Parameter regime of a resonance quantum switch

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The design of a three-terminal resonance quantum switch is suggested in the form of a network consisting of a circular quantum well with four semi-infinite single-mode quantum wires attached to it. In the resonance case, when the Fermi level in the wires is close to an energy level in the well, the magnitude of the governing electric field on the well may be specified such that the switching effect is obtained; the quantum current across the switch from the incoming wire to the outgoing wires (terminals) is controlled via rotation of the orthogonal projection of the field onto the plane of the device. Using an approximate formula for the scattering matrix we define essential details of the design and the parameter regime of the switch depending on the desired working temperature, the Fermi level in the wires, and the effective mass of an electron.

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I. INTRODUCTION

The basic problems of quantum conductance were related to scattering processes long ago[1–3] and the role of scattering in the mathematical design of quantum electronic devices was clearly understood by the beginning of the 1990s.[4–6] Modern experimental techniques already permit observation of resonance effects caused by details in the shape of the resonance wave functions.[7–11] We propose using these effects as efficient tools for manipulation of the transport properties of the three-terminal quantum switch.

The idea of resonance manipulation of the *single-mode* quantum current is based on the following observation from Ref. 12: "The resonance transmission across the quantum system caused by attachment of incoming and outgoing channels is proportional to the products of some local characteristics of the corresponding resonance eigenfunction of the system at the places where the channels are attached."

In this paper we suggest a method for an approximate description of the resonance transport in a quantum network, considering the basic example, the resonance quantum switch[13] formed on the surface R_2 of a semiconductor as a union $\Omega=\cup_{s=0}^{4}\Omega_s$ of the quantum well Ω_0 and four equivalent quantum wires Ω_1, Ω_2, Ω_3, Ω_4, of equal width δ attached to it. See Fig. 1. In this paper the basic quantum well Ω_0 is a disk. Similar analysis can be developed for the ring, but in that case the resonance manipulation of the quantum current is more difficult because of instability of lines of zeroes of the resonance eigenfunction. The role of the Hamiltonian of the relevant quantum system is played by the one-electron Schrödinger operator l. See below in Sec. II. The corresponding potential is constant $V(x)=V_\infty$ in the wires, linear (for the macroscopic electric field) on the well, and zero on the complement $R_2\backslash\Omega:=\Omega'$ of the network. We assume that the Fermi level in the wires with respect to the potential on the complement of the network is deep enough. See the estimates below in the beginning of Sec. II. Then one can replace the matching boundary conditions on the common boundary of the network and the complementary domain by the homogeneous Dirichlet conditions, at least for electrons with energy close to the Fermi level. We do not assume that the wires are thin (see the discussion below in Secs. II B and II C) or that the connection of the wires to the quantum well is weak. But we assume that the dynamics of the electrons in the wires is single mode and ballistic on large intervals of the wires, compared with the size of the geometric details of the construction (the width of the wire or the size of the contacts). Imposing certain weak conditions, see Secs. II B and II C below, on geometrical details of the network Ω, we derive an approximate formula for the corresponding scattering matrix which is used for optimization of transport properties of the switch.

In this paper we replace the transport problem for the quantum switch by the one-body scattering problem, assuming that quantum current is weak and few-body effects are

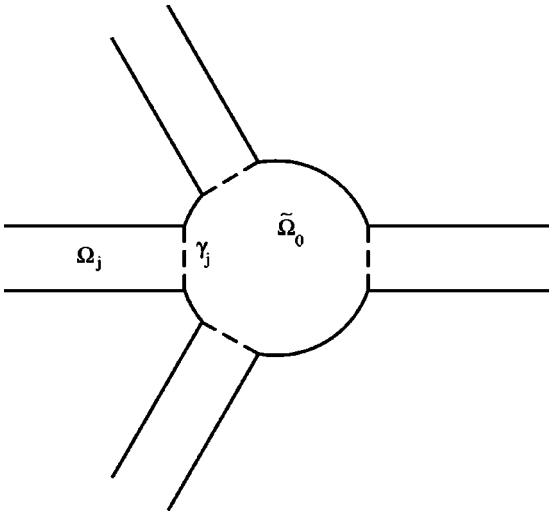

FIG. 1. Resonance quantum switch. The circular quantum well Ω_0 with wires Ω_j, $j=1,2,3,4$ attached. The wires are enumerated in a counterclockwise direction, beginning from the horizontal wire Ω_1 on the left of the well. The modified well $\tilde{\Omega}_0$ is obtained as the complement of the wires in the circular well Ω.

suppressed by scattering by impurities, which is also neglected in this paper. A more realistic analysis of conductance requires estimation of decoherence and taking into account many-body effects, for instance as in Ref. 14. This analysis will be developed in the forthcoming paper.

Solution of the one-particle spectral problem for the Schrödinger equation on the two-dimensional network obviously can't be obtained in explicit form. One usually believes that the explicit formulas can be substituted by the solution of a one-dimensional Schrödinger equation on the corresponding graph. See for instance Refs. 20 and 19. In mathematical papers[15–17] based on Ref. 18 the authors noticed that the (discrete) spectrum of the Laplacian on a system of finite length shrinking waveguides of width δ attached to the shrinking vertex domain of diameter $R=R_1\delta^\alpha$, $0<\alpha<1$ tends to the spectrum of the Laplacian on the corresponding one-dimensional graph, but with different boundary conditions at the vertices, depending on the "speed" α of shrinking. Unlike the papers in Refs. 15–17, our approach to the Schrödinger equation on the "fattened graph" with semi-infinite wires is based on an analysis of the resonance transmission through the quantum well. Assuming that the Fermi level is situated on the first spectral band in the wires, we impose additional "chopping off" boundary conditions on the bottom sections γ_j of the semi-infinite wires. See Fig. 1 above. These conditions split the original Schrödinger operator into the orthogonal sum, $l\to\{\Sigma_{s=1}^4 l_s^r\}\oplus l_0^r$ of the trivial part $\{\Sigma_{n=1}^4 l_s^r\}$ in the open channels and the nontrivial part l_0^r which plays the role of an "intermediate operator." The spectrum of the intermediate operator consists of an absolutely continuous part of varying multiplicity which begins from the second threshold in the wires. See Sec. II A, and a sequence of eigenvalues λ_l^r which can accumulate at infinity. The eigenvalues that are sitting on the first spectral band generically give rise to resonances of the scattering problem on the network and hence define the resonance conductance. Our main tool is the following formula for the scattering matrix:

$$\mathbf{S}(\lambda)=\frac{i\mathbf{p}(\lambda)I+\Lambda^r}{i\mathbf{p}(\lambda)I-\Lambda^r}, \tag{1}$$

where $i\mathbf{p}(\lambda)$ is, in the simplest case, an exponent from the bounded modes $e^{\pm i\mathbf{p}(\lambda)x}e_s$ in the open channels, spanned by the corresponding cross section eigenfunction e_s, and Λ^r is the Dirichlet-to-Neumann map of the intermediate operator l_0^r. See Sec. II B and the Appendix.

For low temperatures the summation and integration over the spectrum $\{\lambda\}$ of the intermediate operator may be reduced, see for instance Ref. 21, to the essential interval of energy $|\hbar^2/2m_0\lambda-E_F|<\kappa_B T$. In our case this gives an approximate scattering matrix. In the most interesting case, when only one resonance eigenvalue λ_0^r is sitting on the essential spectral interval, the above formula (1) gives an important "one-pole approximation" which is used below for the estimation of the parameter regime of the switch.

Here is the plan of our paper. In Sec. II we derive an explicit formula (12) for the scattering matrix in terms of the spectral characteristics of the intermediate operator and describe the geometrical and physical limitations which allow us to replace, in Sec. II B, the exact formula (1) for the scattering matrix by the approximate formula (27). This approximate formula is interpreted as a scattering matrix for a solvable model. In Sec. III we find positions of resonances, and in Sec. IV we optimize the conductance of the switch based on Eq. (27). Useful mathematical facts concerning the Dirichlet-to-Neumann map of the intermediate operator and the analytic perturbation procedure for calculation of the corresponding spectral data are collected in the Appendixes A and B.

II. THE SCHRÖDINGER EQUATION AND THE HAMILTONIAN OF THE SWITCH

Conductance of the network constructed on the surface of the crystallized medium depends on the geometry of the network and on the correspondence between the crystal structure and the form and positions of the wires with respect to the crystal's lattice. For networks of quantum wires and quantum wells formed on the surface of GaAs, InAs, and the narrow-gap semiconductors CdHgTe, and InSb the tensor of effective mass is isotropic, and the value of the effective mass is small, $m^\parallel=m^\perp=m^*\ll m_0$, see Refs. 22 and 23 and Table I in Sec. IV C. If the Fermi level is deep compared with the radius of the well,

$$\frac{2m^*(0-E_F)}{\hbar^2}R^2=0.3136(0-E_f)R^2\gg 1$$

[R is measured in Å and the depth of the Fermi level $(0-E_F)$ in eV], then for the values of the energy E near to the Fermi level E_F the spectral problem

$$-\frac{\hbar^2}{2m^*}\Delta u+V(x)u=Eu \tag{2}$$

on the whole plane can be reduced to the spectral problem for the corresponding Schrödinger operator on the network $\Omega:=\Omega_0\cup\Omega_1\cup\Omega_2\cup\Omega_3\cup\Omega_4$ with zero (Dirichlet homogeneous) boundary condition: $u|_{\partial\Omega_0}=0$.

We assume that the potential $V(x)$ on the quantum well is defined by the macroscopic "governing" electric field $\mathcal{E}\nu$ which is constant inside the well, $V(x)=\mathcal{E}e\langle x,\nu\rangle+V_0$, where e is the electron charge and the unit vector ν shows the direction of the field. The magnitude of the field is specified in Sec. IV to optimize the switching effect for a circular quantum well of a certain radius. In the wires Ω_s: $-\mathbf{l}<x_s<\infty$, $0<y<\delta_s$ the potential is piecewise constant, possibly with a barrier height $\hbar^2H^2/2m^*$ at the beginning,

$$V_s(x,y)=\begin{cases}V_\infty+\dfrac{\hbar^2H^2}{2m^*} & \text{if } -\mathbf{l}<x<0\\ V_\infty & \text{if } x>0,\end{cases}$$

or constant, $V_s(x)=V_\infty$, if $\mathbf{l}=0$. The first case corresponds to the presence of a split gate on the initial part $-\mathbf{l}<x<0$ of the wire. In the second case the split gate is absent or switched off. In the case of the ring-shaped well, see for instance Ref. 7, when the resonance manipulation is unstable, the electron current can be manipulated with the help of a split gate. In

this paper we focus on the resonance manipulation of the electron current.

