{NS}(E,t)]_{he}. \tag{29}$$

Similar to the analysis for the pumped heat current, the joule heat dissipated and the noise power can be expressed in the weak pumping regime as

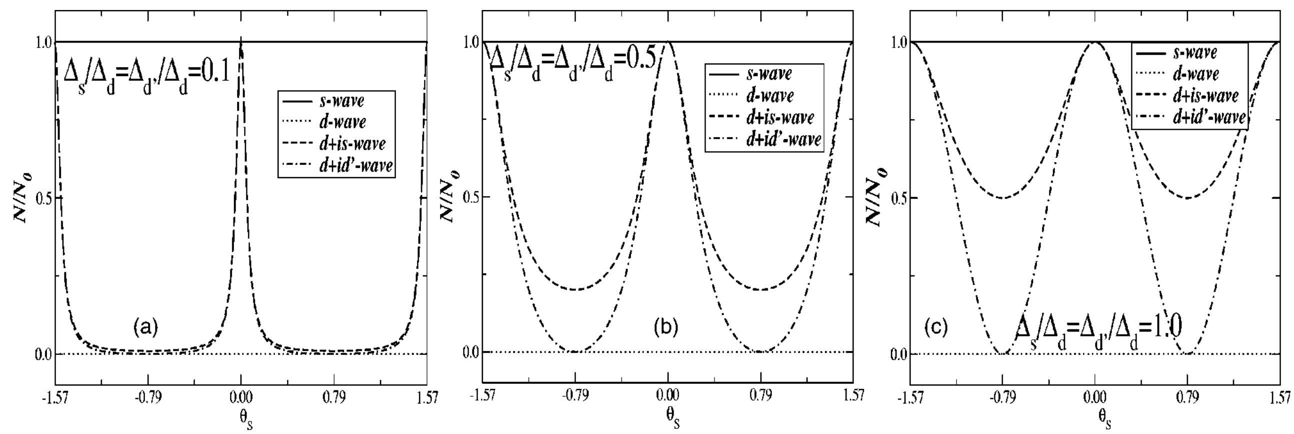

FIG. 3. The noise generated for double-barrier quantum dot at resonance in junction with a high-T_c superconductor. N_0 denotes the noise generated for the s-wave case. (a) Magnitude of the subdominant component in the mixed order-parameter cases is 10% of the dominant component, (b) magnitude of the subdominant component in the mixed order-parameter cases is *half* of the dominant component, and (c) magnitude of the subdominant component in the mixed order-parameter cases is *equal* to the dominant component.

$$
\begin{aligned}
J = \frac{V_p^2 w^2}{16\pi}\Biggl\{ & \Biggl[\sum_{\beta=e,h} |S_{e\beta}\partial_{V_1}S_{e\beta}|^2 + 2\mathcal{R}(S_{ee}^* S_{eh}\partial_{V_1}S_{ee}\partial_{V_1}S_{eh}^*)\Biggr] \\
& + \Biggl[\sum_{\beta=e,h} |S_{e\beta}\partial_{V_2}S_{e\beta}|^2 + 2\mathcal{R}(S_{ee}^* S_{eh}\partial_{V_2}S_{ee}\partial_{V_2}S_{eh}^*)\Biggr] \\
& + 2\cos(\phi)\Biggl[\sum_{\beta=e,h} |S_{e\beta}|^2 \mathcal{R}(\partial_{V_1}S_{e\beta}\partial_{V_2}S_{e\beta}^*) \\
& + \mathcal{R}(S_{eh}^* S_{ee}\partial_{V_1}S_{eh}\partial_{V_2}S_{ee}^*) + \mathcal{R}(S_{ee}^* S_{eh}\partial_{V_1}S_{ee}\partial_{V_2}S_{eh}^*)\Biggr]\Biggr\},
\end{aligned} \tag{30}
$$

while the noise power is given as below

$$
\begin{aligned}
N = \frac{V_p^2 w^2}{16\pi}\Biggl\{ & \Biggl[\sum_{\beta=e,h} |S_{h\beta}\partial_{V_1}S_{e\beta}|^2 + 2\mathcal{R}(S_{he}^* S_{hh}\partial_{V_1}S_{ee}\partial_{V_1}S_{eh}^*)\Biggr] \\
& + \Biggl[\sum_{\beta=e,h} |S_{h\beta}\partial_{V_2}S_{e\beta}|^2 + 2\mathcal{R}(S_{he}^* S_{hh}\partial_{V_2}S_{ee}\partial_{V_2}S_{eh}^*)\Biggr] \\
& + \cos(\phi)\Biggl[\sum_{\beta=e,h} |S_{h\beta}|^2 \mathcal{R}(\partial_{V_1}S_{e\beta}\partial_{V_2}S_{e\beta}^*) \\
& + \mathcal{R}(S_{hh}^* S_{he}\partial_{V_1}S_{eh}\partial_{V_2}S_{ee}^*) \\
& + \mathcal{R}(S_{he}^* S_{hh}\partial_{V_1}S_{ee}\partial_{V_2}S_{eh}^*)\Biggr]\Biggr\}.
\end{aligned} \tag{31}
$$

