Short-time dynamics of spin systems with long-range correlated quenched impurities

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The theoretic renormalization-group approach is applied to the study of short-time critical behavior of the d-dimensional spin systems (model A) in the presence of quenched impurities with a long-range correlations decaying as $r^{-(d-\rho)}$. The asymptotic scaling laws are studied in the frame of a double expansion in $\epsilon=4-d$ and ρ with ρ of order ϵ. In $d<4$, the initial slip exponents θ' of the magnetization and θ of the response function, are calculated up to two-loop order. The crossover between fixed points is obtained. The long-time limit of the fluctuation-dissipation ratio is found in the aging regime, and its connection to equilibrium quantities is discussed. The comparison of our results with those of other systems without long-range correlated quenched impurities is also investigated.

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I. INTRODUCTION

For critical dynamic systems, it has traditionally been believed that universal scaling behavior exists in the long-time regime of dynamic evolution. However, in 1989, it was discovered that starting from macroscopic initial states, the macroscopic short-time stages $0\ll t\ll t_\tau$ (here t_τ is the relaxation time) of dynamic processes display universal behavior dominated by initial slip exponents θ and θ'.[1] In recent years, universal short-time scalings that govern the nonequilibrium relaxation have been found in various models.[2–8] In general, after the system initially at a high temperature T_i with a small magnetization m_0 is suddenly quenched to the critical temperature $T_c\ll T_i$, in the short-time regime, not only does the order parameter show an critical initial increase $m(t)\sim m_0 t^{\theta'}$ before crossing over to the long-time behavior $\sim t^{-\beta/(\nu z)}$, but it also gives the response function $G(r,t,t')\sim(t/t')^{\theta}$ and the correlation function $C(r,t,t')\sim(t/t')^{\theta-1}$ for $t'\to 0$.

The short-time phenomena are also characterized by the nonequilibrium correlation length[3] $\xi(t)$ $(\sim t^{1/z})$. The length scale $\xi(t)$ is initially small and grows as time increases, arriving at its equilibrium value $\xi=\xi(\infty)\sim|T-T_c|^{-\nu}$ for $T\geqslant T_c$. It is believed that the singularity of the temporal correlation is essential to the short-time scaling and the scaling can emerge in the early stage of the evolution even though all correlations are still short ranged. As long as the spatial dimension d is smaller than the critical dimension d_c, the order parameter follows a mean-field ordering process because the mean-field critical temperature $T_c^{(\mathrm{mf})}$ is larger than the actual critical temperature T_c. This ordering causes an amplification of the initial order parameter at short times $t<t_i\ll t_\tau$, where $t_i\sim m_0^{-1/(\theta'+\beta/\nu z)}$.[1] For $d>d_c$, mean-field theory applies and there is no critical increase.

If the system does not reach the equilibrium all the response functions and correlation functions will depend both on the waiting time t' (the "age" of the system) and the observation time t. The distance from equilibrium of an aging system, evolving at a fixed temperature T, may be characterized the breaking of the fluctuation-dissipation theorem in terms of the fluctuation-dissipation ratio[9–11] (FDR)

$$X_r(t,t') = TG(r,t,t')/\partial_{t'}C(r,t,t'). \tag{1}$$

When $t,t'>t_\tau\sim|T-T_c|^{-\nu z}$, dynamics of fluctuations are described by the equilibrium dynamics of the system, and the fluctuation-dissipation theorem holds and thus $X_r(t,t')=1$. However, this is no longer true in the aging time $t,t'<t_\tau$. Consequently, $X_r(t,t')\neq 1$ becomes a nontrivial function of both t and t'.

Physically, $X_r(t,t')=1$ is realized in the high-temperature phase $(T>T_c)$ after the initial quench. Since the relaxation time is small, the system relaxes rapidly to equilibrium.[12] On the other hand, if either $T<T_c$ or $T=T_c$, an infinite spin system does not reach equilibrium on some finite time scale but instead undergoes either phase-order kinetics[13] or nonequilibrium critical dynamics.[1] In recent years, several works[9–12,14–18] have been devoted to the study of FDR for systems exhibiting domain growth.[13] or for aging systems such as glasses and spin glasses, showing that in the low-temperature phase, $X_r(t,t')$ is a nontrivial functions of its two arguments. In particular, for domain-growth systems, analytical and numerical studies indicate that the limit of FDR,

$$X_{r=0}^{\infty} = \lim_{t'\to\infty}\lim_{t\to\infty} X_{r=0}(t,t'), \tag{2}$$

vanishes through the low-temperature phase.[12,14–18] It is the slow motion of the domain boundaries that is responsible for $X_{r=0}^{\infty}=0$. This feature can be understood from the fact that the long-time memory of coarsening systems tends to vanish, unlike in the mean-field glass model in which it does not, so that $X_{r=0}(t,t')>0$ even at long times.[16] However, the situation is different for the quench from the high temperature T_i to the critical temperature $T_c\ll T_i$, as ordered domain does not exist. It has been argued that $X_{r=0}^{\infty}$ is a new nonequilibrium critical quantity characteristic of the different universality class. In exactly solvable cases,[11,12,18,19] in various Monte Carlo studies,[18,20–22] and in field-theoretical calculations,[23–25] $X_{r=0}^{\infty}$ has values ranging between 0 and $1/2$. Using standard renormalization group procedures, and its values of $X_{r=0}^{\infty}$ for

O(n) vector model[23] is in very good agreement with numerical simulations for the two- and three-dimensional Ising model.[18,21,22]

It is interesting whether and how this critical behavior is altered by introducing in the systems a small amount of impurities leading to models with quenched disorder. The theory of second-order phase transitions in the presence of quenched impurities was worked out and developed by the authors[26–29] for the case of short-range correlated quenched impurities (SCQI). It is consistent with the Harris criterion,[30] which states a random transition-temperature system possesses the same critical exponents and critical properties as the corresponding pure system if $\alpha_p=2-d\nu_p$ is negative (where α_p and ν_p are the specific heat exponent and the correlation-length exponent of the pure system, respectively, and d is the dimensionality of the system). However, for $\alpha_p>0$, the impurities can leads to new critical behavior. These impurities are described by the random local transition temperature $T_c(\mathbf{x})$, with short-range correlations in disorder that are proportional to δ function.

In 1983, Weinrib and Halperin[31] extended the theory for SCQI to the case of the long-range correlated quenched impurities (LCQI), which is more relevant to experimental results of interest.[32] The LCQI is characterized by the correlation function of the random local transition temperature $\langle T_c(\mathbf{x})T_c(\mathbf{y})\rangle - \langle T_c(\mathbf{x})\rangle\langle T_c(\mathbf{y})\rangle$, which falls off with distance as a power law $|\mathbf{x}-\mathbf{y}|^{-(d-\rho)}$ (where ρ characterizes the decay rate of the correlations). It was shown that for $\rho>0$, the Harris criterion is modified to be $(d-\rho)\nu_p-2>0$, which means the LCQI is irrelevant.[31] For $\rho\leqslant 0$, the LCQI is reduced as the SCQI, and the normal Harris criterion $d\nu_p-2=-\alpha_p>0$ is recovered.