We assume that the role of the one-electron Hamiltonian on the network is played by the Schrödinger operator,

$$l = -\frac{\hbar^2}{2m^*}\Delta + V(x), \tag{3}$$

with the potential and the tensor of effective mass specified as described above. It is convenient to use the "geometric" form of the corresponding Schrödinger equation with the renormalized spectral parameter $\lambda = p_s^2 = (2m^*/\hbar^2)[E - V_\infty - (\hbar^2/2m^*)(\pi^2/\delta_s^2)]$, and the corresponding effective wave number $p = \sqrt{\lambda}$. Furthermore, we assume that the width of all wires is $\delta_s = \delta$. This allows us to introduce the universal renormalized spectral parameter $\lambda = p^2$.

We use the spectral parameter λ in the Schrödinger equation on the well, after substraction of appropriate correcting terms from the potential,

$$-\frac{1}{m^*}\Delta_x u(x) + \frac{2}{\hbar^2}\left[V(x) - V_\infty - \frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}\right]u = \frac{2}{\hbar^2}\left[E - V_\infty - \frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}\right]u(x) = \frac{1}{m^*}\lambda u, \quad |x| < R. \tag{4}$$

A nondimensional form of the above Schrödinger equation on the well is obtained via change of the space variable $x \to \xi = (\xi_1, \xi_2) = x/R$,

$$-\frac{1}{m^*}\Delta_\xi u(R\xi) + \frac{2R^2}{\hbar^2}\left[V(R\xi) - V_\infty - \frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}\right]u(R\xi) = \frac{2R^2}{\hbar^2}\left[E - V_\infty - \frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}\right]u(R\xi) = R^2\frac{\lambda}{m^*}u(R\xi) := R^2\frac{p^2}{m^*}u, \quad |\xi| < 1. \tag{5}$$

The corresponding change of variables on each wire, $x \to \xi = x/R$ along the wire and $y \to \eta = y/R$ across the wire, $0 < y < \delta$, $x > -\mathbf{l}$ gives the equation (39), see below. We will use, furthermore, both geometric and dimensionless forms of the Schrödinger equation in the wires and on the well assuming that each time the appropriate change of variables is also done for the function u. The roles of the dimensionless spectral parameters are played by $R^2\lambda = \hat{\lambda}$ and $Rp = \hat{p}$.

A. The intermediate operator and the scattering matrix

In Sec. II A we derive the formula for the scattering matrix based on the Dirichlet-to-Neumann map for the intermediate operator. The corresponding elementary calculation is postponed to the Appendix. We use the "geometric form" of the Schrödinger equation on the network, replacing the standard Schrödinger equations, Eq. (5) on the well Ω_0 by

$$-\frac{1}{m^*}\Delta u + \frac{1}{m^*}\mathbf{V}_0(x)u = \frac{1}{m^*}\lambda u, \tag{6}$$

and on the wires Ω_s by

$$-\frac{1}{m^*}u_{xx} - \frac{1}{m^*}u_{yy} + \frac{1}{m^*}\mathbf{V}_s(x)u = \frac{1}{m^*}\lambda u. \tag{7}$$

If the Fermi level in the wires sits on the first spectral band, we use split operator l^r defined by the above Schrödinger differential expressions on the whole network with special boundary conditions on the common boundary $\Gamma = \cup_{s=1}^4 \gamma_s$ of the well and the wires. In the first open channel the standard matching boundary condition is replaced by the *partial* Dirichlet boundary condition, *chopping the first channel off*. We denote by E_+ the four-dimensional *entrance subspace* of the open channel, spanned by the first-order eigenfunctions e_1^s, $s = 1,2,3,4,$ on the bottom sections γ_s, $s = 1,2,\ldots$ of the wires, $e_1 = \sqrt{2/\delta}\sin \pi y/\delta, \ldots$, and by P_+, the corresponding orthogonal projection in $E = L_2(\Gamma)$. We present the partial *chopping-off* boundary condition as

$$P_+ u|_\Gamma = 0, \quad \text{or} \quad \int_0^\delta \sin\frac{\pi y}{\delta}u_s(y)dy = 0, \quad s = 1,2,3,4, \tag{8}$$

both for functions u_s from the domain of the corresponding split operator in the wire and in the well. The partial matching condition in all upper (closed) channels, $l > 1$, with the entrance subspace $E_- = L_2(\Gamma) \ominus E_+$ and the corresponding complementary projection $P_- = I_\Gamma \ominus P_+$ in $L_2(\Gamma) = E$ is presented as

$$P_-[u_s - u_0]|_\Gamma = 0, \qquad P_-\left[\frac{1}{m^*}\frac{\partial u_s}{\partial n} - \frac{1}{m^*}\frac{\partial u_0}{\partial n}\right]\Bigg|_\Gamma = 0. \tag{9}$$

The split operator l^r defined by the above differential expressions [Eqs. (6) and (7)] and the boundary conditions [Eqs. (8) and (9)] can be presented as an orthogonal sum of the *trivial part*

$$\sum_{s=1}^4 l_s^r$$

of the one-dimensional Schrödinger operators l_s, $s = 1,2,3,4$

$$l_s u_s = -\frac{1}{m^*}\frac{d^2u_s}{dx^2} = \frac{1}{m^*}\lambda u_s,$$

on the open channel with zero boundary conditions at the bottom sections, and the *nontrivial part* l_0^r defined in the orthogonal complement in the Hilbert space of all square-integrable functions on the network $l^r = \Sigma_{s=1}^4 l_s^r \oplus l_0^r$. The nontrivial part l_0^r of the split operator l^r plays a role of an *intermediate operator*. We will present the intermediate operator in geometric form introduced above, see Eqs. (4) and (5) and use for it the geometrical spectral parameter $\lambda = p^2$. The continuous spectrum σ_c^r of l_0^r fills the semiaxis $[\lambda_{min}^r, \infty)$, with $\lambda_{min}^r = 3\pi^2\delta^{-2}$. The multiplicity jumps up by four units at the thresholds $(l^2 - 1)\pi^2/\delta^2$, $l = 2,3,\ldots$. There is a finite number of eigenvalues λ_s^r of the intermediate operator on each finite interval and possibly the accumulation point at $+\infty$. We assume that "the resonance eigenvalue" λ_0^r sits close to the scaled Fermi level,

$$\lambda_F = \frac{2m^*}{\hbar^2}\left[E_F - V_\infty - \frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}\right].$$

For the detailed spectral analysis of the intermediate operator see Ref. 26.

The split spectral problem for the operator l^r can be converted back into the original spectral problem for the operator l via replacement of the partial zero boundary condition [Eq. (8)] in the first channel by the corresponding partial matching condition with solutions u_s of the homogeneous problem in the first channel of the wires,

$$P_+[u_s - u_0]|_{\Gamma_s} = 0, \qquad P_+\left[\frac{1}{m^*}\frac{\partial u_s}{\partial n} - \frac{1}{m^*}\frac{\partial u_0}{\partial n}\right]\Bigg|_{\Gamma_s} = 0. \tag{10}$$

The perturbation caused by this one-dimensional change of the boundary condition (8) to (10) transforms the separated branch of the continuous spectrum in the first channel $0 < \lambda < \infty$ into the branch of the continuous spectrum of the original spectral problem. The components $\{u_{1s}\}_s^4 = u_1$ of the scattered waves in the first channel, $l=1$, are combined of Jost solutions, $f_\pm = e^{\pm ipx}$, $x>0$, of the Schrödinger equation with the compactly supported potential which is trivial on the wire

$$[V_s(x) - V_\infty] = 0,$$

for $x>0$, and may have (if $\mathbf{l} \neq 0$) a barrier (split gate) on $(-\mathbf{l}, 0)$,

$$[V_s(x) - V_\infty] = H^2\frac{\hbar^2}{2m^*}, \quad -\mathbf{l} < x < 0.$$

Then the solution $u_1 = \{u_{11}, u_{12}, u_{13}, u_{14}\}$ of the homogeneous equation in the first channel in the wires can be found in form of the scattered wave,

$$-\frac{d^2u_1}{dx^2} + \frac{2m^*}{\hbar^2}[V_1(x) - V_\infty]u_1 = p^2u_1,$$

$$u_1(x) = e^{-ipx}\mathbf{e} + e^{ipx}S_1\mathbf{e}, \quad x>0, \tag{11}$$

with any vector $\mathbf{e} \in E_+$. Components of the scattered wave in the upper channels $l>1$ are exponentially decreasing on the first spectral band,

$$u_l(x) = \exp\left[-\sqrt{\left(\frac{\pi^2 l^2}{\delta^2} - \frac{\pi^2}{\delta^2}\right) - p^2}\,x\right]S_l\mathbf{e}, \quad x>0, \quad l>1.$$

Here S_1—the scattering matrix—and the amplitudes S_l, $l>1$, in upper channels, are defined from the matching condition (10) of u_1 to the solutions of the corresponding homogeneous equation inside the well. To calculate the scattering matrix using the conventional matching technique one should compare the solution of the Schrödinger equation on the well with the solutions in all (open and closed) channels in the wires. See Appendix A. This requires the solution of an infinite linear system. The partial matching procedure (9) requires comparing the solutions of the homogeneous intermediate equations with the oscillating solutions *in open channels of the wires only*, thus replacing the infinite algebraic system by the finite one. The mathematical convenience of this approach lies in the replacement of the unbounded DN map Λ by the finite matrix $\Lambda^r = P_+\Lambda^r P_+$. Using partial matching gives the following explicit formula for the scattering matrix in the general case, when the split gate is present, $\mathbf{l} \neq 0$,

$$S(p) = -\frac{ip\dfrac{\tanh\sqrt{(H^2-p^2)}\mathbf{l}}{\sqrt{H^2-p^2}} + 1}{-ip\dfrac{\tanh\sqrt{(H^2-p^2)}\mathbf{l}}{(\sqrt{(H^2-p^2)}+1)}}\,\frac{P_+\Lambda^r P_+ - P_+\mathbf{Q_l}}{P_+\Lambda^r P_+ - P_+\overline{\mathbf{Q}}_\mathbf{l}}, \tag{12}$$

where the first factor is scalar, the denominator is preceding the numerator in the second factor,

$$\mathbf{Q_l} = \frac{-ip - \sqrt{(H^2-p^2)}\tanh\sqrt{(H^2-p^2)}\mathbf{l}}{i\sqrt{p\dfrac{\tanh\sqrt{(H^2-p^2)}\mathbf{l}}{\sqrt{(H^2-p^2)}} + 1}},$$

$\overline{\mathbf{Q}}_\mathbf{l}$ is the complex conjugate of $\mathbf{Q_l}$ and Λ^r is the Dirichlet-to-Neumann map (DN map) of the intermediate operator l^r. See Appendix A and Refs. 24–26. The corresponding formula, for the case when the split gate is absent is obtained by replacing in (12) the width $\mathbf{l}$ of the barrier by zero, $\mathbf{Q}_1 \to \mathbf{Q}_0 = -ip$.

