Spin transport and resistance due to a Bloch wall

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We study spin transport in a ferromagnet containing Bloch wall. Starting from the kinetic equations in Wigner space, we derive matrix diffusion equation for the accumulation of transverse magnetization driven by the spin polarized electrical current. The resistance produced by this accumulation exhibits damped oscillations as a function of wall thickness with a period $\pi|k_F^{\uparrow}-k_F^{\downarrow}|^{-1}$, where $k_F^{\uparrow}$ and $k_F^{\downarrow}$ are the spin up and spin down Fermi wave numbers. Geometrically constrained walls are suggested for observing the resistance oscillations.

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Recent technological advances in material fabrication led to resurgence of interest in spin-polarized transport in a ferromagnet containing domain walls (DW). In view of potential applications to devices where information is written electrically, there has been strong activity surrounding the problem of a current driven motion of the DW.[1–4]

Moreover, the excess resistance due to DW has been the subject of a number of studies. Experiments have shown either an increase[5–7] or decrease[8] of the resistance due to DW. First, we briefly mention previous theoretical works that are most relevant to our aim. Levy and Zhang[9] obtain a positive DW resistance from a Boltzmann equation by taking into account the mixing of spin states due to the magnetization twist in the DW. Tatara and Fukuyama[10] consider the destruction of the electron weak localization by the dephasing caused by the DW. In principle, this mechanism yields a negative DW resistance. However, since the excess resistance remains experimentally negative up to relatively high temperatures[8] (where localization does not play a role) another mechanism must be at work. Van Gorkom *et al.*[11] consider the modification of the electron band structure of a two-band ferromagnet due to magnetization twist in the DW. They predict an excess resistance that can be either positive or negative dependent upon the sign of the difference between the up and down spin relaxation times.

The suggestion made by Ebels *et al.*,[6] that spin accumulation (SA) around the DW should be the dominant mechanism, motivated the theory of Ref. 12. This theory shows that the DW resistance is determined by transverse SA of the conduction electrons. Equipped with an approach which correctly incorporates the precessional dynamics into the kinetic equations, we revisit in this paper the problem of DW resistance.

The precessional dynamics plays also an essential role in the problem of current-driven DW.[1–4] Thus, we need to take a deeper look into the general problem of transverse spin currents in a ferromagnet. In 1966, Hirst[13] and Kaplan[14] studied the effects of spin precession on diffusive transport of the transverse magnetization density of itinerant electrons. Treating the circularly polarized electron gas within the single band itinerant model, Hirst[13] derives a Fick's law with a complex diffusion coefficient. When combined with the equation of motion for the ferromagnetic magnetization, the real part of this coefficient is found to act as an ordinary diffusion constant while the imaginary part yields a contribution to the exchange stiffness. This implies that the circularly polarized magnetization decays exponentially within a distance that is very short owing to the fast precession about the exchange field.

Subsequently, Berger[15] investigated the injection of a spin polarized electron beam into a ferromagnet whose magnetization is not collinear with the beam polarization. Then the transverse polarization of the electron is found to oscillate in the ferromagnet with a wavelength $\pi|k_F^{\uparrow}-k_F^{\downarrow}|^{-1}$. This is because an electron with transverse spin polarization is in a quantum state that is a superposion of spin eigenstates associated with Fermi wave vectors $k_F^{\uparrow}$ and $k_F^{\downarrow}$. The same wavelength appears in the oscillation of transverse spin current calculated by Stiles and Zangwill[16] near the interface of a ferromagnet and nonmagnet.

Recently, Hitchon *et al.*[17] and Rebei *et al.*[18] addressed the problem of spin diffusion in nonhomogeneous ferromagnet using an effective action functional. Using the relaxation time approximation, they derive a Fick's law in which the diffusion constant becomes a matrix. This result is consistent with the complex Fick's law of Hirst.[13] However, in contrast to Ref. 13, the transverse magnetization of the conduction electrons is found to exhibit, as a function of position, damped oscillations with a period $\pi|k_F^{\uparrow}-k_F^{\downarrow}|^{-1}$.

Motivated by Refs. 17 and 18, we revisit in this work the previously studied problem of SA around the DW. In Ref. 12, the transverse SA is found to satisfy an ordinary diffusion equation with a diffusion length drastically reduced compared with the longitudinal diffusion length as a result of the fast precession about the exchange field. However, a simple argument shows that the dependence of the transverse magnetization on distance should contain a strong oscillatory component: As the spin polarized conduction current passes through the wall, its polarization is noncollinear with the local magnetization in the wall. Thus, we have a situation similar to that considered in Refs. 15 and 16 and oscillations with period $\pi|k_F^{\uparrow}-k_F^{\downarrow}|^{-1}$ are expected.