The renormalization-group (RG) approach can be used to investigate the case with LCQI, provided that $d-\rho$ and d are close to 4. For the static properties, it is found that a new fixed-point characteristic of LCQI describes a second-order phase transition, with the correlation-length exponent ν evaluated as $\nu=2/(d-\rho)$,[31] which is exact and holds in all orders in perturbation theory.[33] For the dynamical critical properties, the results of Refs. 34–36 have shown that the LCQI affects the equilibrium critical dynamics in one-loop approximation, leading to new values of the dynamic exponent z.

In this work we will analyze the short-time critical behavior and the aging properties of spin systems with the LCQI. Although the static and equilibrium dynamical critical properties of the systems with LCQI have been under intensive theoretical and experimental study,[31–36] its nonequilibrium dynamics is less investigated. The aim of the present paper, based on the theoretic RG approach, is to make a more detailed investigation of the critical behavior of the systems with LCQI, to give a check of the expected scaling laws, and to predict the universal dynamical quantities (such as θ' and $X^{\infty}_{r=0}$), which could be measured in Monte Carlo simulations and experiments and could be used to identify a universality class. Using a double expansion in ϵ and ρ, we calculate initial slip exponents θ' and θ in the two-loop approximation. In particular, for $n=1$, θ' is found to take the nontrivial value instead of vanishing[35] at first order in ρ. The discussion of the crossover between fixed points modifies one-loop results in Refs. 31 and 35. The limit FDR $X^{\infty}_{r=0}$ is also calculated in the aging regime. It is argued that the relation between $X^{\infty}_{r=0}$ and the static Parisi function $P(q)$[37] is dependent on the impurities.

The paper is organized as follows. In Sec. II, the dynamics of the model with LCQI is defined. In Sec. III, using the theoretic RG approach, the asymptotic scaling laws are obtained. In Sec. IV, the initial slip exponents are studied in the frame of a double expansion in $\epsilon=4-d$ and ρ, with ρ of order ϵ. The scaling behavior of the two-time response and correlation functions for zero momentum is obtained as well as the long-time limit FDR in the aging time in Sec. V. The crossover between fixed points is discussed in Sec. VI. Finally, Sec. VII contains some discussion and conclusions.

II. THE MODEL

In equilibrium at temperature T, the O(n) symmetric Hamiltonian describing the spin systems with LCQI is given by

$$H[s] = \int d^dx\left\{\frac{1}{2}(\nabla s)^2 + \frac{\tau}{2}s^2 + \frac{g}{4!}(s^2)^2 + \frac{1}{2}\phi s^2\right\}, \qquad (3)$$

where $s=(s^i)$ are n-component order parameter fields, and $s^2=\sum_{\alpha=1}^{n}s^is^i$. $\phi(x)$ describes the static random-temperature impurity with the mean $\langle\phi(x)\rangle_\phi=0$ and the long-range correlations $\langle\phi(x)\phi(x')\rangle_\phi=[g_1+g_2(-\nabla^2)^{-\rho/2}]\delta(x-x')$. The angular bracket $\langle\cdots\rangle_\phi$ indicates the Gaussian configuration average with the impurities. In momentum space, the Fourier transform of the LCQI correlations is $g_1+g_2p^{-\rho}$. Note that in the case of $g_2=0$ or $\rho\leqslant 0$, the problem reduces to the description of the SCQI.[26–28] In particular the correlation function for LCQI with $\rho=1$ describes straight lines of impurities or straight dislocation lines of random orientation, whereas random planes of impurities would give to $\rho=2$.

In the absence of impurities [i.e., $\phi(x)=0$ or $g_1=g_2=0$], the O(n) symmetric Hamiltonian (1) for $n=4$ can be used to describe the spiral magnets Tb, Dy, and Ho, which belong to the same universality as NbO_2, DyC_2, and $TbAu_2$.[38] In the presence of LCQI, the O(4) model was applied to x-ray and neutron critical scattering experiments [such as Ho (see Ref. 39) and Tb (see Ref. 40)], which revealed two different length scales for critical fluctuations. It is argued that the emergence of the longer of the two length scales is a consequence of the presence of the LCQI in the neighborhood of the sample surface.[32] In this part of the sample, a crossover to critical behavior (corresponding to the longer length scale) dominated by the LCQI fixed point[31] takes place, while the bulk displays the critical behavior of the pure systems[38,41] (corresponding to the shorter length scale).

In this paper, the dynamics to be discussed has no conservation law, and is called the model A dynamics,[42] which is controlled by the Langevin equation

$$\partial_t s^i(x,t) = -\lambda \frac{\delta H[s]}{\delta s^i(x,t)} + \xi^i(x,t), \tag{4}$$

where λ is the kinetic coefficient. The random forces $\xi = (\xi^i)$ are assumed to be Gaussian with a mean of zero and correlations $\langle \xi^i(x,t)\xi^j(x',t')\rangle_\xi = 2\lambda\delta^{ij}\delta(x-x')\delta(t-t')$. The angular bracket $\langle\cdots\rangle_\xi$ indicates an average with the thermal noise.

The equilibrium critical dynamics of the O(n) model can be generated by the Langevin equation (4) and the Hamiltonian (3). In the following, we are interested in the nonequilibrium relaxation from the initial state $s_0(x)=s(x,t=0)$, which is macroscopically prepared at some very high temperature $T_i \gg T_c$. This initial state with short-range correlations corresponds to a distribution $P[s_0] \propto \exp\{-\tau_0 \int d^dx [s_0(x)-m_0]^2/2\}$. Here m_0 is a homogeneous initial order parameter. τ_0 has a physically interesting fixed point $\tau_0^* = +\infty$, which corresponds to a sharply prepared initial state with initial order m_0 and zero correlation length.[1] An initial condition with long-range correlations may lead to different universality class, e.g., shown for the sphere model.[12]

As shown in Ref. 43, the dynamics expressed in Eqs. (3) and (4) can be cast in field theoretical form in terms of a path integral that involves a set of conjugated variables s and $\tilde{s}$. The variable $\tilde{s}$ has a simple physical interpretation in terms of the response field, sometimes called Martin-Siggia-Rose response field.[44] All correlation and response functions can then be obtained by the path integral over phase space variables s and $\tilde{s}$. The generating functional for all the nonequilibrium connected correlation and response functions is now given by

$$W[h,\tilde{h}] = \ln \int \mathcal{D}(i\tilde{s},s)\exp\left[-\mathcal{L}[\tilde{s},s] - \int d^dx \frac{\tau_0}{2}(s_0-m_0)^2 + \int_0^\infty dt \int d^dx (hs + \tilde{h}\tilde{s})\right], \tag{5}$$

where h and $\tilde{h}$ are the source fields for the fields s and $\tilde{s}$, respectively. The effective action functional $\mathcal{L}[\tilde{s},s]$ is given by

$$\mathcal{L}[\tilde{s},s] = \int_0^\infty dt \int d^dx \left\{\tilde{s}\left[\dot{s} + \lambda(\tau - \nabla^2)s + \frac{\lambda g}{6}ss^2\right] - \lambda\tilde{s}^2\right\} - \frac{\lambda^2}{2}\int d^dx [g_1 + g_2(-\nabla^2)^{-\rho/2}]\left[\int_0^\infty dt \tilde{s}(x,t)s(x,t)\right]^2. \tag{6}$$

Here we have used a prepoint discretization with respect to time so that the step function $\Theta(t=0)=0$. The contribution [proportional to $\Theta(0)$] to $\mathcal{L}[\tilde{s},s]$ arising from the functional determinant $\det[\delta\xi(x,t)/\delta s(x,t)]$ then vanishes.