B. Geometrical and physical limitations

Resonance properties of the scattering matrix (12) are revealed when substituting in it the spectral representation for the $\Lambda^r = P_+\Lambda^r P_+$ in the form of the sum and/or integral over the spectrum $\{\lambda^s\} \cup \sigma_c^r$ of the intermediate operator,

$$\Lambda^r = \sum_{\lambda_s} \frac{P_+\dfrac{\partial\varphi_s^r}{\partial n}\Big\rangle\Big\langle P_+\dfrac{\partial\varphi_s^r}{\partial n}}{\lambda - \lambda_s^r} + \int_{\sigma>3\pi^2/\delta^2} \frac{P_+\dfrac{\partial\varphi_2^r(\sigma)}{\partial n}\Big\rangle\Big\langle P_+\dfrac{\partial\varphi_2^r(\sigma)}{\partial n}}{\lambda - \sigma}d\sigma + \int_{\sigma>8\pi^2/\delta^2} \frac{P_+\dfrac{\partial\varphi_3^r(\sigma)}{\partial n}\Big\rangle\Big\langle P_+\dfrac{\partial\varphi_3^r(\sigma)}{\partial n}}{\lambda - \sigma}d\sigma + \cdots. \tag{13}$$

The corresponding eigenstates φ_s^r minimize the Rayleigh ratio $\langle u,u\rangle^{-1}\langle l_0^r u, u\rangle$ of the intermediate operator l_0^r.

1. Comparison of spectral data of the intermediate operator and the operator on the well

It is shown in Appendix A that the DN map of the intermediate operator is connected with the conventional DN map of the Schrödinger operator on the quantum well. See Eq. (A7). In fact, the "renormalized" eigenvalues λ^r of the intermediate operator that sit on the first spectral band in the wires, between the first and second thresholds, are obtained

by minor shifts Δ_s of the eigenvalues λ_s of the Schrödinger operator on the quantum well with a zero boundary condition on the whole boundary,

$$\lambda_s \to \lambda_s^r = \lambda_s + \Delta_s.$$

The deviations of the eigenvalues of the dimensionless intermediate operator from the eigenvalues of the corresponding Schrödinger operator on the well with zero boundary conditions are estimated theoretically in Ref. 27, discussed in the following Sec. II C, and verified numerically in Appendix B for the most interesting case of the circular well. It is shown there that the "resonance entrance vectors" $P_+ \partial\varphi_l/\partial n|_\Gamma$ combined of eigenfunctions φ_l of the intermediate operator on the bottom sections of the wires, coincide, in the first order of the analytic perturbation procedure, with the corresponding data of eigenfunctions of the Schrödinger operator on the quantum well with zero boundary conditions. See Appendix B. In particular in the remaining part of this section we may neglect the difference between spectral data $\lambda_s^r, \varphi_s^r, \Lambda^r$ of the intermediate operator and the corresponding spectral data of the Schrödinger operator on the well with zero boundary conditions. We assume further in this section that split gates are absent, or switched off, $\mathbf{l}=0$.

2. Temperature

We assumed that the total number of electrons participating in conductance is rather small, so that many-body effects are suppressed by scattering on impurities and decoherence, which are also neglected in this paper. We take into account only the statistics of electrons, assuming, similar to Ref. 21, that observable values of dynamical variables are obtained via averaging of the corresponding theoretical data on a Fermi-Dirac distribution. Practically we take into account only an essential interval of energy length $2\kappa_B T$ centered at the Fermi level. We say that *the radius R of the quantum well is relatively small, for the given temperature*, if the spacing $\rho(E_F)=\min_{s\neq 0}|E_s-E_0|$ of energy levels of the intermediate operator at the resonance energy level $E_0 \approx E_F$ is large compared with the temperature,

$$\kappa_B T < \frac{1}{2}\inf_{E_s \neq E_F}|E_F - E_s| = \rho(E_0). \tag{14}$$

Generally, there may be several eigenvalues

$$\lambda_s^r = \frac{2m^*}{\hbar^2}\left[E_s - V_\infty - \frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}\right]$$

of the intermediate operator (or the Schrödinger operator on the well) situated on the essential interval of energy $[E_F - \kappa_B T, E_F + \kappa_B T]$,

$$E_F - \kappa_B T \leqslant E_s \leqslant E_F + \kappa_B T. \tag{15}$$

Appropriate domination conditions can be formulated also for the resonance group Λ^r_{approx} of terms in Λ^r, if the spacing ρ_Δ between the resonance group and the remaining part of the discrete spectrum is greater than the $\kappa_B T$. Under these conditions the nonresonance terms, which correspond to eigenvalues outside the essential interval, can be neglected together with the contribution to DN map from the continuous spectrum, if it does not overlap with the essential spectral interval,

$$\mu_2 \frac{3\pi^2\hbar^2}{2m^*\delta^2} > \kappa_B T. \tag{16}$$

3. Leading terms of the DN map

Assume that the essential spectral interval centered at the Fermi level does not overlap with the continuous spectrum of the intermediate operator and neglects the contribution from the continuous spectrum in the expression (13). The contribution to (13) from the nonresonance terms with $\lambda_s^r \neq \lambda_0^r$ will be estimated.

Denote by $\rho^r(\lambda_0^r)$ the spacing on the resonance level $\lambda = \lambda_0^r$ and by $C_R = \hat{C}R^3$ the square norm of the operator,

$$P_+\frac{\partial\varphi_s^r}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_s^r}{\partial n} \quad .$$

We estimate the contribution to Λ^r [see Eq. (13)] from the nonresonance polar term, $s \neq 0$,

$$\left\| \frac{P_+\frac{\partial\varphi_s^r}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_s^r}{\partial n}}{\lambda_0^r - \lambda_s^r} \right\| = \frac{C_R}{|\lambda_0^r - \lambda_s|} \leqslant \frac{\hat{C}R^3}{\rho^r(\lambda_0^r)}. \tag{17}$$

The dimensionless constant $\hat{C}$ in the case of a circular quantum well (see the example in Sec. II C) is estimated as $\hat{C} < 10$. We assume that the whole nonresonance contribution to the DN map is dominated by the contribution from the nearest to λ_0^r nonresonance eigenvalue and estimated as $\hat{C}R^3/\rho^r(\lambda_0^r)$ again, with $\hat{C} \leqslant 10$. In Sec. III we will show that the most interesting case is $\mathbf{l}=0$ (the split gates open). Then the numerator of the scattering matrix is presented as

$$\frac{P_+\frac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_0^r}{\partial n}}{\lambda - \lambda_0^r} + O\left(\frac{C}{\rho^r(\lambda_0^r)}\right) + iP_+p, \tag{18}$$

where the whole nonresonance contribution is denoted by

$$O\left(\frac{C}{\rho^r(\lambda_0^r)}\right) = \frac{1}{R}O\left(\frac{\hat{C}R^3}{\rho^r(\lambda_0^r)}\right).$$

Leading terms in the numerator near the resonance eigenvalue λ_0^r of the intermediate operator are the polar term

$$\frac{P_+\frac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_0^r}{\partial n}}{\lambda - \lambda_0^r}$$

and the last term iP_+p containing the effective wave number p. Both of them are homogeneous functions of degree -1 of the space variable. The middle term defining the nonresonance contribution $O(C/\rho^r(\lambda_0^r))$ is also a homogeneous operator-function degree -1 and can be neglected if the condition (24) below is fulfilled. On the other hand, one can also

develop a perturbation technique for the calculation of the zeros of the numerator of the scattering matrix using the small parameter, which is chosen below. See Eq. (22). The small parameter is estimated numerically in the example in Sec. II C. To introduce the small parameter, let us neglect the contribution to the DN map from the nonresonance terms and upper branches of the absolutely continuous spectrum. This results in the resonance term only,

$$\Lambda^r(\lambda) \approx \frac{P_+\frac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_0^r}{\partial n}}{\lambda-\lambda_0^r} := \Lambda^r_{appr}.$$

Then we separate the group of leading terms in Eq. (18) as

$$\frac{P_+\frac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_0^r}{\partial n}}{\lambda-\lambda_0^r} + iP_+p := \mathbf{D}. \tag{19}$$

The operator **D** is invertible, hence the numerator of the scattering matrix can be factorized as a product,

$$\frac{P_+\frac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_0^r}{\partial n}}{\lambda-\lambda_0^r} + O\left(\frac{C}{\rho^r(\lambda_0^r)}\right) + iP_+p = \mathbf{D}\left[I+\mathbf{D}^{-1}O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\right]. \tag{20}$$

The term Λ^r_{appr} gives a convenient "one-pole" approximation for the DN map of the intermediate operator. The corresponding one-pole approximation for the scattering matrix in the case when the split gates are open (or absent) can be obtained via neglecting the second term in square brackets

$$S(\lambda) = -\left[I+[\mathbf{D}^+]^{-1}O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\right]^{-1}[\mathbf{D}^+]^{-1}\mathbf{D} \times\left[I+\mathbf{D}^{-1}O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\right] \approx -[\mathbf{D}^+]^{-1}\mathbf{D} := S_{approx}(\lambda). \tag{21}$$

The approximation (21) is quantitatively consistent if the contribution from the nonresonance terms is dominated by the group of leading terms **D**. See the discussion in Secs. II B and II C. In that case $[\mathbf{D}^+]^{-1}\,O(C/\rho^r(\lambda_0^r))$ can play the role of the small parameter,

$$\left\|\mathbf{D}^{-1}O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\right\| \ll 1, \quad \left\|[\mathbf{D}^+]^{-1}O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\right\| \ll 1. \tag{22}$$

If similar weaker conditions like $\|\mathbf{D}^{-1}O(C/\rho^r(\lambda_0^r))\|<1$ are fulfilled, then an analytic perturbation procedure can be developed for the calculation of the scattering matrix, beginning from the one-pole approximation $S_{approx}(\lambda)=-[\mathbf{D}^+]^{-1}\,\mathbf{D}$. We will suggest below, see (24)–(26), some practical conditions for the domination (22).

Assume that the Fermi level E_F is situated on the first spectral band,

$$\left[\frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}+V_\infty, \frac{\hbar^2}{2m^*}\frac{4\pi^2}{\delta^2}+V_\infty\right],$$

in the wires dividing it in the ratio $\mu_1:\mu_2, \mu_1+\mu_2=1$,

$$\mu_1:\mu_2 = \left[E_F - V_\infty - \frac{\hbar^2\pi^2}{2m^*}\right]:\left[V_\infty + 4\frac{\hbar^2\pi^2}{2m^*} - E_F\right]. \tag{23}$$

Then one can estimate the distances of the Fermi level to the first $(\hbar^2\pi^2/2m^*\delta^2)+V_\infty$ and the second $(4\hbar^2\pi^2/2m^*\delta^2)+V_\infty$ thresholds in the wires, respectively, as $\mu_{1,2}3(\hbar^2\pi^2/2m^*\delta^2)$ and the "effective wave number" on the Fermi level as $p_F=\sqrt{3\mu_1\pi^2(\delta^2)^{-1}}$.