Now for our considered system, i.e., a double-barrier quantum dot at resonance, we have seen in Sec. III that $\partial_{V_j}S_{he}=\partial_{V_j}S_{eh}=0$ regardless of the order parameter symmetry of the high-T_c superconductor, and hence the expressions for the pumped heat, noise, and joule heat dissipated reduce to

$$
H = \frac{V_p^2 w^2}{16\pi}[|\partial_{V_1}S_{ee}|^2 + |\partial_{V_2}S_{ee}|^2 + 2\cos(\phi)\mathcal{R}(\partial_{V_1}S_{ee}\partial_{V_2}S_{ee}^*)] \tag{32}
$$

$$
J = \frac{V_p^2 w^2}{16\pi}|S_{ee}|^2[|\partial_{V_1}S_{ee}|^2 + |\partial_{V_2}S_{ee}|^2 + 2\cos(\phi)\mathcal{R}(\partial_{V_1}S_{ee}\partial_{V_2}S_{ee}^*)] \tag{33}
$$

$$
N = \frac{V_p^2 w^2}{16\pi}|S_{he}|^2[|\partial_{V_1}S_{ee}|^2 + |\partial_{V_2}S_{ee}|^2 + 2\cos(\phi)\mathcal{R}(\partial_{V_1}S_{ee}\partial_{V_2}S_{ee}^*)]. \tag{34}
$$

For our chosen system, i.e., the double-barrier quantum dot at resonance in junction with the high-T_c superconductor when $\epsilon=0$, we have, $|S_{ee}|^2=0$ and $|S_{he}|^2=1$, therefore, $J=0$ and $H=N$.

Now analyzing the above expressions for the different order parameters, we have the following.

s-wave superconductor: In the *s*-wave case as we have already seen $\partial_{V_1}S_{ee}=2\partial_{V_1}S_{11}=-i/k$ and $\partial_{V_2}S_{ee}=2\partial_{V_2}S_{11}=-(i/k)(S_{12})$.[2] With this, the expression for the heat current pumped, which is equal to the noise power, reduces to

$$
H = N = \frac{V_p^2 w^2}{8\pi k^2}[1 + \cos(\phi)\cos(4ka)]. \tag{35}
$$

Thus as is evident from the expression for the pumped noise, the quantum pump is nonoptimal[42] (or nonnoiseless), only in the special case when $4ka=(2n+1)\pi$ and $\phi=2n\pi$, with $n=0,1,\ldots$ is the optimality condition met. Of course, $\phi=2n\pi$ implies that in this case there is no charge current as well.

$d_{x^2-y^2}$-wave superconductor: In this case as also seen earlier, we have $\partial_{V_1}S_{ee}=0$ and $\partial_{V_2}S_{ee}=0$. Thus there is no heat pumped and neither any noise generated nor any joule heat dissipated. Thus the pump in conjunct with a $d_{x^2-y^2}$-wave superconductor is cent-percent optimal for any configuration of the parameters and under any condition.

$d_{x^2-y^2}+is$-wave superconductor: From the previous section, we have $\partial_{V_1}S_{ee}$ and $\partial_{V_2}S_{ee}$ for the order parameter in this

TABLE I. A comparative analysis of pumped charge, heat and noise in cases of s-wave, $d_{x^2-y^2}$-wave, $d_{x^2-y^2}+is$-wave, and $d_{x^2-y^2}+id_{xy}$-wave superconductors in conjunct with a normal metal double barrier structure, $[H_0 = V_p^2 w^2/8\pi k^2[1+\cos(\phi)\cos(4ka)]$.,