For $g=g_1=g_2=0$, the generating functional (6) becomes Gaussian and can be easily evaluated in momentum space. The free response function $G_p^{(o)}(t,t') = \langle s_p(t)\tilde{s}_{-p}(t')\rangle_G$ and the free correlation function $C_p^{(o)}(t,t') = \langle s_p(t)s_{-p}(t')\rangle_G$ are, respectively,

$$G_p^{(o)}(t,t') = \Theta(t-t')\exp[-\lambda(p^2+\tau)(t-t')], \tag{7}$$

$$C_p^{(o)}(t,t') = \frac{1}{\tau+p^2}e^{-\lambda(p^2+\tau)|t-t'|} + \left(\tau_0^{-1} - \frac{1}{\tau+p^2}\right)e^{-\lambda(p^2+\tau)(t+t')}. \tag{8}$$

III. RENORMALIZATION AND CRITICAL SCALING

With the help of Eqs. (7) and (8), one now sets a perturbation expansion ordered by the number of loops in the Feynman diagrams. It is convenient to consider the Dirichlet boundary conditions $\tau_0 = +\infty$ and $m_0 = 0$. The general case is recovered by treating the parameters τ_0^{-1} and m_0 as additional perturbations. The model (6) with Dirichlet boundary conditions must be renormalized. For this purpose, notice that the free correlation function simplifies to

$$C_p^{(D)}(t,t') \equiv \frac{1}{\tau+p^2}\{\exp[-\lambda(p^2+\tau)|t-t'|] - \exp[-\lambda(p^2+\tau)(t+t')]\}.$$

The relations $\dot{s}_0(x) = 2\lambda\tilde{s}_0(x)$ and $s_0(x) = \tilde{s}_0(x)/\tau_0$ are invariant under renormalization.

A dimensional analysis of Eq. (6) allows us to carry out a double expansion in $\epsilon = 4-d$ and ρ, with ϵ and ρ of the same order to calculate the connected Green functions $G_{N\tilde{N}}^M = \langle s^N \tilde{s}^{\tilde{N}} \tilde{s}_0^M\rangle$. However, the calculation results in integrals divergent at the upper critical dimension $d_c = 4$. To obtain a meaningful theory, the divergence must be absorbed into the renormalizations of the model parameter and the fields. We will adopt the dimensional regularization with minimal subtraction scheme,[45] and introduce renormalized quantities through some multiplicative factors

$$s_b = Z_s^{1/2}s, \quad \tilde{s}_b = Z_{\tilde{s}}^{1/2}\tilde{s}, \quad \tilde{s}_{0b} = (Z_{\tilde{s}}Z_0)^{1/2}\tilde{s}_0,$$
$$\lambda_b = (Z_s/Z_{\tilde{s}})^{1/2}\lambda, \quad \tau_b = Z_s^{-1}Z_\tau\tau,$$
$$g_b = K_d^{-1}\mu^\epsilon Z_s^{-2}Z_u u, \quad g_{1b} = K_d^{-1}\mu^\epsilon Z_s^{-2}Z_{u_1}u_1, \tag{9}$$
$$g_{2b} = K_d^{-1}\mu^{\epsilon+\rho}Z_s^{-2}Z_{u_2}u_2.$$

Here the subscript b denotes the bare quantity $K_d = 2^{1-d}\pi^{-d/2}[\Gamma(d/2)]^{-1}$.

As usual, the RG equation is derived by exploiting the fact that the unrenormalized Green functions $G_{N\tilde{N}b}^M = \langle s_b^N \tilde{s}_b^{\tilde{N}} \tilde{s}_{0b}^M\rangle$ are independent of the external momentum scale μ. This leads to the RG equation

$$\left\{\mu\partial_\mu + \zeta\lambda\partial_\lambda + \kappa\tau\partial_\tau + \beta_u\partial_u + \beta_{u_1}\partial_{u_1} + \beta_{u_2}\partial_{u_2} + \tfrac{1}{2}[N\gamma_s + \tilde{N}\gamma_{\tilde{s}} + M(\gamma_{\tilde{s}} + \gamma_0)]\right\}G_{N\tilde{N}}^M = 0 \tag{10}$$

for the renormalized Green functions $G_{N\tilde{N}}^M = \langle s^N \tilde{s}^{\tilde{N}} \tilde{s}_0^M\rangle$. Here $\beta_w = \mu\partial_\mu w|_0$ (for $w = u, u_1, u_2$) and $A = \mu\partial_\mu \ln B|_0$ (for $A = \gamma_s$, $\gamma_{\tilde{s}}$, γ_0, κ, ζ, and $B = Z_s, Z_{\tilde{s}}, Z_0, \tau, \lambda$, respectively) are Wilson functions. The symbol $|_0$ means that μ-derivatives are calculated at fixed bare parameters.

At the fixed points $w^*=(u^*,u_1^*,u_2^*)$, using dimensional analysis and the solution of Eq. (10), we derive the scaling laws

$$G^M_{N\tilde{N}}(\{x,t\},\tau,\lambda,w^*,\mu) = l^{(d-2+\eta_s)N/2+(d+2+\eta_{\tilde{s}})(\tilde{N}+M)/2+\eta_0 M/2} \times G^M_{N\tilde{N}}(\{lx,l^z t\},\tau l^{-1/\nu},\lambda,w^*,\mu), \tag{11}$$

where $\eta_s=\gamma_s(w^*)$, $\eta_{\tilde{s}}=\gamma_{\tilde{s}}(w^*)$, and $\eta_0=\gamma_0(w^*)$ are the anomalous dimensions. The long-time critical exponents are determined by the relations $\eta=\eta_s$, $z=2+(\eta_{\tilde{s}}-\eta)/2$ and $1/\nu=2-\kappa(w^*)$.