We say that the width of the wires is *relatively small compared with the radius R of the quantum well*, if the nonresonance contribution to the DN map is dominated on the Fermi level by the effective wave number. This condition can be presented as the domination of the inverse "renormalized" spacing on Fermi level by the dimensionless, effective wave number $p_F=Rp$,

$$\frac{\hat{C}}{\hat{\rho}_0(\lambda_0^r)} = \frac{\hbar^2\hat{C}}{2m^*R^2\rho^r(E_F)} \ll R\sqrt{\frac{2m^*[E_F-V_\infty]}{\hbar^2}-\frac{\pi_F^2}{\delta^2}} := \sqrt{\mu_1}p_F = \frac{\pi R}{\delta}\sqrt{\mu_1 3}. \tag{24}$$

Here the renormalized spacing $\hat{\rho}_0 := \hbar^2 2m^*R^2\rho_0$ is actually equal to the spacing on resonance level E_0,

$$\frac{\hbar^2}{2m^*R^2}\hat{\lambda}_0 = E_0 - V_0 - \frac{\hbar^2\pi^2}{2m^*\delta^2} \approx E_F - V_0 - \frac{\hbar^2\pi^2}{2m^*\delta^2}$$

of the corresponding dimensionless Schrödinger equation on the well. The domination condition [Eq. (24)] implies the estimates for the leading terms in the numerator and the denominator of the scattering matrix from below or the corresponding inverse from above,

$$\|\mathbf{D}^{-1}\| \leqslant \left(\pi\frac{R}{\delta}\sqrt{3\mu_1}\right)^{-1}, \quad \|[\mathbf{D}^+]^{-1}\| \leqslant \left(\pi\frac{R}{\delta}\sqrt{3\mu_1}\right)^{-1}. \tag{25}$$

Together with the previous estimate for the nonresonance terms,

$$\left\|O\left(\frac{\hat{C}}{\hat{\rho}^r(\lambda_0^r)}\right)\right\| \approx \frac{10}{\hat{\rho}^r(\lambda_0^r)}$$

this implies estimates for the "small parameters" in the formulas (21) and (22). A dimensionless version of the estimate is, for instance,

$$\left\|\mathbf{D}^{-1}O\left(\frac{\hat{C}}{\hat{\rho}^r(\lambda_0^r)}\right)\right\| \leqslant \frac{10}{\hat{\rho}^r(\lambda_0^r)}\left(\pi\frac{R}{\delta}\sqrt{3\mu_1}\right)^{-1}. \tag{26}$$

A similar condition of domination can be formulated if some "resonance group" of several polar terms in the DN map Λ^r is included in the "leading group." See the corresponding discussion in Sec. IV. The role of spacing in that case is played by the minimal distance between the eigenvalues of the selected resonance group and the rest of the spectrum.

4. One-pole and few-pole approximations

Assuming that the above domination conditions are satisfied, we can neglect in the formula for the scattering matrix the contribution to the DN map from the nonresonance eigenvalues. This way we obtain the convenient *one-pole approximations* for expressions in the numerator and denominator of the scattering matrix which take into account only the leading terms. Assuming that the split gates are open, $\mathbf{l}=0$, we consider an approximate expression for the scattering matrix (21), a combination of the leading terms (19) of the numerator and denominator only,

$$S(\lambda) \approx -[\mathbf{D}^+]^{-1}\mathbf{D} = -\frac{\dfrac{P_+\dfrac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\dfrac{\partial\varphi_0^r}{\partial n}}{\lambda-\lambda_0} + ipI}{\dfrac{P_+\dfrac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\dfrac{\partial\varphi_0^r}{\partial n}}{\lambda-\lambda_0} - ipI} := \mathbf{S}_{approx}(\lambda). \tag{27}$$

The corresponding dimensionless formula has the same form. Here and in following sections we denote the dimensionless variables $\hat{\lambda},\hat{p}$ just by λ,p. Note that the zeros of the function $\mathbf{S}_{approx}(\lambda)$ can be found from an elementary algebraic equation. See the next section. Their deviations from the zeros of the scattering matrix can be estimated rigorously due to the presence of the small parameters and the operator version of Rouche's theorem. See Ref. 28. One can use the above *one-pole approximation* (27) of the scattering matrix for the approximate description of the electron transport across a quantum well, if the conditions (14) and (24) are fulfilled. The above one-pole approximation (27) and even similar "few-poles" approximations (1) of the scattering matrix of the switch,

$$\frac{ipI+\Lambda^r_{approx}}{ipI-\Lambda^r_{approx}} := \mathbf{S}_{approx}(\lambda), \tag{28}$$

with rational "essential DN map,"

$$\Lambda^r_{approx}(\lambda) = \sum_{l=1}^{N_T} \frac{P_+\dfrac{\partial\varphi_l}{\partial n}\Big\rangle\Big\langle P_+\dfrac{\partial\varphi_l}{\partial n}}{\lambda-\lambda_l}, \quad \lambda_l \in \Delta_T, \tag{29}$$

constructed as a sum of terms with poles λ_l, $l=1,2,\ldots,N_T$ on the essential interval

$$\Delta_T = \left(\lambda_F - \frac{2m^*R^2}{\hbar^2}\kappa_B T, \lambda_F + \frac{2m^*R^2}{\hbar^2}\kappa_B T\right)$$

can be interpreted as scattering matrices for solvable models. Construction of the solvable model of the switch is a partial case of the solvable model of the general quantum network which is described in Ref. 26. Note that these solvable models allow appropriate fitting of all free parameters.

C. Example

In Sec. II C we apply the analysis of geometrical limitations developed above in a general case to the most interesting example of a switch, based on a circular quantum well with quantum wires of width $\delta=R/2$. If the shift potential V_0 in the well is selected such that $V_0-V_\infty-\hbar^2/2m^*\delta^2=0$, then the potential of the corresponding dimensionless Schrödinger equation on the well is just proportional to $\langle\xi,\nu\rangle u$. We also assume that the renormalized electric field is selected as $\epsilon=18.86$, which corresponds to the appropriate shape of the resonance eigenfunction. See Sec. IV A. The resonance eigenfunction of the dimensionless Schrödinger operator on the quantum well satisfies the equation

$$-\Delta_\xi u - \epsilon\langle\xi,\nu\rangle u = R^2p^2u, \quad u|_{\partial\Omega_0}=0,$$

which corresponds (see Sec. IV A) to the (dimensionless) resonance eigenvalue,

$$\hat{\lambda}_0 = \frac{2m^*R^2}{\hbar^2}\left[E_F - V_0 - \frac{\hbar^2\pi^2}{2m^*\delta^2}\right] = R^2\lambda_0 = 14.62.$$

It appeared that the eigenfunctions and eigenvalues of the Schrödinger operator l_0 on the quantum well with zero boundary conditions are very close to the eigenfunctions and eigenvalues of the intermediate operator. See Appendix B. Hence, the spectral data l_0 can be used in the above approximate formulas for the DN map and the scattering matrix. An alternative direct calculation of the dimensionless eigenvalues of the intermediate operator was done[29] for $\delta=R/2$. The deviation $\Delta_0=-0.07$ of the resonance eigenvalue $\hat{\lambda}_0^r$ from the corresponding eigenvalue $\hat{\lambda}_0=14.62$ of the Schrödinger operator on the well is small and is dominated by the nonperturbed spacing 2.30—the distance to the nearest nonresonance eigenvalue 12.32 on the well. The estimation of the deviation via analytic perturbation procedure is given in Appendix B. The above condition (24) is verified for the Schrödinger operator on the well. See Eqs. (30) and (31) below. It remains valid for the intermediate operator, too.

Small parameter

If the Fermi level divides the first spectral band in ratio $\mu_1:\mu_2$, then the one-pole approximation is applicable to the switch with "relatively narrow" wires if condition (24) is fulfilled. In the actual case of a switch based on a circular quantum well this condition takes the form

$$\frac{10}{2.3} < \frac{R}{\delta}\pi\sqrt{3\mu_1}. \tag{30}$$

It obviously holds if the width δ of the wires does not exceed $R/2$, and the Fermi level sits in the middle of the first spectral band $\mu_1=\mu_2=1/2$. It may be reduced in this case to

$$10 \ll 20, \tag{31}$$

which is not restrictive. Moreover, this reveals the "natural" small parameter $1/2$,

$$\|\mathbf{D}^{-1}\| \leqslant \left[8.63\frac{R}{\delta}\right]^{-1}, \quad \left\|\mathbf{D}^{-1}O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\right\| \leqslant \frac{1}{2}, \tag{32}$$

if $\delta \leqslant R/2$. This small parameter shows the "degree of domination" of the nonresonance contribution to the DN map.

According to Eqs. (22) and (21) this gives an estimation for the approximation rate $|\mathbf{S}_{approx}-S|$ in the example considered and allows us to develop the perturbation procedure for the scattering matrix. For $\delta/R=10$ the analog of the above estimate (32) has $1/10$ in the right side.

For the wires of width 2 nm this condition estimates the temperature in K as $T<312K\mu_2$, which means that the nitrogen temperature 77 K is low enough to allow us to neglect in the approximate formula for the scattering matrix the contribution from the upper branches of the continuous spectrum, if the Fermi level divides the first spectral band in ratio $\mu_1:\mu_2=2:1$. For wires of width 3 nm one can neglect the contribution from the upper branches of the continuous spectrum of the intermediate operator for the nitrogen temperature if the Fermi level is situated in the middle of the first spectral band.

We do not estimate here admissible nonaccuracies of geometrical details of the construction which would not affect the transport properties of the switch. This important question is postponed to the following publication, as well as discussion of the important case of the ring-shaped quantum well. Here we just mention that the geometrical details can be already controlled to a precision of 2 nm,[8] which is less than 10% of the diameter of the well for most prospective materials. See the discussion in Sec. IV.

In Sec. IV we will discuss the geometrical limitations in connection with the high-temperature regime of the switch and the choice of materials for manufacturing of the switch.