In what follows, we verify this conclusion using kinetic equations for the Wigner function in a way that allows to include the physics of precession missing in Ref. 12. The transverse component of the magnetization is found to satisfy a matrix diffusion equation yielding a damped oscillation of the SA with a period $\pi|k_F^{\uparrow}-k_F^{\downarrow}|^{-1}$. We also derive an expression for the resistance due to the wall which exhibits damped oscillations as a function of the wall thickness. This is not surprising since, in the rotated frame of reference, the electron sees a potential well of the same thickness produced by the magnetization twist.

We adopt the *sd* model of a ferromagnet. The conduction electrons interact with the local *d*-electron magnetization via the term $J_{sd}\vec{\sigma}\cdot\vec{M}(\vec{r})$, where $\vec{M}(\vec{r})$ is a unit vector in the direction of the magnetization, J_{sd} is the *s-d* exchange integral, and $\vec{\sigma}$ is the Pauli matrix. In addition, there is an interaction with an electric field $\vec{E}_o$, and scattering by impurities, phonons and magnons.

We consider transport in the presence of a pinned 180° wall. In the laboratory coordinate system (X,Y,Z), $\vec{M}(-\infty,Y,Z)$ is parallel and $\vec{M}(\infty,Y,Z)$ is antiparallel to the axis OZ. The X-dependent angle between $\vec{M}$ and OZ-axis is $\alpha(X)$. The term $J_{sd}\vec{\sigma}\cdot\vec{M}(X)$ is diagonalized by going to a new coordinate system $(x=X,y,z)$, where the Oz axis is parallel to $\vec{M}(x)$ and the OX axis is not changed. In the rotated system, the transport is conveniently formulated using the Wigner function $\hat{F}(x,\vec{p})=1/2[f_1(x,\vec{p})\hat{I}+\vec{f}(x,\vec{p})\cdot\hat{\vec{\sigma}}]$. For the steady state and $\vec{E}_0\|Ox$, its components satisfy the following system of equations:[12]

$$v_x\frac{\partial f_1}{\partial x}-\frac{\hbar}{2m}\alpha'\frac{\partial f_x}{\partial x}+ev_xE_o\frac{\partial f_1}{\partial \epsilon}=\left(\frac{\partial f_1}{\partial t}\right)_{coll}, \quad (1)$$

$$v_x\frac{\partial f_x}{\partial x}-\frac{\hbar}{2m}\alpha'\frac{\partial f_1}{\partial x}+ev_xE_0\frac{\partial f_x}{\partial \epsilon}+\omega_e f_y=\left(\frac{\partial f_x}{\partial t}\right)_{coll}, \quad (2)$$

$$v_x\frac{\partial f_y}{\partial x}-v_x\alpha' f_z+ev_xE_0\frac{\partial f_y}{\partial \epsilon}-\omega_e f_x=\left(\frac{\partial f_y}{\partial t}\right)_{coll}, \quad (3)$$

$$v_x\frac{\partial f_z}{\partial x}-v_x\alpha' f_y+ev_xE_o\frac{\partial f_z}{\partial \epsilon}=\left(\frac{\partial f_z}{\partial t}\right)_{coll}, \quad (4)$$

where $\alpha'=d\alpha(x)/dx$ and $\omega_e=2J_{sd}/\hbar$. Equations (1)–(4) represent a generalization of the multilayer theory of Valet and Fert[19] to a ferromagnet containing the Bloch wall. They were derived under the assumption of a slowly varying angle $\alpha(x)$ such that $k_F d\gg 1$, where d is the thickness of the wall.

The electrical resistance due to spin accumulation is determined by the magnetization component $m_y(x)$ given by momentum average of the component f_y.[12] From Eqs. (1) and (2) we see that f_y couples to f_x which in turn is coupled to the "charge" component f_1. In distinction from Ref. 12, we decouple Eq. (1) from Eq. (2). This is possible since the coupling terms in Eqs. (1) and (2) are second order in α' and can be neglected as long as we calculate the quantity $m_y(x)$ to first order in $\alpha'(x)$.