According to the general scaling law (11) and $s_0=\tau_0^{-1}\tilde{s}_0$, we find the autocorrelation function $C(t)=\langle s(x,t)s_0(x)\rangle$ displaying the scaling form

$$C(t) = t^{\theta'-d/z} f_a(\tau t^{1/(\nu z)}), \tag{12}$$

where the initial slip exponent θ' is defined by $\theta'=-(\eta_s+\eta_{\tilde{s}}+\eta_0)/(2z)$. The RG analysis of nonequilibrium critical relaxation also yields the scaling form of the order parameter $m(t)\equiv\langle s(x,t)\rangle|_{\tilde{h}=h=0}$, which is expanded in powers of m_0; i.e.,

$$m(t) = m_0 t^{\theta'} f_m(m_0 t^{\theta'+\beta/(\nu z)},\tau t^{1/(\nu z)}), \tag{13}$$

where the function $f_m(0,0)$ is finite, while for $x\to\infty$, $f_m(x,0)\sim 1/x$, which leads to the long-time behavior $m(t)\sim t^{-\beta/(\nu z)}$.[4,42]

The short-time scaling behavior of correlation and response functions can be obtained by a short-time expansion of the fields $s(x,t)$ and $\tilde{s}(x,t)$, as done in Ref. 1. By means of Green functions (11), one will find for $t\to 0$

$$s(x,t) = t^{1-\theta}\varphi(t/\xi^z)\tilde{s}_0(x) + \cdots, \tag{14}$$

$$\tilde{s}(x,t) = t^{-\theta}\tilde{\varphi}(t/\xi^z)\tilde{s}_0(x) + \cdots, \tag{15}$$

where $\varphi(0)$ and $\tilde{\varphi}(0)$ are finite quantities. The exponent θ is defined by $\theta=-\eta_0/(2z)$ and satisfies the scaling relation $z(1+\theta'-\theta)=2-\eta$. By means of the Green functions (11), one will find that two-point response function $G(\mathbf{x}-\mathbf{x}',t,t')=G^0_{11}(\mathbf{x},\mathbf{x}';t,t')$ and two-point correlation function $C(\mathbf{x}-\mathbf{x}',t,t')=G^0_{20}(\mathbf{x},\mathbf{x}';t,t')$ are given by, respectively,

$$G(r,t,t') = r^{-(d-2+\eta+z)}\left(\frac{t}{t'}\right)^{\theta} f_G(r\tau^{\nu},t\tau^{1/(\nu z)}), \tag{16}$$

$$C(r,t,t') = r^{-(d-2+\eta)}\left(\frac{t}{t'}\right)^{\theta} f_C(r\tau^{\nu},t\tau^{1/(\nu z)}), \tag{17}$$

for $t'\to 0$, neglecting corrections due to τ_0^{-1}.

As already mentioned in Sec. I, the violation of the fluctuation-dissipation theorem out of thermal equilibrium can be characterized by the FDR $X_r(t,t')$, defined in Eq. (1). In the present theoretical representation, Eq. (1) is rewritten as

$$X_r(t,t') = \lambda G(r,t,t')/\partial_{t'}C(r,t,t'). \tag{18}$$

For $r=0$, and using Eq. (11) in the above equation, one has

$$X_{r=0}(t,t') = f_x(l^z t,l^z t',l^{-1/\nu}\tau) = f_x(t/t',1,\tau t'^{1/(\nu z)}), \tag{19}$$

where $f_x(\infty,1,0)$ is finite. In $T=T_c$, it is easy to show that $\lim_{t'\to 0} X_{r=0}(t,t')=X^{\infty}_{r=0}$, which has been conformed in Ref. 23. If we insert the expansions (14) and (15) in the Green functions G^0_{11} and G^0_{20}, and take into account Eq. (11), we find immediately the FDR for $t'\to 0$, $X_{r=0}(t,t')=\tilde{f}_x(\tau t'^{1/(\nu z)})$. It is independent of t if $t>t'$. This suggests that $X^{\infty}_{r=0}$ not only appears for $t'\to 0$ but also for all $t/t'\gg 1$.

In momentum space, the following quantity related to the FDR,

$$\tilde{X}_p(t,t') = \lambda G_p(t,t')/\partial_{t'}C_p(t,t'), \tag{20}$$

is introduced,[23] where $G_p(t,t')$ and $C_p(t,t')$ are the Fourier transforms of $G(r,t,t')$ and $C(r,t,t')$, respectively. When the model is not at its critical point, i.e., $\tau\neq 0$, the limit of this ratio for $t'\to\infty$ is 1 for all values of p, according to the idea that in the high-temperature phase, all modes have a finite equilibration time, so that equilibrium is approached quickly and the fluctuation-dissipation theorem holds; i.e., $\tilde{X}_p(t,t')=1$. For the critical model, i.e., $\tau=0$, the nonequilibrium dynamics consists in the growth of the dynamical correlation length, $\xi(t)\sim t^{1/z}$. Critical fluctuation of large wave vectors, $p\xi(t)\gg 1$, are almost equilibrated, while those with small wave vectors, $p\xi(t)\ll 1$, still retain their nonequilibrium initial condition.[22] As a consequence $\tilde{X}_{p\neq 0}(t,t')=1$ in large-time limit. It is argued that its zero-momentum long-time behavior, $\tilde{X}^{\infty}_{p=0}=\lim_{t'\to\infty}\lim_{t\to\infty}\tilde{X}_{p=0}(t,t')$, is equal to the same limit of the FDR for $r=0$; i.e.,

$$\tilde{X}^{\infty}_{p=0} = X^{\infty}_{r=0} \tag{21}$$

to all orders.[23] This fact allows an easier perturbative computation in momentum space of the universal quantity $X^{\infty}_{r=0}$.

From RG arguments it is expected the functions $G_p(t,t')$ and $C_p(t,t')$ scale, for $p=0$ and $T=T_c$, as[1,23]

$$G_{p=0}(t,t') = \left(\frac{t}{t'}\right)^{\theta}(t-t')^{(2-\eta-z)/z}A_G F_G(t'/t), \tag{22}$$

$$C_{p=0}(t,t') = \lambda^{(2-\eta)/z}t'\left(\frac{t}{t'}\right)^{\theta}(t-t')^{(2-\eta-z)/z}A_C F_C(t'/t). \tag{23}$$

Here A_G and A_C are the nonuniversal amplitudes, satisfying the relation $(1-\theta)X^{\infty}_{r=0}=A_G/A_C$. The functions F_G and F_C are universal with $F_G(0)=F_C(0)=1$. Whereas the theory of local scale transformations show $F_G(x)=1$,[46] RG calculations at two-loop order give small corrections to this sample behavior.[23]

IV. INITIAL SLIP EXPONENTS

In this section, using the double (ϵ,ρ) expansion, we compute the Feynman graphs and determine the Z factors for the model A with the LCQI. As a result, we obtain the Wilson functions in the two-loop approximation in the form of the expansion series in the renormalized vertices u, u_1, u_2. At the two-loop level, all Wilson functions are given by

$$\beta_u = -\epsilon u + \frac{n+8}{6}u^2 - 6(u_1+u_2)u - \frac{3n+14}{12}u^3 + \frac{11n+58}{6}(u_1+u_2)u^2 - \frac{41}{2}(u_1+u_2)^2u + \frac{\rho}{2}u_2u, \tag{24}$$

$$\beta_{u_1} = -\epsilon u_1 + \frac{n+2}{3}uu_1 - 2(u_1+u_2)u_1 - 2(u_1+u_2)^2 - 8(u_1+u_2)^3 - \frac{5}{2}(u_1+u_2)^2u_1 - \frac{5(n+2)}{36}u^2u_1 + (n+2)u(u_1+u_2)^2 + \frac{5(n+2)}{6}u(u_1+u_2)u_1 + \frac{\rho}{2}u_2u_1, \tag{25}$$