III. CALCULATION OF RESONANCES

The 4×4 scattering matrix (12) in the first channel is an analytic matrix function in the complex plane of the dimensionless effective wave number p and may have zeros—resonances $\{p_s\}$—in the upper half plane and complex-conjugate poles in the lower half plane. In particular, when the split gate is absent, $\mathbf{l}=0$, the resonances can be found as vector zeros of the dimensionless numerator (18),

$$\frac{P_+\dfrac{\partial\varphi_0^r}{\partial n}\Big\rangle\Big\langle P_+\dfrac{\partial\varphi_0^r}{\partial n},\mathbf{e}}{\lambda-\lambda_0^r}+O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\mathbf{e}+ip\mathbf{e}=0, \quad (33)$$

with a normalized vector $\mathbf{e}\in E_+$. The contribution $O(C/\rho^r(\lambda_0^r))$ from the nonresonance terms of the DN map is estimated as $10[\rho^r(\lambda_0^r)]^{-1}$. Multiplying by the orthogonal projection $P_0=\phi_0\rangle|\phi_0|^{-2}\langle\phi_0$ onto the "resonance entrance vector,"

$$\phi_0^r:=P_+\frac{\partial\varphi_0^r}{\partial n}:=\phi_0=\{\phi_0^1,\phi_0^2,\phi_0^3,\phi_0^4\},$$

we may reduce the equation to the pair of equations,

$$\frac{|\phi_0|^2}{\lambda-\lambda_r^0}+\left\langle P_0O\left(\frac{C}{\rho^r(\lambda_0^r)}\right),\mathbf{e}\right\rangle+ip=0,$$

$$-ip(P_+-P_0)\mathbf{e}=(P_+-P_0)O\left(\frac{C}{\rho^r(\lambda_0^r)}\right)\mathbf{e}.$$

The first equation is used to estimate the position of the resonance p. See Sec. II B, and the second can be used to estimate the deviation of the corresponding zero vector e from the direction of the resonance entrance vector $\mathbf{e}_0=|\phi_0|^{-1}\phi_0$. For a given resonance p and the corresponding null-vector e the resonance solution u_0 of the Schrödinger equation exists, with exponential asymptotic in the wires,

$$u_0(x)=\{e^1,e^2,e^3,e^4\}e^{-ipx}.$$

The corresponding solution of the nonstationary dimensional Schrödinger equation,

$$\frac{\hbar}{i}\frac{\partial u}{\partial t}+\frac{\hbar^2}{2m^*}\Delta u-V(x)u,$$

$$u(x,t)=e^{i(\hbar/2m^*)p^2t}u_0(x):=e^{-t/r}e^{i(\hbar/2m^*R^2)\,\mathrm{Re}\,p^2t}u_0(x), \quad (34)$$

is exponentially decreasing with the exponent, $1/\tau=(\hbar/2m^*)\mathrm{Im}p^2$. The inverse exponent τ is called the lifetime of the resonance. The lifetime is defined similarly for a closed split gate as zeros of the numerator of the expression (12). Note that for the dynamics associated with the wave equation (see Ref. 30), the lifetime of the resonance is usually measured by the inverse imaginary part of the resonance in the plane p of the wave number. The lifetime estimates the duration of transition processes defined by the geometry of the switch.

We calculate the resonances and estimate the corresponding lifetime approximately, neglecting the nonresonance terms, in cases when the split gates are absent (or switched off) and the wires are attached straight to the quantum well. We assume that the radius of the well is 230 Å and the width of the wires is 20 Å. The dimensionless spacing on the resonance level is 2.3 and the corresponding dimensionless coefficient in front of the maximal nonresonance term is estimated as $C<10$. In this case the equation for the resonances may be presented in dimensionless form as

$$\left\{\left[\frac{\phi_0\rangle\langle\phi_0}{\lambda-\lambda_0^r}+O\left(\frac{C}{\rho(\lambda_0^r)}\right)\right]+iRp\right\}\mathbf{e}=0,$$

or, with numerical data inserted, as

$$0=\frac{|\phi|^2}{\lambda-\lambda_0^r}+O(4.3)+i\times230\times0.372.$$

The contribution to the DN map from the neighboring nonresonance eigenvalues is dominated by the wave number $|O(4.3)|\ll230\times0.372=86$. The dimensionless resonance $\hat{\lambda}$ is calculated from the one-pole approximation of DN map as

$$\lambda=\lambda_0^r-\frac{|\phi|^2}{4.3+86\,i}=\lambda_0^r-5.5\times10^{-3}+i0.11,$$

and the lifetime of the resonance is found for $R=230$ Å as $\tau=(2m^*/\hbar)(R^2/\mathrm{Im}\lambda)$. This gives for InSb $\tau\approx0.13$ ps. In

cases when the split gates are present, with the same material and geometry of the switch and the height of the barrier over the Fermi level 1 eV, the lifetime is much longer $\tau = 10^{-4}$ s. See the relevant calculation in Ref. 27.

IV. TRANSPORT PROPERTIES OF THE SWITCH

A. The shape of the resonance eigenfunction and the switching effect

One can see from Eq. (27) that the transmission from the input wire Ω_1 to the terminal Ω_s is blocked if the component $\int_{\gamma_s}(\partial\varphi_0^r/\partial n)(y)\sin(\pi y/\delta)dy$ of the resonance entrance vector $\phi_0^r = P_+(\partial\varphi_0^r/\partial n)$ in the entrance subspace of the open channel in the wire Ω_s vanishes. It is true if the zero of the normal derivative of the resonance eigenfunction sits near to the middle point a_s of the bottom cross section γ_s. This statement is in full agreement with the observation (Ref. 12) that the single-mode transmission of an electron across the quantum system (the quantum well in our case) is implemented via excitation of the resonance mode φ_0^r inside the quantum well Ω_0.

It was noticed in Refs. 31 and 32 that the design of a network and the magnitude of the constant field inside the basic domain may be selected so that the zeros of the normal derivative of the resonance eigenfunction are sitting at the entrances of the two wires simultaneously, leaving the incoming wire and only one of the outgoing wires (terminals) nonblocked. One can show that the resonance entrance vector $\phi_0^r = P_+(\partial\varphi_0^r/\partial n)$ produced from the resonance eigenfunction of the intermediate operator (with the first channel "chopped off") coincides with the corresponding portion $\phi_0 = P_+(\partial\varphi_0/\partial n)$ of the eigenfunction φ_0 of the Dirichlet problem in the quantum well. See Appendix B. In this section again we do not distinguish the resonance entrance vectors obtained from the eigenfunctions of the intermediate operator from ones calculated based on eigenfunctions of the inner Dirichlet problem on the well. Our calculations with the Dirichlet problem in Ref. 31 show that for the special choice of the magnitude $\mathcal{E}$ of the macroscopic electric field $\mathcal{E}e\langle\nu,x\rangle$ inside the quantum well, such that $\epsilon = (2m^*R^3e/\hbar^2)\mathcal{E} = 18.86$, the eigenfunction φ_0 corresponding to the dimensionless *resonance* eigenvalue $\lambda_0 = 14.62$ inside the well *has two zeros of its normal derivative on the unit circle at the points forming angles* $\pm\pi/3$ *with the direction of the unit vector* ν. This eigenfunction is even with respect to reflection in the line spanned by the vector ν. The nearest eigenvalues in the well for the corresponding linear potential sit at 12.32 and 25.82. Hence, the dimensionless spacing on the resonance level is 2.3, which corresponds to the data in the above example found in Sec. III. The resonance eigenfunction on the quantum well rotates by an angle $\pm\pi/3$ when the direction vector ν is rotated by this angle in the plane parallel to the device. This observation allows us *to block alternatively any two of three outgoing wires* for electrons with energy close to the resonance eigenvalue—to the Fermi level.

B. Transmission coefficients

The dimensionless resonance entrance vector ϕ_0 computed with the use of the normalized eigenfunction φ_0 of the dimensionless intermediate operator (or Schrödinger operator on the well) with the potential defined by the vector ν directed to the point a_1 (to the entrance of the input wire) has the components (Ref. 33)

$$\phi_0 = (1, 0.1, 3, 0.1). \tag{35}$$

Hence, $\|\phi_0\|^2 = C \approx 10$. Then the transmission coefficients can be calculated from the one-pole approximation [Eq. (27)] as

$$|S_{12}| = |S_{14}| = 0.02, \quad |S_{13}| = 0.6.$$

Really, using the one-pole approximation for the scattering matrix presented as a function of the geometric spectral parameter $\lambda = p^2$ near to resonance λ_0,

$$S(\lambda) \approx -\frac{\dfrac{\phi_0\rangle\langle\phi_0}{\lambda-\lambda_0} + ip}{\dfrac{\phi_0\rangle\langle\phi_0}{\lambda-\lambda_0} - ip} = I - 2\frac{\phi_0\rangle\langle\phi_0}{|\phi_0|^2}\frac{1}{1 + i\dfrac{p(\lambda_0-\lambda)}{|\phi_0|^2}},$$

gives the transmission coefficients as nondiagonal elements of the scattering matrix and implies the announced result at $\lambda = \lambda_0$. This allows us to calculate the ratio of the amplitudes of the signal in the closed and open wires as 1:30 and calculate the conductance from the input wire to the open wire Ω_3 just from the Landauer formula (see Ref. 1) since other wires $\Omega_{2,4}$ are closed,

$$\sigma_{13} \approx \frac{e^2}{h}\frac{S_{13}^2}{1 - S_{13}^2} = \frac{e^2}{h}\frac{0.36}{0.64}. \tag{36}$$

This result holds for zero absolute temperature. The transmission coefficient at the resonance energy for nonzero absolute temperature should be obtained via averaging over the Fermi distribution on the essential interval of energy $(E_F - \kappa_B T, E_F + \kappa_B T)$, similarly to Ref. 21, and may give a result close to the previous one (36), or close to zero in the two limiting cases,

$$\kappa_B T \ll \frac{\hbar}{\tau} \quad \text{or} \quad \kappa_B T \gg \frac{\hbar}{\tau}, \tag{37}$$

respectively.

The above formulas show that in a certain range of temperatures the transmission is proportional to the product of components $\langle\partial\varphi_0/\partial n_s, e_s\rangle$ of the resonance entrance vector on the bottom sections of the corresponding wires, in complete agreement with the basic observation in Ref. 12 quoted in the Introduction. A similar fact for the switch based on the quantum well with Neumann boundary conditions was noticed in Ref. 31. An analog of it remains true for scattering on the quantum ring. See the corresponding solvable model in Ref. 34.

One can design a *triadic* (three-terminal) switch (RQS-3) based on selection of the magnitude of the governing electric field as suggested above. The corresponding resonance eigenfunction has the two zeros of the normal derivative that divide the boundary of the well in the ratio 1:2. Taking into account that the zeros on the boundary of the well rotate by the same angle together with the rotation of the vector ν, one can see that the resonance transmission is manipulated via

the rotation of the electric field $\mathcal{E}$ which defines the corresponding rotation of the resonance eigenfunction. Directing the vector ν opposite to the contact point a_s shifts the zeroes of the normal derivative to the complementary contact points and thus blocks the corresponding open channels.

Design of a *dyadic* (one input and two terminals) resonance quantum switch, based on the above observation, does not require any special selection of the magnitude of the electric field.