Let us focus our attention on Eqs. (2) and (3). In Eq. (3) we retain the coupling to f_z since it provides the driving torque for the component m_y. This torque comes from the longitudinal magnetization current density j_z related to the density of spin polarized electric current $j_z^{(e)}$ as follows:

$$j_z=\frac{\mu_B}{e}j_z^{(e)}=\frac{\mu_B E_o\beta}{e(1-\beta^2)\rho_F^*}, \quad (5)$$

where β is the spin asymmetry coefficient, and ρ_F^* is the bulk resistivity of the ferromagnet.[19] The right-hand side follows by writing $j_z^{(e)}=j_{\uparrow}^{(e)}-j_{\downarrow}^{(e)}=eE_0(\sigma_+-\sigma_-)$ where $\sigma_s=\sigma_0/2(1-s\beta)^{-1}$ with $\sigma_0^{-1}=\rho_F^*$. Since we calculate m_y to first order in α', and the coupling term in Eq. (3) is $v_x\alpha' f_z$, the quantity f_z should be obtained from Eq. (4) with $\alpha'=0$ yielding $m_z(x)=m_z^{(0)}$. This implies, in conjunction with the continuity equation, an x-independent j_z consistent with Eq. (5). Note that in rotated coordinates, the exchange field along the z axis is also independent of x.

In view of Eq. (5), the second term on the left-hand side of Eq. (3) is of order $\alpha' E_0$. Since we need m_y to order E_0, we can set $E_0=0$ in the Eqs. (2) and (3). Performing the momentum averaging of these equations, we obtain the continuity equations for the transverse magnetization

$$\frac{\partial j_x}{\partial x}=-\omega_e m_y-\frac{2}{\tau}m_x, \quad (6)$$

$$\frac{\partial j_y}{\partial x}=\omega_e m_x-\frac{2}{\tau}m_y+\alpha' j_z, \quad (7)$$

where j_x and j_y are the densities of the magnetization current flowing along the x axis and polarized in the x and y direction, respectively, and τ is the transverse spin relaxation time. The relaxation terms in these equations originate by writing, for $i=(x,y)$, $(\partial f_i/\partial t)_{coll}=-(2/\tau)f_i^{(1)}(x)-(1/2\tilde{T})f_i^{(1)}\times(x,\vec{p})$, where $\tilde{T}$ is the momentum relaxation time, and noting that the momentum average of $f_i^{(1)}(x,\vec{p})$ vanishes.[12]

Next we multiply by v_x Eqs. (2) and (3), taken in the same approximate form as used to derive Eqs. (6) and (7). Subsequent momentum average yields the following system of linear equations in the variables $j_x(x)$ and $j_y(x)$:

$$\frac{1}{2\tilde{T}}j_x+\omega_e j_y=-\frac{1}{3}v_F^2\frac{\partial m_x}{\partial x} \quad (8)$$

$$-\omega_e j_x+\frac{1}{2\tilde{T}}j_y=-\frac{1}{3}v_F^2\frac{\partial m_y}{\partial x}+\frac{1}{3}v_F^2\alpha' m_z. \quad (9)$$

Using Cramer's rule, we solve this system for j_x and j_y and obtain Fick's law in a matrix form

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix}=-\tilde{D}\begin{pmatrix} 1 & -2\tilde{T}\omega_e \\ 2\tilde{T}\omega_e & 1 \end{pmatrix}\begin{pmatrix} \dfrac{\partial m_x}{\partial x} \\ \dfrac{\partial m_y}{\partial x} \end{pmatrix}-\tilde{D}\alpha' m_z\begin{pmatrix} 2\tilde{T}\omega_e \\ -1 \end{pmatrix}, \quad (10)$$

where $\tilde{D}=2D(1+4\omega_e^2\tilde{T}^2)^{-1}$ and $D=1/3v_F^2\tilde{T}$.

The diffusion matrix seen in the first term of Eq. (10) first appeared in the work by Hirst.[13] Using an adaptation of the kinetic formulation of transport by Chambers,[20] Hirst derives an equation for the divergence of complex spin current density j_x+ij_y. Inserting this divergence into the Landau-Lifshitz-Gilbert (LLG) equation,[21] he obtains a diffusion term with a diffusion constant given (in the present notation) by $\tilde{D}$, and an additional exchange stiffness given by $2\omega_e\tilde{T}\tilde{D}$. This gives an impression that the transverse components of the magnetization satisfy an ordinary diffusion with exponentially decaying spatial solutions. This is clearly in disagreement with the conclusion of Refs. 15–18. The problem is that on going from the LLG equation to his Eq. (3), Hirst[13] does not include the torque $-\gamma(\vec{m}\times\vec{H}_{exch})$. In our approach this corresponds to neglecting in Eqs. (6) and (7) the terms $-\omega_e m_y$ and $\omega_e m_x$, respectively. It turns out that these terms are essential to yield correct differential equations for the transverse magnetization.