$$\beta_{u_2} = -(\epsilon+\rho)u_2 + \frac{n+2}{3}uu_2 - 2(u_1+u_2)u_2 - \frac{5(n+2)}{36}u^2u_2 - \frac{5}{2}(u_1+u_2)^2u_2 + \frac{5(n+2)}{6}u(u_1+u_2)u_2 + \frac{\rho}{2}u_2^2, \tag{26}$$

$$\gamma_s = \frac{n+2}{72}u^2 - \frac{n+2}{12}u(u_1+u_2) + \frac{1}{4}(u_1+u_2)^2 + \frac{\rho}{4}u_2, \tag{27}$$

$$\gamma_{\tilde{s}} = 2(u_1+u_2) + \frac{n+2}{72}\left(12\ln\frac{4}{3}-1\right)u^2 - \frac{n+2}{4}u(u_1+u_2) + \frac{11}{4}(u_1+u_2)^2 - \frac{\rho}{4}u_2, \tag{28}$$

$$\gamma_0 = -\frac{n+2}{6}u - \frac{n+2}{6}\left(\ln 2 - \frac{1}{2}\right)u^2 - \frac{n+2}{6}[2+\ln 2 - \sqrt{3}\ln(2+\sqrt{3})]u(u_1+u_2), \tag{29}$$

$$\kappa = \frac{n+2}{6}u - u_1 - u_2 - \frac{5(n+2)}{72}u^2 - \frac{5}{4}(u_1+u_2)^2 + \frac{5(n+2)}{12}u(u_1+u_2) + \frac{\rho}{4}u_2 \tag{30}$$

and $\zeta = (\gamma_{\tilde{s}} - \gamma_s)/2$.

Equation (6) allows us to study the infrared asymptotic properties of the Green functions which are dominated by the scaling solution of the RG equations (24)–(26) at the stable fixed points $w^* = (u^*, u_1^*, u_2^*)$. Here we are only interested in the behavior is governed by the fixed point characteristic of the LCQI system. Since Wilson functions have the combinations of u_1+u_2 and ρu_2, to second order in ϵ and ρ it is convenient for the LCQI fixed point to take the form

$$u^* = \frac{6(3\rho+2\epsilon)}{5n+4} + \frac{3}{2(5n+4)^3}[9(16-8n-5n^2)\rho^2 + 8(32+8n+5n^2)\rho\epsilon + (80+8n+5n^2)\epsilon^2], \tag{31}$$

$$u_1^* + u_2^* = \frac{(n+8)\rho + (4-n)\epsilon}{2(5n+4)} + \frac{1}{16(5n+4)^3}[9(64+16n + 76n^2 + 15n^3)\rho^2 + 16(64+48n+72n^2+5n^3)\rho\epsilon + (192-272n-236n^2-215n^3)\epsilon^2], \tag{32}$$

$$\rho u_2^* = [(n+8)\rho + (4-n)\epsilon][4(n-1)\rho + (n-4)\epsilon]/[2(5n+4)^2]. \tag{33}$$

To second order in ϵ and ρ, initial slip exponents are given by

$$\theta' = \frac{2(n-1)\rho + 3n\epsilon}{4(5n+4)} + \frac{1}{32(5n+4)^3}[n(272+104n+65n^2)\epsilon^2 - 6(32-32n+58n^2+5n^3)\rho\epsilon - (64-336n+156n^2 + 35n^3)\rho^2] + \frac{(n+2)(3\rho+2\epsilon)}{8(5n+4)^2}\left\{[(n+8)\rho + (4-n)\epsilon] \times[\ln 2 - \sqrt{3}\ln(2+\sqrt{3})] + 12(3\rho+2\epsilon)\ln\frac{3}{2}\right\}, \tag{34}$$

$$\theta = \frac{(n+2)(3\rho+2\epsilon)}{4(5n+4)} - \frac{n+2}{16(5n+4)^3}[(16+136n+25n^2)\epsilon^2 - (16-248n+25n^2)\rho\epsilon + 216n^2\rho^2] + \frac{(n+2)(3\rho+2\epsilon)}{8(5n+4)^2}\{[(n+8)\rho+(4-n)\epsilon][\ln 2 - \sqrt{3}\ln(2 + \sqrt{3})] + 12(3\rho+2\epsilon)\ln 2\}, \tag{35}$$

which are valid for $n>1$. Their values in the first order in ϵ and ρ, has already been obtained in the Ref. 35. Up to two-loop order, for $n>1$ the values of the long-time exponents η, z and ν are consistent with the existing results.[34]

For the special case of the Ising-like ($n=1$), the inclusion of the term $u_2 \neq 0$ breaks the accidental degeneracy[26,27,31] in the fixed point equations when $u_2=0$. However, the Ising LCQI fixed point is unstable for $\rho = O(\epsilon)$. For $\rho = O(\epsilon^{1/2})$, the LCQI fixed point has the values of

$$u^* = \frac{2}{3}(2\epsilon+3\rho) + \frac{1}{18}\rho^2, \tag{36}$$

$$u_1^* = \frac{1}{6}(2\epsilon+3\rho) - \frac{17}{72}\rho^2, \tag{37}$$

$$u_2^* = -\frac{\epsilon}{6} + \frac{53}{144}\rho^2. \tag{38}$$

The corresponding exponents for $n=1$ are then given by

$$\eta = -\rho^2/48, \tag{39}$$

$$\nu \equiv 2/(4-\epsilon-\rho), \tag{40}$$

$$z = 2 + \frac{1}{6}(\epsilon + 3\rho) + \left(\frac{1}{36} + \ln\frac{4}{3}\right)\rho^2, \tag{41}$$

$$\theta' = \epsilon/12 + [1/36 - 3\ln 2 + 4\ln 3 - \sqrt{3}\ln(2+\sqrt{3})]\rho^2/8, \tag{42}$$

$$\theta = \rho/4 + \epsilon/6 - [4/9 - 5\ln 2 + \sqrt{3}\ln(2+\sqrt{3})]\rho^2/8, \tag{43}$$

to second order in $\sqrt{\epsilon}$.

V. THE LIMIT FDR

The relaxation of the system from the nonequilibrium initial state is characterized by two different regimes:[23] a transient one with nonequilibrium evolution for $t<t_\tau$, and a stationary one with equilibrium evolution of fluctuations for $t>t_\tau$. For $t<t_\tau$, the fluctuation-dissipation theorem does not hold, and a dependence of the behavior of the system on initial condition is expected.