C. Parameter regime of the triadic resonance quantum switch

The quantum dynamics in the triadic resonance quantum switch is defined by several geometrical and physical parameters: the radius R of the well, the width δ of the wires, the effective mass of the electron, the magnitude of the basic shift potentials V_0, V_∞ on the well and in the wires, the magnitude of the classical electric field $\mathcal{E}$, the positions of contacts, and the temperature T and the Fermi level in the wires. The switching effect described above is observed only in cases when all these parameters are properly selected. The working point T, E_F, $\mathcal{E}$, $\{a_s\}_1^4 V_0$, V_∞, m^*, R, δ of the triadic resonance quantum switch (RQS) has to be chosen in the multidimensional space of the parameters. It cannot be done via straightforward experimental scanning, and it is very time consuming to obtain via direct computations. The above approximate formula (1) for the scattering matrix allows us to reduce the region of the search in the space of parameters and optimize the switching effect not only in the case of a single resonance eigenvalue, but also in the more general case when the resonance group of eigenvalues of the intermediate operator is selected.

In this section we will estimate the parameter regime of the triadic resonance quantum switch via the reduction of the transport problem for the switch to the scattering problem for the dimensionless Schrödinger operator on the well,

$$-\Delta_\xi u_0 + \epsilon\langle \xi, \nu\rangle u_0 + \frac{2m^*}{\hbar^2}\left[V_0 - V_\infty - \frac{\hbar^2}{2m^*}\frac{\pi^2}{\delta^2}\right] = \hat{\lambda} u_0 = p^2 u_0, \tag{38}$$

and on the wires,

$$-\frac{d^2 u_s}{d\xi^2} - \left[\frac{d^2 u_s}{d\eta^2} - \frac{\pi^2}{\delta^2}\right] = p^2 u_s. \tag{39}$$

We consider the quantum network as consisting of three terminals Ω_s, $s=2,3,4$ of width δ/R attached to the quantum well of radius 1 at the points with azimuth $a_s=(2\pi/3)(s-2)$, and one incoming wire Ω_1 attached at the azimuth $a_1=\pi$. (See Fig. 1.) We assume here that the split gates are switched off or absent. As before, we substitute the spectral data for the intermediate operator for the corresponding spectral data for the Dirichlet problem on the well. (See Sec. II B.)

To apply already developed theory one should verify the following basic conditions formulated above:

(1) The Fermi level in the wires divides the first spectral band in the wires in ratio $\mu_1:\mu_2$. [See Eq. (23).]

(2) The basic potentials V_0, V_∞ on the well and in the wires should be adjusted such that the potential in the corresponding dimensionless equation on the well,

$$-\Delta_\xi u + \frac{2m^* e\mathcal{E}R^3}{\hbar^2}\langle \xi, \nu\rangle u + \frac{2m^* R^2}{\hbar^2}\left[V_0 - V_\infty - \frac{\hbar^2}{2m^*}\left(\frac{\pi}{\delta}\right)^2\right]u = p^2 = \lambda u \tag{40}$$

is purely linear, without the constant term $V_0 - V_\infty - (\hbar^2/2m^*)(\pi/\delta)^2 = 0$.

(3) The resonance eigenvalue 14.62 of the Dirichlet problem on the well radius 1 corresponds to the Fermi level in the wires,

$$E_F - V_\infty - \frac{\hbar^2}{2m^*}\left(\frac{\pi}{\delta}\right)^2 = 14.62\frac{\hbar^2}{2m^* R^2}. \tag{41}$$

(4) The temperature is small compared with the spacing on the resonance level of the intermediate operator,

$$2\kappa_B T < 2.3\frac{\hbar^2}{2m^* R^2}. \tag{42}$$

(5) The domination condition in either form [Eqs. (24), (26), (31), and (32)] should be fulfilled.

(6) Combining condition (1) with Eqs. (41) and (42) we also obtain

$$3\frac{\hbar^2}{2m^*}\left(\frac{\pi}{\delta}\right)^2 > E_F - V_\infty - \frac{\hbar^2}{2m^*}\left(\frac{\pi}{\delta}\right)^2 = 14.62\frac{\hbar^2}{2m^* R^2} > \frac{2\kappa_B T}{2.3}. \tag{43}$$

This means, in particular, that R, δ should satisfy the condition

$$\frac{R^2}{\delta^2} > \frac{14.62}{3\pi^2}. \tag{44}$$

The roles of the above conditions (1)–(6) were discussed previously. Now we will estimate the stability of the regime of the switch. We assume that the parameter regime is stable at the resonance energy if the bound states in the well corresponding to the neighboring nonresonance eigenvalues are not excited at the temperature T. See Eq. (42). This condition may be formulated in terms of the *scaled temperature* $\hat{T} = (\hbar^2)^{-2} 2m^* R^2 T$ as

$$\kappa_B \hat{T} < \frac{\hat{\rho}_0}{2} = \frac{2.3}{2}. \tag{45}$$

The temperature which fulfills the above condition we call *low temperature*, for the given device. If the radius R of the corresponding quantum well is small enough, then the condition (45) can be fulfilled for some (absolutely) high temperature, which corresponds to the relative *low* scaled temperature. For instance, the effective mass m_0 of electron *in the well* (for a narrow-gap semiconductor) may be small. See Refs. 35 and 36. Even room temperature may be "low" enough for the appropriate construction of the device.

The importance of developing technologies for manufacturing devices of small size with rather high potential barriers is systematically underlined when discussing the pros-

TABLE I. Calculation of the radius of the quantum well for different materials based on the average effective mass.

Material	m/m_0	$R_{300\ \mathrm{K}}$ (Å)	$R_{77\ \mathrm{K}}$ (Å)	$\mathcal{E}_{300\ \mathrm{K}}$ (V/M)	$\mathcal{E}_{77\ \mathrm{K}}$ (V/M)
$Cd_{0.15}Hg_{0.85}Te$	0.0069	160	310	1.75×10^5	2.41×10^4
InSb	0.013	110	230	5.4×10^5	5.89×10^4
InAs	0.023	90	170	9.86×10^5	1.46×10^5
GaAs	0.067	50	100	5.75×10^6	7.2×10^5

pects of nanoelectronics. (See, for instance, Ref. 37.) Use of narrow-gap materials may open a way to room-temperature devices of relatively large size.

We assume that the Fermi level sits in the middle of the first spectral band $\mu_1=\mu_2=\frac{1}{2}$. Then we obtain the following estimate for the radius R of the domain and the width of the wires from (42),

$$R^2 \leqslant \frac{2.3}{2\kappa_B T}\frac{\hbar^2}{2m^*}, \quad \delta < \frac{R}{2}. \tag{46}$$

Assuming that $E_F - V_\infty = 1$ eV, we derive from condition (3) that R and δ are connected as

$$\frac{2m^*[E_F - V_\infty]}{\hbar^2} = \frac{14.62}{R^2} + \pi^2\frac{1}{\delta^2}. \tag{47}$$

Then we obtain

$$R^2 = 14.62\left[\frac{2m^*[E_F - V_\infty]}{\hbar^2} - \frac{\pi^2}{\delta^2}\right].$$

Once R is chosen, this equation allows us to select δ satisfying domination condition (6). For given $E_F - V_\infty$ the basic shift potential V_0 in the well is found from condition (2). Finally, the electric field $\mathcal{E}$ is found from the condition

$$\epsilon = 18.86 = e\mathcal{E}\frac{2m^*R^3}{\hbar^2},$$

where e is the absolute value of the electron charge. The electric field obtained from this condition is strong enough to guarantee the proper shape of the resonance wave function, but not yet destructive for standard semiconductors. (See Table I.)

One can see from the above calculations that switches manufactured of appropriate materials will work at room temperature, if the radius R of the quantum well is small enough and the geometric details are exact. We postpone to a forthcoming publication the important questions on the relative magnitude of admissible inaccuracy in the manufacturing of the geometrical details of the switch.

The calculation of the radius of the quantum well for different materials based on the average effective mass gives the results found in Table I (Refs. 22 and 23). The De-Broglie wavelengths of the materials for nitrogen temperature are 1300, 970, 730, and 430 Å, respectively.

V. CONCLUSION

Working parameters of the switch were estimated based on the one-pole approximation of the scattering matrix which coincides with an exact scattering matrix for some solvable models of the quantum switch. The developed approach, based on the observation from Ref. 12 quoted in the Introduction, can be used not only for devices designed to manipulate the current, but also for analysis of the corresponding nonstationary problems, like quantum pumping (see Refs. 38–40) and even for spin-filtering (Ref. 41), based on the Rashba spin-orbital Hamiltonian (Refs. 42 and 43). Considering the details of the shape of the wave function encoded in the corresponding Dirichlet-to-Neumann map, rather than just the "overlapping integrals," may help us to better understand the mechanism of conductance in synthetic metals.

Use of the solvable model for the quantum switch mentioned above in Sec. II B and general solvable models constructed in Ref. 26 allows us to suggest a quantitatively consistent description of transport phenomena on quantum networks, substituting the corresponding two-dimensional Schrödinger equation for the appropriate ordinary differential equation on the quantum graphs with the resonance nodes. We anticipate that an appropriate fitting would allow one to extend the field of applications of the *qualitative* results for zero-range models obtained in Refs. 16, 19, 20, 44–53, and 56–61, transforming them into efficient tools of *quantitative* analysis of quantum systems.

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APPENDIX A: DN MAP AND SCATTERING MATRIX

In this section we collected some mathematical facts which were used in the main text of the paper. We quote them in a standard mathematical form.

1. Standard DN map

The Dirichlet-to-Neumann map for the Schrödinger equation presented in geometric form is a map of the boundary values u_Γ of the solution,

$$-\Delta u + \mathbf{V}u = \lambda u, \quad u\big|_{\partial\Omega} = u_\Gamma,$$

into the boundary values of its normal derivative,

$$\Lambda{:}u_\Gamma \to \left.\frac{\partial u}{\partial n}\right|_{\partial\Omega}.$$

In electrodynamics (with $\mathbf{V}=0$) Λ defines the connection of the potential on the boundary with the normal current. Detailed description of general features of the DN map and its relations to the scattering matrix may be found in Refs. 24 and 25, respectively. We will review here only the basic features of the standard DN map.