Order Parameter→	s-wave	$d_{x^2-y^2}$-wave	$d_{x^2-y^2}+is$-wave	$d_{x^2-y^2}+id_{xy}$-wave
Pumped↓				
Charge	$\frac{I(NS)}{I(N)}=4$	0	$\frac{I(NDs)}{I(N)}=\frac{\Delta_s^2}{\Delta_d^2\sin^2(2\theta_s)+\Delta_s^2}$	$\frac{I(NDd')}{I(N)}=\frac{\Delta_d'^2\cos^2(2\theta_s)}{\Delta_d^2\sin^2(2\theta_s)+\Delta_d'^2\cos^2(2\theta_s)}$
Heat	H_0	0	$H_0\frac{\Delta_s^2}{\Delta_d^2\sin^2(2\theta_s)+\Delta_s^2}$	$H_0\frac{\Delta_d'^2\cos^2(2\theta_s)}{\Delta_d^2\sin^2(2\theta_s)+\Delta_d'^2\cos^2(2\theta_s)}$
Noise	Nonoptimal	Cent-percent optimal	Nonoptimal	Nonoptimal[a]

[a]Optimal for injection angles $\theta_s = \pm\pi/4$ (see Sec. IV)].

case, with this the pumped heat, which is same as the noise generated in the pumping process, reduces to

$$H = N = \frac{V_p^2 w^2}{8\pi k^2}\frac{\Delta_s^2}{\Delta_d^2\sin^2(2\theta_s)+\Delta_s^2}[1+\cos(\phi)\cos(4ka)].$$

Denoting the noise generated in the s-wave case by N_0, we have the noise generated for this case becoming just $N_0\Delta_s^2/(\Delta_d^2\sin^2(2\theta_s)+\Delta_s^2)$.

$d_{x^2-y^2}+id_{xy}$-wave superconductor: From the previous section, we have $\partial_{V_1}S_{ee}$ and $\partial_{V_2}S_{ee}$ for this case, too, with this the pumped heat, which is same as the noise generated in the pumping process, reduces to

$$H = N = \frac{V_p^2 w^2}{8\pi k^2}\frac{\Delta_d'^2\cos^2(2\theta_s)}{\Delta_d^2\sin^2(2\theta_s)+\Delta_d'^2\cos^2(2\theta_s)} \times[1+\cos(\phi)\cos(4ka)].$$

Denoting the noise generated in the s-wave case by N_0, we have the noise generated for this case becoming just $N_0\Delta_d'^2\cos^2(2\theta_s)/[\Delta_d^2\sin^2(2\theta_s)+\Delta_d'^2\cos^2(2\theta_s)]$.

To end this section we have seen that the pumped heat and noise generated in the pumping process can also show marked differences for the various order parameters considered. In the s-wave, the $d_{x^2-y^2}+is$-wave, and the $d_{x^2-y^2}+id_{xy}$-wave cases the system is nonoptimal, whereas in the $d_{x^2-y^2}$-wave case it is cent-percent optimal. Furthermore, in the $d_{x^2-y^2}+id_{xy}$-wave case the pump may be turned optimal in some special situations as seen in Fig. 3. These situations would help in differentiating between the order parameters for the mixed parameter cases. Especially for Figs. 3(b) and 3(c) as it is quite clear that the pump is optimal (or noiseless) in case of the $d_{x^2-y^2}+id_{xy}$-wave superconductor at injection angles $\theta_s = \pm\pi/4$.

V. EXPERIMENTAL REALIZATION

Although theoretical examples in quantum pumping phenomena are quite abundant, experiments in this field are very much lacking. To date there have been notably four experiments in Refs. 24, 43, and 44 and a quantum spin pump in Watson *et al.*[25] Watson *et al.*[25] deals with a quantum dot, which with application of an in-plane magnetic field can pump a pure spin current. One can suitably modify these experiments and place the quantum dot in junction with a high-T_c superconductor. The resonant condition of the quantum dot can be easily established by applying a suitable gate voltage that will enable resonant transport through the quantum dot. After this, the two δ barriers can be two gates that control the charge on the dot; modulating these two gates in time will enable a pumped charge (also heat and noise) current to flow. This setup can easily establish the results arrived at in this work and hopefully give more clues into building a correct theory for high-T_c superconductors.

VI. CONCLUSIONS

To conclude we have given a simple procedure to distinguish various order parameters proposed in the context of high-T_c superconductivity. In Table I above we juxtapose the results obtained in this work. The pumped charge current, heat pumped, and noise generated for the four cases considered (the s-wave, $d_{x^2-y^2}$-wave, $d_{x^2-y^2}+is$-wave, and $d_{x^2-y^2}+id_{xy}$-wave) vary markedly, which easily reveals the differences among three.

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