The aim of this section is the computation of the critical nonequilibrium two-point response function $G_p(t,t')$ and correlation function $C_p(t,t')$. Since we are only interested in the limit of FDR, we set $\tau=0$ and the external momentum $p=0$ in the following. In order to cancel the dimensional poles in ϵ, ρ expansion, we has to use Eq. (9) to render the Green function finite. In the following, we set $\lambda=1$ to lighten the notation. The dependence on λ of the final formulas may be simply obtained by $t\to\lambda t$, where t is the generic time variable. Using one-loop renormalization, the calculation shows that the response function $G_{p=0}(t,t')$ and the correlation function $C_{p=0}(t,t')$ are given by, respectively,

$$G_{p=0}(t,t') = 1 + \frac{n+2}{24}u^*\ln\frac{t}{t'} - \frac{1}{2}(u_1^*+u_2^*)[\ln(t-t') + \gamma_E], \tag{44}$$

$$C_{p=0}(t,t') = 2t' + \frac{n+2}{12}u^*t'\left(\ln\frac{t}{t'}+2\right) + (u_1^*+u_2^*)t'\left[1-\gamma_E-\ln(t-t') + \frac{t+t'}{2t'}\ln\frac{t-t'}{t+t'}\right], \tag{45}$$

which are fully compatible with the expected scaling forms (22) and (23) with one-loop exponents $(2-\eta-z)/z=-(u_1^*+u_2^*)/2$ and $\theta=[(n+2)/24]u^*$,[35] and the new results

$$A_G = 1 - \frac{1}{2}(u_1^*+u_2^*)\gamma_E, \tag{46}$$

$$A_C = 2 + \frac{n+2}{6}u^* - (u_1^*+u_2^*)\gamma_E, \tag{47}$$

$$F_C(x) = 1 + \frac{1}{2}(u_1^*+u_2^*)\left(1+\frac{1+x}{2x}\ln\frac{1-x}{1+x}\right), \tag{48}$$

and $F_G(x)=1$. Here $\gamma_E=0.577\ldots$ is Euler's constant. Using Eq. (20), the fluctuation-dissipation ratio for the long-time limit and for finite times, are, respectively,

$$\tilde{X}^\infty_{p=0} = \frac{1}{2} - \frac{n+2}{48}u^* \tag{49}$$

$$\tilde{X}_{p=0}(t,t') = \frac{1}{2} - \frac{n+2}{48}u^* - \frac{1}{8}(u_1^*+u_2^*)\ln\frac{t-t'}{t+t'}. \tag{50}$$

Here u^*, u_1^*, u_2^* all take one-loop values. Equation (49) is also obtained from the relation $(1-\theta)X^\infty_{r=0}=A_G/A_C$. For one-loop LCQI fixed point in Eqs. (31)–(33) for $n>1$, or Eqs. (36)–(38) for $n=1$, the limit FDR is given by

$$\tilde{X}^\infty_{p=0} = \frac{1}{2} - \frac{n+2}{8(5n+4)}(2\epsilon+3\rho), \quad \text{if} \quad n\neq 1,$$

$$=1/2-\rho/8, \quad \text{if} \quad n=1. \tag{51}$$

In experiments or simulations, instead of measuring $G_{p=0}(t,t')$, one considers the integrated responses, i.e., the zero-field-cooled susceptibility $\chi_{zfc}(t,t')=\lambda\int_{t'}^{t}dt''G_{p=0}(t,t'')$ and the thermoremanent susceptibility $\chi_{trm}(t,t')=\lambda\int_0^{t'}dt''G_{p=0}(t,t'')$. At one-loop order, they are given by, respectively,

$$\chi_{zfc}(t,t') = t - t' + \frac{n+2}{24}u^*\left(t-t'-t'\ln\frac{t}{t'}\right) - \frac{1}{2}(u_1^*+u_2^*)(t-t')[\ln(t-t')+\gamma_E-1], \tag{52}$$

$$\chi_{trm}(t,t') = t' + \frac{n+2}{24}u^*t'\left(1+\ln\frac{t}{t'}\right) - \frac{1}{2}(u_1^*+u_2^*)[(\gamma_E-1)t' - (t-t')\ln(t-t') + t\ln t]. \tag{53}$$

For $t/t'\gg 1$ or $t'\to 0$, $\chi_{trm}(t,t')\approx X^\infty_{r=0}C_{p=0}(t,t')$, and $\chi_{zfc}(t,t')\approx X^\infty_{r=0}[C_{p=0}(t,t)-C_{p=0}(t,t')]$, which agree with the Refs. 18 and 22.

VI. THE CROSSOVER

As mentioned in Sec. IV, the zeros of the functions $\beta_w=0$ for $w=u,u_1,u_2$ give the fixed points u^*, u_1^*, u_2^*. Eigenvalues of the fixed points are obtained by diagonalizing the matrix of the derivatives of β_u, β_{u_1}, β_{u_2} with respect to u, u_1, u_2 about each of the fixed points. The critical behavior of the system is stable for positive eigenvalues. In this section we pay attention to the stability properties of three nontrivial physical fixed points (i.e., the pure, SCQI, and LCQI fixed points), and their related crossovers.

When $\rho=\rho_p$ (or $\rho=\rho_s$) with

$$\rho_p = \frac{n-4}{n+8}\epsilon + \frac{(n+2)(13n+44)}{(n+8)^3}\epsilon^2, \tag{54}$$

$$\rho_s = \frac{4-n}{4(n-1)}\epsilon + \frac{n(55n^2-500n-32)}{512(n-1)^3}\epsilon^2, \quad (55)$$

for $n \neq 1$, the LCQI scaling behavior associated with the critical exponents and the limit FDR, crosses over to the pure[1,23] [or SCQI (see Ref. 28)] behavior. For $\rho < \rho_p$ (or $\rho < \rho_s$), the scaling regime governed by the pure fixed point (or the SCQI fixed point) is stable. If $\rho > \max(\rho_s, \rho_p)$ and $\rho > 0$, the LCQI is relevant and the behavior controlled by the LCQI fixed point is expected. These behaviors are consistent with the modified Harris criterion.[31]

For the special case of the Ising-like ($n=1$), the inclusion of the term $u_2 \neq 0$ breaks the accidental degeneracy[26,27,31] in the fixed point equations when $u_2=0$. However, the Ising LCQI fixed point is unstable for $\rho = O(\epsilon)$; only when $\rho > 2(6\epsilon/53)^{1/2} - 36\epsilon/53$ it is stable. At $\rho = 2(6\epsilon/53)^{1/2} - 36\epsilon/53$, the Ising LCQI and SCQI fixed points are coincident, and the critical exponents as well as the limit FDR change continuously to their SCQI values.[24,27,28]

The calculation of the fixed-point stability also shows that in $d<4$, the pure (or SCQI) fixed point is stable when n is greater (or less) than a critical value n_1 (or n_2), given by

$$n_1 = n'\left(1 + \frac{3n'\epsilon}{16} - \frac{(n'+2)(n'^2+23n'+60)}{144n'}(\epsilon - \rho)\right), \quad (56)$$

$$n_2 = n''\left(1 - \frac{3n''\epsilon}{16} + \frac{127n''^2 - 572n'' - 32}{1152}(\epsilon + 4\rho)\right), \quad (57)$$

with $n' = 4(\epsilon+2\rho)/(\epsilon-\rho)$ and $n'' = 4(\epsilon+\rho)/(\epsilon+4\rho)$. The LCQI fixed point is stable only in the region $n_2 < n < n_1$, while for $n > n_1$ or $n < n_2$ the LCQI is irrelevant. The crossover to the LCQI fixed point from the pure fixed point is expected at $n=n_1$, and a further crossover to the SCQI fixed point from the LCQI fixed point is expected at $n=n_2$. When $\rho=0$ (i.e., the LCQI is irrelevant), one finds crossover between the SCQI and pure fixed points at $n=n_1=n_2=4(1-\epsilon)$, which agrees with the result of Ref. 26.