We denote by L the self-adjoint operators defined in $L_2(\Omega)$ by the above differential expression $Lu=-\Delta u+\mathbf{V}u$ with homogeneous Dirichlet. The corresponding Green functions $G(x,y,\lambda)$ and the Poisson kernel,

$$\mathcal{P}_\lambda(x,s) = -\frac{\partial G(x,s,\lambda)}{\partial n_s}, \quad s \in \partial\Omega,$$

exist if λ is not an eigenvalue of L (is "regular"). Solution of the Dirichlet boundary problem is represented for regular λ by the renormalized double-layer potential,

$$u(x) = \int_\Gamma \mathcal{P}_D(x,s,\lambda)u_\Gamma(s)d\Gamma. \tag{A1}$$

Generally the standard DN map is represented for regular points λ of L_D as a generalized integral operator with a singular kernel,

$$[\Lambda(\lambda)u_\Gamma](x_\Gamma) = \left.\frac{\partial}{\partial n}\right|_{x=x_\Gamma}\int_{\partial\Omega}\mathcal{P}_D(x,s,\lambda)u_\Gamma d\Gamma, \tag{A2}$$

and exists as an operator in the appropriate Sobolev classes. (See Ref. 54.) In particular for operators defined on $W_2^2(\Omega)$ it acts from $W_2^{3/2}(\Gamma)$ to $W_2^{1/2}(\Gamma)$. See, for instance, Refs. 24 and 25. One can see from the straightforward integration by parts that the DN map is an analytic function of the spectral parameter λ with a negative imaginary part (for an interior problem, with an outer positive normal on the boundary). The DN map can be presented as a formal integral operator in $L_2(\Gamma)$ with the generalized kernel,

$$\Lambda(x,x',\lambda) = -\frac{\partial G}{\partial n_x \partial n_{x'}}(x,x',\lambda) = \sum_{\lambda_s}\frac{\frac{\partial\varphi_s^r}{\partial n}(x)\Big\rangle\Big\langle\frac{\partial\varphi_s^r}{\partial n}(x')}{\lambda-\lambda_s^r} + \int_{\sigma>3\pi^2/\delta^2}\frac{P_+\frac{\partial\varphi_2^r(\sigma,x)}{\partial n}\Big\rangle\Big\langle P_+\frac{\partial\varphi_2^r(\sigma,x')}{\partial n}}{\lambda-\sigma}d\sigma + \cdots. \tag{A3}$$

The corresponding spectral series and/or integral is divergent, but can be regularized. (See Ref. 25.)

The standard DN map permits us to formalize the procedure of matching solutions of the partial differentials equation on the inner boundary $\cup_{s=0}^4\Gamma_s$ of the composite domain $\cup_{s=0}^4\Omega_s$. In the text below we distinguish again the dimensional λ, p, x and dimensionless $\hat{\lambda}$, $\hat{p}$, $\hat{x}$ variables introduced in Sec. II. We denote by E_+ the 4-dimensional subspace in $\mathrm{E}:=L_2(\Gamma)$ spanned by the cross section eigenvectors e_s^1, $s=1,2,3,4$, of the open channel, $0<p^2<(3\pi^2/\delta^2)$. The orthogonal complement of it $E\ominus E_+=E_-$ is the entrance subspace of the closed channels. On the first spectral band $0\leqslant\lambda\leqslant 3\pi^2/\delta^2$ there are two bounded exponential modes of the first order based on the cross section eigenfunction $e_s^1=\sqrt{2/\delta}\sin\pi y/\delta$ in the wire Ω_s with exponentials defined by the dimensional effective wave number p,

$$f_s^\pm(x,y) = e_s^1 e^{\pm ipx} \quad \text{if } x>0,$$

and only one bounded exponential mode order l in upper channels, $l>1$,

$$f_s^l(x,y) = e_s^l e^{-\sqrt{[\pi^2(l^2-1)/\delta^2]-p^2}x} \quad \text{if } x>0.$$

The corresponding scattering Ansatz in the wires Ω_s is combined as

$$\Psi_s(x) = \delta_{s1}f_{1s}^- + f_{1s}^+S_{s1}^1 + \sum_{l=2}^\infty S_{s1}^l f_{s1}^l, \tag{A4}$$

with coefficients S_{s1}^l to be defined from the matching of the scattering ansatz to the corresponding solution of the above Schrödinger equation inside the quantum well Ω_0. We find them from the condition (10) of continuation of the scattering ansatz inside the domain,

$$\frac{\partial\Psi_\gamma}{\partial n_\gamma} = \frac{\partial\Psi_0}{\partial n_\gamma} = \Lambda^0\Psi_\gamma. \tag{A5}$$

We denote by K^+, $\overline{K^+}$, K^- the operators in E_+ which compute the components of the normal derivatives of the exponential modes on the bottom sections of the wires Ω_s in the open and closed channels,

$$K^+ = ipI, \quad \overline{K^+} = -ipI, \quad K_l^- = \sqrt{\frac{(l^2-1)\pi^2}{\delta^2}-p^2}I,$$

$$l=2,3,\ldots, \quad K^- = \mathrm{diag}\{K_l^-\}_{l=2}^\infty,$$

with a positive square root, and by $P_\pm$ the orthogonal projections onto the subspaces $E_\pm\subset E$. Here K^+ is a 4×4 matrix

proportional to the unit matrix, since the potentials on the wires are equivalent. Then the above Eq. (A5) may be presented as a matrix equation with respect to the components $\Psi_s^{\pm}$ of the above decomposition of $E=E_+\oplus E_-$. Elements of the subspace E_+ belong to the Sobolev class $W_2^{3/2-\varepsilon}(\Gamma)$, for each positive ε. Hence, the operators $P_+\Lambda^0P_+$, $P_+\Lambda^0P_-$, $P_-\Lambda^0P_+$, $P_-\Lambda^0P_-$ constructed via framing of the standard DN map Λ^0 of the Schrödinger operator on the well by projections onto the entrance subspaces of open and closed channels exist as operators in the appropriate Sobolev classes. We denote them by Λ^0_{++}, Λ^0_{+-}, Λ^0_{-+}, Λ^0_{--}, respectively, and by S^1, the 4×4 matrix with elements S^1_{st} and set $(\Psi_s^+)_{s=1}^4(0)=\Psi^+$, $(\Psi_s^-)_{s=1}^4(0)=\Sigma_{l=2}^{\infty}S^l\Psi^+$.

Theorem A1. The scattering matrix on the whole network $\Omega_0\cup\Omega_1\cup\Omega_2\cup\Omega_3\cup\Omega_4$ may be presented in terms of the Dirichlet-to-Neumann-map Λ^0 of the quantum well Ω_0 as

$$S(\lambda)=-\frac{\Lambda^0_{++}-\Lambda^0_{+-}\dfrac{I}{K^-+\Lambda^0_{--}}\Lambda^0_{-+}-\overline{K^+}}{\Lambda^0_{++}-\Lambda^0_{+-}\dfrac{I}{K^-+\Lambda^0_{--}}\Lambda^0_{-+}-K^+}. \tag{A6}$$

The proof is obtained based on an orthogonal decomposition of the whole space $L_2(\Gamma)$ into the orthogonal sum of open and closed channels, followed by a straightforward calculation. (See Ref. 26.)

2. DN map of the intermediate operator

The DN map of the intermediate operator, associated with the part Γ of the boundary for regular λ, is an operator transforming the boundary data u_Γ from E_+ into the projection of the normal derivative of the corresponding square-integrable solution u of the Schrödinger equation $-\Delta u+Vu=\lambda u$ onto E_+. To obtain the formula for DN map of the intermediate operator associated with the E_+ one should consider for regular λ the square-integrable solution of the Schrödinger equation $lu=\lambda u$ with the boundary data $u_\Gamma\in E_+$ on Γ presented via the corresponding Poisson integral, $u(x)=-\int_\Gamma(\partial G^r/\partial n_y)(x,y)u_\Gamma(y)dy$, and calculate $\partial u/\partial n|_\Gamma$. The projection of the normal derivative of the solution of the Dirichlet problem on Γ onto E_+ gives the DN map as an integral transform $\Lambda^r:E_+\to E_+$, with the kernel:

$$\Lambda^r(x_\Gamma,x'_\Gamma,\lambda)=-P_+\frac{\partial^2G^r}{\partial n_x\partial n'_x}P_+, \tag{A7}$$

which is equivalent to the formula in Eq. (13). This gives the DN map Λ^r in terms of appropriate eigenfunctions and eigenvalues φ_l^r, λ_l^r. Though there is no standard software for calculation of eigenfunctions and eigenvalues of the intermediate operator, one can present the DN map of the intermediate operator as the Schur complement (see Ref. 55), of the DN map Λ^0 of the Schrödinger operator l_0 on the well. Indeed, the scattering matrix (A6) contains a special combination of matrix elements of the DN map $\hat\Lambda^0$ of the quantum well,

$$\Lambda^0_{++}-\Lambda^0_{+-}\frac{I}{K^-+\Lambda^0_{--}}\Lambda^0_{-+}. \tag{A8}$$

This function has negative imaginary part in upper half-plane $\Im\lambda>0$ and singularities at the vector zeros of the denominator $(K^-+\Lambda^0_{--})\,\mathbf{e}=0$. (See Appendix B.) The following theorem (see Ref. 27) gives an interpretation of this function in terms of the intermediate operator.

Theorem A2. The split operator $l^r=l_0^r\oplus\Sigma_{s=1}^4l_s^r$ defined in $L_2(\Omega)$ by the Schrödinger differential expression (4) and the boundary conditions (9) and (8) is self-adjoint. The continuous spectrum of the nontrivial part l_0^r of it in the orthogonal complement of the open channels $L_\Omega\ominus\Sigma_{s=1}^4L_2(\Omega_s)$ consists of the branch $\lambda\geqslant3\pi^2/\delta^2$ with a countable sequence of thresholds. The eigenvalues of l_0^r below the threshold $3\pi^2/\delta^2$ coincide with the vector—zeros λ_s^r of the denominator $(K^-+\Lambda^0_{--})$. The DN map of the operator l_0^r on the whole network with a chopped-off first channel coincides with the operator-function Eq. (A8).

This statement is verified in Ref. 26. Note that the DN map of the intermediate operator is actually a 4×4 matrix.

3. Scattering matrix via partial DN map

To calculate the scattering matrix in terms of the partial DN map of the intermediate operator we should match the restriction of the scattering anzatz onto the sum $\Gamma=\cup_{s=1}^4\Gamma_s$ of the bottom sections in the entrance subspace E_+ of the open channels *only*,

$$\Psi_s=\delta_{s1}F_{in,s}+F_{out,s}S^1_{s1},\quad s=1,2,3,4,$$

$$\boldsymbol{\Psi}=\{\Psi_s^+\}_{s=1}^4=F_{in}e_1+F_{out}Se_1,$$

with the Jost matrices $F_{in,out}$, to the solution Ψ_0 of the intermediate homogeneous equation,

$$P_+\Psi_0=\boldsymbol{\Psi},\quad \frac{\partial\Psi_0}{\partial n_s}=\frac{\partial\Psi_s}{\partial n_s}=(\Lambda^r\boldsymbol{\Psi})_s,\quad s=1,2,3,4. \tag{A9}$$

We use further the above expression (A7) for the partial DN map. The Jost matrices are proportional to the unit matrix in open channels, since all wires are equivalent. Then we obtain the following equation for the scattering matrix:

$$\Lambda^rF_{in}-F'_{in}=-(\Lambda^rF_{out}-F'_{out})S,$$

and the solution of it

$$S=-\frac{F_{in}}{F_{out}}\frac{\Lambda^r-\dfrac{F'_{in}}{F_{in}}}{\Lambda^r-\dfrac{F'_{out}}{F_{out}}},$$

where the denominator is preceding the numerator, and the fractions

$$\frac{F_{in}}{F_{out}} = P_+ \frac{F_{in}}{F_{out}}, \quad \frac{F'_{in}}{F_{in}} = P_+ \frac{F'_{in}}{F_{in}} \quad \text{and} \quad \frac{F'_{out}}{F_{out}} = P_+ \frac{F'_{out}}{F_{out}},$$

are proportional to the unit matrix in E_+. Substituting here the explicit expression for the Jost solutions in the open channel we obtain the expression (12) for the scattering matrix in terms of DN map of the intermediate operator. Note that the scattered waves constructed with use of the partial DN map of the intermediate operator automatically satisfy the matching conditions in closed channels.