We now show that, in the present model, the transverse magnetization exhibits damped fast oscillations. Differentiating Eq. (10) with respect to x, and eliminating $\partial j_x/\partial x$ and $\partial j_y/\partial x$ via Eqs. (6) and (7), we obtain

$$\tilde{D}\frac{\partial^2}{\partial x^2}(m_x - 2\tilde{T}\omega_e m_y) - \frac{2}{\tau}m_x - \omega_e m_y = 0, \qquad (11)$$

$$\tilde{D}\frac{\partial^2}{\partial x^2}(m_y + 2\tilde{T}\omega_e m_x) - \frac{2}{\tau}m_y + \omega_e m_x = -j_z\alpha'(x). \qquad (12)$$

The driving terms on the right-hand side of these equations are obtained, to first order in α', by using $\partial m_z/\partial x=0$ as implied by the absence of the longitudinal spin accumulation for $\alpha'=0$. The key step, that distinguishes the present approach from Ref. 13 is that the exchange torque is included not only in the Fick's law but also in the continuity equations [see Eqs. (6) and (7)].

Fourier transforming Eqs. (11) and (12), we obtain coupled algebraic equations for $m_x(q)$ and $m_y(q)$ yielding

$$\int_{-\infty}^{\infty} m_y(x)e^{iqx}dx = m_y(q) = \frac{j_z(q^2 - a + 2b\omega_e\tilde{T})\alpha'(q)}{2D(q-q_1)(q-q_2)(q-q_3)(q-q_4)}, \qquad (13)$$

where $a=3\omega_e^2/v_F^2-1/D\tau$ and $b=(\omega_e/2D)(1+4\tilde{T}/\tau)$. The oscillatory nature of $m_y(x)$ is revealed by examining the poles of expression (13). They are $q_1=(a^2+b^2)^{1/4}\exp(i\phi/2)$, $q_2=-q_1$, $q_3=q_1^*$, and $q_4=q_2^*$, where $\phi=-\tan^{-1}(b/a)$. For $\omega_e\tilde{T}\gg 1$ and $\tau\gg\tilde{T}$, we have $q_1\approx\chi-i\gamma$ with $\chi=\sqrt{3}\omega_e/v_F$ and $\gamma=\sqrt{3}/4l$ where $l=v_F\tilde{T}$ is the mean free path. Since $\chi/\gamma=4\omega_e\tilde{T}\gg 1$, the poles of Eq. (13) are dominated by the real part, implying slightly damped fast oscillations of $m_y(x)$.

Evaluation of the inverse transform of Eq. (13) is facilitated by adopting a simple "square well" model for $\alpha'(x)$. Thus, we let $\alpha'(x)=\pi/d$ for $-1/2d\leqslant x\leqslant 1/2d$ and zero otherwise. Then the solution for $m_y(x)$ is given by

$$m_y(x) \approx \frac{\sqrt{3}\pi j_z}{2dv_F\chi}[I(x)+I(-x)], \qquad (14)$$

where

$$I(x) = \exp\left[-\gamma\left(x+\frac{d}{2}\right)\right]\sin\chi\left(x+\frac{d}{2}\right). \qquad (15)$$

Thus $m_y(x)$ exhibits damped oscillations with wavelength of order $\pi v_F/\omega_e$. Interestingly, this quantity can be written as $\pi|k_F^{\uparrow}-k_F^{\downarrow}|^{-1}$ corresponding to the wavelength of the transverse spin oscillations associated with the injection of a polarized electron beam into the ferromagnet.[15,16]

Now we consider the excess resistance per unit area due to Bloch wall

$$r_\omega = \frac{\Delta V_I}{j^{(e)}} = \frac{\Delta V_I\rho_F^*(1-\beta^2)}{E_o}, \qquad (16)$$

where $j^{(e)}$ is the net electrical current and ΔV_I is the voltage drop due to spin accumulation[12]

$$\Delta V_I = \frac{2\epsilon_F\beta}{3n\mu_B e}\int_{-\infty}^{\infty} m_y(x)\alpha'(x)dx. \qquad (17)$$

The integral on the right-hand side of this equation is evaluated with use of Eqs. (14) and (15) where Eq. (5) is substituted for j_z. Using the resulting ΔV_I in Eq. (16), we obtain

$$r_w \approx \frac{2\sqrt{3}\pi^2\beta^2\rho_F^* v_F\tilde{T}}{3d^2\chi^3}[\chi - \exp(-\gamma d)(\chi\cos\chi d + \gamma\sin\chi d)]. \qquad (18)$$

We see that r_w exhibits damped fast oscillations as a function of the wall thickness d with the same period as $m_y(x)$ of Eqs. (14) and (15). The amplitude of the oscillations decays exponentially with a characteristic length given by the mean free path $l=v_F\tilde{T}$.