In dimensions $d=3$, the SCQI is relevant only for the Ising model since the specific-heat exponent α_p of the pure systems is positive for $n=1$, and is negative for $n \geq 2$.[30] Unfortunately, due to the slow crossover in dilute Ising sytems,[28] it is probably difficult to measure the symptotic exponents θ' and θ in simulations or real experiments. However, if a system with the LCQI could be made, and if the correlations of the impurities are sufficiently long ranged, the critical behavior governed by the LCQI fixed point can be well observed. As pointed out in Ref. 32, the correspondence between the nature of defects and the value of ρ could be established by the polishing procedure. Using the surface treatment, the behavior described by the LCQI fixed point may be displayed in some crystalline mixtures of the Ising-like uniaxial antiferromagnet (e.g., FeF_2, MnF_2) with a nonmagnetic material (e.g., ZnF_2). The occurrence of the LCQI in these mixtures is due to the elastic interaction of defects.[36]

TABLE I. The values of η, γ, ν, z, θ', and $\widetilde{X}^{\infty}_{p=0}$ for $d=3$, $\rho=1$, and $n=1,2,3,4$.

	η	γ	ν	z	θ'	$\widetilde{X}^{\infty}_{p=0}$
$n=1$	−0.021	2.021	1.0	2.982	0.091	0.375
$n=2$	0.010	1.990	1.0	2.737	0.207	0.321
$n=3$	0.019	1.981	1.0	2.494	0.210	0.336
$n=4$	0.021	1.971	1.0	2.371	0.214	0.344

For $n \geq 2$, the SCQI is irrelevant to the critical behavior, but the crossover between the LCQI and pure critical scaling behaviors may be observed. It has been argued that the LCQI is an origin[32] to this kind of crossover in Ho and Tb. Like the two length scales revealed in static properties of Ho and Tb, dynamical scalings should be observed in the dynamical critical scattering experimemts.

Some values of the exponents obtained in this work for $d=3$ and $\rho=1$, are listed in Table I. The values of η, γ, ν, z for $n>1$ agree with Ref. 34, but disagree with Ref. 36. For $n=4$ and $\epsilon=\rho=1$, the exponent $\nu=1.0$ supports the experimental values for the longer length scale, which range around 1.0 in Ho and Tb.[32,39,40] $\gamma=1.971$ is close to the lower limit of its measured value, which ranges between 2 and 5 in Ho.[39] For $n=\epsilon=\rho=1$, θ' is smaller than its corresponding pure value 0.131 (see Ref. 1) and Monte Carlo value 0.108(2),[47] but is larger than its SCQI value 0.087.[28] In the Ising case, the exponent $\widetilde{X}^{\infty}_{p=0}=0.375$ is different from the pure one (0.458) and the SCQI one (0.416).[23,24] Those values show that the critical properties are changed by the LCQI.

VII. DISCUSSION AND CONCLUSIONS

In the following, we focus on the pole the LCQI plays in the short-time scaling behavior. Let us first notice that both the response and the correlation functions measure the fluctuations of the order parameter fields. For the short time after quench, the LCQI scaling behaviors are governed by the initial slip exponents θ and θ'. Since the initial slip exponents θ and θ' are positive, one expects, according to Eqs. (13), (16), and (17), an initial increase of the fluctuations. The stronger the fluctuations, the bigger the values of θ and θ'. Of course, the increase depends upon ρ, d, and n. Since fluctuations are reduced as the dimension becomes larger or the decay rate of the LCQI correlations ρ becomes smaller, θ and θ' decrease when d increases or ρ decreases. For fixed ρ and d, more internal degrees of freedom (larger n) help the fluctuations increase, which leads to the increase of θ and θ' with increasing n.

It is interesting to compare the initial critical increase affected by the LCQI with that by the SCQI, long-range interaction (LRI), and cubic anisotropy. The LRI decaying as $r^{-d-\sigma}$ corresponds to the added term $(a/2)(\nabla^{\sigma/2}s)^2$ in Eq. (3). The stable regions of the LCQI, SCQI, long-range interaction (LRI), and cubic anisotropy are $n_2 < n < n_1$, $n<4$, $\sigma < 2-\eta$, and $n > n_c$, respectively. Here $2 < n_c < 4$. The effect of the pure LRI on initial critical increase[6] is similar to

TABLE II. The values of θ' for $d=3$ and $n=1$ together θ'_{sr} in Refs. 8 and 26 and θ'_p in Refs. 1 and 6.

$\rho=2\sigma-d$	0.1	0.3	0.6	0.9	1
θ'	0.0834	0.0840	0.0861	0.0896	0.0911
θ'_{sr}	0.0117	0.0330	0.0611	0.0863	0.0868
θ'_p	0.0202	0.0548	0.0954	0.1261	0.1306

that of the LCQI for smaller σ. However, for large σ, when n exceeds some threshold value, the effect of the mean fields is much stronger than in the case with LCQI, and then leads to the decreasing of the fluctuations of the initial increase. For given d and n, θ' increases slowly as ρ increases. While in the LRI systems without[6] or with[8] SCQI or with cubic anisotropy,[7,49] it increases more quickly to its corresponding short-range values[1,7,28] if keeping $0<\rho=2\sigma-d<1$. For instance, in $d=3$, the values of θ' corresponding to $\rho=0.1,0.3,0.6,0.9,1$ for $n=1,5$ are listed in Table II and Table III, respectively, with their corresponding nonrandom values θ'_p,[1,6] SCQI values θ'_{sr},[8,28] and cubic values θ'_{cub}.[7,49] For fixed d and σ, the exponents θ' and θ'_{cub} are all smaller than θ'_p, as in Table II and III. That is because the SCQI and cubic anisotropy impede the formation and development of the order parameter, and then decrease the initial critical increase. However, in the presence of the LCQI, the situation seems delicate. If $\rho=2\sigma-3$, $\theta'>\theta'_p$ for large n. When $n=1$, $\theta'>\theta'_p$ for small ρ, but for large ρ one can reach the opposite effect. This could be explained by assuming that the LCQI suppresses the effect of the mean fields due to large n. As to $n=1$, in Eq. (6) the SCQI g_1 is generated by the LCQI g_2 even if $g_1=0$. In the case with $g_2\neq 0$, the fact that g_1 has a tendency to impede the growing of the order parameter leads to slowly increasing of θ'.