APPENDIX B: SPECTRAL DATA OF THE INTERMEDIATE OPERATOR VIA ANALYTIC PERTURBATION PROCEDURE

Zeros and poles of the scattering matrix sit near the resonance eigenvalues of the intermediate operator, and they are shifted with respect to the corresponding eigenvalues of the Schrödinger operator with Dirichlet boundary conditions on the boundary of the well. The shift generically is small and can be estimated via the analytical perturbation procedure which is developed in this subsection. This important fact was used previously in Secs. II B and II C. It is based on the following statement.

Theorem A3. The pole λ_0 of the DN map Λ^0 of the quantum well, which is the singularity of the first addendum Λ_{++} of Eq. (A8), is compensated by the pole of the second addendum and disappears as a singularity of the whole function Λ^r, so that the whole expression (A8) is, generically, regular at the point λ_0. A new pole appears as a zero of the denominator $K^- + \Lambda^0_{--}$ and coincides with the eigenvalue of the intermediate operator. The corresponding residue is a combination of the root vectors which correspond to this new pole and coincide, in the first order of the perturbation procedure, with the resonance entrance vector $P_+ \partial\varphi_0/\partial n = \phi_0$ of the Dirichlet problem on the quantum well Ω_0.

The proof is presented in Ref. 27. Here we will verify the corresponding dimensionless statement numerically for the special case of a switch based on a circular quantum well. We use the above representation [Eq. (A8)] for the dimensionless DN map,

$$\hat{\Lambda}^r = \hat{\Lambda}^0_{++} - \hat{\Lambda}^0_{+-} \frac{I}{\hat{K}^- + \hat{\Lambda}^0_{--}} \hat{\Lambda}^0_{-+}.$$

Assuming that $R/2 > \delta$ denotes the projections of the normal derivatives of the resonance eigenfunctions onto subspaces $E_\pm$ on the sum Γ of the bottom sections of the wires as

$$\hat{\phi}^\pm_s = P_\pm \frac{\partial \hat{\varphi}_s}{\partial n},$$

and separate the resonance term in the DN map framed by projections onto $E_\pm$,

$$P_+ \hat{\Lambda} P_+ = \frac{\hat{\phi}^+_0 \rangle\langle \hat{\phi}^+_0}{\hat{\lambda} - \hat{\lambda}_0} + \sum_{s \neq 0} \frac{\hat{\phi}^+_s \rangle\langle \hat{\phi}^+_s}{\hat{\lambda} - \hat{\lambda}_s} := \frac{\hat{\phi}^+_0 \rangle\langle \hat{\phi}^+_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{++},$$

$$P_+ \hat{\Lambda} P_- = \frac{\hat{\phi}^+_0 \rangle\langle \hat{\phi}^-_0}{\hat{\lambda} - \hat{\lambda}_0} + \sum_{s \neq 0} \frac{\hat{\phi}^+_s \rangle\langle \hat{\phi}^-_s}{\hat{\lambda} - \hat{\lambda}_s} := \frac{\hat{\phi}^+_0 \rangle\langle \hat{\phi}^-_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{+-},$$

$$P_- \hat{\Lambda} P_+ = \frac{\hat{\phi}^-_0 \rangle\langle \hat{\phi}^+_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{-+}, \quad \hat{\mathcal{K}}_{-+} = (\hat{\mathcal{K}}_{+-})^+,$$

$$P_- \hat{\Lambda} P_- = \frac{\hat{\phi}^-_0 \rangle\langle \hat{\phi}^-_0}{\hat{\lambda} - \hat{\lambda}_0} + \sum_{s \neq 0} \frac{\hat{\phi}^-_s \rangle\langle \hat{\phi}^-_s}{\hat{\lambda} - \hat{\lambda}_s} := \frac{\hat{\phi}^-_0 \rangle\langle \hat{\phi}^-_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{--}.$$

Then, dimensionless expression [Eq. (A8)] may be presented as

$$R\Lambda^r = \hat{\Lambda}^r = \frac{\hat{\phi}^+_0 \rangle\langle \hat{\phi}^+_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{+-} - \left[\frac{\hat{\phi}^+_0 \rangle\langle \hat{\phi}^-_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{+-} \right] \times \frac{I}{\frac{\hat{\phi}^-_0 \rangle\langle \hat{\phi}^-_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{--} + RK^-} \left[\frac{\hat{\phi}^-_0 \rangle\langle \hat{\phi}^+_0}{\hat{\lambda} - \hat{\lambda}_0} + \hat{\mathcal{K}}_{-+} \right]. \tag{B1}$$

Assume, that the Fermi level is situated in the middle of the first spectral band in the wires, $\mu_1 = \mu_2 = 1/2$. Then the effective wave number p at the Fermi level is equal to $\sqrt{\frac{3}{2}(\pi^2/\delta^2)}$ and the corresponding dimensionless wave number $\hat{p} = Rp$ is estimated as

$$\hat{p} = Rp = \pi \frac{R}{\delta} \sqrt{\frac{3}{2}} \approx 4 \frac{R}{\delta}, \tag{B2}$$

since $m_0/m^\perp = 5.2$ (for Si). Recall (see Sec. II C) that the term $-\hat{K}^+$ in the dimensionless expression for the scattering matrix may be estimated as

$$-\hat{K}^+ = iRpI = i\pi \frac{R}{\delta} \sqrt{3/2} I \approx i4(R/\delta)I.$$

It contains a "large" parameter compared with dimensionless inverse spacing $|\hat{\lambda}_0 - \hat{\lambda}_1|^{-1} = 1/2.3$ already for $R > \delta/2$: $10/2.3 = 4.3 \ll 8$. Note that for $R = 10\delta$ the corresponding inequality is $4.3 \ll 40$.

Similarly, the operator $RK^- = \hat{K}^-$ for the selected value of energy is positive and may be estimated from below by the dimensionless distance from the Fermi level to the second threshold with (dimensionless) coefficient R/δ,

$$\hat{K}^- \geqslant \sqrt{\frac{3}{2}} \frac{R}{\delta} \pi I > 8I.$$

Then, estimating the contribution $\mathcal{K}$ from the nonresonance terms to DN map as $\|\hat{\mathcal{K}}\| \leqslant \hat{C}/\rho(\hat{\lambda}_0)$, we conclude that, in agreement with Eq. (32),

$$\|[\hat{K}^-]^{-1} \hat{\mathcal{K}}\| \leqslant \sqrt{\frac{2}{3}} \frac{\hat{C}}{\pi \hat{\rho}(\hat{\lambda}_0)} \frac{\delta}{R} = 1.1 \frac{\delta}{R}. \tag{B3}$$

Assuming that $\hat{C} = 10$, $\hat{\rho}(\hat{\lambda}_0) = 2.3$, $\delta = 2$ nm, $R = 10$ nm, we obtain in the right side of Eq. (B3) the small dimensionless parameter $\delta(R)^{-1} \approx 0.22$. Then we can calculate the inverse $(\hat{\mathcal{K}}_{--} + \hat{K}^-)^{-1} := k(\delta, R) := k$ via the perturbation series and obtain the estimate,

$$\|k\| \leqslant \frac{\delta}{R}\frac{1}{\pi}\sqrt{\frac{2}{3}}, \quad \frac{1}{1-1/4} \approx 0.16\frac{\delta}{R}.$$

This gives an explicit expression for the inverse operator,

$$\left[\frac{\hat{\phi}_0^-\rangle\langle\hat{\phi}_0^-}{\hat{\lambda}-\hat{\lambda}_0} + \hat{\mathcal{K}}_{--} + \hat{K}^-\right]^{-1}$$

in the form

$$u = \left[\frac{\hat{\phi}_0^-\rangle\langle\hat{\phi}_0^-}{\hat{\lambda}-\hat{\lambda}_0} + \hat{\mathcal{K}}_{--} + \hat{K}^-\right]^{-1} f = kf - \frac{1}{\boldsymbol{\mathcal{D}}} k\hat{\phi}_0^- \Big\rangle\langle\hat{\phi}_0^-, kf\rangle,$$

where $\boldsymbol{\mathcal{D}} = \hat{\lambda}-\hat{\lambda}_0 + \langle\hat{\phi}_0^-, k\hat{\phi}_0^-\rangle$. Substituting that expression into Eq. (B1), we notice that all terms containing $(\hat{\lambda}-\hat{\lambda}_0)$ in the denominator are cancelled and we obtain

$$\hat{\Lambda}^r = \frac{\hat{\phi}_0^r\rangle\langle\hat{\phi}_0^r}{\hat{\lambda}-\hat{\lambda}_0^r} = \frac{\hat{\phi}_0^+\rangle\langle\hat{\phi}_0^+}{\boldsymbol{\mathcal{D}}},$$

with the residue of the DN map proportional to $\hat{\phi}_0^+\rangle\langle\hat{\phi}_0^+$, as announced. The dimensionless eigenvalues $\hat{\lambda}^r$ of the intermediate operator $\hat{l}_0^r$ can be obtained from the equation $\boldsymbol{\mathcal{D}}(\hat{\lambda}_r) = 0$. Due to $\|\hat{\Phi}_0^-\| \approx 1$ using Eq. (31) we obtain for $\delta = R/2$, in the first order of the perturbation procedure, $|\hat{\lambda}_0^r - \hat{\lambda}_0| < 0.08 \ll 2.3$. It is in full agreement with the corresponding direct calculation[27] $|\hat{\lambda}_0^r - \hat{\lambda}_0| = 0.07$ used above in Secs. II–IV. Actually the above calculation gives an analytical foundation for substitution, in practical calculations with the scattering matrix, of the leading terms of the DN map of the intermediate operator with the leading terms of the DN map of the corresponding Schrödinger operator on the quantum well.

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