In the diffusive limit, $\gamma d\gg 1$, the first term on the right-hand side of Eq. (18) dominates yielding

$$r_w \approx 2\beta^2\rho_F^* l_{eff}, \qquad (19)$$

where $l_{eff}=(4\sqrt{3}/9)\xi^2 l$ and $\xi=\pi v_F/2d\omega_e$ is the mistracking parameter.[5] This result contrasts with the effective length $l_{eff}=8/3\xi^2 d(\tilde{T}/\tau)$ obtained in Ref. 12. The suppression of the spin accumulation contribution to r_ω is characterized by the ratio l_{eff}/l_{sf} where $l_{sf}=(D\tau_{sf})^{1/2}$, and τ_{sf} is the spin-flip relaxation time.[19] For a domain wall in cobalt, we have $d\approx 1.5\times 10^{-6}$ cm, $\omega_e\approx 1.5\times 10^{15}$ s^{-1}, and $v_F\approx 1.4\times 10^8$ cm/s yielding $\xi\approx 0.1$. With these values, the effective length in Eq. (19) is $l_{eff}\approx 7.7\times 10^{-3}\,l$. Assuming $\tau_{sf}\simeq 10\tilde{T}$ and $\omega_e\tilde{T}=3$, we have $l_{sf}\approx 5\times 10^{-7}$ cm and $l_{eff}\approx 2.2\times 10^{-9}$ cm yielding $l_{eff}/l_{sf}\approx 4.4\times 10^{-3}$. Hence, for cobalt, the spin accumulation contribution to r_ω is suppressed in the diffusive limit.

Let us compare the expression (19) with the theory of Levy and Zhang.[9] Expressing the quantities $\rho_0^{\uparrow}$ and $\rho_0^{\downarrow}$ in terms of the parameter β, their equation (20) yields an excess resistance $r_w^{LZ} = (12\xi^2 \rho_F^* \beta^2 d/5)(1+5/3\sqrt{1-\beta^2})$. For a wide range of the parameter β, the ratio $r_w^{LZ}/r_w \approx 2d/l$. Thus, in the diffusive limit the DW resistance is dominated by the mechanism of Ref. 9.

In the ballistic limit, $\gamma d \ll 1$, the wall resistance of Eq. (18) reduces to

$$r_\omega^{(b)} \approx 2\beta^2 \rho_F^* l_{eff}^{(b)} \sin^2 \frac{\chi d}{2}, \qquad (20)$$

where $l_{eff}^{(b)} \approx (8\sqrt{3}/9)\xi^2 l$. Oscillations of the excess resistance, due to the Bloch wall, may be observable in extremely clean samples with a relatively large value of the parameter ξ. Also, low temperatures may be required to minimize the electron-phonon and electron-magnon relaxation rates. A promising candidate for observing the resistance oscillations is DW in a constriction between two wider sections. In this case, the thickness d can be varied by changing the length of the constriction.[22]

In summary, we have studied the accumulation of the transverse spin density of conduction electrons around the Bloch wall in the presence of a spin-polarized electric current. We solve perturbatively the coupled kinetic equations in Wigner space and find that the transverse spin accumulation exhibits damped spatial oscillations with wavelength $\pi |k_F^\uparrow - k_F^\downarrow|^{-1}$. The electrical resistance due to the spin accumulation is found to contain damped oscillations as a function of the wall thickness. In the diffusive limit, the resistance is suppressed as a result of adiabatic tracking of the wall by fast precessing electron spin. The oscillatory part of the DW resistance becomes pronounced in the ballistic limit owing to the off-diagonal terms in the diffusion equations. Experiments on geometrically constrained DW are suggested for a possible observation of the resistance oscillations. The present theory is also relevant to the problem of current driven motion of the DW[4] and magnetization switching in magnetic multilayers.[23]

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