Our static and dynamical results, in principle, may be tested by some experiments in the magnetic systems such as Ho and Tb, or random Ising-like uniaxial antiferromagnets, in particular, in these systems with straight dislocation lines of random orientation in a sample. Examination of the initial exponent θ' may be carried out by short-time Monte Carlo simulations.[5] The numerical evaluation of the limit FDR $X^\infty_{r=0}$ may be obtained as the slope of the integrated reponse function when plotted versus the correlation $C(r=0,t,t')$ or $C_{p=0}(t,t')$ by the large-scale Monte Carlo simulations.[18,22] In addition the simulation of the Ising-Glauber model may give a direct measurement of the FDR $X_{r=0}(t,t')$ for any time t and $t'<t$.[21] To one-loop order, since the LCQI fixed point is stable when $\rho>2(6\epsilon/53)^{1/2}$ for the Ising system, the value of $X^\infty_{r=0}$ is smaller than its pure[23] and SCQI ones.[24] The universal scaling function $F_C(x)$ in Eq. (48) has received a greater contribution than its pure and SCQI ones[23,24] for any given x, which should be observable in the simulations.

TABLE III. The values of θ' for $d=3$ and $n=5$ together θ'_{cub} in Ref. 7 and θ'_p in Refs. 1 and 6.

$\rho=2\sigma-d$	0.1	0.3	0.6	0.9	1
θ'	0.1758	0.1889	0.2045	0.2152	0.2178
θ'_{cub}	0.0183	0.0573	0.1176	0.1763	0.1895
θ'_p	0.0186	0.0589	0.1220	0.1838	0.1980

Unlike the case below the critical temperature T_c, at criticality, for the pure ferromagnet systems, the FDR $X_{r=0}(t,t')$ is not a function of the correlation function $C(r=0,t,t')$.[18] It is instead a function of the ratio t/t'. In the presence of impurities, our results confirm this; i.e., $\tilde{X}_{p=0}(t,t')$ is only dependent of the ratio t/t'. However, since the correlation and response functions are expected to scale as[1,23,48] $C_{p=0}(t,t')=t'^{(2-\eta)/z}\tilde{f}_c(t/t')$ and $G_{p=0}(t,t')=t'^{(2-\eta-z)/z}\tilde{f}_g(t/t')$ [where the scaling functions $\tilde{f}_c(x)$ and $\tilde{f}_g(x)$ both vary as $\sim x^{\theta'}$ for $x\gg 1$], respectively, and if the normalized two-point correlation

$$\tilde{C}(t,t') = C_{p=0}(t,t')/C_{p=0}(t,t) = (t'/t)^{(2-\eta)/z}\tilde{f}_c(t/t')/\tilde{f}_c(1)$$

is introduced, $\tilde{X}_{p=0}(t,t')$ can be expressed as a function of $\tilde{C}$; i.e., $\tilde{X}_{p=0}(t,t')=\tilde{X}_{p=0}[\tilde{C}_{p=0}(t,t')]$. Thus, $X_{r=0}(t,t')=X_{r=0}[\tilde{C}(r=0,t,t')]$ if $\tilde{C}(r=0,t,t')=C(r=0,t,t')/C(r=0,t,t')$.

Is the amplitude ratio $X_{r=0}(t,t')$ or $\tilde{X}_{p=0}(t,t')$ related to equilibrium quantities? This remains an interesting open question.[23,50] In Ref. 37, Franz *et al.* have shown the FDR $X_{r=0}(t,t')$ is linked to the Parisi function $P(q)$ through the relation $X_{r=0}(q)=\int_0^q dq' P(q')$ if $X_{r=0}(t,t')=X_{r=0}[C(r=0,t,t')]$, where $P(q)$ is the overlap probability distribution in equilibrium state. This relation holds for some domain-growth models and continuous spin-glass models (note that it does not hold for models with a threshold as the p spin) at low temperature.[50] For the pure ferromagnetic model [which is described by the Hamiltonian (3) without impurities], the fact that the system has two degenerate minima of the potential is responsible for the nontrivial $P(q)$ at the low-temperature phase.[17] Whereas for the critical dynamics, since the double well potential does not exist, it is not easy to find some $P(q)$ related to $X_{r=0}$. Only for two-dimensional XY model without impurities, a nontrivial Parisi function is found, but it has to be generalized to finite-time and finite-size dependencies.[20]

However, the situation is different if impurities are introduced in the systems. As it was shown in Refs. 51 and 52, at the critical point the system with SCQI has a macroscopic number of the local minima solutions of the saddle-point equation corresponding to the Hamiltonian (3) for $\tau+\phi(x)<0$. In each of these solutions, one finds two local minimum configurations of the field $s(x)$, which is different in the pure case, wherein the solution of the saddle-point equation is unique. The fluctuations of the SCQI $\phi(x)<0$ can lead to realization in a system of numerous regions with $s(x)\neq 0$ displaying through the numerous local minimal energy configurations separated from the ground state by finite barriers. In order to get the Parisi function, one should integrate in an RG way over fluctuations around the local minima configurations. For the case with LCQI, the situation is similar to the case with SCQI, but may be more complicated. As a result, for $T=T_c$, it is expected that $X_{r=0}(t,t')$ or $\tilde{X}_{p=0}(t,t')$ will be corresponding to the combination of many nontrivial Parisi functions that may be dependent of local minima configurations.

The model discussed here also helps to investigate the scaling properties of polymers. In the limit $n \to 0$, the equilibrium behavior of the model discussed here is mapped to the statistical properties of polymers in porous media with LCQI,[53] which described a model of self-avoiding walks in a randomly diluted lattice. From Eqs. (54) and (55), in the polymer limit $n \to 0$, it is found that the SCQI is irrelevant for self-avoiding walks or polymers as long as $\rho \leqslant -\epsilon$. If $\rho > -\epsilon/2 + 11\epsilon^2/64$ and $\rho > 0$, the LCQI is relevant for polymers. For the case of $\rho = 0$, which corresponds to the SCQI, the model reproduces the well-known values of the critical exponent for the pure self-avoiding walk model.[45]

Further, our results also suggest many interesting lines for the future investigation. An important question is what are the short-time scalings and the limit FDR in disordered electron systems with LCQI (see Ref. 54) and whether the spin systems with anisotropic LCQI are totally correlated in ϵ_d dimensions and randomly distributed in the remaining d-ϵ_d space directions.[55]

In summary, short-time critical dynamics of spin systems with LCQI is studied in double (ϵ, ρ) expansion. The exponents θ' and θ governing the scaling behavior characteristic of the LCQI in the short-time regime are computed to second order in ϵ and ρ. Up to the first order in ϵ, ρ, the limit FDR $X_{r=0}^{\infty}$, which is the universal characteristic of nonequilibrium dynamics, is computed at criticality. We also obtain the (ϵ, ρ) expansion for the response and correlation functions, and the integrated responses for vanishing external momentum. Our results show that the LCQI exerts a strong influence on the short-time dynamics for $\rho > \max(\rho_p, \rho_s)$ (or $n_2 < n < n_1$). In $d < 4$, the pure (or SCQI) fixed point is stable only when n is greater (or less) than a critical value n_1 (or n_2), or is stable for $\rho < \rho_p$ (or $\rho < \rho_s$). Our results are compared with those obtained previously for the models without the LCQI, and it is found that although these models have some similar properties in initial critical increase, the LCQI leads to new properties and modifies further the short-time behavior.

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